DIGITAL COMMUNICATION LAB

Electrical Engineering Department

Experiment 8: Generation of orthonormal basis function and signal space representation by using Gram-Schmidt Orthogonalization Procedure in Matlab

AIM: To implement Gram-Schmidt Orthogonalization Procedure for the generation of orthonormal basis function and signal space representation in MATLAB.

Theory

Gram-Schmidt orthogonalization is a method used to create orthogonal basis functions for signals, enhancing the efficiency and reliability of transmission. This technique enables better separation of symbols, minimizes interference, and maximizes bandwidth utilization, making it essential for modern communication systems like QAM, PSK, and OFDM.

To proceed with the formulation of this procedure,suppose we have a set of M energy signals denoted by $s_1(t), s_2(t), \ldots, s_M(t)$.Starting with $s_1(t)$ chosen from this set arbitrarily, the first basis function is defined by

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

where E_1 is the energy of the signal $s_1(t)$.

Then, clearly, we have

$$\begin{aligned} s_1(t) &= \sqrt{E_1}\,\phi_1(t) \\ &= s_{11}\,\phi_1(t) \end{aligned}$$

where the coefficient $s_{11} = \sqrt{E_1}$ and $\phi_1(t)$ has unit energy as required.

Next, using the signal $s_2(t)$, we define the coefficient s_{21} as

$$s_{21} = \int_0^T s_2(t)\phi_1(t)\,dt$$

We may thus introduce a new intermediate function

$$g_2(t) = s_2(t) - s_{21}\phi_1(t)$$

which is orthogonal to $\phi_1(t)$ over the interval $0 \leq t \leq T$ by virtue of the definition of s_{21} and the fact that the basis function $\phi_1(t)$ has unit energy. Now, we are ready to define the second basis function as

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)\,dt}}$$

By bserving and simplifying the above equations, we get the desired result

$$\phi_2(t) = \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{E_2 - s_{21}^2}}$$

where E_2 is the energy of the signal $s_2(t)$. From above we readily see that

$$\int_0^T \phi_2^2(t)\,dt = 1$$

in which case above equation yields

$$\int_0^T \phi_1(t)\phi_2(t)\,dt = 0$$

That is to say, $\phi_1(t)$ and $\phi_2(t)$ form an orthonormal pair as required. Continuing the procedure in this fashion, we may, in general, define

$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\phi_j(t)$$

where the coefficients s_{ij} are themselves defined by

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)\,dt, \quad j = 1, 2, \ldots, i-1$$

For $i = 1$, the function $g_1(t)$ reduces to $s_1(t)$.

Given the $g_i(t)$, we may now define the set of basis functions

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)\,dt}}, \quad j = 1, 2, \ldots, N$$

which form an orthonormal set. The dimension N is less than or equal to the number of given signals, M, depending on one of two possibilities:

- The signals $s_1(t), s_2(t), \ldots, s_M(t)$ form a linearly independent set, in which case $N = M$.

- The signals $s_1(t), s_2(t), \ldots, s_M(t)$ are not linearly independent, in which case $N < M$ and the intermediate function $g_i(t)$ is zero for $i > N$.

Task:

Consider three vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \text{and} \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

Apply the Gram-Schmidt process to find an orthonormal basis for the space spanned by these vectors. article amsmath

Note: The two experiments which you performed in lab should also be attached with this mannual

Conclusion: Study and analysis Gram-Schmidt Orthogonalization Procedureof using MATLAB.

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1 S. Haykin, Communication Systems, 4th ed. Hoboken, NJ: Wiley, 2001.