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Non-linear flow modes of identified particles in Pb–Pb collisions at $\sqrt{s_{\text{NN}}} = 5.02 \text{ TeV}$

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Abstract

The p_{T} -differential non-linear flow modes, $v_{4,22}$, $v_{5,32}$, $v_{6,33}$ and $v_{6,222}$ for ~~identified particles~~ π^{\pm} , K^{\pm} , $p + \bar{p}$, K_S^0 , $\Lambda + \bar{\Lambda}$ and ϕ -meson have been measured for the first time in Pb–Pb collisions at $\sqrt{s_{\text{NN}}} = 5.02 \text{ TeV}$ with the ALICE detector at the Large Hadron Collider. The results were obtained with a multi-particle ~~correlation technique, namely the generic framework,~~ technique, correlating the identified hadrons with ~~the~~ reference particles from a different pseudorapidity region. ~~The second and third order anisotropic flow coefficients are mostly related to their corresponding initial spatial anisotropy. It has been shown that higher harmonics ($n > 3$) have significant contribution from lower order flow harmonics giving rise to~~ These non-linear flow modes in the higher flow harmonics. ~~Thus, these non-linear~~ observables probe the contribution of the second and third order symmetry plane angles in higher flow harmonics. ~~Interestingly, all~~ All the characteristic features observed in previous p_{T} -differential measurements (e.g. v_2 and v_3) for various particle species are also present in the measurement of the non-linear flow modes, i.e. increase of magnitude ~~by~~ with increasing the centrality percentile, mass ordering at low p_{T} and particle type grouping in the intermediate p_{T} range. Hydrodynamical calculations (iEBE-VISHNU) that use different initial conditions and values of shear and bulk viscosity to entropy density ratios are confronted with the data at low transverse momenta. This comparison provides increased discriminatory power in the study of initial conditions as well as a new stringent ~~constraint~~ constraints to hydrodynamical calculations.

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1 Introduction

Lattice quantum chromodynamics (QCD) calculations [1, 2] suggest that at extremely high temperature and energy density a state of matter is produced in which quarks and gluons are no longer confined ~~to~~ into hadrons. This state of matter is called quark-gluon plasma (QGP) [3–5]. The main goal of heavy-ion collision experiments is to study the properties of the QGP, such as the speed of the sound ~~in this medium~~, the equation of state and its ~~transport properties, e.g.~~ shear and bulk ~~viscosity~~ viscosities.

One of the observables sensitive to the properties of the QGP is the azimuthal angular distribution of particles emitted in the plane perpendicular to the beam axis. In a heavy ion collision, the overlap region of the colliding nuclei exhibits an irregular shape [6–10]. This spatial irregularity is a superposition of the geometry, i.e. centrality of the collision reflected in the value of the impact parameter, and the ~~anisotropic initial~~ density profile of nucleons participating in the collision. Through interactions between partons and at later stages between the produced particles, this spatial irregularity is transferred into ~~the an~~ anisotropy in momentum space ~~which~~. The latter is usually expressed by ~~the a~~ Fourier expansion of the azimuthal particle distribution [11, 12] according to

$$\underline{E \frac{d^3N}{dp^3} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T d\eta} 1 + 2 \frac{dN}{d\phi} \propto \sum_{n=1}^{\infty} v_n(p_T, \eta) \cos[n(\phi - \Psi_n)]}, \quad (1)$$

where ~~E, N, p, p_T, ϕ~~ and η are the ~~energy~~, particle yield, ~~total momentum~~, transverse momentum, azimuthal angle and pseudorapidity of particles, respectively, and Ψ_n is the azimuthal angle of the symmetry plane of the n^{th} -order coefficient [6–9]. The parameter v_n is the magnitude of the n^{th} -order complex flow coefficient V_n , defined as $V_n = v_n e^{in\Psi_n}$, and can be calculated according to

$$v_n = \langle \cos[n(\phi - \Psi_n)] \rangle, \quad (2)$$

where the brackets denote an average over all particles in all events. Since the symmetry planes are not accessible experimentally, the flow coefficients are estimated solely from the azimuthal angles of the particles emitted in the transverse plane. Measurements of different anisotropic flow coefficients at both RHIC [13–16] and the LHC [17–19] have not only confirmed the production of a strongly coupled quark gluon plasma (sQGP) but they have also constrained the value of its shear viscosity over entropy density (η/s) very close to the ~~conjectured~~ lower limit of $1/4\pi$ ~~from conjectured by~~ AdS/CFT [20]. In addition, ~~both viscous hydrodynamical calculations and measurements [21]~~ the comparison between experimental data [21] and both viscous hydrodynamical calculations show that higher order flow coefficients and more importantly their transverse momentum dependence are more sensitive probes than lower order coefficients, i.e. v_2 and v_3 , of the initial spatial irregularity and its fluctuations [9].

~~The~~ This initial state spatial irregularity can be quantified with the standard (moment-based) anisotropy coefficients, ϵ_n . Together with their corresponding initial symmetry planes, Φ_n , ~~the initial anisotropy coefficients ϵ_n~~ can be calculated from the transverse positions of the ~~participating nucleons as [8, 9]~~ nucleons participating in a collisions according to [22, 23]

$$\epsilon_n e^{in\Phi_n} = \frac{\langle r^n e^{in\phi} \rangle}{\langle r^n \rangle} \text{ (for } n > 1), \quad (3)$$

where the brackets denote ~~the an~~ average over the transverse position of all participating nucleons ~~, ϕ is azimuthal angle and that have an azimuthal angle ϕ and a polar distance from the centre r the polar distance~~. Model calculations show that V_2 and to a large extent, V_3 are determined by their corresponding initial spatial anisotropy coefficients, ϵ_2 and ϵ_3 , respectively [8]. It has been ~~recently realised that~~

V_n ($n > 3$) realised that for $n > 3$, V_n are not linearly correlated with their corresponding ϵ_n [24]–[9, 24]. In fact, a cumulant-based definition of initial anisotropic coefficient suggests additional terms in the definition of ϵ_n for higher order flow coefficients ($n > 3$). As an example, the fourth order spatial anisotropy is given by [23, 25]

$$\epsilon'_4 e^{i4\Phi'_4} = \epsilon_4 e^{i4\Phi_4} + \frac{3\langle r^2 \rangle^2}{\langle r^4 \rangle} \epsilon_2^2 e^{i4\Phi_2}, \quad (4)$$

where ϵ'_4 is the cumulant-based spatial anisotropy coefficient [23, 25]. This dependence on lower order initial anisotropies gives rise to additional terms in the higher order flow coefficients. As a result, V_n ($n > 3$) obtain contributions not only from the linear response of the system to ϵ_n , but also a non-linear response proportional to the product of lower order initial spatial anisotropies [24, 26].

It was shown in [27] that in particular, it was shown that for a single event, V_n with $n = 4, 5, 6$ are decomposed to the linear (V_n^L) and non-linear (V_n^{NL}) modes according to

$$\begin{aligned} V_4 &= V_4^L + V_4^{\text{NL}} = V_4^L + \chi_{4,22}(V_2)^2, \\ V_5 &= V_5^L + V_5^{\text{NL}} = V_5^L + \chi_{5,32}V_3V_2, \\ V_6 &= V_6^L + V_6^{\text{NL}} = V_6^L + \chi_{6,222}(V_2)^3 + \chi_{6,33}(V_3)^2 + \chi_{6,42}V_2V_4^L, \end{aligned} \quad (5)$$

where $\chi_{n,mk}$, known as non-linear flow mode coefficients, quantify the contributions of the non-linear modes to the total V_n [27]–[26, 27]. The magnitude of the p_T -differential non-linear modes in higher order flow coefficients, v_n^{NL} , can be written as:

$$v_{4,22}(p_T) = \frac{\langle v_4(p_T)v_2^2 \cos(4\Psi_4 - 4\Psi_2) \rangle}{\sqrt{\langle v_2^4 \rangle}} \approx \langle v_4(p_T) \cos(4\Psi_4 - 4\Psi_2) \rangle, \quad (6)$$

$$v_{5,32}(p_T) = \frac{\langle v_5(p_T)v_3v_2 \cos(5\Psi_5 - 3\Psi_3 - 2\Psi_2) \rangle}{\sqrt{\langle v_3^2 v_2^2 \rangle}} \approx \langle v_5(p_T) \cos(5\Psi_5 - 3\Psi_3 - 2\Psi_2) \rangle, \quad (7)$$

$$v_{6,33}(p_T) = \frac{\langle v_6(p_T)v_3^2 \cos(6\Psi_6 - 6\Psi_3) \rangle}{\sqrt{\langle v_3^4 \rangle}} \approx \langle v_6(p_T) \cos(6\Psi_6 - 6\Psi_3) \rangle, \quad (8)$$

$$v_{6,222}(p_T) = \frac{\langle v_6(p_T)v_2^3 \cos(6\Psi_6 - 6\Psi_2) \rangle}{\sqrt{\langle v_2^6 \rangle}} \approx \langle v_6(p_T) \cos(6\Psi_6 - 6\Psi_2) \rangle, \quad (9)$$

where brackets denote an average over all events. The approximation is valid assuming a weak correlation between the lower ($n = 2, 3$) and higher ($n > 3$) order flow coefficients.

Various measurements of total flow coefficients the p_T -differential anisotropic flow, $v_n(p_T)$, of charged particles at the LHC [17–19, 34] and RHIC [13–16] as a function of transverse momentum [28–31] have provided a testing ground for hydrodynamical calculations that attempt to describe the dynamical evolution of the system created in heavy-ion collisions. Early predictions showed that p_T -differential anisotropic flow for different particle species have can reveal more information about the equation of state and hadronic rescattering phase [32, 33] as well as particle production mechanisms. In order to test these predictions, $v_n(p_T)$ have been measured for different particle species at the LHC [17–19, 34] and RHIC

[13–16]. These measurements have revealed that an interplay between radial flow and ~~the total flow coefficients anisotropic flow~~ leads to a characteristic mass dependence in the low transverse momentum (p_T) region ($p_T < 3$ GeV/c). For higher values of p_T (up to 6 GeV/c) results ~~for all total flow coefficients~~ indicate a particle type grouping where baryons have a larger magnitude than the one of mesons. This feature was explained in a dynamical model where flow develops at the partonic level followed by quark coalescence into hadrons [35, 36]. This model assumes that the invariant spectrum of produced particles is proportional to the product of the spectra of their constituents and ~~in turn flow~~, in turn, the flow coefficients of produced particles is ~~a the~~ sum of the flow v_n values of its constituents. ~~Previous measurements~~ Measurements of lower order total flow coefficients exhibit number of constituent quarks (NCQ) scaling at RHIC [37] and the LHC [17, 18] at an approximate level of $\pm 20\%$ for $p_T > 3$ GeV/c.

The measurements of non-linear flow modes in different collision geometries challenge hydrodynamic models and have the potential to further constrain both the initial conditions of the collision system and its transport properties [27]. The p_T -dependent non-linear flow modes of identified particles are important observable for studying the characteristics of QGP. They not only put a stringent constraint on both the initial conditions of the collision system and its transport properties, i.e. η/s and ζ/s , but also allow to test the effect of late-stage interactions in the hadronic rescattering phase as well as the effect of particle production via the coalescence mechanism to the development of the mass ordering and particle type grouping [19, 28].

In this article, we report the first results of the p_T -differential non-linear flow modes, $v_{4,22}$, $v_{5,32}$, $v_{6,33}$ and $v_{6,222}$ for π^\pm , K^\pm , $p + \bar{p}$, K_S^0 , $\Lambda + \bar{\Lambda}$ and ϕ measured in Pb–Pb collisions at ~~the a~~ centre of mass energy per nucleon pair $\sqrt{s_{NN}} = 5.02$ TeV, recorded with the ALICE detector [38] at the LHC. The detectors and the selection criteria used in this analysis are described in Sec. 2 and 3, respectively. The ~~results are obtained with a generic framework described analysis methodology and technique is presented~~ in Section 4 and in detail in [50]. In this article, the identified hadron under study and the charged reference particles are obtained from different, non-overlapping pseudorapidity regions. The correlations not related to the common symmetry plane (known as non-flow), like those arising from jets, resonance decays and quantum statistics correlations, are suppressed by using multi-particle correlations as explained in Section 4 and the residual effect is assigned as a systematic uncertainty, described in Section 5. All coefficients for charged particles were measured separately for particles and anti-particles and were found to be compatible within statistical uncertainties. The reported measurements are therefore an average of the results for the opposite charges. The results are reported within the pseudorapidity range $|\eta| < 0.8$ at different collision centralities between 0–60% range of Pb–Pb collisions.

2 Experimental setup

ALICE [38, 39] is one of the four large experiments at the LHC, particularly designed to cope with the large charged-particle densities present in central Pb–Pb collisions [40]. By convention, the z -axis is parallel to the beam direction, the x -axis is horizontal and points towards the centre of the LHC, and the y -axis is vertical and points upwards. The apparatus consists of a set of detectors located in the central barrel, positioned inside a solenoidal magnet which generates a 0.5 T field parallel to the beam direction, and a set of forward detectors.

The Inner Tracking System (ITS) [38] and the Time Projection Chamber TPC [41] are the main tracking detectors of the central barrel. The ITS consists of six layers of silicon detectors employing three different technologies. The two innermost layers, positioned at $r = 3.9$ cm and 7.6 cm, are Silicon Pixel Detectors (SPD), followed by two layers of Silicon Drift Detectors (SDD) ($r = 15$ cm and 23.9 cm). Finally, the two outermost layers are double-sided Silicon Strip Detectors (SSD) at $r = 38$ cm and 43 cm. The TPC has a cylindrical shape with an inner radius of about 85 cm, an outer radius of about 250 cm, and a length of 500 cm and it is positioned around the ITS. It provides full azimuthal coverage in the pseudorapidity

range $|\eta| < 0.9$.

Charged particles were identified using the information from the TPC and the TOF detectors [38]. The TPC allows for a simultaneous measurement of the momentum of a particle and its specific energy loss $\langle dE/dx \rangle$ in the gas. The detector provides a separation more than 2 standard deviations for the hadron species at $p_T < 0.7$ GeV/c and the possibility to identify particles on a statistical basis in the relativistic rise region of dE/dx (i.e. $2 < p_T < 20$ GeV/c) [39]. The dE/dx resolution for the 5% most central Pb–Pb collisions is 6.5% and improves for more peripheral collisions. The TOF detector is situated at a radial distance of 3.7 m from the beam axis, around the TPC and provides a 3σ separation between π –K and K–p up to $p_T = 2.5$ GeV/c and $p_T = 4$ GeV/c, respectively [39]. This is done by measuring the flight time of particles from the collision point with a resolution of about 80 ps. The start time for the TOF measurement is provided by the T0 detectors, two arrays of Cherenkov counters positioned at opposite sides of the interaction points covering $4.6 < \eta < 4.9$ (T0A) and $-3.3 < \eta < -3.0$ (T0C). The start time is also determined using a combinatorial algorithm that compares the timestamps of particle hits measured by the TOF to the expected times of the tracks, assuming a common event time t_{ev} [39]. Both methods of estimating the start time are fully efficient for ~~the 50% most central~~ Pb–Pb collisions up to 80% centrality interval.

A set of forward detectors, the V0 scintillator arrays [42], were used in the trigger logic and for the determination of the collision centrality. The V0 consists of two detectors, the V0A and the V0C, ~~that are~~ positioned on each side of the interaction point ~~and cover~~, covering the pseudorapidity ranges of $2.8 < \eta < 5.1$ and $-3.7 < \eta < -1.7$, respectively.

For more details on the ALICE apparatus and the performance of the detectors, see Refs. [38, 39].

