DIGITAL COMMUNICATION LAB

Electrical Engineering Department

Experiment 9: Simulation of BER of BPSK in an AWGN channel in Matlab

AIM: To implement the BER of BPSK in an AWGN channel in MATLAB.

Theory

BPSK (Binary Phase Shift Keying) is a simple and widely used modulation technique in digital communications. It represents data using the phase of a carrier signal, which is a sinusoidal wave. article amsmath amssymb

In a **binary PSK system**, the pair of signals $s_1(t)$ and $s_2(t)$ used to represent binary symbols 1 and 0, respectively, is defined by:

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t),$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t + \pi\right) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t),$$

where T_b is the *bit duration* and E_b is the *transmitted signal energy per bit.* We

find it convenient, although not necessary, to assume that each transmitted bit contains an integral number of cycles of the carrier wave; that is, the carrier frequency f_c is chosen equal to n_c/T_b for some fixed integer n_c.
A pair of sinusoidal waves that differ only in a relative phase-shift of 180°, is referred to as an *antipodal signal*.

Error Probability of Binary PSK Using Coherent Detection

To make an optimum decision on the received signal $x(t)$ in favor of symbol 1 or symbol 0 (i.e., estimate the original binary sequence at the transmitter input), we assume that the receiver has access to a *locally generated replica of the basis function* $\phi(t)$. In other words, the receiver is *synchronized* with the transmitter, . We may identify two basic components in the binary PSK receiver:

1. **Correlator**, which correlates the received signal $x(t)$ with the basis function $\phi(t)$ on a bit-by-bit basis.

2. **Decision device**, which compares the correlator output against a zero-threshold, assuming that binary symbols 1 and 0 are equiprobable. If the threshold is exceeded, a decision is made in favor of symbol 1; if not, the decision is made in favor of symbol 0. Equality of the correlator with the zero-threshold is decided by the toss of a fair coin (i.e., in a random manner).

With coherent detection in place, we may apply the decision rule . Specifically, we partition the signal space of Figure 7.13 into two regions:

- the set of points closest to message point 1 at $+\sqrt{E_b}$; and

- the set of points closest to message point 2 at $-\sqrt{E_b}$.

This is accomplished by constructing the midpoint of the line joining these two message points and then marking off the appropriate decision regions. hese two decision regions are marked Z_1 and Z_2, according to the message point around which they are constructed.
The decision rule is now simply to decide that signal $s_1(t)$ (i.e., binary symbol 1) was transmitted if the received signal point falls in region Z_1 and to decide that signal $s_2(t)$ (i.e., binary symbol 0) was transmitted if the received signal point falls in region Z_2. Two kinds of errors are possible:

1. **Error of the first kind.** Signal $s_2(t)$ is transmitted but the noise is such that the received signal point falls inside region Z_1; so the receiver decides in favor of signal $s_1(t)$.

2. **Error of the second kind.** Signal $s_1(t)$ is transmitted but the noise is such that the received signal point falls inside region Z_2; so the receiver decides in favor of signal $s_2(t)$. article amsmath amssymb amsthm

   To calculate the probability of making an error of the first kind, we note that the decision region associated with symbol 1 or signal $s_1(t)$ is described by:

   $$Z_1 : 0 < x_1 < \infty$$

   where the observable element x_1 is related to the received signal $x(t)$ by:

   $$x_1 = \int_0^{T_b} x(t)\phi_1(t)$$

The conditional probability density function of random variable X_1, given that symbol 0 (i.e., signal $s_2(t)$) was transmitted, is defined by:

$$f_{X_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}\left(x_1 + \sqrt{E_b}\right)^2\right]$$

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The conditional probability of the receiver deciding in favor of symbol 1, given that symbol 0 was transmitted, is therefore:

$$P_{10} = \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{1}{N_0}\left(x_1 + \sqrt{E_b}\right)^2\right] dx_1$$

Putting:

$$z = \sqrt{\frac{2}{N_0}}\left(x_1 + \sqrt{E_b}\right)$$

and changing the variable of integration from x_1 to z, we may compactly rewrite above equation in terms of the Q-function:

$$P_{10} = \frac{1}{\sqrt{2\pi}} \int_0^\infty \exp\left(-\frac{z^2}{2}\right) dz \tag{0}$$

Using the above formula , we get:

$$P_{10} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Consider next an error of the second kind. We note that the signal space is symmetric with respect to the origin. It follows, therefore, that P_{01}, the conditional probability of the receiver deciding in favor of symbol 0, given that symbol 1 was transmitted.

Thus, averaging the conditional error probabilities P_{10} and P_{01}, we find that the *average probability of symbol error*, or, equivalently, the *BER for binary PSK* using coherent detection and assuming equiprobable symbols is given by:

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

As we increase the transmitted signal energy per bit E_b for a specified noise spectral density $N_0/2$, the message points corresponding to symbols 1 and 0 move farther apart, and the average probability of error P_e is correspondingly reduced , which is intuitively satisfying.

Note: The experiments which you performed in lab should also be attached with this mannual.

Conclusion: We have successfully implemented the BER of BPSK in an AWGN channel and its changes with SNR in MATLAB .

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1 S. Haykin, Communication Systems, 4th ed. Hoboken, NJ: Wiley, 2001.