Basic Algorithms for Digital Image Analysis

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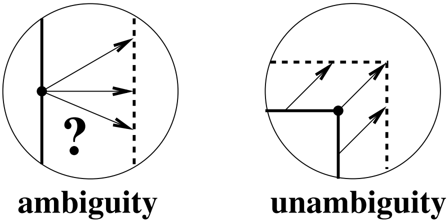
Corner detection

Corners in curves and images

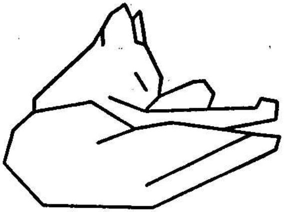
- Corner is sharp turn of contour
- Corners are used in shape analysis and motion analysis
- They are dominant in human perception of 2D shapes
- Two types of corner detection:
  - in digital curves
    - $\Rightarrow$ assumes extracted contours
  - in digital images
    - $\Rightarrow$ does not assume extracted contours
- We only consider corner detection in greyscale images

Corners in motion analysis: the aperture problem



- Motion vectors are **ambiguous** at edge
  - locally, normal component can only be determined
  - tangential component cannot be determined
- This problem is **aperture problem**
  - the problem arises from local character of observation
- Motion vectors are **unambiguous** at corner

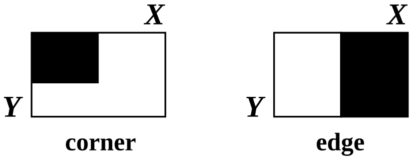
Importance of corners in shape perception



The Attneave's Cat
(Attneave, 1955)

- Original smooth shape restored based on small number of high curvature points
- Cat is easy to recognise despite big loss of data

Image corners and edges



- **Corners** are locations where variations of intensity function $f(x, y)$ in both X **and** Y are high
  - $\Rightarrow$ both partial derivatives f_x and f_y are large
- **Edges** are locations where variation of $f(x, y)$ in certain direction is high, while variation in the orthogonal direction is low
  - $\Rightarrow$ when edge is oriented along Y, f_x is large, f_y small

Two selected corner detectors

- Many corner detectors exist, but we will only consider two of them
  - **Kanade-Lucas-Tomasi** (KLT) operator (1994)
  - **Harris** operator (1988)
- Reasons
  - most frequently used
  - select corners and other **interest points**
  - many application areas

    $\Rightarrow$ motion tracking, stereo matching, image database retrieval
  - relatively simple but still efficient and reliable
- KLT and Harris are closely related
  - both based on the **local structure matrix** (tensor)

Local structure matrix C_{str}

In every position in image $f(x, y)$, calculate

$$C_{str} \doteq w_G(r;\sigma) * \begin{bmatrix} (f_x)^2 & f_x f_y \\ f_x f_y & (f_y)^2 \end{bmatrix} = \begin{bmatrix} \widehat{(f_x)^2} & \widehat{f_x f_y} \\ \widehat{f_x f_y} & \widehat{(f_y)^2} \end{bmatrix}$$

- Derivatives of $f(x, y)$ are first calculated using gradient masks
  - if necessary, image is smoothed before doing this
- Then, entries of C_{str} are obtained: $(f_x)^2$, $(f_y)^2$, $f_x f_y$

  $\Rightarrow$ **Not** f_{xx}, f_{yy}, f_{xy}!
- Finally, each entry is smoothed (integrated) by Gaussian filter $w_G(r;\sigma)$
  - parameter σ controls fineness of edges
  - often, box filter is used instead of Gaussian
  - smoothing is denoted by hat: $\widehat{\ }$

Eigenvalues of local structure matrix C_{str} 1/2

- Matrix C_{str} is **symmetric**

$\Rightarrow$ Matrix R, $R^T R = RR^T = \mathbf{I}$, exists that diagonalises C_{str}