3 Event sample, track selection and particle identification

3.1 Trigger selection and data sample

The analysis is performed on the minimum bias Pb–Pb collision data at $\sqrt{s_{NN}} = 5.02$ TeV collected by ALICE in 2015. These events were triggered by the coincidence between signals from both V0A and V0C detectors. An offline event selection, exploiting the signal arrival time in V0A and V0C, measured with a 1 ns resolution, was used to discriminate beam induced-background (e.g. beam gas events) from collision events. This led to a reduction of background events in the analysed samples to a negligible fraction ($< 0.1\%$) [39]. Events with multiple reconstructed vertices were rejected by comparing multiplicity estimates from the V0 detector to tracking detectors at mid-rapidity, exploiting the difference in readout times between the systems. The fraction of pileup events left after applying the dedicated pileup removal criteria is found to be negligible. All events selected for the analysis had a reconstructed primary vertex position along the beam axis (z_{vtx}) within 10 cm from the nominal interaction point. After all the selection criteria, a filtered data sample of approximately 40 million Pb–Pb events in 0-60% centrality interval were analysed to produce the results presented in this article.

Events were classified according to fractions of the inelastic cross section. The 0-5% interval represents the most central interactions (i.e. smallest impact parameter) and is referred to as most central collisions. On the other hand, the 50-60% centrality interval corresponds to the most peripheral (i.e. largest impact parameter) collisions in the analysed sample. The centrality of the collision was estimated using the energy deposition measured in the V0 detectors. Details about the centrality determination can be found in Ref. [43].

3.2 ~~Track selection~~ Selection of primary π^\pm , K^\pm and $p + \bar{p}$

In this analysis, tracks are reconstructed using the information from the TPC and the ITS detectors. The tracking algorithm, based on the Kalman filter [44, 45], starts from a collection of space points (referred to as clusters) inside the TPC, and provides the quality of the fit by calculating its χ^2 value. Each space point is reconstructed at one of the TPC padrows, where the deposited ionisation energy is also measured. The specific ionisation energy loss $\langle dE/dx \rangle$ is estimated using a truncated mean, excluding the 40% highest-charge clusters associated to the track. The obtained $\langle dE/dx \rangle$ has a resolution, which we later refer to as σ_{TPC} . The tracks are propagated to the outer layer of the ITS, and the tracking algorithm attempts to identify space points in each one of the consecutive layers, reaching the innermost ones (i.e. SPD). The track parameters are then updated using the combined information from both the TPC and the ITS detectors.

Primary charged pions, kaons and (anti-)protons were required to have at least 70 reconstructed space points out of the maximum of 159 in the TPC. The average of the track fit per TPC space point per degree of freedom (see [39] for details) was required to be below 4. These selections reduce the contribution from short tracks, which are unlikely to originate from the primary vertex. To further reduce the contamination by secondary tracks from weak decays or from the interaction with the material, only particles within a maximum distance of closest approach (DCA) between the tracks and the primary vertex in both the transverse plane ($DCA_{xy} < 0.0105 + 0.0350(p_T \text{ c/GeV})^{-1.1}$ cm) and the longitudinal direction ($DCA_z < 2$ cm) were analysed. Moreover, the tracks were required to have at least two associated ITS clusters in addition to having a hit in either of the two SPD layers. This selection leads to an efficiency of about 80% for primary tracks at $p_T < 0.6$ GeV/c and a contamination from secondaries of about 5% at $p_T = 1$ GeV/c [46]. These values depend on particle species and transverse momentum [46]. [These selection criteria are listed in Tab. 1.](#) Relevant selection criteria for tracks used for the reconstruction of K_S^0 , $\Lambda + \bar{\Lambda}$ and ϕ -meson are given in Sec. ?? and ??.

3.3 Identification of, and

The particle identification (PID) for pions (π^\pm), kaons (K^\pm) and protons ($p + \bar{p}$) used in this analysis relies on the two-dimensional correlation between the number of standard deviations in units of the resolution from the expected signals of the TPC and the TOF detectors similar to what was reported in [17–19]. In this approach particles were selected by requiring their standard deviations from the $\langle dE/dx \rangle$ and t_{TOF} values to ~~not only be the minimum among other candidate species and lie within~~ be less than a p_T -dependent ~~maximum combined standard deviations for a given particle species and~~ transverse momentum value, maintaining a minimum purity of 90% for π^\pm , 75% for K^\pm and 80% for $p + \bar{p}$. In order to further reduce the contamination from other species, the standard deviation of a given track was required to be the minimum among other candidate species.

In addition, for the systematics (see section 5) the minimum purity was required to be 80%, a condition that becomes essential with increasing transverse momentum where the relevant detector response for different particle species starts to overlap.

3.3 Reconstruction of $K_S^0 \Lambda + \bar{\Lambda}$ and ϕ meson

In this analysis, the K_S^0 and $\Lambda + \bar{\Lambda}$ are reconstructed via the following fully hadronic decay channels: $K_S^0 \rightarrow \pi^+ + \pi^-$ and $\Lambda(\bar{\Lambda}) \rightarrow p(\bar{p}) + \pi^-(\pi^+)$ with branching ratios of 69.2% and 63.9% [47], respectively. The reconstruction is performed, as shown in Tab. 2, by identifying the candidates of secondary vertices, denoted as V^0 s, from which two oppositely-charged decay products originate. Such candidates are obtained during data processing by looking for characteristic V-shaped decay topology among reconstructed tracks.

The daughter tracks were reconstructed within $|\eta| < 0.8$, while the criteria on the number of TPC space points, the number of crossed TPC padrows, and the percentage of the expected TPC space points used to reconstruct a track are identical to those applied for primary particles. In addition, the minimum DCA of daughter tracks to the primary vertex is 0.1 cm. Furthermore, the maximum DCA of daughter tracks to the secondary vertex is 0.5 cm to ensure that they are products of the same decay. To suppress the combinatorial background, ~~the~~ PID is applied for the daughter particles in the whole p_T region by requiring the particle to be within $3\sigma_{\text{TPC}}$ for a given species hypothesis.

To reject secondary vertices arising from decays into more than two particles, the cosine of the pointing angle, θ_p , is required to be larger than 0.998. This angle is defined as the angle between the momentum vector of the V^0 candidate assessed at its decay vertex and the line connecting the V^0 decay vertex to the primary vertex and has to be close to 1 as a result of momentum conservation. In addition, only the candidates reconstructed between 5 and 100 cm from the nominal primary vertex in radial direction are accepted. The lower value is chosen to avoid any bias from the efficiency loss when secondary tracks are being wrongly matched to clusters in the first layer of the ITS. To assess the systematic uncertainty related to the contamination from $\Lambda + \bar{\Lambda}$ and electron-positron pairs coming from γ -conversions to the K_S^0 sample, a selection in the Armenteros-Podolanski variables [48] is applied for the K_S^0 candidates, rejecting ones with $q \leq 0.2|\alpha|$. Here q is the momentum projection of the positively charged daughter track in the plane perpendicular to the V^0 momentum and $\alpha = (p_L^+ - p_L^-)/(p_L^+ + p_L^-)$ with p_L^\pm the projection of the positive or negative daughter tracks' momentum onto the momentum of the V^0 .

~~To obtain the differential yield of and (which, together with background yields), are used for the signal extraction in Eq. 14, invariant mass distributions at various intervals are parametrized as a sum of two Gaussian distribution and a third-order polynomial function. The latter is introduced to account for residual contamination (background yield) that are present in the and signals after the topological and daughter track selections. The and yields are extracted by integration of the Gaussian distribution.~~

3.4 Reconstruction of ϕ meson

The reconstruction of ϕ meson candidates is done via ~~its following the~~ hadronic decay channel: $\phi \rightarrow K^+ + K^-$ with a branching ratio of 48.9% [47]. The ϕ meson candidates were reconstructed from the charged tracks passing all criteria for charged kaons, listed in Table 1. Kaon daughters are identified by utilising Bayesian PID approach [49] with a minimum probability threshold of 85% using TPC and TOF detectors. Additionally, to reduce combinatorial background of ϕ candidates, a track is identified as kaon if it has the highest probability among all considered species ($e, \mu, \pi^\pm, K^\pm, p + \bar{p}$). The vector sum of all possible pairs of charged kaons are called ϕ candidates. The invariant mass distribution ($M_{inv}^{K^+K^-}$) of ϕ candidates is then obtained in various p_T intervals by subtracting a combinatorial background yield from the candidate yield. This combinatorial background yield is estimated from like-sign kaon pairs (unphysical ϕ state with total charge of ± 2) normalised to the candidate yield. ~~To obtain the differential yield of ϕ mesons, the invariant mass distributions of the candidate yield is, after the aforementioned subtraction, parametrized as a sum of a Breit-Wigner distribution and a third-order polynomial function, the latter introduced to account for residual contamination.~~

4 Analysis method

~~The magnitude of the p_T~~ In this article the p_T -differential non-linear ~~modes in higher order flow coefficients, v_n^{NL} , can be written as:-~~

$$\begin{aligned} v_{4,22}(p_T) &= \frac{\langle v_4(p_T) v_2^2 \cos(4\Psi_4 - 4\Psi_2) \rangle}{\sqrt{\langle v_2^4 \rangle}} \approx \langle v_4(p_T) \cos(4\Psi_4 - 4\Psi_2) \rangle, \\ v_{5,32}(p_T) &= \frac{\langle v_5(p_T) v_3 v_2 \cos(5\Psi_5 - 3\Psi_3 - 2\Psi_2) \rangle}{\sqrt{\langle v_3^2 v_2^2 \rangle}} \approx \langle v_5(p_T) \cos(5\Psi_5 - 3\Psi_3 - 2\Psi_2) \rangle, \\ v_{6,33}(p_T) &= \frac{\langle v_6(p_T) v_3^2 \cos(6\Psi_6 - 6\Psi_3) \rangle}{\sqrt{\langle v_3^4 \rangle}} \approx \langle v_6(p_T) \cos(6\Psi_6 - 6\Psi_3) \rangle, \\ v_{6,222}(p_T) &= \frac{\langle v_6(p_T) v_2^3 \cos(6\Psi_6 - 6\Psi_2) \rangle}{\sqrt{\langle v_2^6 \rangle}} \approx \langle v_6(p_T) \cos(6\Psi_6 - 6\Psi_2) \rangle, \end{aligned}$$

~~where brackets denote an average over all events. The approximation is valid assuming a weak correlation between the lower ($n=2,3$) and higher ($n>3$) order flow coefficients. In this analysis, each flow modes are calculated according to Eqs. 10-13 with a method explained in [50]. Each~~ event is divided into two subevents “A” and “B”, covering the ranges $-0.8 < \eta < 0.0$ and $0.0 < \eta < 0.8$, respectively. Thus $v_{n,mk}(p_T)$ is a weighted average of $v_{n,mk}^A(p_T)$ and $v_{n,mk}^B(p_T)$. The measured $v_{n,mk}^A(v_{n,mk}^B)$ coefficients are calculated using $d_{n,mk}^{A(B)}(p_T)$ and c_{mkmk} multi-particle correlators. ~~These correlators are calculated with a method generic framework. Details of this multi-particle correlation technique can be found in [50].~~ In this analysis, $d_{n,mk}^{A(B)}(p_T)$ is measured by selecting the identified hadrons (POIs) from subevent “A”(“B”) and the reference particles from subevent “B”(“A”) and c_{mkmk} by selecting half of the reference particles from subevent “A” and the other half from “B”. Thus, Eqs.6 to 9 for $v_{n,mk}^A(p_T)$ translate to

$$v_{4,22}^A(p_T) = \frac{d_{4,22}^A(p_T)}{\sqrt{c_{2222}}} = \frac{\langle \langle \cos(4\phi_1^A(p_T) - 2\phi_2^B - 2\phi_3^B) \rangle \rangle}{\sqrt{\langle \langle \cos(2\phi_1^A + 2\phi_2^A - 2\phi_3^B - 2\phi_4^B) \rangle \rangle}}, \quad (10)$$

$$v_{5,32}^A(p_T) = \frac{d_{5,32}^A(p_T)}{\sqrt{c_{3232}}} = \frac{\langle \cos(5\phi_1^A(p_T) - 3\phi_3^B - 2\phi_2^B) \rangle}{\sqrt{\langle \cos(3\phi_1^A + 2\phi_2^A - 3\phi_3^B - 2\phi_4^B) \rangle}}, \quad (11)$$

$$v_{6,33}^A(p_T) = \frac{d_{6,33}^A(p_T)}{\sqrt{c_{3333}}} = \frac{\langle \cos(6\phi_1^A(p_T) - 3\phi_2^B - 3\phi_3^B) \rangle}{\sqrt{\langle \cos(3\phi_1^A + 3\phi_2^A - 3\phi_3^B - 3\phi_4^B) \rangle}}, \quad (12)$$

$$v_{6,222}^A(p_T) = \frac{d_{6,222}^A(p_T)}{\sqrt{c_{222222}}} = \frac{\langle \cos(6\phi_1^A(p_T) - 2\phi_2^B - 2\phi_3^B - 2\phi_4^B) \rangle}{\sqrt{\langle \cos(2\phi_1^A + 2\phi_2^A + 2\phi_3^A - 2\phi_4^B - 2\phi_5^B - 2\phi_6^B) \rangle}}, \quad (13)$$

where $\langle \rangle$ denotes an average over all particles and events.

For inclusive charged hadrons, i.e. π^\pm , K^\pm and $p + \bar{p}$, the $d_{n,mk}$ correlators are calculated on a track-by-track basis as a function of p_T per centrality percentile.

For particle species reconstructed on statistical basis from decay products, i.e. K_S^0 , $\Lambda + \bar{\Lambda}$ and ϕ meson, the selected sample contains both signal and the combinatorial background. Therefore, for the aforementioned particle species, the $d_{n,mk}$ correlators are measured as a function of invariant mass (M_{inv}) and p_T per centrality percentile. The $d_{n,mk}$ vs. m_{inv} method is based on the additivity of correlations and is a weighted sum of the $d_{n,mk}^{\text{sig}}$ and $d_{n,mk}^{\text{bkg}}$ as described in according to

$$d_{n,mk}(M_{\text{inv}}) = \frac{N^{\text{sig}}}{N^{\text{sig}} + N^{\text{bkg}}} (M_{\text{inv}}) d_{n,-m,-k}^{\text{sig}} + \frac{N^{\text{bkg}}}{N^{\text{sig}} + N^{\text{bkg}}} (M_{\text{inv}}) d_{n,-m,-k}^{\text{bkg}}(M_{\text{inv}}), \quad (14)$$

where N^{sig} and N^{bkg} are signal and background yields obtained for each p_T interval and centrality percentile from fits to K_S^0 , $\Lambda + \bar{\Lambda}$ and ϕ meson invariant mass distributions described in ?? and ?. To obtain the p_T -differential yield of K_S^0 and $\Lambda + \bar{\Lambda}$ (which, together with background yields), are used for the signal extraction in Eq. 14, invariant mass distributions at various p_T intervals are parametrized as a sum of two Gaussian distribution and a third-order polynomial function. The latter is introduced to account for residual contamination (background yield) that are present in the K_S^0 and $\Lambda + \bar{\Lambda}$ signals after the topological and daughter track selections. The K_S^0 and $\Lambda + \bar{\Lambda}$ yields are extracted by integration of the Gaussian distribution. Similarly, to obtain the p_T -differential yield of ϕ -mesons, the invariant mass distributions of the candidate yield is parametrized as a sum of a Breit-Wigner distribution and a third-order polynomial function, the latter introduced to account for residual contamination.

To extract $d_{n,mk}^{\text{sig}}$, $d_{n,mk}(M_{\text{inv}})$ is fitted together with the fit values from the invariant mass distribution and parametrising $d_{n,mk}^{\text{bkg}}(M_{\text{inv}})$ with a first order polynomial function. Figure 1 illustrates this procedure for the ϕ -meson, with the invariant mass distribution for ϕ -meson in the upper panel and the measurement of $d_{4,22}(M_{\text{inv}})$ in the lower panel.

