CSE 317 Numerical Analysis
Assignment 1

Nagihan BAZ
171805024

① $f(x) = \pi + e^x \qquad x = 2$

$\pi + e^2 = 10.5306$
$e^2 = 7,38905$
$f(2) = \pi + e^2$
$f'(2) = e^2$
$f''(2) = e^2$
$f'''(2) = e^2$
$f^4(2) = e^2$
$f^5(2) = e^2$

$$f(x) = f(2) + f'(2)(x-2) + f''(2)\frac{(x-2)^2}{2!} + f'''(2)\frac{(x-2)^3}{3!} + f^4(2)\frac{(x-2)^4}{4!} + f^5(2)\frac{(x-2)^5}{5!}$$

$$= 10,5306 + 7,38905(x-2) + 7,38905\frac{(x-2)^2}{2} + 7,38905\frac{(x-2)^3}{6} + 7,38905\frac{(x-2)^4}{24} + 7,38905\frac{(x-2)^5}{120}$$

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(2)

$2x+y-4z=-7$
$x-y+z=-2$
$-x+3y-2z=6$

$$\left[\begin{array}{ccc|c} 2 & 1 & -4 & -7 \\ 1 & -1 & 1 & -2 \\ -1 & 3 & -2 & 6 \end{array}\right] \xrightarrow[\frac{1}{2}R_1+R_3 \to R_3]{-\frac{1}{2}R_1+R_2 \to R_2} \left[\begin{array}{ccc|c} 2 & 1 & -4 & -7 \\ 0 & -3/2 & 3 & 3/2 \\ 0 & 7/2 & -4 & 5/2 \end{array}\right]$$

$$\xrightarrow{\frac{7}{3}R_2+R_3 \to R_3} \left[\begin{array}{ccc|c} 2 & 1 & -4 & -7 \\ 0 & -3/2 & 3 & 3/2 \\ 0 & 0 & 3 & 6 \end{array}\right]$$

$3z=6$
$z=2$

$-\frac{3}{2}y+3z=\frac{3}{2}$
$-\frac{3}{2}y+6=\frac{3}{2}$
$-\frac{3}{2}y=\frac{3}{2}-6$
$y=3$

$2x+y-4z=-7$
$2x+3-8=-7$
$2x=-2$
$x=-1$

$(x,y,z)=(-1,3,2)$

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3.

$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$A = LU$

$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \times \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12}+U_{22} & L_{21}U_{13}+U_{23} \\ L_{31}U_{11} & L_{31}U_{12}+L_{32}U_{22} & L_{31}U_{13}+L_{32}U_{23}+U_{33} \end{bmatrix}$$

$U_{11} = 3$
$U_{12} = 1$
$U_{13} = 2$

$L_{21}U_{11} = 6 \Rightarrow L_{21} \times 3 = 6 \Rightarrow L_{21} = 2$

$L_{21}U_{12} + U_{22} = 3 \Rightarrow 2 \times 1 + U_{22} = 3 \Rightarrow U_{22} = 1$

$L_{21}U_{13} + U_{23} = 4 \Rightarrow 2 \times 2 + U_{23} = 4 \Rightarrow U_{23} = 0$

$L_{31}U_{11} = 3 \Rightarrow L_{31} \times 3 = 3 \Rightarrow L_{31} = 1$

$L_{31}U_{12} + L_{32}U_{22} = 1 \Rightarrow 1 \times 1 + L_{32} \times 1 = 1 \Rightarrow L_{32} = 0$

$L_{31}U_{13} + L_{32}U_{23} + U_{33} = 5 \Rightarrow 1 \times 2 + 0 \times 0 + U_{33} = 5 \Rightarrow U_{33} = 3$

$A = L \times U$

$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$Ax = B$

$A = LU$

$LUx = B$

$Ux = y$

$Ly = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$y_1 = 0$

$2y_1 + y_2 = 1$

$2 \times 0 + y_2 = 1$

$y_2 = 1$

$y_1 + y_3 = 3$

$0 + y_3 = 3$

$y_3 = 3$

$Ux = y$

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$x_2 = 1$

$3x_3 = 3$

$x_3 = 1$

$3x_1 + x_2 + 2x_3 = 0$

$3x_1 + 1 + 2 = 0$

$x_1 = -1$

$x_1 = -1$

$x_2 = 1$

$x_3 = 1$

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(4)

$u-8v-2w=1$

$u+v+5w=4$

$3u-v+w=-2$

$3u-v+w=-2$

$u-8v-2w=1$

$u+v+5w=4$

I. $u=\frac{-2+v-w}{3}$ $\quad v=0,\ w=0 \quad u=\frac{-2}{3}$

$v=\frac{-1+u-2w}{8}$ $\quad u=0,\ w=0 \quad v=\frac{-1}{8}$

$w=\frac{4-u-v}{5}$ $\quad u=0,\ v=0 \quad w=\frac{4}{5}$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} -2/3 \\ -1/8 \\ 4/5 \end{pmatrix}$$

II.

$$u=\frac{-2-\frac{1}{8}-\frac{4}{5}}{3}=\frac{-117}{120}$$

$$v=\frac{-1-\frac{2}{3}-\frac{8}{5}}{8}=\frac{-49}{120}$$

$$w=\frac{4+\frac{2}{3}+\frac{1}{8}}{5}=\frac{115}{120}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} -39/40 \\ -49/120 \\ 23/24 \end{pmatrix}$$

III.

$$u = \frac{-2 \; \frac{-49}{120} \; \frac{-23}{24}}{3} = \frac{-404}{360} = \frac{-101}{90}$$

$$v = \frac{-1 \; \frac{-39}{40} \; \frac{-46}{24}}{8} = \frac{-467}{960}$$

$$w = \frac{4 \; \frac{+39}{40} \; \frac{+49}{120}}{5} = \frac{646}{600} = \frac{323}{300}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} -101/90 \\ -467/960 \\ 323/300 \end{pmatrix}$$