Experiment 1O: Simulation of BER of BFSK in an AWGN Channel in MATLAB

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Aim

To implement the BER of BFSK in an AWGN channel in MATLAB.

Binary FSK

In **binary FSK**, symbols 1 and 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount. A typical pair of sinusoidal waves is described by:

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t), & 0 \le t \le T_b, \\ 0, & \text{elsewhere}, \end{cases}$$

where $i = 1, 2$, and E_b is the transmitted signal energy per bit. The transmitted frequency is set at

$$f_i = \frac{n_c + i}{T_b}, \quad \text{for some fixed integer } n_c \text{ and } i = 1, 2.$$

Symbol 1 is represented by $s_1(t)$ and symbol 0 by $s_2(t)$. The FSK signal described here is known as *Sunde's FSK*. It is a continuous-phase signal, in the sense that phase continuity is always maintained, including the inter-bit switching times.

The signals $s_1(t)$ and $s_2(t)$ are orthogonal, but not normalized to have unit energy. The most useful form for the set of orthonormal basis functions is described by:

$$\phi_i(t) = \begin{cases} \sqrt{\frac{2}{T_b}} \cos(2\pi f_i t), & 0 \le t \le T_b, \\ 0, & \text{elsewhere}, \end{cases}$$

where $i = 1, 2$.

Correspondingly, the coefficient s_{ij} for where $i = 1, 2$ and $j = 1, 2$ is defined by:

$$s_{ij} = \int_0^{T_b} s_i(t)\phi_j(t)\, dt.$$

Substituting $s_i(t)$ and $\phi_j(t)$:

$$s_{ij} = \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_j t)\, dt.$$

Carrying out the integration , the formula for s_{ij} simplifies to:

$$s_{ij} = \begin{cases} \sqrt{E_b}, & i = j, \\ 0, & i \neq j. \end{cases}$$

Thus, unlike binary PSK, binary FSK is characterized by having a signal-space diagram that is two-dimensional (i.e., $N = 2$) with two message points (i.e., $M = 2$). The two message points are defined by the vectors:

$$\mathbf{s}_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix}$$

$$\mathbf{s}_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$$

The Euclidean distance $\|\mathbf{s}_1 - \mathbf{s}_2\|$ is equal to $\sqrt{2E_b}$.

Error Probability of Binary FSK

The observation vector $\mathbf{x}$ has two elements x_1 and x_2 that are defined by, respectively,

$$x_1 = \int_0^{T_b} x(t)\phi_1(t)\,dt$$

and

$$x_2 = \int_0^{T_b} x(t)\phi_2(t)\,dt$$

where $x(t)$ is the received signal, whose form depends on which symbol was transmitted. Given that symbol 1 was transmitted, $x(t)$ equals $s_1(t) + w(t)$, where $w(t)$ is the sample function of a white Gaussian noise process of zero mean and power spectral density $N_0/2$. If, on the other hand, symbol 0 was transmitted, $x(t)$ equals $s_2(t) + w(t)$. Now, applying the decision rule assuming the use of coherent detection at the receiver, we find that the observation space is partitioned into two decision regions, labeled Z_1 and Z_2 The decision boundary, separating region Z_1 from region Z_2, is the perpendicular bisector of the line joining the two message points. The receiver decides in favor of symbol 1 if the received signal point represented by the observation vector $\mathbf{x}$ falls inside region Z_1. This occurs when $x_1 > x_2$. If, on the other hand, we have $x_1 < x_2$, the received signal point falls inside region Z_2 and the receiver decides in favor of symbol 0. On the decision boundary, we have $x_1 = x_2$, in which case the receiver makes a random guess in favor of symbol 1 or 0.

To proceed further, we define a new Gaussian random variable Y whose sample value y is equal to the difference between x_1 and x_2; that is,

$$y = x_1 - x_2$$

The mean value of the random variable Y depends on which binary symbol was transmitted. Given that symbol 1 was sent, the Gaussian random variables X_1 and X_2, whose sample values are denoted by x_1 and x_2, have mean values equal to $\sqrt{E_b}$ and zero, respectively. Correspondingly, the conditional mean of the random variable Y given that symbol 1 was sent is

$$\mathbb{E}[Y|1] = \mathbb{E}[X_1|1] - \mathbb{E}[X_2|1] = +\sqrt{E_b}$$

On the other hand, given that symbol 0 was sent, the random variables X_1 and X_2 have mean values equal to zero and $\sqrt{E_b}$, respectively. Correspondingly, the conditional mean of the random variable Y given that symbol 0 was sent is

$$\mathbb{E}[Y|0] = \mathbb{E}[X_1|0] - \mathbb{E}[X_2|0] = -\sqrt{E_b}$$

The variance of the random variable Y is independent of which binary symbol was sent. Since the random variables X_1 and X_2 are statistically independent, each with a variance equal to $N_0/2$, it follows that

$$\mathrm{var}[Y] = \mathrm{var}[X_1] + \mathrm{var}[X_2] = N_0$$

Suppose we know that symbol 0 was sent. The conditional probability density function of the random variable Y is then given by

$$f_Y(y|0) = \frac{1}{\sqrt{2\pi N_0}} \exp\left[-\frac{\left(y+\sqrt{E_b}\right)^2}{2N_0}\right]$$

Since the condition $x_1 > x_2$, or, equivalently, $y > 0$ corresponds to the receiver making a decision in favor of symbol 1, we deduce that the conditional probability of error given that symbol 0 was sent is

$$p_{10} = \mathbb{P}(y > 0|\text{symbol 0 was sent}) = \int_0^\infty f_Y(y|0)\,dy$$

$$= \frac{1}{\sqrt{2\pi N_0}} \int_0^\infty \exp\left[-\frac{\left(y+\sqrt{E_b}\right)^2}{2N_0}\right] dy$$

To put the integral in a standard form involving the Q-function, we set

$$z = \frac{y+\sqrt{E_b}}{\sqrt{N_0}}$$

Then, changing the variable of integration from y to z, we may rewrite (7.165) as

$$p_{10} = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{E_b/N_0}}^\infty \exp\left(-\frac{z^2}{2}\right) dz = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Similarly, we may show the p_{01}, the conditional probability of error given that symbol 1 was sent, has the same value . Accordingly, averaging p_{10} and p_{01} and assuming equiprobable symbols, we find that the average probability of bit error, or equivalently, the BER for binary FSK with coherent detection is

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Conclusion

The experiment successfully simulated the BER of BFSK in an AWGN channel and observed its dependence on SNR using MATLAB.

References

S. Haykin, *Communication Systems*, 4th ed., Hoboken, NJ: Wiley, 2001.