5 Systematic uncertainties

The systematic uncertainties are estimated by varying the selection criteria for all particle species as well as topological reconstruction requirements for K_S^0 , $\Lambda + \bar{\Lambda}$ and ϕ . The contributions from different sources are extracted from the relative ratio of the p_T -differential $v_{n,mk}$ between the default selection criteria described in Section 3 and their variations summarised in Tabs. 1 and 2. Each source with a statistically significant contribution, i.e. $|x_1 - x_2| / \sqrt{(|\sigma_1^2| + |\sigma_2^2|)} > 1$ (known as barlow check [51]) was fitted to create a smooth change along p_T and then the value of these fits were added in quadrature to form the final value of the systematic uncertainties on the non-linear flow modes. An overview of the magnitude of the relative systematic uncertainties per particle species is given in Tab. 3 for π^\pm , K^\pm and $p + \bar{p}$ and Tab. 4 for K_S^0 , $\Lambda + \bar{\Lambda}$ and ϕ -meson. Systematic uncertainties are grouped into five categories,

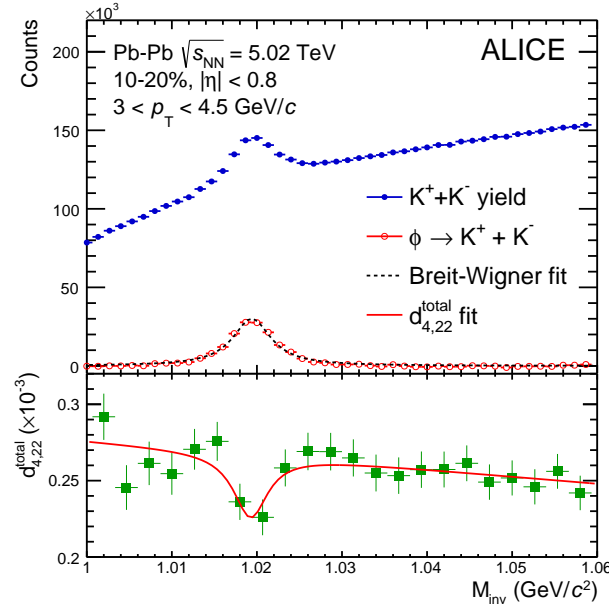


Figure 1: Reconstruction and $d_{4,22}$ measurement of ϕ -meson. Upper panel: extraction of N^{sig} and N^{bkg} by fitting the invariant mass (m_{inv}) distribution for ϕ -meson for $3 < p_T < 4.5$ GeV/ c at 10-20% centrality interval, lower panel: extraction of $d_{4,22}^{\text{sig}}$ by fitting Eq. 14 to the invariant mass dependence of $d_{4,22}$

| Selection requirement | Default | Variations |
|------------------------------|-------------------|---|
| Primary vx_z | $\pm 10\text{cm}$ | $\pm 6\text{cm}, \pm 8\text{cm}$ |
| Centrality estimator | V0M | CL0, CL1 |
| Magnetic field polarity | both fields | ++, -- |
| pile-up rejection | strict | loose |
| Tracking mode | global (96) | hybrid (768) |
| Number of TPC space-points | 70 | 80, 90, 100 |
| χ^2 per TPC space-point | 4 | 2 |
| DCA _{xy} cm | p_T dependant | 0.2, 0.15 cm |
| DCA _z cm | 2 cm | 0.2, 0.3 cm |
| PID method | p_T -dependent | tight p_T -dependent, Bayesian prob. >80% |
| POI vs. RFP charges | All | ++, -- |
| η gap | 0.0 | 0.4 |

Table 1: List of the selection criteria and the corresponding variations used for the estimation of the systematic uncertainties of π^\pm , K^\pm and $p + \bar{p}$

i.e. event selection, tracking, particle identification, topological cuts and non-flow contribution and are described below.

~~Effect of event selection~~ The effects of event selection criteria on the measurements are studied by: (i) varying the primary vertex position along the beam axis (z_{vtx}) from a nominal ± 10 cm to ± 8 cm and ± 6 cm; (ii) changing the centrality estimator from the signal amplitudes in the V0 scintillator detectors to the multiplicity of TPC tracks or the number of SPD clusters; (iii) analysing events recorded for different magnetic field polarities independently; (iv) not rejecting events with tracks caused by pileup.

~~Effect of tracking cuts are then studies by~~ Systematic uncertainties induced by the selection criteria imposed at the track level were investigated by: (i) changing the tracking from global mode where combined track information from both TPC and ITS detectors are used to hybrid mode in which track parameters from TPC are used if the algorithm is unable to match the track reconstructed in the TPC with

| Selection requirements | Default | Variations |
|---|-------------------------|-------------------|
| Reconstruction method (V^0 finder) | offline | online |
| Decay vertex (radial position) | $5 < r < 100$ cm | $10 < r < 100$ cm |
| Cosine of pointing angle | > 0.998 | > 0.99 |
| Number of crossed TPC clusters | > 70 | > 90 |
| Number of TPC clusters used for PID | > 70 | > 90 |
| Number of findable TPC clusters | > 1 | – |
| Ratio of crossed to findable TPC clusters | > 0.8 | > 1.0 |
| DCA decay products to primary vertex | > 0.1 cm | > 0.3 cm |
| DCA among daughters | < 0.5 cm | < 0.3 cm |
| Daughter p_T acceptance | – | > 0.2 GeV/c |
| TPC PID on daughters | $< 3\sigma$ | – |
| Armenteros-Podolanski (K_S^0) | $q > 0.2 \alpha $ | – |
| Daughter η acceptance | $ \eta < 0.8$ | – |
| Mother η acceptance | $ \eta < 0.8$ | – |
| Competing inv. mass rejection (K_S^0) | < 5 MeV ² | – |
| Competing inv. mass rejection ($\Lambda + \bar{\Lambda}$) | < 10 MeV ² | – |

Table 2: List of topological reconstruction requirements and cuts applied on V^0 candidates including variations for systematical uncertainty study where applicable.

associated ITS clusters; (ii) increasing the number of TPC space points from 70 up to 100 and (iii) the decreasing the value of the χ^2 per TPC space point per degree of freedom from 4 to 2; (iv) varying the selection criteria on both the longitudinal and transverse components of the DCA to estimate the impact of secondary particles from a ~~strict~~ p_T -dependent cut to 0.15 cm and 2 cm to 0.2 cm, respectively.

Systematic uncertainties associated with the ~~particle identification~~ $\text{particle identification}$ procedure were studied by varying the PID method from a p_T -dependent ~~method described in ??~~ $\text{one described in 3.2}$ to a stricter version where the purity increases to 95% (π^\pm), 80% (K^\pm) and 80% ($p + \bar{p}$) across the entire p_T range of study. The second ~~method used is~~ $\text{approach used relied on the}$ Bayesian method with a probability of at least 80% which gives an increase in purity to 97% (π^\pm), 87% (K^\pm) and 90% ($p + \bar{p}$) across the entire p_T range of study.

~~The~~ In addition, the non-flow contribution is studied by (i) selecting like sign pairs of particles of interest and reference particles to decrease the effect from decay of resonance particles; (ii) applying pseudorapidity gaps between the two subevents from $|\Delta\eta| > 0.0$ to $|\Delta\eta| > 0.4$.

The ~~topological cuts~~ topological cuts were also varied to account for the V^0 and ϕ -meson reconstruction. The default V^0 finding method is described in Sec. ~~??3.3~~. These selection criteria are varied by (i) changing the reconstruction method for V^0 particles from offline to online ~~method~~; (ii) varying the minimum radial distance to the primary vertex at which the V^0 can be produced from 5 cm to 10 cm; (iii) changing the minimum value of cosine of pointing angle from 0.998 to 0.99; (iv) varying the minimum number of TPC space points crossed by the V^0 daughter tracks from 70 to 90; (v) changing the requirement on the minimum number of TPC space points that are used in the reconstruction of the V^0 daughter tracks from 70 to 90; (vi) requesting a minimum ratio of crossed to findable TPC clusters from 0.8 to 1.0; (vii) changing the minimum DCA of the V^0 daughter tracks to the primary vertex from 0.1 cm to 0.3 cm; (viii) changing the maximum DCA of the V^0 daughter tracks to the secondary vertex from 0.5 cm to 0.3 cm; (ix) requiring a minimum p_T of the V^0 daughter tracks of 0.2 GeV/c.

$\text{The contributions from each source were added in quadrature to form the total systematic uncertainties. This will be represented in all plots of this article as a box around each data point while the statistical uncertainty will be shown by the error bars.}$

| Error source | $v_{4,22}$ | | | $v_{5,32}$ | | | $v_{6,33}$ | | | $v_{6,222}$ | | |
|------------------------------|------------|---------|---------------|------------|---------|---------------|------------|---------|---------------|-------------|---------|---------------|
| | π^\pm | K^\pm | $p + \bar{p}$ | π^\pm | K^\pm | $p + \bar{p}$ | π^\pm | K^\pm | $p + \bar{p}$ | π^\pm | K^\pm | $p + \bar{p}$ |
| Primary z_{vtx} | 0-2% | 1-3% | 0-3% | 0-3% | 1-3% | 1-4% | 3-5% | 2-5% | 3-5% | 2-7% | 2-7% | 4-7% |
| Centrality estimator | 0-4% | 1-4% | 1-5% | 0-4% | 1-3% | 2-4% | 4-10% | 4-10% | 5-10% | 3-10% | 5-10% | 4-10% |
| Magnetic field polarity | 0-2% | 0-3% | 0-3% | 0-4% | 0-5% | 0-5% | 0-10% | 0-10% | 0-10% | 0-10% | 0-10% | 0-10% |
| Pileup rejection | 0-4% | 0-4% | 0-4% | 0-5% | 1-5% | 0-5% | 5-7% | 5-10% | 5-8% | 4-10% | 4-10% | 2-10% |
| Tracking mode | 1-4% | 1-5% | 1-4% | 2-6% | 3-5% | 2-8% | 0-8% | 0-7% | 3-8% | 1-10% | 4-10% | 2-10% |
| Number of TPC space-points | 1-2% | 0-2% | 0-2% | 0-3% | 1-3% | 1-3% | 4-8% | 3-8% | 3-8% | 2-8% | 4-8% | 4-8% |
| χ^2 per TPC space-point | 0-2% | 1-2% | 1-3% | 1-3% | 1-3% | 2-4% | 3-5% | 3-6% | 3-6% | 2-6% | 4-7% | 4-7% |
| DCAxy | 0-2% | 0-2% | 1-3% | 0-3% | 1-3% | 1-3% | 2-7% | 2-8% | 4-8% | 2-8% | 4-8% | 3-8% |
| DCAz | 0-3% | 0-2% | 1-2% | 1-2% | 1-3% | 2-3% | 3-7% | 3-7% | 5-7% | 2-7% | 4-8% | 2-8% |
| Particle identification | 1-5% | 1-5% | 1-3% | 1-5% | 2-5% | 1-5% | 5-10% | 5-10% | 6-12% | 4-12% | 6-15% | 4-15% |
| POI vs. RFP charges | 0-2% | 0-3% | 2-3% | 0-4% | 0-4% | 2-4% | 0-4% | 0-6% | 0-6% | 0% | 0% | 0% |
| η gap | 1-3% | 1-4% | 1-2% | 1-4% | 1-4% | 1-5% | 0-5% | 0-5% | 0-5% | 0% | 0% | 0% |

Table 3: List of the maximum systematic uncertainties from each individual source for $v_{n,mk}$ of π^\pm , K^\pm and $p + \bar{p}$. The uncertainties depend on the transverse momenta and centrality interval. Hence here maximum and minimum values are listed.

| Error source | $v_{4,22}$ | | | $v_{5,32}$ | | $v_{6,33}$ | |
|---|------------|---------------------------|--------|------------|---------------------------|------------|---------------------------|
| | K_S^0 | $\Lambda + \bar{\Lambda}$ | ϕ | K_S^0 | $\Lambda + \bar{\Lambda}$ | K_S^0 | $\Lambda + \bar{\Lambda}$ |
| Primary z_{vtx} | 0% | 0-2% | 1% | 0% | 0-3% | 0% | 1-3% |
| Tracking mode | - | - | 2% | - | - | - | - |
| Number of TPC space-points | 0-3% | 1-2% | 2% | 0% | 2% | 0% | 2% |
| Particle identification | - | - | 4-6% | - | - | - | - |
| Reconstruction method (V^0 finder) | 3-5% | 2-3% | N/A | 5% | 1% | 5% | 1% |
| Decay radius | 3-5% | 1-3% | N/A | 5-6% | 0-2% | 5% | 2% |
| Ratio of crossed to findable TPC clusters | 0-2% | 0-3% | N/A | 0% | 1-2% | 0% | 3% |
| DCA decay products to primary vertex | 2-5% | 2-4% | N/A | 4-5% | 2-3% | 5% | 2-3% |
| DCA between decay products | 0-3% | 1-2% | N/A | 0-4% | 0-4% | 0% | 0-4% |
| Pointing angle $\cos(\theta_p)$ | 3-4% | 0-2% | N/A | 3-4% | 0-3% | 3% | 1% |
| Minimum p_T of daughter tracks | 1-3% | 0-1% | N/A | 2-3% | 2-3% | 0% | 0-3% |

Table 4: List of the maximum systematic uncertainties from each individual source for $v_{n,mk}$ of K_S^0 , $\Lambda + \bar{\Lambda}$ and ϕ -meson. The uncertainties depend on the transverse momenta and centrality interval. Hence here maximum and minimum values are listed. "N/A" indicates that a certain check was not applicable to the given particle of interest. If a source was checked and proved to be of negligible effect, the field is marked with "-".

6 Results and discussion

In this section, the results for the p_T -dependent non-linear flow modes $v_{4,22}$, $v_{5,32}$, $v_{6,33}$ and $v_{6,222}$ of identified particles are presented for various centrality intervals in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. We first present the centrality and p_T dependence of $v_{n,mk}$ in Sec. 6.1. [Sealing The scaling](#) properties of non-linear flow modes are [discussed in Sec. ?? and comparisons also discussed in this section](#). [Comparisons](#) to two model calculations are shown in Sec. 6.2. Note that the same data are used in different representations to highlight the physics implications of the measurements in each section.

6.1 Centrality and p_T dependence of non-linear flow modes

Higher order flow coefficients ($n > 3$) are mainly generated by inhomogeneities in the initial density profile, the collision geometry as well as the non-linear hydrodynamic response of the system. Figure 2 presents the non-linear term for the fourth order flow coefficient, i.e. $v_{4,22}(p_T)$, of π^\pm , K^\pm , $p + \bar{p}$, $\Lambda + \bar{\Lambda}$, K_S^0 and ϕ -meson for a wide range of centrality intervals, i.e. 0-60%. For the ϕ -meson, the results are reported in the 10-50% centrality interval, where $v_{4,22}$ can be measured accurately. The magnitude of $v_{4,22}$ rises steeply with increasing centrality interval from 0-5% to 40-50% for all particle species. This increase is expected as $v_{4,22}$ measures the contribution of the second order eccentricity, ϵ_2 , in v_4 [and the eccentricity of the overlap region of the colliding nuclei which](#) increases for peripheral collisions [8]. For more peripheral collisions (i.e. 50-60%), the magnitude of $v_{4,22}$ is smaller than in the previous centrality intervals for all particle species. This [effect that was observed also in \$v_n\$ measurements](#) is probably due

to the shorter lifetime of the produced system in more peripheral collisions, which ~~stops~~ prevents $v_{4,22}$ from developing further.

