Assignment-5(PS_MA2102)

1. The JPDF of (X,Y) is given by $f_{XY}(x,y) = \begin{cases} 4xy & 0<x<1, 0<y<1 \\ 0, & otherwise \end{cases}$. Find the marginal PDFs an verify whether X,Y are independent. Also find
   i. $P\left(0 < X < \frac{1}{2}, \frac{1}{4} < Y < 1\right)$
   ii. $P(X+Y<1)$

2. If the JPDF of (X,Y) $f_{XY}(x,y) = \begin{cases} cx^2y & 0<x<y<1 \\ 0 & otherwise \end{cases}$
   i. Find c
   ii. Find marginal PDF's of X,Y
   iii. $P(X+Y \le 1)$

3. If the JPDF of (X,Y) $f_{XY}(x,y) = \begin{cases} xe^{-x(1+y)} & x>0, y>0 \\ 0 & otherwise \end{cases}$, Find
   i. conditional PDF of X given $Y=y$
   ii. conditional PDF of Y given $X=x$
   iii. Find PDF of $S=XY$

4. Suppose $f_{X/Y}(x/y) = \begin{cases} \frac{cx}{y^2} & 0<x<y \\ 0 & otherwise \end{cases}$ and $f_Y(y) = \begin{cases} dy^4 & 0<y<1 \\ 0 & otherwise \end{cases}$
   i. Find c and d
   ii. Find JPDF of (X,Y)
   iii. Compute $P(0.25 < X < 0.5)$ and $P\,(0.25 < X < 0.5/Y = 0.625)$

5. Suppose $f_X(x) = \begin{cases} 4x(1-x^2) & 0<x<y \\ 0 & otherwise \end{cases}$ and $f_{Y/X}\,(y/x) = \begin{cases} \frac{2y}{1-x^2} & 0<y<1 \\ 0 & otherwise \end{cases}$, Find
   i. $f_{X/Y}(x/y)$
   ii. $E(X/Y = \frac{1}{2})$
   iii. $Var(X/Y = \frac{1}{2})$

6. He JPDF of (X,Y) $f_{X,Y}(x,y) = \begin{cases} 2 & 0<x<y<1 \\ 0 & otherwise \end{cases}$, Find $E(X+Y)$ and $Var(X+Y)$

7. Let X_1, X_2, X_3 be three independent random variables each with variance σ^2. Define the new random variables $W_1 = X_1$, $W_2 = \frac{\sqrt{3}-1}{2}X_1 + \frac{\sqrt{3}-1}{2}X_2$,$W_3 = (\sqrt{2}-1)X_2 + (2-\sqrt{2})X_3$, then find $\rho_{W1,W2}$, $\rho_{W2,W3}$ and $\rho_{W1,W3}$

8. $(X,Y) \sim BVN(0,0,4,3,\frac{\sqrt{3}}{2})$ Find $P(X-2Y > -4/X+Y=10\,)$

9. If the JPDF of (X,Y), $f_{X,Y}(x,y) = \begin{cases} \frac{1}{x^2y^2} & x\ge 1, y \ge 1 \\ 0 & otherwise \end{cases}$
   i. Find JPDF of $U=XY$, $V=X/Y$
   ii. Find marginal PDF's of U,V

10. Let $Y/X \sim Poisson(X)$,$X \sim Gamma(\alpha,\lambda)$ then find the $E(Y)$, and $Var(Y)$

11. Suppose $X_1, X_2, \ldots, X_6$ are i.i.d random variables with uniform distribution on (0,1), and $W = \frac{X_{(1)}}{X_{(6)}}$. Find (i) $f_W(w)$ (ii) $F_W(w)$ (iii) $E(W)$ (iv) $Var(W)$

12. The PDF of random variable X is $f(x) = \begin{cases} 1/x^2 & x \geq 1 \\ 0 & otherwise \end{cases}$,
    Consider a random sample of size 72 from population having the above PDF, compute, approximately, the probability that more than 50 of these observations are less than 3.

13. Let $X_1, X_2, \ldots, X_{20}$ are independent Poisson random variables with mean 1.
    (i) Use computer to find the exact value of $P(\sum_{i=1}^{20} X_i > 15)$
    (ii) Use the Markov's inequality to obtain a bound on $P(\sum_{i=1}^{20} X_i > 15)$
    (iii) Use Central Limit theorem to approximate $P(\sum_{i=1}^{20} X_i > 15)$

14. Each computer chip made in a certain plant will, independently be defective with probability 0.25. If a sample of 1000 chips is tested, what is the approximate probability that fewer than 200 chips will be defective?

15. If the temperature at which a thermostat goes off is normally distributed with variance σ^2. If the thermostat is to be tested five times, find
    i. $P(S^2/\sigma^2 \leq 1.8)$
    ii. $P(0.85 \leq S^2/\sigma^2 \leq 1.15)$
    Where S^2 is the sample variance of the five data values.

******** GOOD LUCK********