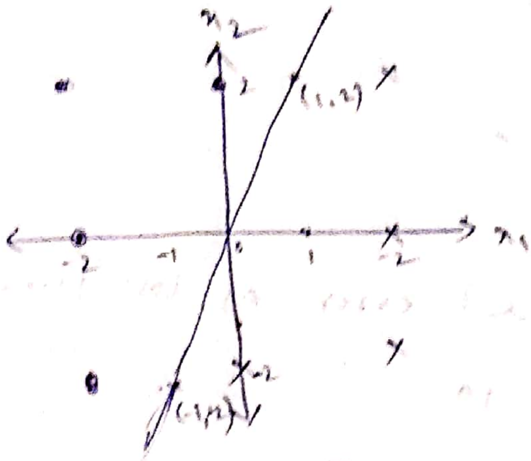
1)

a)

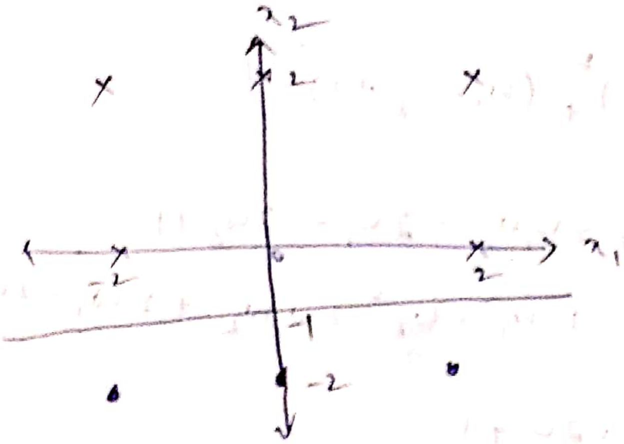


The linear classifier that separates both classes passes through $(0,0)$, $(1,2)$ and $(-1,-2)$.

So, the equation of line is $x_2 - 2x_1 = 0$

The weights are $\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and bias is 0.

b)



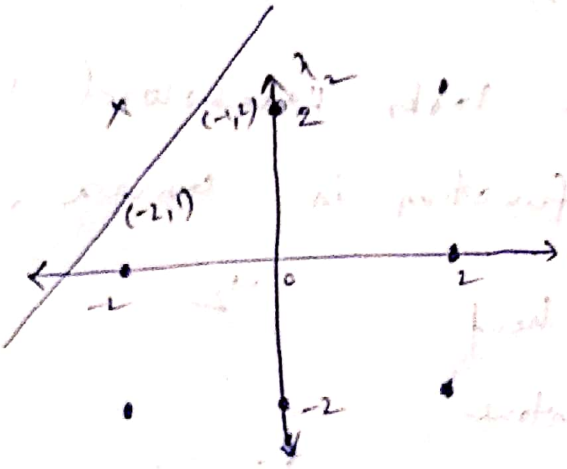
Here, the linear classifier equation is

$x_2 = -1 \Rightarrow 0x_1 + 1x_2 + 1 = 0$

So, the weights are

$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and bias $= 1$

c)



Here, the boundary line that separates both classes goes through $(-2,1)$ and $(-1,2)$

So, the equation of line

$(x_2 - 1) = \frac{1}{1} \cdot (x_1 + 2)$

$\Rightarrow x_1 - x_2 + 3 = 0.$

So, corresponding weights are

$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and bias $= 3$.

2) The given data ~~looks~~ is:

	x_1	x_2	y
$x^{(1)}$	1	1	1
$x^{(2)}$	1	-1	-1

With LMS (Least mean square) error as loss function the total empirical loss is

$$L = \frac{1}{m} \sum_{i=1}^{m} L(f(\theta), y^{(i)})$$

$$L(f(\theta), y^{(i)}) = (W x^{(i)} - y^{(i)})^2 \quad (\because \text{without bias})$$

$$\Rightarrow L = \frac{1}{2}\left((w_1 + w_2 - 1)^2 + (w_1 - w_2 + 1)^2 \right)$$

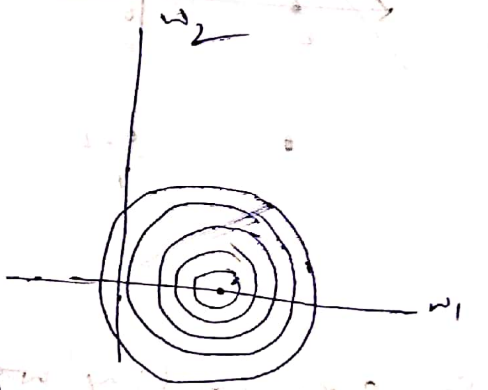
$$= \frac{1}{2}\left(w_1^2 + w_2^2 + 2w_1 w_2 - 2w_1 - 2w_2 + 1 + w_1^2 + w_2^2 - 2w_1 w_2 + 2w_1 - 2w_2 + 1 \right)$$

$$= w_1^2 + w_2^2 - 2w_2 + 1$$

$$= w_1^2 + (w_2 - 1)^2$$

a) The error surface looks paraboloid in 3-D graph and the function is convex.

The figure in right hand side shows the contour plot of error surface.



The curvature of error surface is uniform in both the axis w_1, w_2.

As the error surface is strictly convex, we get global minimum at $\Delta L_{(W)} = 0$

$$\Rightarrow \begin{bmatrix} w_1^* \\ w_2^* \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

b) The Hessian matrix of error function is

$$H = \begin{bmatrix} \frac{\partial^2 L}{\partial w_1} & \frac{\partial L}{\partial w_1 \partial w_2} \\ \frac{\partial L}{\partial w_2 \partial w_1} & \frac{\partial^2 L}{\partial w_2^2} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

The eigen values of $H = 2$ always

→ As eigenvalues of H are positive ~~and~~, we can say the error surface is convex.

→ we can also say curvature is uniform ~~be can~~ in both the axis because eigen values is same in both the directions of w_1 and w_2.