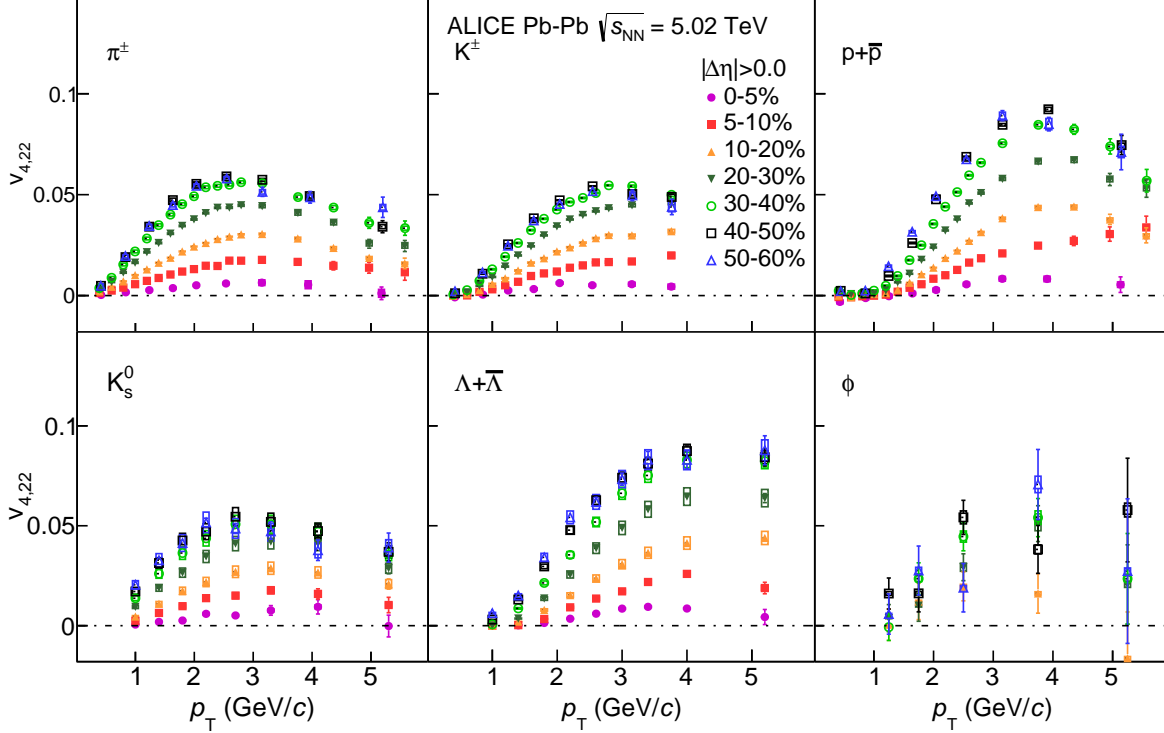


Figure 2: The p_T -differential $v_{4,22}$ for different centrality intervals of Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV grouped by particle species.

Figure 3 presents the non-linear term for the fifth order flow coefficient, i.e. $v_{5,32}(p_T)$, of π^\pm , K^\pm , $p + \bar{p}$, $\Lambda + \bar{\Lambda}$ and K_S^0 for the same range of centrality intervals, i.e. 0-60%. Statistical precision limits extending the measurements of non-linear flow modes of ϕ -meson for $n > 4$. The measurements show a significant increase in the magnitude of this non-linear flow mode with increasing centrality percentile. ~~Similar to $v_{4,22}$, this trend was expected. This is~~ due to the ~~contribution of the second order eccentricity, fact that $v_{5,32}(p_T)$ has a contribution from both ϵ_2 and ϵ_3 . It is shown in MC studies that both ϵ_2 and ϵ_3 increase for peripheral collisions [8]. Although, this increase is less pronounced for ϵ_3 .~~

Figure 4 and 5 present the non-linear terms for the sixth order flow coefficient, i.e. $v_{6,33}(p_T)$ for π^\pm , K^\pm , $p + \bar{p}$, $\Lambda + \bar{\Lambda}$ and K_S^0 at 0-50% centrality intervals and $v_{6,222}(p_T)$ for π^\pm , K^\pm , $p + \bar{p}$ at 0-60% centrality intervals. As expected, measurements of $v_{6,222}(p_T)$ show an increase in the magnitude of this non-linear flow mode with increasing centrality percentile, whereas, $v_{6,33}(p_T)$ presents little to no dependence on centrality.

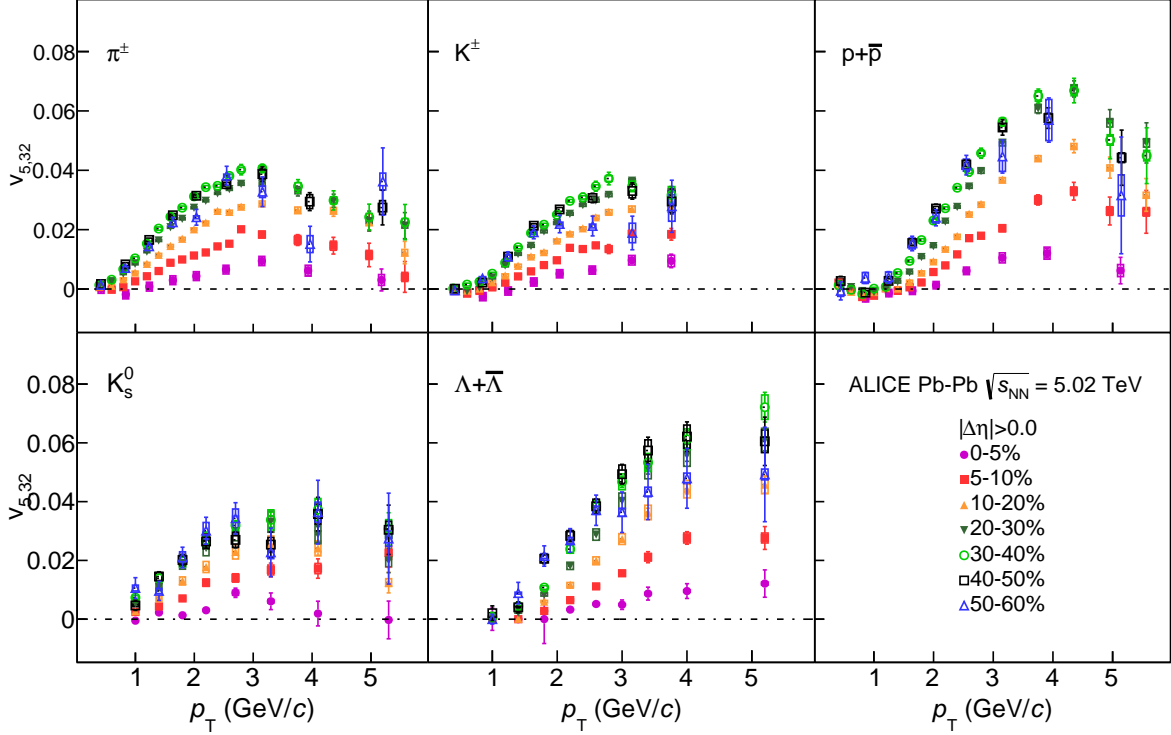


Figure 3: The p_T -differential $v_{5,32}$ for different centrality intervals of Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV grouped by particle species.

In Fig. 6 the same data points are grouped by centrality interval to highlight how $v_{4,22}$ develops for a given centrality for various particle species as a function of p_T . A clear mass ordering can be seen in the low p_T region (i.e. $p_T < 2.5$ GeV/c) at all collision centralities. This mass dependence arises from the interplay between the ~~non-linear response of the system~~ anisotropic flow and radial flow. Radial flow creates a depletion in the particle spectra at lower p_T values which leads to lower $v_{4,22}$ for heavier particles [52].

Similarly, Figs. 7, 8 and 9 show the p_T -differential $v_{5,32}$, $v_{6,33}$ and $v_{6,222}$ respectively, of different particle species for each centrality interval. A clear mass ordering is seen in the low p_T region, (i.e. $p_T < 2.5$ GeV/c), for $v_{5,32}(p_T)$, $v_{6,33}(p_T)$ and $v_{6,222}(p_T)$, which similarly arises from the interplay between the non-linear response of the system and radial flow.

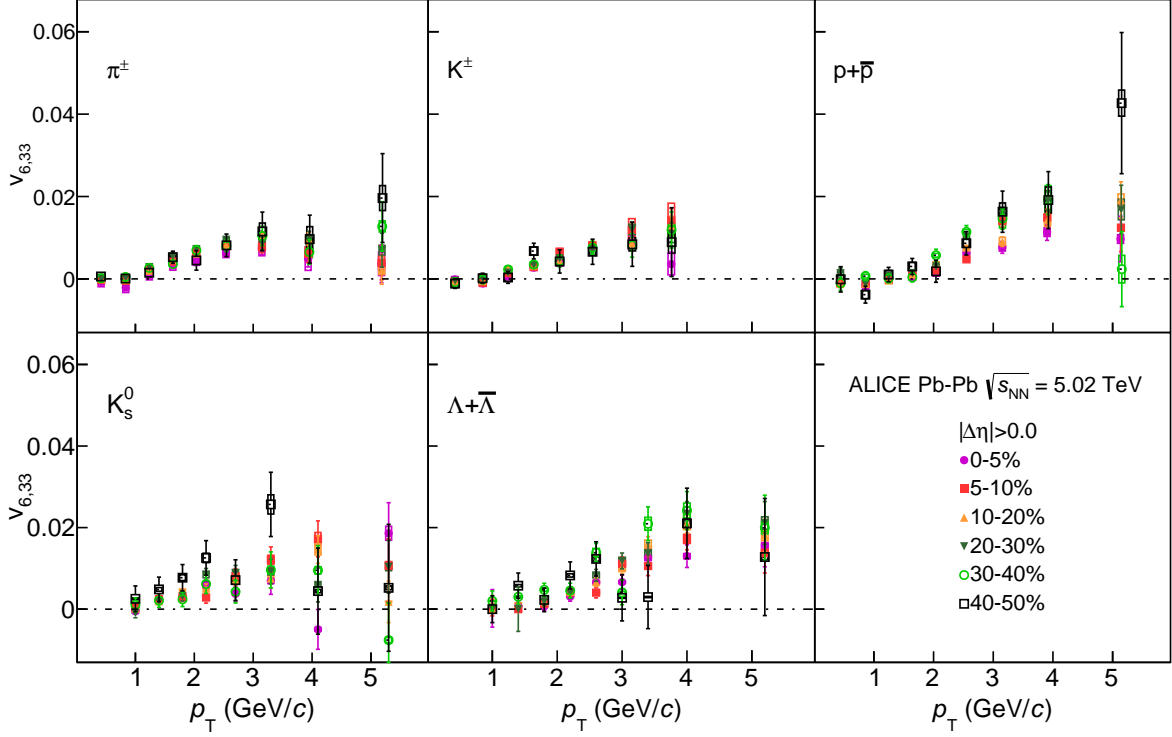


Figure 4: The p_T -differential $v_{6,33}$ for different centrality intervals of Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV grouped by particle species.

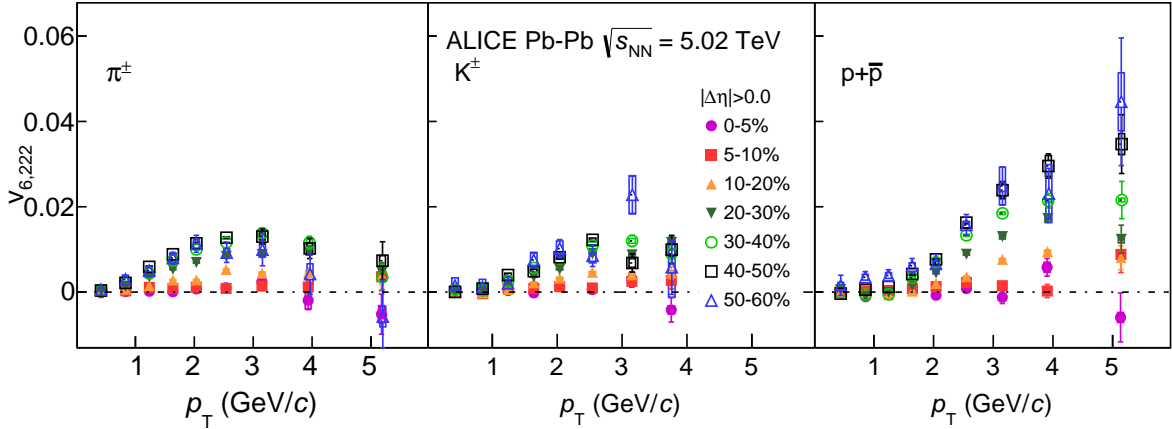


Figure 5: The p_T -differential $v_{6,222}$ for different centrality intervals of Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV grouped by particle species.

396 In addition, in the intermediate p_T region (for $p_T > 2.5$ GeV/c) the data points of Figs. 6-9 exhibit a par-
 397 ticle type grouping. In particular, mesons (π^\pm , K^\pm , K_S^0 and ϕ) and baryons ($p + \bar{p}$ and $\Lambda + \bar{\Lambda}$) group based
 398 on their type with $v_{n,mk}$ of baryons having a larger magnitude. This particle type grouping was previously
 399 seen in the anisotropic flow measurements [13–19] [of various particle species](#) triggering the development

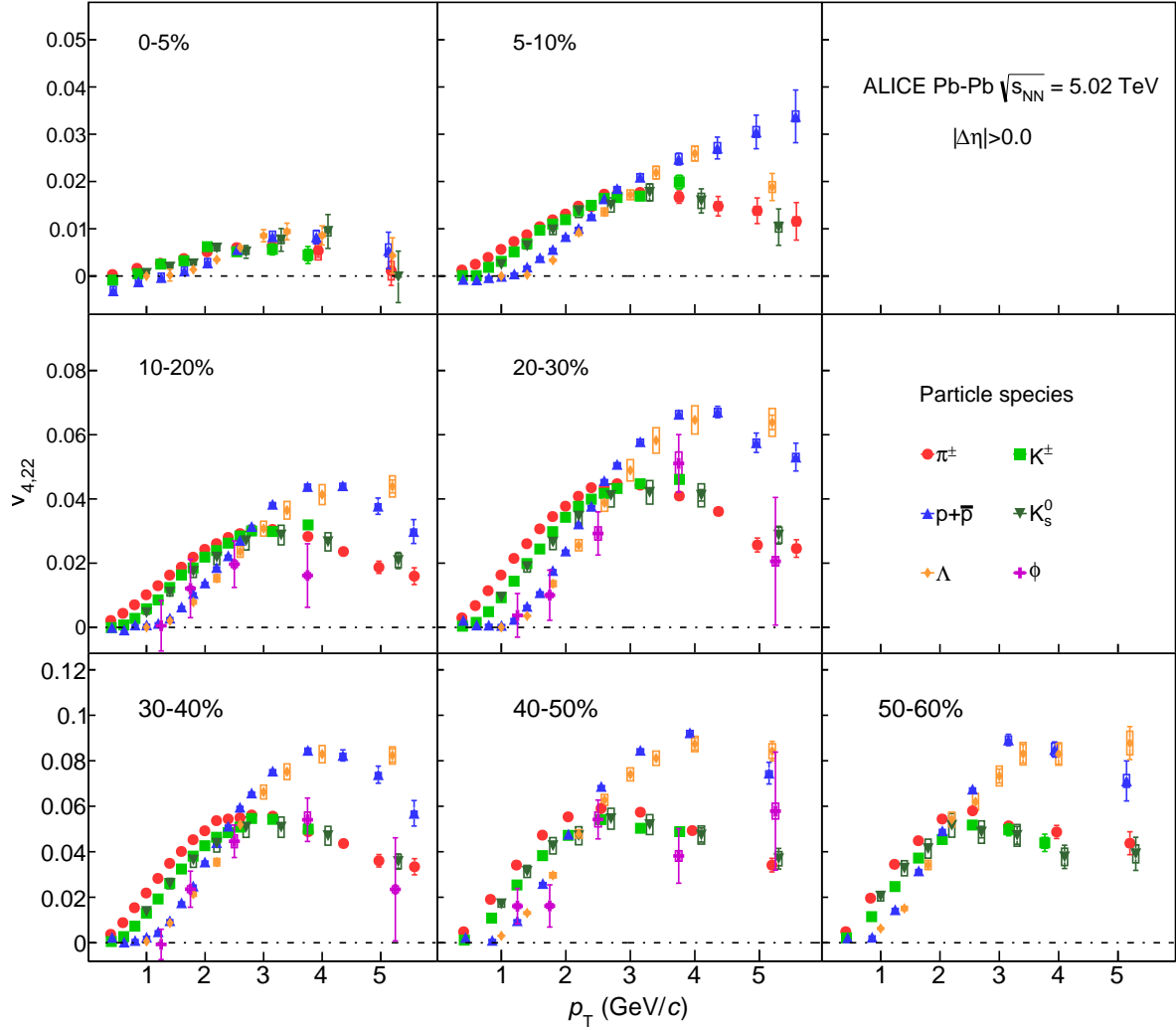


Figure 6: The p_T -differential $v_{4,22}$ for different particle species grouped into different centrality intervals of Pb–Pb collisions $\sqrt{s_{NN}} = 5.02$ TeV

of calculations relying on coalescence. This suggests that flow develops at the partonic stage and if so, combining two or three quarks to form hadronic states might result into hadrons inheriting the transverse momentum and subsequently, v_n of their constituents. As a next step it was suggested to use a form of number of constituent quark (NCQ) scaling in which both flow coefficients and p_T were scaled by the number of constituent quarks (n_q). This scaling, worked initially at RHIC energies, although later measurements revealed sizeable deviations from a perfect scaling [13–16]. Recently, ALICE measurements showed that the NCQ scaling at LHC energies holds at best at an approximate level of 20% for v_n [17–19]. Various theoretical ideas were created to address the origin of possible scaling by requiring quark coalescence to be the dominant particle production mechanism in the intermediate p_T region, where the hydrodynamic evolution of the fireball is not the driving force behind the development of anisotropic flow [35, 36].