  - $\mathbf{I}$ is 2×2 unit matrix
  - R is rotation of local coordinate system

- Diagonal entries λ_1, λ_2 are **eigenvalues** of C_{str}:

$$C_{str} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

- Eigenvalues are solutions of characteristic equation

$$\det\left(C_{str} - \lambda\mathbf{I}\right) = 0$$

Eigenvalues of local structure matrix C_{str} 2/2

- To obtain eigenvalues, we can use properties

$$D \doteq \det C_{str} = \prod_i \lambda_i = \lambda_1 \lambda_2$$

$$T \doteq \text{trace } C_{str} = \sum_i \lambda_i = \lambda_1 + \lambda_2$$

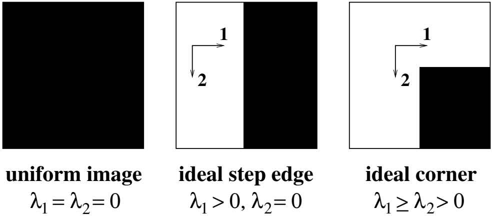
- Eigenvalues are given by the derivatives as

$$\lambda_{1,2} = \frac{1}{2}\left(T \pm \sqrt{T^2 - 4D}\right)$$

$$= \frac{1}{2}\left(\widehat{f_x^2} + \widehat{f_y^2} \pm \sqrt{\left(\widehat{f_x^2} - \widehat{f_y^2}\right)^2 + 4\left(\widehat{f_x f_y}\right)^2}\right)$$

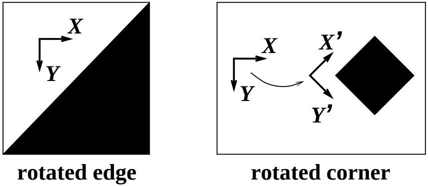
$$\Rightarrow \lambda_1 \geq \lambda_2 \geq 0$$

Geometric interpretation of eigenvalues



- For perfectly **uniform image**:
  - $C_{str} = 0$ and $\lambda_1 = \lambda_2 = 0$
- For perfectly black-and-white **step edge**:
  - $\lambda_1 > 0,\ \lambda_2 = 0$
  - first eigenvector is orthogonal to edge
- For **corner** of black square against white background:
  - $\lambda_1 \geq \lambda_2 > 0$
  - higher contrast $\longrightarrow$ larger eigenvalue

Meaning of structure matrix diagonalisatio



- Eigenvectors encode edge directions
  - eigenvalues encode edge magnitudes
- Corner is identified by two strong edges

  $\Rightarrow$ *smaller eigenvalue*, λ_2, is *large enough*
- Diagonalisation of C_{str} means aligning local coordinate axes with the two edge directions
  - this ensures orientation-invariance
  - simply setting threshold on $\min(f_x, f_y)$ would not work!

Kanade-Lucas-Tomasi (KLT) corner detector

Similarly to edge detectors, corner detectors have two steps:

1. Corner filtering
2. Post-processing, i.e., corner localisation

KLT corner detector has two **parameters**:

- Neighbourhood size D that has two roles:
  - in corner filtering, it defines the size of mean filter in definition of C_{str}
  - in post-processing, it defines the smallest distance between two corners to be detected separately
- Threshold λ_{thr} on the smaller eigenvalue λ_2
  - corner can only be in locations where $\lambda_2 > \lambda_{thr}$

KLT corner detection algorithm

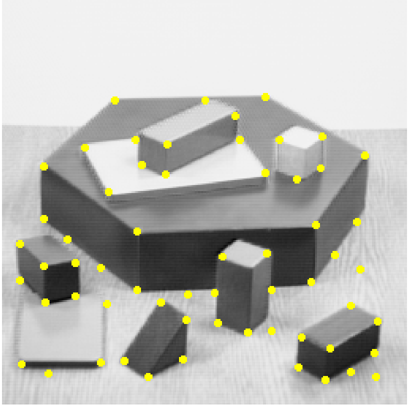
Algoritmus: KLT corner detector

1. Compute f_x and f_y over entire image $f(x, y)$
2. For each image point p:
   (a) form matrix C_{str} over $D \times D$ neighbourhood of p
   (b) compute λ_2, the smaller eigenvalue of C_{str}
   (c) if $\lambda_2 > \lambda_{thr}$, save point p into list L
3. Sort L in decreasing order of λ_2, obtain sorted list L_S
4. Scan L_S from top to bottom; for each current point, p_i
   - consider all points p_k, $k > i$
   - delete p_k from L_S if $p_k \in D \times D$ neighbourhood of p_i

Output and parameters of KLT corner detector

- Output is list of feature points with following properties:
  - in these points, $\lambda_2 > \lambda_{thr}$
  - D-neighbourhoods of these points do not overlap
- λ_{thr} controls detector sensitivity depending on contrast:
  - larger contrast $\Rightarrow$ stronger corners $\Rightarrow$ larger λ_{thr}
- D controls detector fineness is two ways:
  - larger $D \Rightarrow$ larger mean filter $\Rightarrow$ corner tends to move away from actual position
  - larger $D \Rightarrow$ larger min distance between detected corners $\Rightarrow$ corner close to another one may be lost
  - for smal D, spurious (noisy) responses are possible
  - typical values: $D_w = 5$–19

