PS_MA2102: Assignment-1

1. Let $(\Omega,\ \mathscr{F}, P)$ be a probability space and let A and B be two events in $\mathscr{F}$ with $P(A) = 0.2,\ \ P(B) = 0.4$ and $P(A \cap B) = 0.1$. Find the probability that:

   a. Exactly one of the event A or B will occur.

   b. At least one of A or B occur

   c. None of A and B will occur.

2. Let Ω={0,1,2,3....}. and $\mathscr{F}$= 2^{Ω} (power set of Ω), P: $\mathscr{F} \to \mathbb{R}$, for A∈ $\mathscr{F}$

   a. $P(A) = \sum_{x \in A} \dfrac{e^{-\lambda}\lambda^x}{x!}$, λ>0

   b. $P(A) = \sum_{x \in A} p(1-p)^x,\ 0 < p < 1$

   c. $P(A) = \begin{cases} 1 & if A \, is finite \\ 0 & otherwise \end{cases}$

Determine in each of the above cases whether P is probability function.

3. Consider the sample space Ω = {0,1,2,3....}. and $\mathscr{F}$=2^{Ω} (power set of Ω).We assigns
$P(\{j\}) = c\dfrac{2^j}{j!}$, $j = 0,1,2...$

   a. Determine the constant c.

   b. Define the events A, B and C by A={j: 2≤j≤4}, B={j: j≥3}, and C={j: j is an odd integer}. Evaluate $P(A),\ P(B), P(C),\ P(A \cap B), P(A \cap C), P(B \cap C), P(A \cap B \cap C)$ and verify the formula for $P(A \cup B \cup C)$

4. An urn contains r red balls and b blue balls. A ball is chosen at random from the urn, its colour is noted, and it is returned together with d more balls of the same colour. This is repeated indefinitely. What is the probability that?

   i. The second ball drawn is blue?

   ii. The first drawn ball is blue given that second ball drawn is blue?

5. For A, B, and C such that P(C)>0, prove that

   a. $P((A \cup B)/C) = P(A/C) + P(B/C) - P((A \cap B)/C)$

   b. $P(A^c/C) = 1 - P(A/C)$

6. Consider the two events A and B such that $P(A)=\dfrac{1}{4}$, $P(B/A)=\dfrac{1}{2}$ and $P(A/B)=\dfrac{1}{4}$

Which of the following statements are true? Justify your answer.

a. A and B are mutually exclusive events

b. $A \subset B$

c. $P(A^c/B^c)=\dfrac{3}{4}$

d. $P(A/B)+P\left(A/B^c\right)=1$

7. 98% of all babies survive delivery. However, 15% of all births involve caesarean(C) sections, and when a C section is performed the baby survives 96% of time. If a randomly chosen pregnant woman does not have C section, what is the probability that her baby survives?

8. Two litters of a particular rodent species have been born, one with two brown-haired and one gray-haired (litter 1), and the other with three brown-haired and two gray-haired (litter 2). We select a litter at random and then select an offspring at random from the selected litter.

a. What is the probability that the animal chosen is brown-haired?

b. Given that a brown-haired offspring was selected, what is the probability that the sampling was from litter 1?

9. A recent college graduate is planning to take the first three actuarial examinations in the coming summer. He will take the first actuarial exam in June. If he passes that exam, then he will take the second exam in July, and if he also passes that one, then he will take third exam in September. If he fails an exam, then he is not allowed to take any others. The probability that he passes the first exam is 0.9. If he passes the first exam, then the conditional probability that he passes the second one is 0.8, and if he passes both the first and the second exams, then the conditional probability that he passes the third exam is 0.7.

a. What is the probability that he passes all three exams?

b. Given that he did not pass all three exams, what is the conditional probability that he failed the second exam.

10. A worker has asked his supervisor for a letter of recommendation for a new job. He estimates that there is an 80% chance that he will get the job if he receives a strong recommendation, 40% chance if he receives moderately good recommendation, and a 10 percent chance if he receives a weak recommendation. He further estimates that the probabilities that the recommendation will be strong, moderate, or weak are 0.7, 0.2, and 0.1 respectively.

a. How certain is he that he will receive the new job offer?

b. Given that he does receive the offer, how likely should he feel that he received a

   i. strong recommendation.

   ii. moderate recommendation

   iii. weak recommendation.

c. Given that he does not receive the job offer, how likely should he feel that he received a

   iv. strong recommendation.

   v. moderate recommendation.

   vi. weak recommendation.

11. If $A_1,\ A_2,\ \ldots A_n$ be n independent events, then show that $P(\cap_{i=1}^{n} A_i^c) \leq e^{-\sum_{i=1}^{n} P(A_i)}$

[Hint: $1 - x \leq e^{-x}$].

12. Each of four persons fires one shot at a target. Let C_k denote the event that the target is hit by person k, k=1,2,3,4. If the events C_1, C_2, C_3, C_4 are independent and if $P\left(C_1\right) = P\left(C_2\right) = 0.7$, $P\left(C_3\right) = 0.9$ and $P\left(C_4\right) = 0.4$, compute the probability that

a. All of them hit target.

b. No one hits the target.

c. Exactly one hits the target.

d. At least one hit target

13. Suppose we turn over cards simultaneously from two well shuffled decks of ordinary playing cards. We say we obtain an exact match on a particular turn if the same card appears from each deck. Let p_M equal the probability of at least one exact match. Then show that $p_M = 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \ldots. - \frac{1}{52!} \cong 0.632$. Also Write a Python code to check if the answer is consistent with relative frequency of an event of having at least one match for 20000 number of trails and print the relative frequency of 20000th trial, also plot the relative frequency curve.

14. Prove each of the following statements(Assume any conditioning event has positive probability).

    a. If $P(B) = 1$, then $P(A/B) = P(A)$ for any A

    b. If $A \subset B$, then $P(B/A) = 1$ and $P(A/B) = \dfrac{P(A)}{P(B)}$

    c. If A and B are mutually exclusive, then $P(A/A \cup B) = \dfrac{P(A)}{P(A) + P(B)}$

15. Prove that if $P(A) > 0$ and $P(B) > 0$ then,

    a. If A and B are mutually exclusive, they cannot be independent.

    b. If A and B are independent, they cannot be mutually exclusive

**** GOOD LUCK****