6.2 Test of scaling properties

Figures 10, 11, 12 and 13 present $v_{4,22}$, $v_{5,32}$, $v_{6,33}$ and $v_{6,222}$ respectively, scaled by the inverse of number of constituent quarks (n_q) as a function of p_T/n_q for π^\pm , K^\pm , $p + \bar{p}$, K_s^0 , $\Lambda + \bar{\Lambda}$ and ϕ -meson grouped

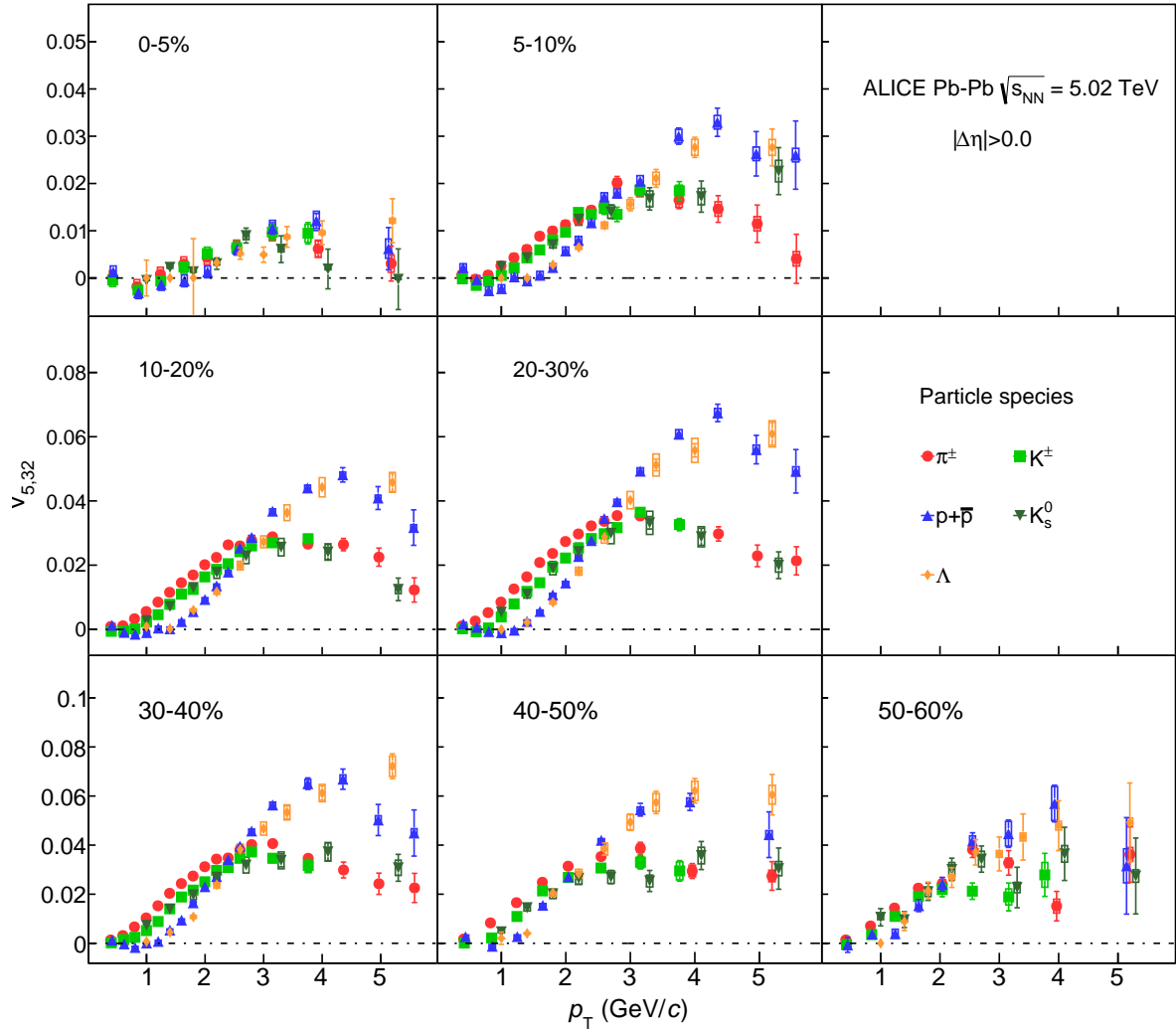


Figure 7: The p_T -differential $v_{5,32}$ for different particle species grouped into different centrality intervals of Pb–Pb collisions $\sqrt{s_{NN}} = 5.02$ TeV

in different centrality intervals. The scaling is consistent with the observations reported for higher order total flow coefficients [19]. Similarly, for non-linear modes this scaling hold at best at an approximate level ($\pm 20\%$).

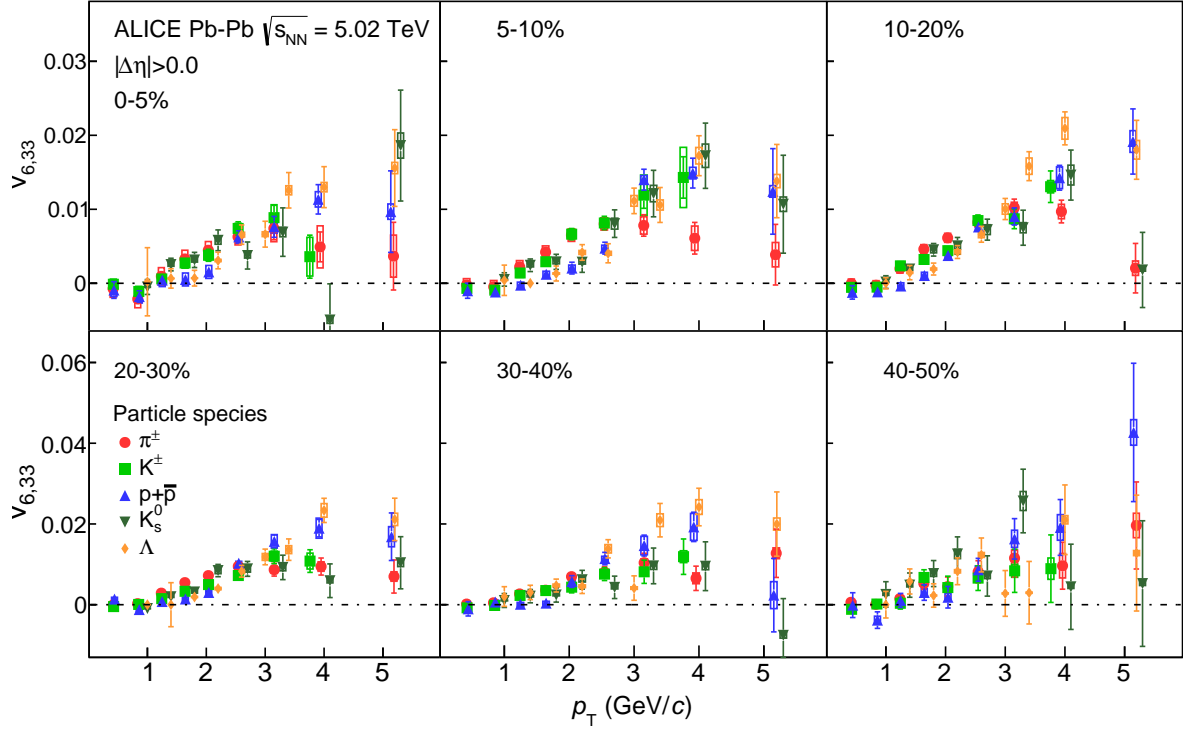


Figure 8: The p_T -differential $v_{6,33}$ for different particle species grouped into different centrality intervals of Pb–Pb collisions $\sqrt{s_{NN}} = 5.02$ TeV

6.2 Comparison with models

Measurements of total flow coefficients at RHIC and LHC are described well by the hydrodynamic calculations [53–55]. Recently, it was shown that the integrated non-linear flow modes are good observables to constrain a recent comparison between total flow measurements at ALICE [19] and two hydrodynamic calculations from [55] shed new light on the initial conditions and the transport properties of the system [27]. To further test the validity of these hydrodynamic models a comparison is performed between the measured non-linear flow modes and two hydrodynamical calculations from [55], created system in Pb–Pb collisions. Both calculations are based on iEBE-VISHNU [56], an event-by-event version of the VISHNU hybrid model [57] coupling 2 + 1 dimensional viscous hydrodynamics (VISH2+1) [58] to a hadronic cascade model (UrQMD). One of these models uses AMPT [59] initial conditions with the initial conditions used for these calculations are described by AMPT [59] and TRENTo [60] both with $\tau_0=0.6$ fm/c and $T_{sw}=148$ MeV [61]. These values are obtained by using Bayesian statistics from a simultaneous fit of final charged-particle density, mean transverse momentum, and integrated total flow coefficients v_n in Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. For AMPT initial conditions, constant values of specific shear viscosity ($\eta/s = 0.08$, the lower limit conjectured by AdS/CFT) and bulk viscosity ($\zeta/s = 0$), and the other model incorporates are utilised, and TRENTo [60] initial conditions with incorporates a temperature dependent specific shear and bulk viscosity. For simplicity in the rest of this article the model with AMPT initial conditions, $\eta/s = 0.08$ and $\zeta/s = 0$ is referred to as AMPT and the model with TRENTo initial conditions¹ The comparison between the total flow measurements and these two calculations illustrates a qualitative agreement. This agreement between the data and the models depends on the particle species, transverse momentum range and centrality percentile and overall the

¹For simplicity in the rest of this article the model with AMPT initial conditions, $\eta/s = 0.08$ and $\zeta/s = 0$ is referred to as AMPT and the model with TRENTo initial conditions, $\eta/s(T)$ and $\zeta/s(T)$ is referred to as TRENTo.

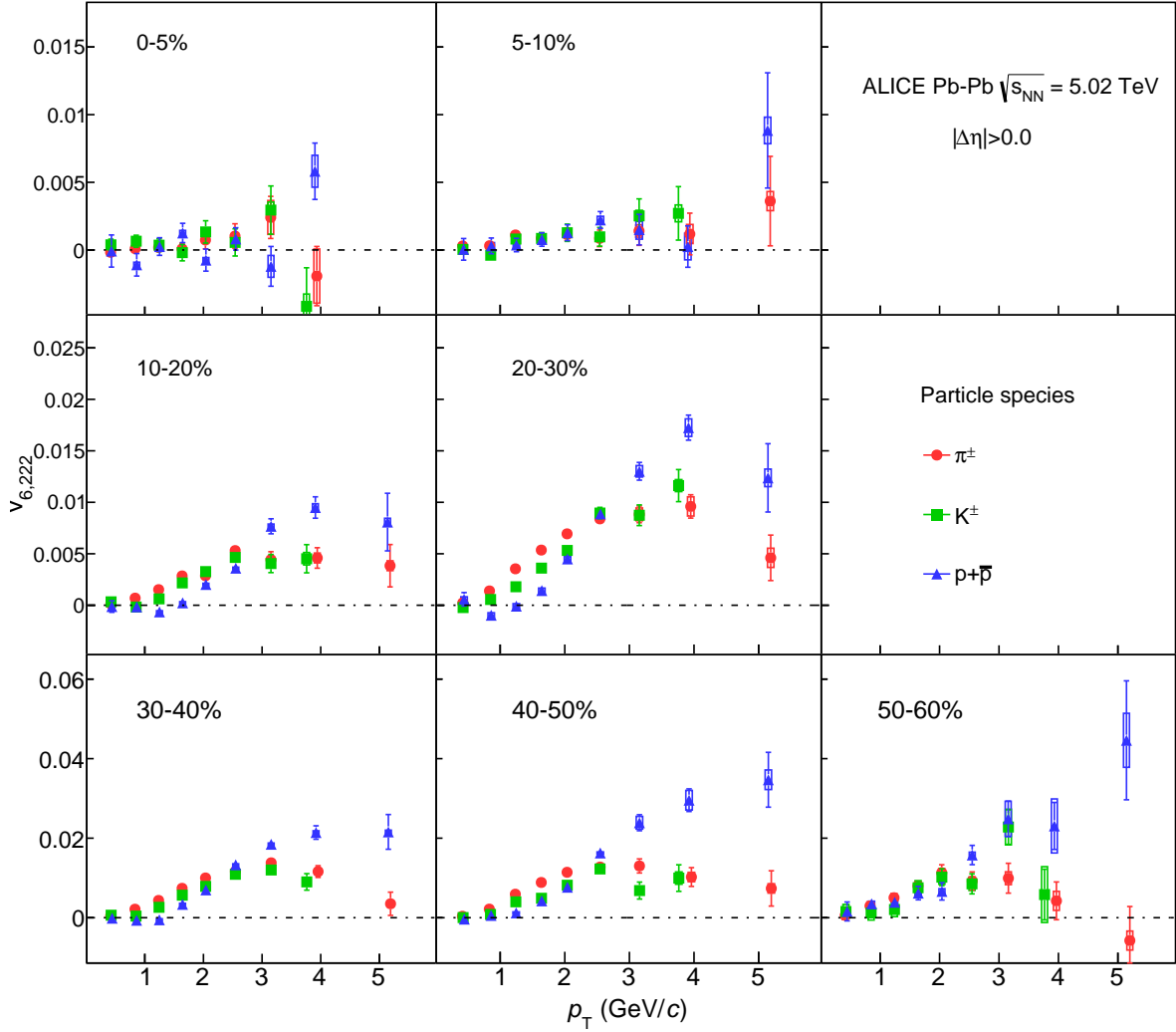


Figure 9: The p_T -differential $v_{6,222}$ for different particle species grouped into different centrality intervals of Pb–Pb collisions $\sqrt{s_{NN}} = 5.02$ TeV

AMPT model reproduces these measurements more accurately than TRENTo [19].