Example of corner detection by KLT operator



- Most of corners are detected.
  - there are lost ones, as well
- False detections can be observed.
  - other well-textured regions also detected

Harris measure of corner strength

- Harris corner detector appeared before KLT
  - KLT is *different interpretation* of same idea
- Harris measure of **corner strength**:

$$H(x, y) = \det C_{str} - \alpha \left(\mathrm{trace}\ C_{str}\right)^2$$

  - α is a parameter
  - $H \geq 0$ if $0 \leq \alpha \leq 0.25$
- **Detect corner** when strength is large: $H(x, y) > H_{thr}$
  - H_{thr} is another parameter

  $\Rightarrow$ threshold on corner strength
- As in KLT, use D-neighbourhoods for post-processing
  - remove weak corners in neighbourhood of strong corner

  $\Rightarrow$ only local maxima of H considered

Parameter of Harris operator

Introduce $\lambda_1 = \lambda$, $\lambda_2 = \kappa\lambda$, $0 \leq \kappa \leq 1$.
Using relations between eigenvalues, determinant and trace

$$\det A = \prod_i \lambda_i \qquad \text{trace } A = \sum_i \lambda_i,$$

we obtain

$$H = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2 = \lambda^2 \left[\kappa - \alpha(1+\kappa)^2\right]$$

Assuming that $H \geq 0$, we have

$$0 \leq \alpha \leq \frac{\kappa}{(1+\kappa)^2} \leq 0.25$$

and, for small κ,

$$H \approx \lambda^2 (\kappa - \alpha), \alpha \lesssim \kappa$$

Relation between Harris and KLT operators

- α of Harris is similar to λ_{thr} of KLT
  - larger $\alpha \Rightarrow$ smaller $H \Rightarrow$ **less sensitive** detector
  - $\Rightarrow$ less corners detected
  - smaller $\alpha \Rightarrow$ larger $H \Rightarrow$ **more sensitive** detector
  - $\Rightarrow$ more corners detected
- Usually, H_{thr} is set close, or equal, to 0 and *fixed*
  - α is variable **parameter**

original image

$\alpha = 0.05$

$\alpha = 0.10$

$\alpha = 0.20$

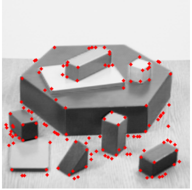
$\alpha = 0.22$

$\alpha = 0.24$

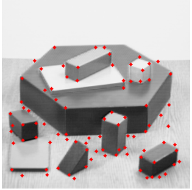
Example of corner detection by Harris operator

original corner strength

thresholded strength



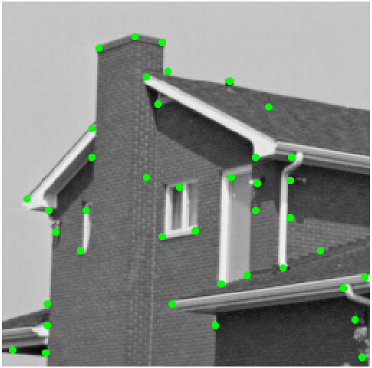
original corners



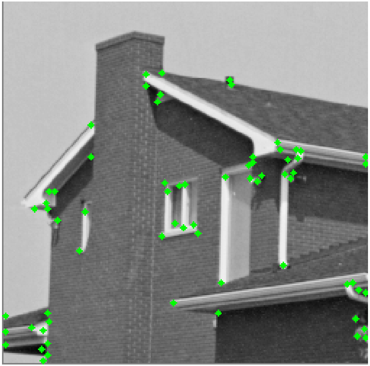
after post-processing

Outline

Comparison of the two operators



KLT, 40 corners



Harris, $\alpha = 0.03$

Summary of corner detection

- KLT and Harris corner detectors are conceptually related
  - based on local structure matrix C_{str}
  - search for points where variations in two orthogonal directions are large
- Difference between the two detectors
  - KLT sets **explicit** threshold on diagonalised C_{str}
  - Harris sets **implicit** threshold via corner magnitude $H(x, y)$
- KLT detector
  - result usually closer to human perception of corners
  - used for **motion tracking** in popular KLT Tracker
- Harris detector
  - good **repeatability** under varying rotation and illumination
  - used in stereo matching and image database retrieval
- Both may detect **interest points** other than corners
  - small well-textured regions