MA2102: PS (Assignment-2)

1. Let $p_X(x)$ be the PMF of a random variable X. Find the CDF $F_X(x)$ of X and sketch its graph.
   i. $p_X(x) = \frac{1}{3}, x = -1, 0, 1$
   ii. $p_X(x) = \frac{x}{15}, , x = 1, 2, 3, 4, 5$
2. Suppose F_1, F_2 are two CDF's then verify whether the following are CDF's?
   i. $G(x) = F_1(x) + F_2(x)$
   ii. $G(x) = \frac{1}{3}F_1(x) + \frac{2}{3}F_2(x)$
   iii. $G(x) = a_1F_1(x) + a_2F_2(x)$ where $a_1 \geq 0, a_2 \geq 0 \; and \; a_1 + a_2 = 1$

3. Which of the following are valid PMF's?
   i. $p_X(x) = \frac{(x-2)}{2}, x = 1, 2, 3, 4$
   ii. $p_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, 3 \ldots where \; \lambda > 0$
   iii. $p_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 1, 2, 3 \ldots \; where \; \lambda > 0$
   iv. $p_X(x) = \binom{x+r-1}{r-1} p^r(1-p)^x, x = 0,1, 2, 3 \ldots \; where \; 0 < p < 1$
4. Find the c value so that the following functions are valid PDF's?
   i. $f_X(x) = \begin{cases} c\,\lambda^\alpha x^{\alpha-1} \; e^{-\lambda x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$
   ii. $f_X(x) = c\, e^{-\frac{x^2}{2}} \quad , -\infty < x < \infty$
   iii. $f_X(x) = \frac{c}{1+x^2} \quad , -\infty < x < \infty$
5. Find the value of constant c such that the following function is PDF, then find the CDF associated with PDF.

$$f(x) = \begin{cases} c(x+1)e^{-\lambda x} & if \; x \geq 0 \\ 0 & if \; x < 0 \end{cases} \quad \text{where } \lambda > 0$$

6. Let us select five cards at random and without replacement from an ordinary deck of playing cards.
   i. Find the PMF of X, the number of hearts in the five cards
   ii. Determine $P(X \leq 1)$.

7. For some constant c, the random variable X has PDF

$$f_X(x) = \begin{cases} c(1-x^2) & -1 < x < 1 \\ 0 & otherwise \end{cases}$$

   i. What is the value of c
   ii. What is CDF of X

8. A battery cell is labelled as good if it works for at least 300 days in a clock, otherwise it is labelled as bad. Three manufacturers, A, B and C make cells with probability of making good cell as 0.95, 0.90 and 0.80 respectively. Three identical clocks are selected and ells made by A, B and C are used in clock-1, clock-2, clock-3 respectively. Let X be the number of clocks working

after 300 days. Find the probability mass function of X , then find and sketch the corresponding CDF.

9. Let X is a continuous random variable with PDF $f_X(x) = \begin{cases} 1 \ \ if \ 0 < x < 1 \\ 0, \ \ otherwise \end{cases}$, then find the PDF of $Y = \sqrt{X}$
10. Let X is a continuous random variable with PDF $f_X(x) = \begin{cases} k \frac{x^{p-1}}{(1+x)^{p+q}} \ \ if \ x > 0 \\ 0, \ \ otherwise \end{cases}$, $p, q > 0$ then find the PDF of $Y = \frac{1}{(1+X)}$
11. Let X is a continuous random variable with PDF $f_X(x) = \begin{cases} k \, x^{\beta-1} \, e^{-\alpha x^{\beta}} \ \ if \ x > 0 \\ 0, \ \ otherwise \end{cases}$ where $\alpha > 0, \beta > 0$,the find the PDF of $Y = X^{\beta}$
12. Let X is a continuous random variable with support $S_X = (a, b)$,PDF f_X, and CDF F_X, then find the PDF of $Y = -\log(F_X(X))$
13. Let X is a continuous random variable with PDF $f_X(x) = \frac{1}{2} e^{-|x|}$, $-\infty < x < \infty$. then find the PDF of $Y = |X|$

****** GOOD LUCK ******