Recently, it was shown that the p_T -integrated non-linear flow modes are good observables to constrain the initial conditions and transport properties of the system [27]. To further constrain the initial conditions and transport properties of the system and test the validity of these hydrodynamic models a comparison is performed between the measured p_T -dependent non-linear flow modes for π^\pm , $\eta/s(T)$ and $\zeta/s(T)$ is referred to as TRENToK $^\pm$ and $p + \bar{p}$ with two hydrodynamical calculations from [55] as were used in comparison to the total flow measurements [19].

Figures 14, 15, 16 and 17 present the comparison between the measurements (data points in the plots) and both models for the p_T -differential $v_{4,22}$, $v_{5,32}$, $v_{6,33}$ and $v_{6,222}$, respectively, for π^\pm , K^\pm and $p + \bar{p}$ at 0-10% up to 50-60% centrality interval (40-50% centrality interval for $v_{6,33}$) of Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. The solid bands show the AMPT model and the hatched bands represent the TRENTo calculations. The bottom panels in each plot in Figs. 14, 15, 16 and 17 present the difference between the models and the measurement. Both models produce a mass ordering in $p_T < 2.5$ GeV/c. The comparison between the models and the the measurements of $v_{4,22}$ reveals that in more central collisions

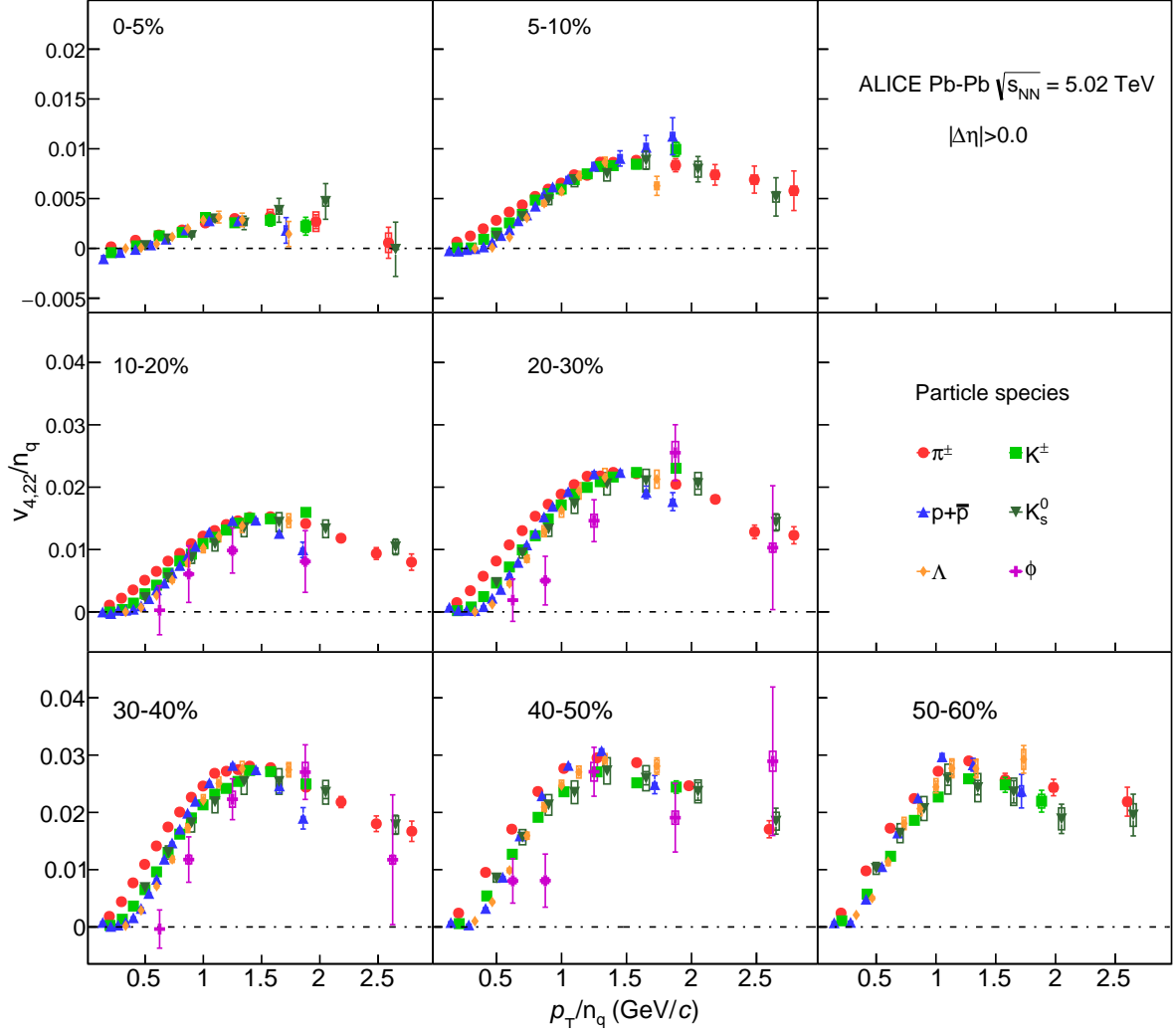


Figure 10: The p_T/n_q -dependence of $v_{4,22}/n_q$ for different particle species grouped into different centrality intervals of Pb–Pb collisions $\sqrt{s_{NN}} = 5.02$ TeV

they reproduce the data however as centrality decreases the models start to deviate from the data. This pattern repeats for $v_{5,32}$ in which AMPT reproduces the data relatively better. The comparison between the measurements and the models for $v_{6,222}$ and $v_{6,33}$ shows that the models are able to reproduce the data only up to 30-40% and 20-30% centrality intervals, respectively.

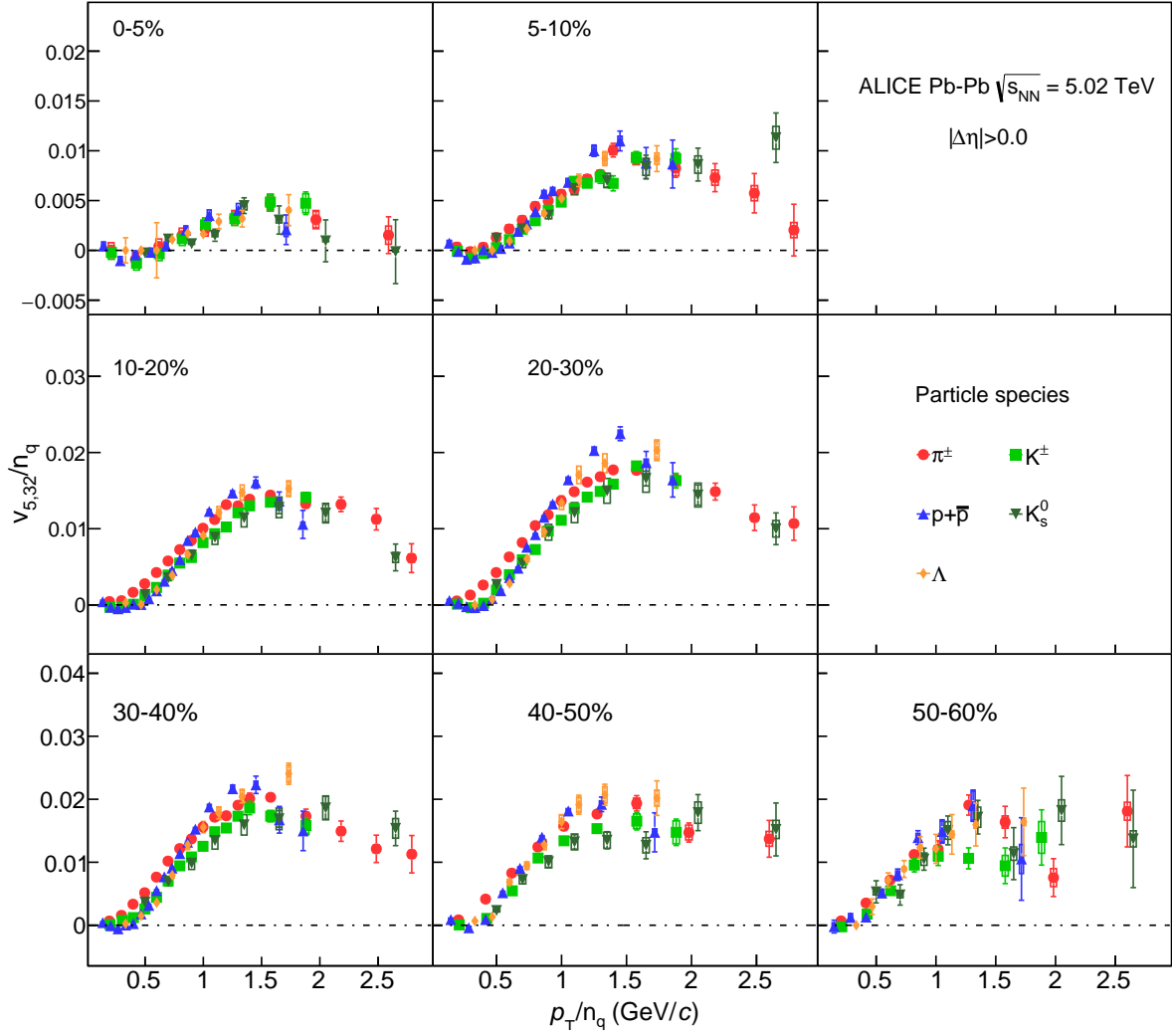


Figure 11: The p_T/n_q -dependence of $v_{5,32}/n_q$ for different particle species grouped into different centrality intervals of Pb–Pb collisions $\sqrt{s_{NN}} = 5.02$ TeV

7 Summary

In this article, a measurement of non-linear flow modes, $v_{4,22}$, $v_{5,32}$, $v_{6,222}$ and $v_{6,33}$ as a function of transverse momentum for different particle species, i.e. π^\pm , K^\pm , $p + \bar{p}$, K_S^0 , $\Lambda + \bar{\Lambda}$ and ϕ -meson are reported for a wide range of centrality intervals from 0-5% up to 50-60% in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. The non-linear flow modes, $v_{n,mk}$, are calculated with a multi-particle correlation technique, namely the generic framework, selecting the identified hadron under study and the reference flow particles from different, non-overlapping pseudorapidity regions. This multi-particle correlation technique by nature removes majority of non-flow correlations. In order to reduce non-flow contributions further, a non-zero gap was applied between the two pseudorapidity regions as well as selecting like sign particles of interest and reference particles. These variations did not affect the results significantly but any variation was included in the final systematics.

The magnitude of $v_{4,22}$, $v_{5,32}$ and $v_{6,222}$ exhibit a clear centrality dependence. This centrality dependence originates from the contribution of second order flow harmonic, as shown in Eq. 5, and reflects the dependence of v_2 on the anisotropy of the collision geometry. As expected, $v_{6,33}$ does not exhibit a

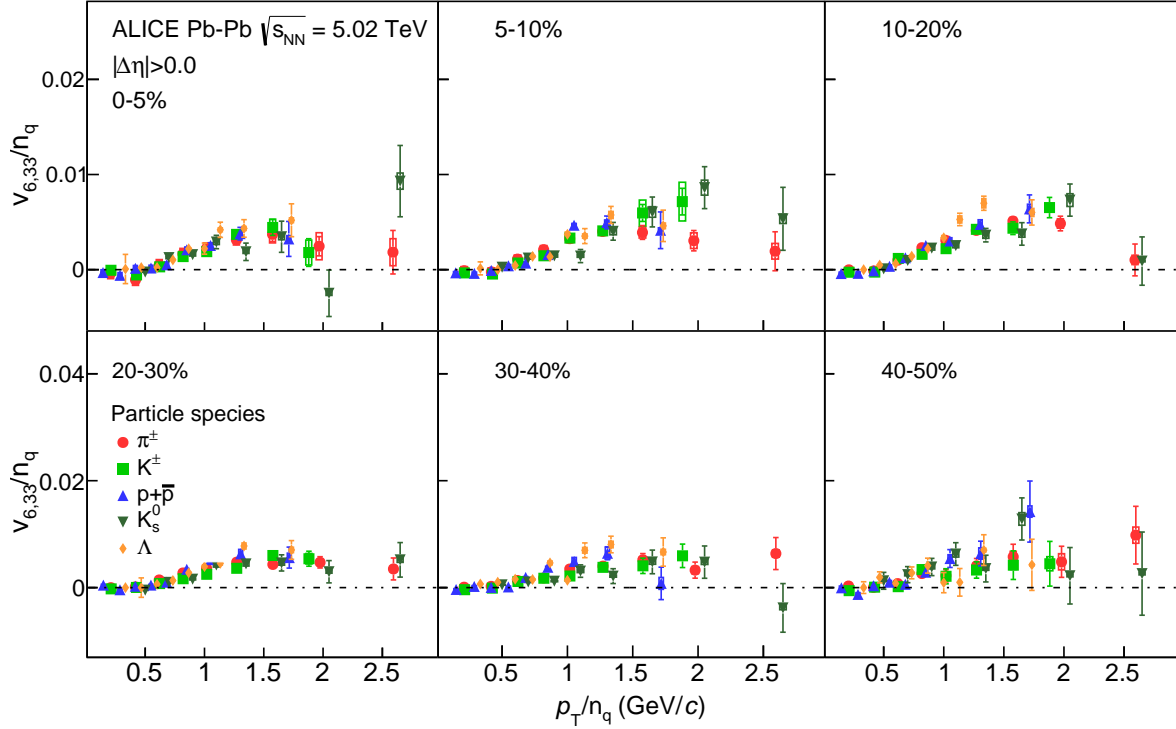


Figure 12: The p_T/n_q -dependence of $v_{6,33}/n_q$ for different particle species grouped into different centrality intervals of Pb–Pb collisions $\sqrt{s_{NN}} = 5.02$ TeV

considerable centrality dependence since v_3 is primarily generated by event-by-event fluctuations of the initial energy density profile. This is supported by the relatively large magnitude of $v_{6,33}$ in the most-central collisions (0-5%). A clear mass ordering is observed in the low p_T region ($p_T < 2.5$ GeV/c). This observation is associated with the interplay between the **non-linear flow modes** **anisotropic flow** and radial flow. In the intermediate p_T region ($p_T > 2.5$ GeV/c), a particle type grouping is observed where the magnitude of non-linear modes for baryons are larger than for mesons. The NCQ scaling holds at best in an approximate level of $\pm 20\%$ within the current level of statistical and systematic uncertainties similar to that of total flow coefficients [19].

The comparison of two models based on the iEBE-VISHNU hybrid model, and with two different initial conditions (AMPT and TRENTo) and transport properties show that neither of the models are able to fully describe the measurements. This varies depending on the centrality percentile. Measurements are better predicted by the models in more central collisions. All in all, the model using AMPT initial conditions ($\eta/s = 0.08$ and $\zeta/s = 0$) exhibits a magnitude and shape closer to the measurements. As a result, in order to further constrain the values of transport properties and the initial conditions of the system, it is necessary to tune the input parameters of future hydrodynamic calculations attempting to describe these measurements.

