

Linear Regression based on Guassian model

input data : Slump test for concrete (<http://archive.ics.uci.edu/ml/datasets/Condition+Based+Maintenance+of+Naval+Propulsion+Plants#> (<http://archive.ics.uci.edu/ml/datasets/Condition+Based+Maintenance+of+Naval+Propulsion+Plants#>))

```
In [1]: import pandas as pd
import numpy as np
from sklearn.model_selection import train_test_split
from numpy.linalg import inv
from numpy.linalg import matrix_rank
from sklearn.metrics import explained_variance_score
```

Read input data:

```
In [2]: ip_data = pd.read_csv("slump_test.data")
ip_data = ip_data.set_index("No")
```

Divide the input data into two sets as samples for input and output random vectors :

Output random vector represents last three columns and input is the remaining columns

```
In [3]: x_size = 7
y_size = 3
total_vars = x_size+y_size
input_dist = ip_data.iloc[:, 0:x_size]
output_dist = ip_data.iloc[:, x_size:total_vars]

X_train, X_test, Y_train, Y_test = train_test_split(input_dist.values, output_dist.values, test_size=0.2, random_state=1)
```

Get the parameters for the conditional Gaussian $p(Y/X=x)$:

```
In [4]: #First get the parameters for joint gaussian using method of moments
mu_Y = Y_train.mean(axis=0)
mu_X = X_train.mean(axis=0)
input_cov_matrix = np.cov(np.transpose(np.concatenate([X_train,Y_train], axis=1)))
cov_XX = input_cov_matrix[0:x_size, 0:x_size]
cov_XY = input_cov_matrix[0:x_size, x_size:total_vars]
cov_YX = input_cov_matrix[x_size:total_vars, 0:x_size]
cov_YY = input_cov_matrix[x_size:total_vars, x_size:total_vars]
```

Covariance parameter for $p(Y/X=x)$:

```
In [5]: cov_Y_x = cov_YY - np.dot(np.dot(cov_YX, inv(cov_XX)), cov_XY)
```

Mean parameter for $p(Y/X=x)$:

```
In [6]: def estimate_Y_given_x_mean(x):  
        '''  
        When sample x is given , find the estimated mean of y/x with formula  
         $\mu(y/x) = \mu(y) + \text{Cov}(y,X) * \text{Cov}(X,X)^{-1} * (x - \mu(X))$   
        '''  
        global cov_XX, cov_YX, mu_Y, mu_X  
        return (mu_Y + np.dot(np.dot(cov_YX, inv(cov_XX)), (x-mu_X)))
```

Estimate parameters for test data using the $p(Y/X=x)$:

```
In [7]: Y_predicted = np.apply_along_axis(estimate_Y_given_x_mean, 1, X_test)  
        for i in range(Y_test.shape[1]):  
            print("Explained variance for Y{0} : {1}".format(i,explained_variance_score(  
                Y_test[:,i], Y_predicted[:,i])))
```

```
Explained variance for Y0 : 0.3635419587805233  
Explained variance for Y1 : 0.5255239777932958  
Explained variance for Y2 : 0.8784326202684619
```