Acknowledgements

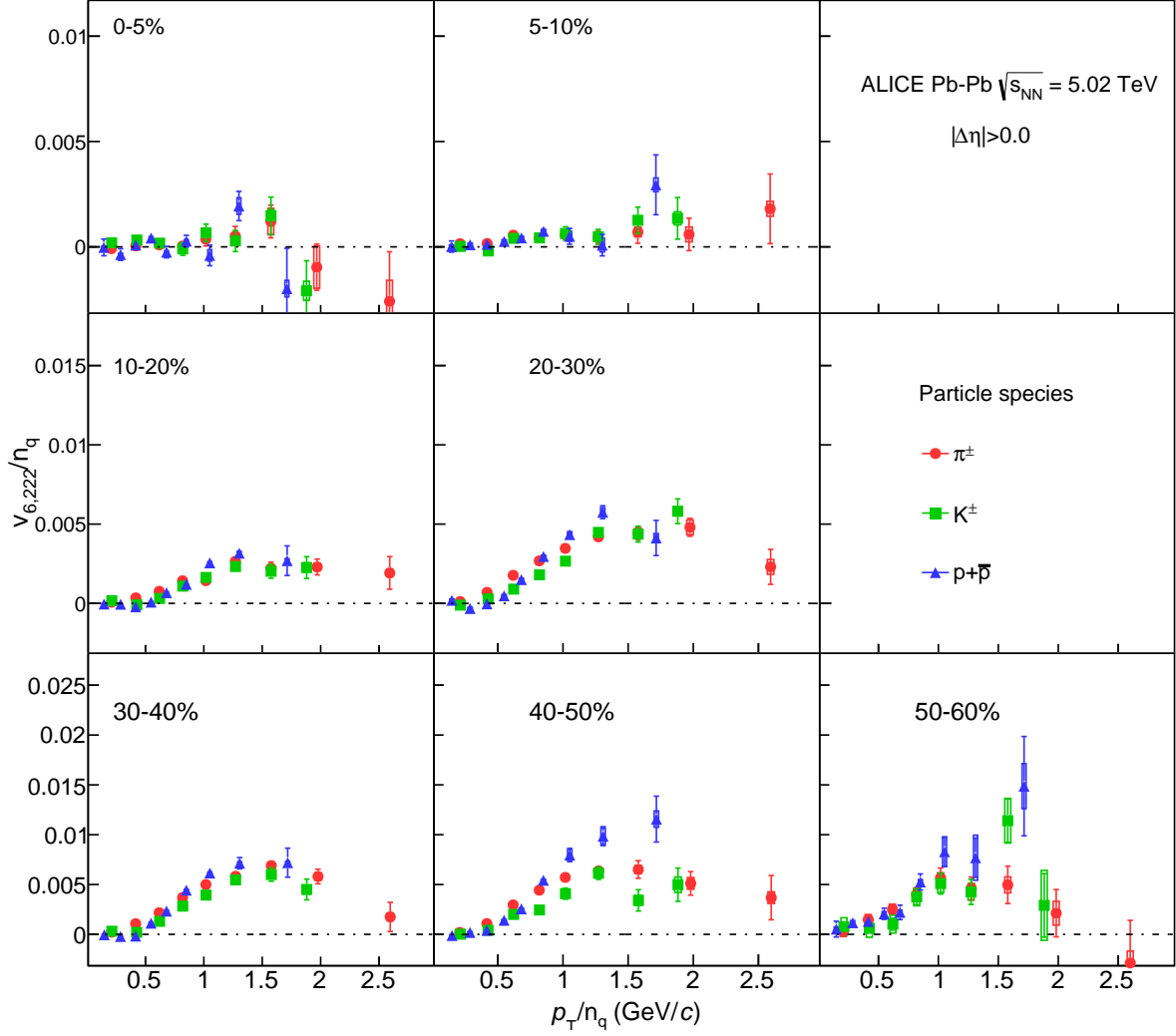


Figure 13: The p_T/n_q -dependence of $v_{6,222}/n_q$ for different particle species grouped into different centrality intervals of Pb–Pb collisions $\sqrt{s_{NN}} = 5.02$ TeV

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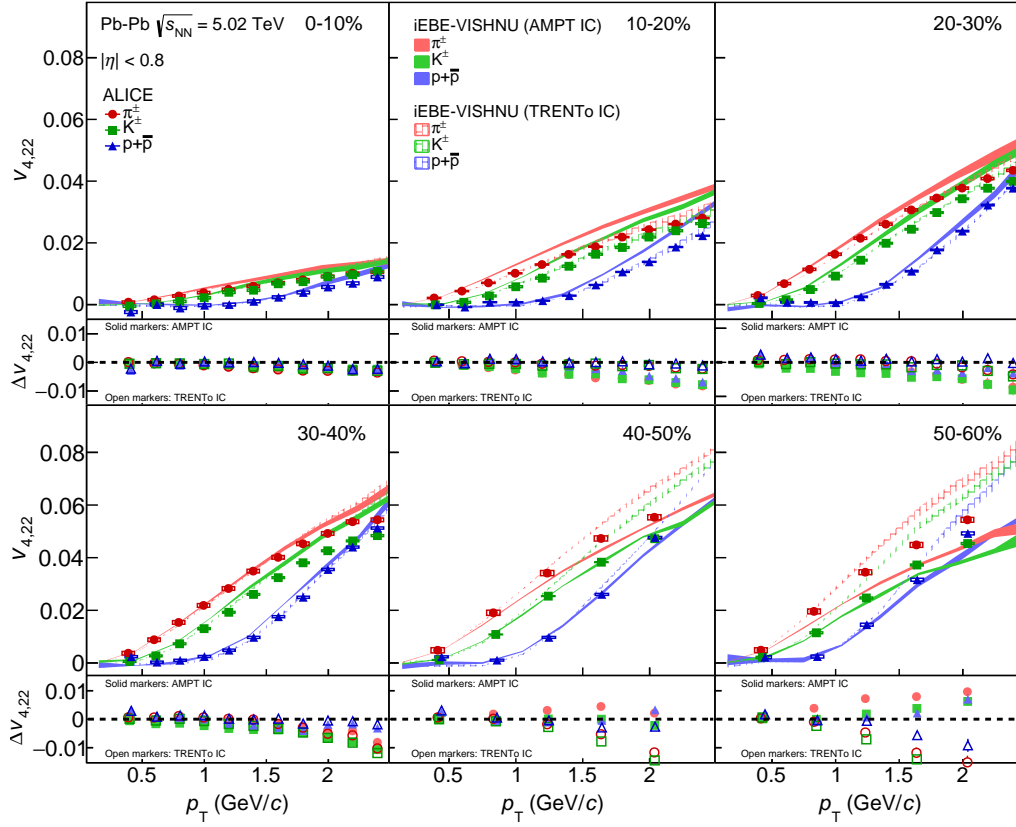


Figure 14: The p_T -differential $v_{4,22}$ for different particle species in 10-20% up to 50-60% centrality intervals of Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV compared with iEBE-VISHNU hybrid models with two different sets of initial parameters: AMPT initial conditions ($\eta/s = 0.08$ and $\zeta/s = 0$) shown in solid bands and TRENTo initial conditions ($\eta/s(T)$ and $\zeta/s(T)$) in hatched bands. The bottom panels show the difference between the measurements and each model.

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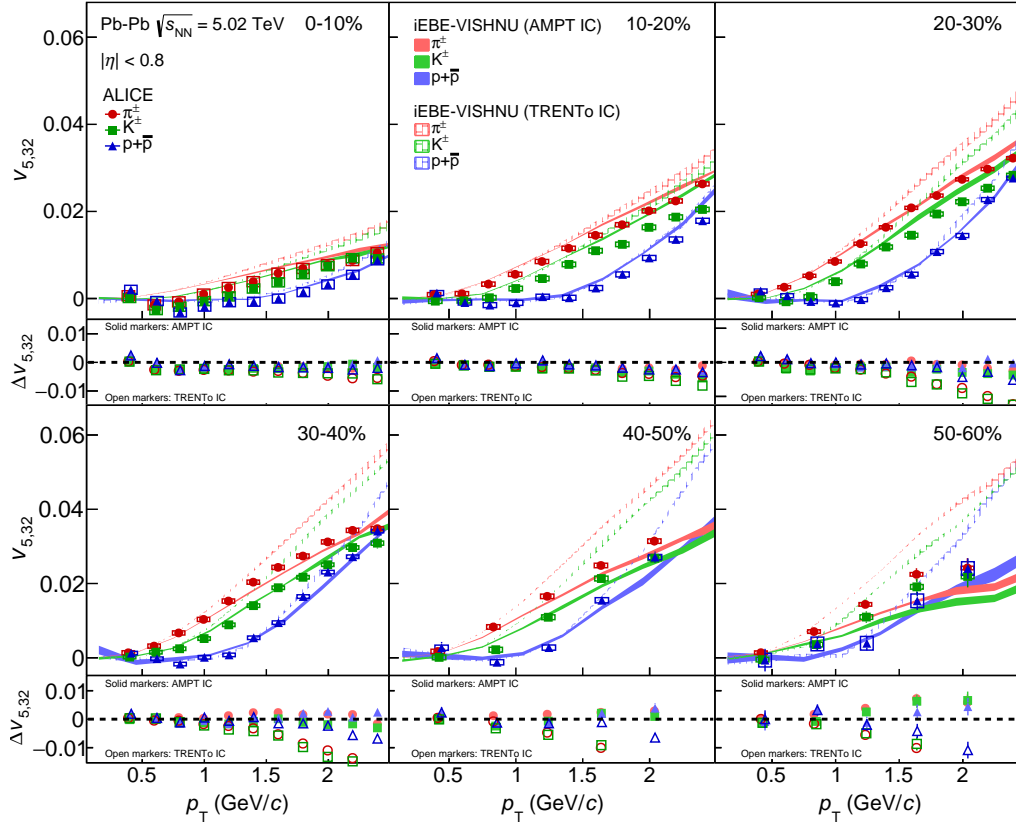


Figure 15: The p_T -differential $v_{5,32}$ for different particle species in 10-20% up to 50-60% centrality intervals of Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV compared with iEBE-VISHNU hybrid models with two different sets of initial parameters: AMPT initial conditions ($\eta/s = 0.08$ and $\zeta/s = 0$) shown in solid bands and TRENTo initial conditions ($\eta/s(T)$ and $\zeta/s(T)$) in hatched bands. The bottom panels show the difference between the measurements and each model.

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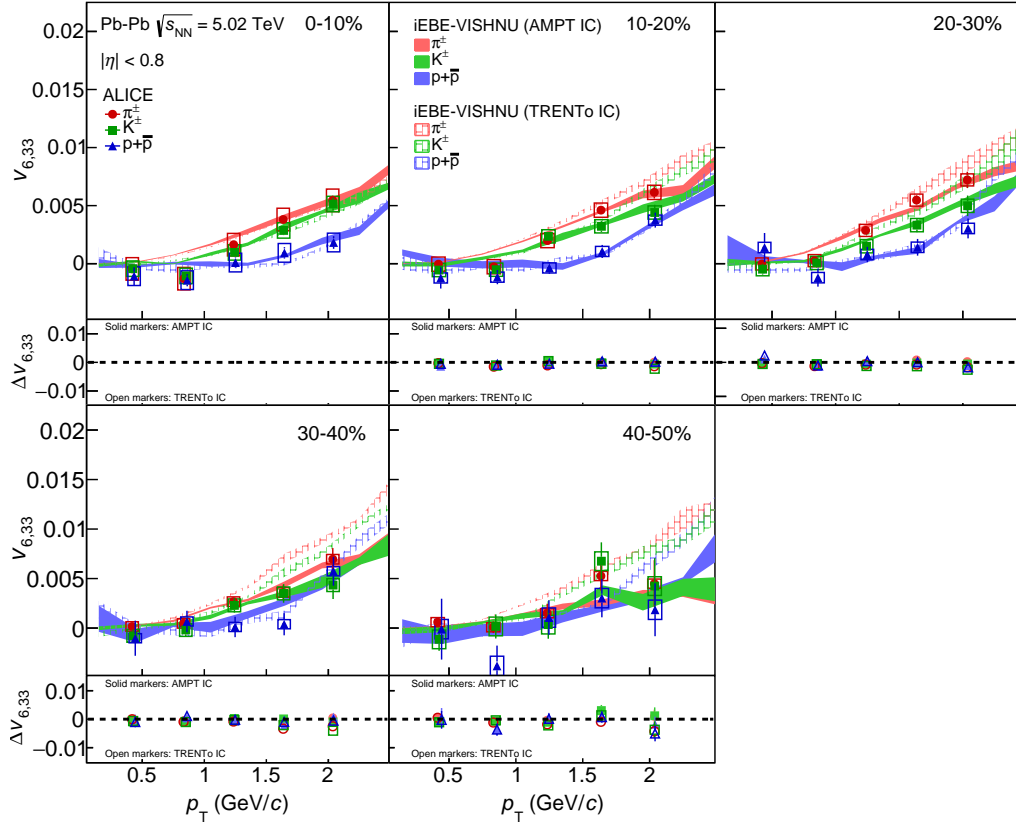


Figure 16: The p_T -differential $v_{6,33}$ for different particle species in 10-20% up to 40-50% centrality intervals of Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV compared with iEBE-VISHNU hybrid models with two different sets of initial parameters: AMPT initial conditions ($\eta/s = 0.08$ and $\zeta/s = 0$) shown in solid bands and TRENTo initial conditions ($\eta/s(T)$ and $\zeta/s(T)$) in hatched bands. The bottom panels show the difference between the measurements and each model.

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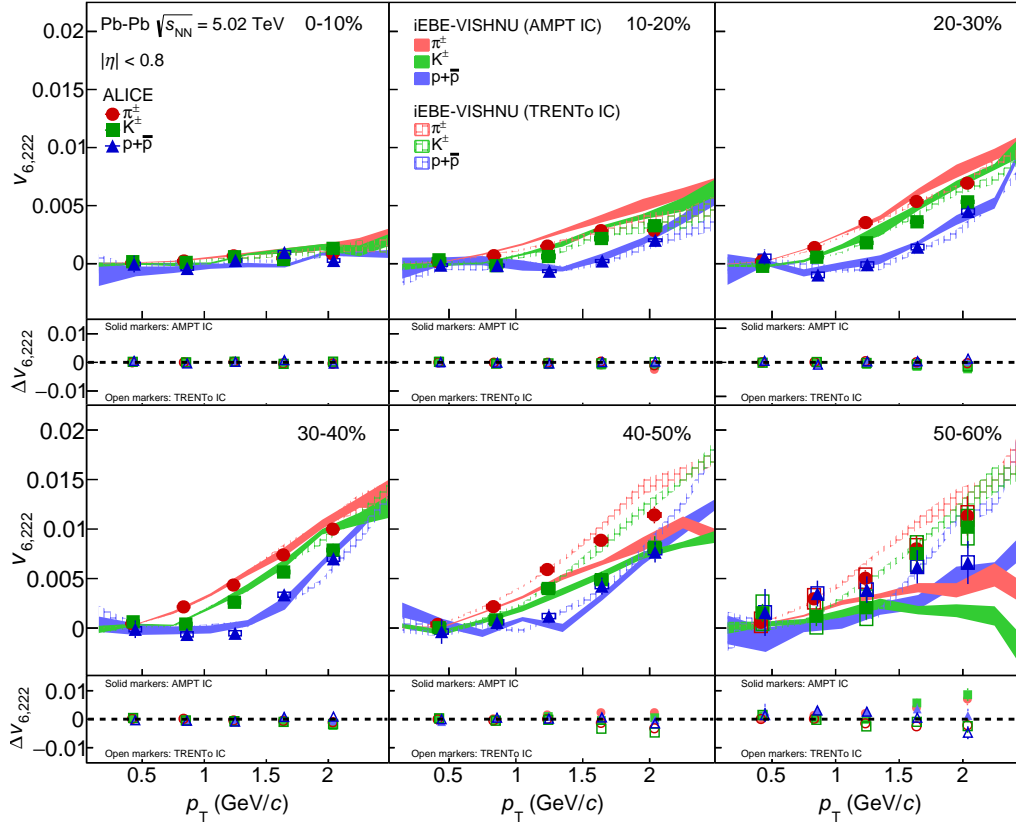


Figure 17: The p_T -differential $v_{6,222}$ for different particle species in 10-20% up to 50-60% centrality intervals of Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV compared with iEBE-VISHNU hybrid models with two different sets of initial parameters: AMPT initial conditions ($\eta/s = 0.08$ and $\zeta/s = 0$) shown in solid bands and TRENTo initial conditions ($\eta/s(T)$ and $\zeta/s(T)$) in hatched bands. The bottom panels show the difference between the measurements and each model.

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A Additional figures

A.1 KE_T scaling

One suggestion to further study the scaling properties of flow coefficients was to extend the scaling to lower p_T values by studying the transverse kinetic energy dependence of anisotropic flow harmonics. Transverse kinetic energy is defined as $KE_T = m_T - m_0$, where $m_T = \sqrt{m_0^2 + p_T^2}$ is the transverse mass. Figures A.1, A.2, A.3 and A.4 present KE_T scaling for $v_{4,22}$, $v_{5,32}$, $v_{6,33}$ and $v_{6,222}$ respectively, for π^\pm , K^\pm , $p + \bar{p}$, K_S^0 , $\Lambda + \bar{\Lambda}$ and ϕ -meson grouped in different centrality intervals.

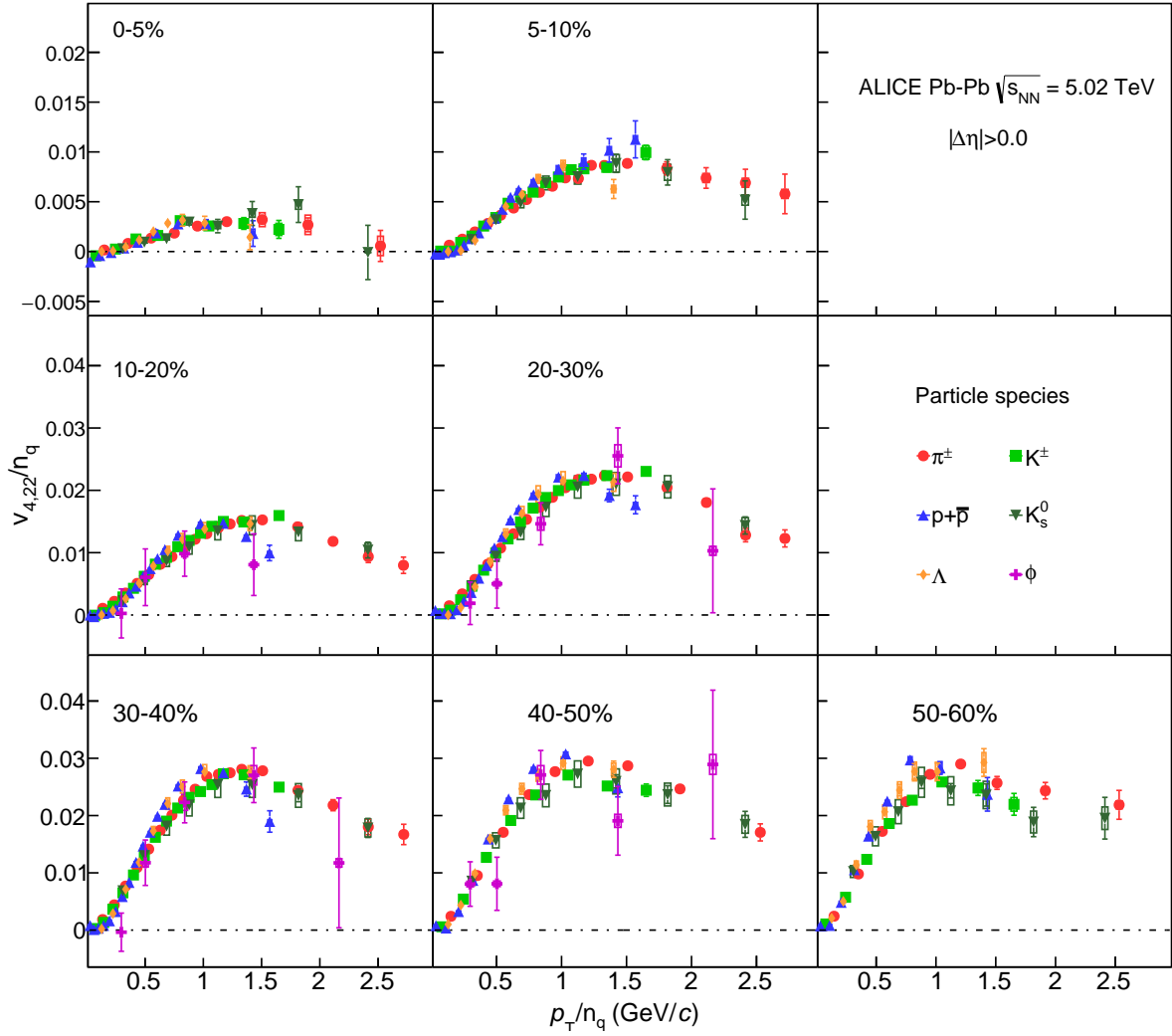


Figure A.1: The $(m_T - m_0)/n_q$ -dependence of $v_{4,22}/n_q$ for different particle species grouped into different centrality intervals of Pb–Pb collisions $\sqrt{s_{NN}} = 5.02$ TeV

B The ALICE Collaboration

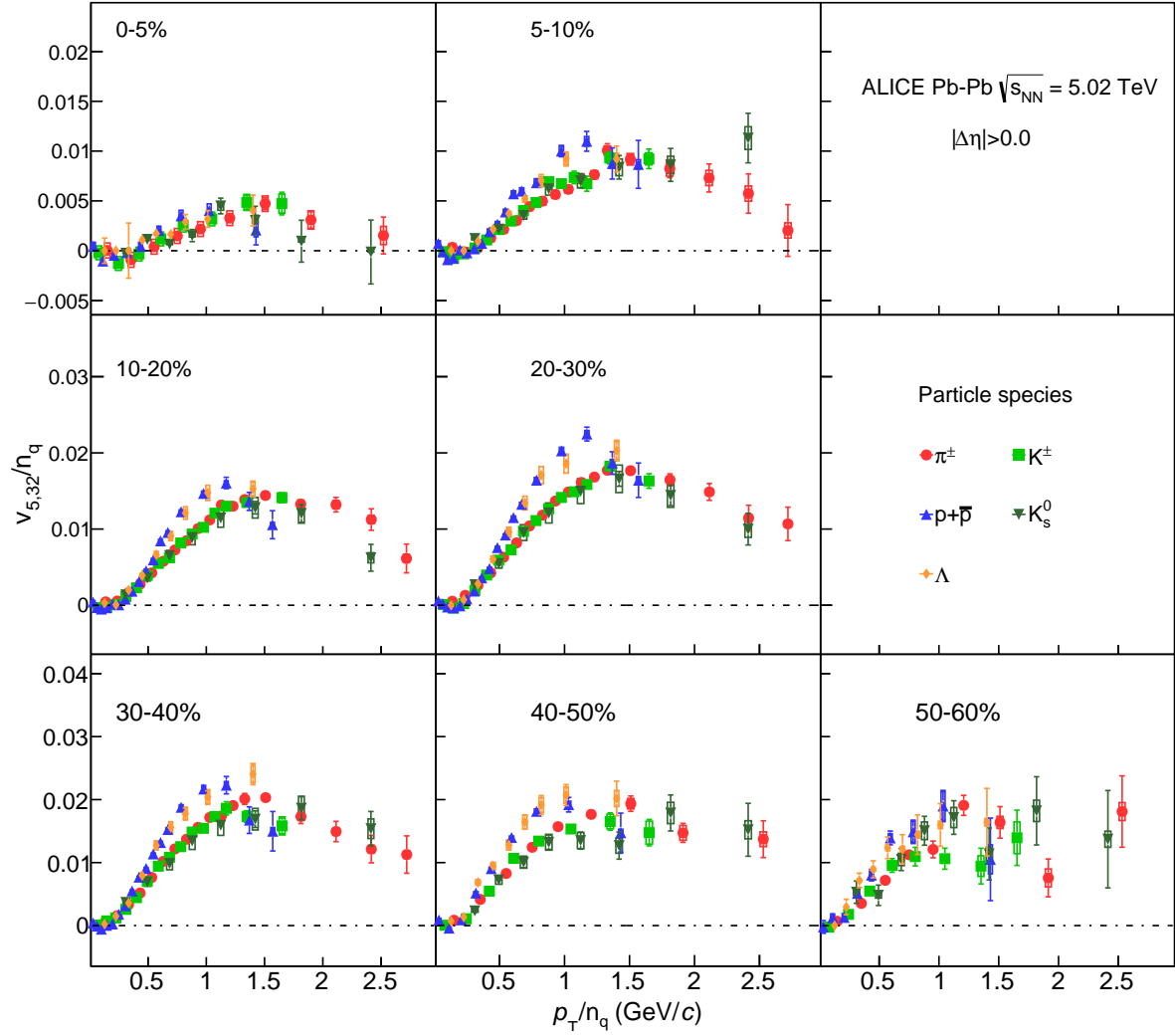


Figure A.2: The $(m_T - m_0)/n_q$ -dependence of $v_{5,32}/n_q$ for different particle species grouped into different centrality intervals of Pb–Pb collisions $\sqrt{s_{NN}} = 5.02$ TeV

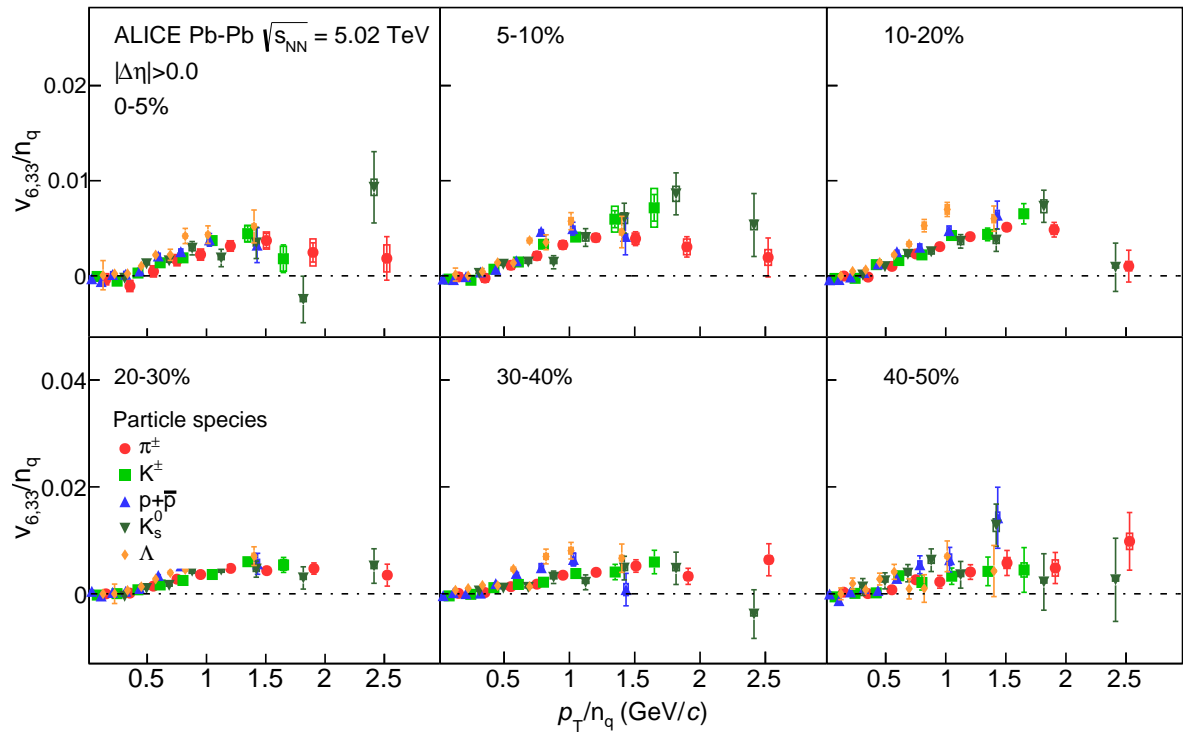


Figure A.3: The $(m_T - m_0)/n_q$ -dependence of $v_{6,33}/n_q$ for different particle species grouped into different centrality intervals of Pb–Pb collisions $\sqrt{s_{NN}} = 5.02$ TeV

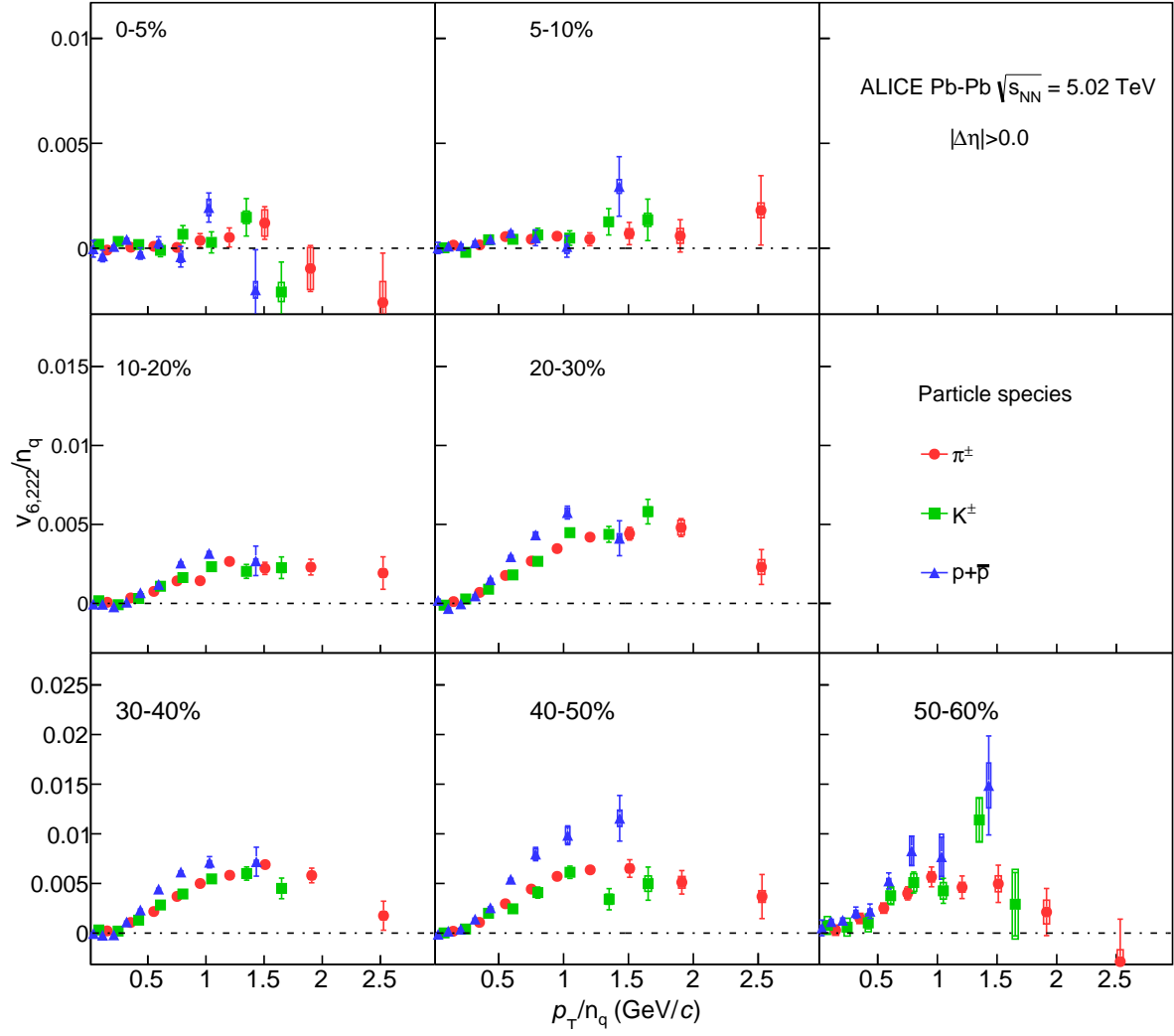


Figure A.4: The $(m_T - m_0)/n_q$ -dependence of $v_{6,222}/n_q$ for different particle species grouped into different centrality intervals of Pb–Pb collisions $\sqrt{s_{NN}} = 5.02$ TeV