CSE 317 Numerical Analysis - Assignment#2

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$$\frac{y_1-y_0}{x_1-x_0} \quad \frac{y_2-y_1}{x_2-x_1} \quad \frac{y_3-y_2}{x_3-x_2} \quad \frac{y_4-y_3}{x_4-x_3} \quad \frac{y_5-y_4}{x_4-x_3}$$

$$\Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow$$

$$[x_0 x_1] \quad [x_1, x_2] \quad [x_2, x_3] \quad [x_3, x_4] \quad [x_4, x_5]$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$1 \quad 19 \quad 11 \quad 5 \quad 11$$

$$\frac{19-1}{4-(-2)} \quad \frac{11-19}{(-1)-1} \quad \frac{5-11}{3-4} \quad \frac{11-5}{-4-(-1)}$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$3 \quad 4 \quad 6 \quad -2$$

$$\frac{4-3}{(-1)-(-2)} \quad \frac{6-4}{3-1} \quad \frac{-2-6}{(-4)-4}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$1 \quad 1 \quad 1$$

$$0 \quad 0$$

(2)

$f(x) = 3x^3 + x^2 - x - 5$

x	0	1	2
$f(x)$	-5	-2	21

1. iteration =

$f(1) = -2 \Rightarrow -2 < 0$
$f(2) = 21 \Rightarrow 21 > 0$

$x_0 = \frac{1+2}{2} = 1,5$

$f(x_0) = f(1,5) = 3.(1,5)^3 + (1,5)^2 - 1,5 - 5 = 5,875 \Rightarrow 5,875 > 0$

2. iteration =

$f(1) = -2 \Rightarrow -2 < 0$
$f(1,5) = 5,875 \Rightarrow 5,875 > 0$

$x_1 = \frac{1+1,5}{2} = 1,25$

$f(x_1) = f(1,25) = 3(1,25)^3 + (1,25)^2 - 1,25 - 5 = 1,719 \Rightarrow 1,719 > 0$

3. iteration =

$f(1) = -2 \Rightarrow -2 < 0$
$f(1,25) = 1,719 \Rightarrow 1,719 > 0$

$x_2 = \frac{1+1,25}{2} = 1,125$

$f(x_2) = f(1,125) = 3(1,125)^3 + (1,125)^2 - 1,125 - 5 = -0,5879 \Rightarrow -0,5879 < 0$

4. iteration =

$f(1,125) = -0,5879 \Rightarrow -0,5879 < 0$
$f(1,25) = 1,719 \Rightarrow 1,719 > 0$

$x_3 = \frac{1,125+1,25}{2} = 1,1875$

$f(x_3) = f(1,1875) = 3.(1,1875)^3 + (1,1875)^2 - 1,1875 - 5 = 0,2463 \Rightarrow 0,2463 > 0$

(3)

$h = 0{,}01$

$$f'(2{,}36) = \frac{-3f(2{,}36) + 4f(2{,}37) - f(2{,}38)}{2(0{,}01)}$$

$$f'(2{,}36) = 0{,}424$$

$$f''(2{,}36) = \frac{-2f(2{,}36) - 5f(2{,}37) + 4f(2{,}38) - f(2{,}39)}{(0{,}01)^2}$$

$$f''(2{,}36) = -0{,}2$$

(4)

$f(x) = x.\cos x \quad , \quad a = 0 \quad , \quad b = \pi$

- Using two panels $\Rightarrow n = 2$

$$\Delta x = \frac{\pi - 0}{2} = \frac{\pi}{2}$$

$$f(x_0) = f(0) = 0$$

$$f(x_1) = f\left(\frac{\pi}{2}\right) = 0$$

$$f(x_2) = f(\pi) = -\pi$$

$$\int_0^{\pi} x.\cos x\, dx = \frac{\frac{\pi}{2}}{3}\left[0 + 4(0) + (-\pi)\right] = \frac{-\pi^2}{6} \cong -1{,}6449$$

- Using four panels $\Rightarrow n = 4$

$$\Delta x = \frac{\pi - 0}{4} = \frac{\pi}{4}$$

$$f(x_0) = f(0) = 0$$

$$f(x_1) = f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$$

$$f(x_2) = f\left(\frac{\pi}{2}\right) = 0$$

$$f(x_3) = f\left(\frac{3\pi}{4}\right) = \frac{-3\pi}{4\sqrt{2}}$$

$$f(x_4) = f(\pi) = -\pi$$

$$\int_0^{\pi} x.\cos x.\, dx = \frac{\frac{\pi}{4}}{3}\left[0 + \frac{4\pi}{4\sqrt{2}} + 2(0) + 4\left(\frac{-3\pi}{4\sqrt{2}}\right) + (-\pi)\right] \cong -1{,}9856$$

- Using six panels $\Rightarrow n=6$

$$\Delta x = \frac{\pi - 0}{6} = \frac{\pi}{6}$$

$$\int_0^{\pi} x.\cos x.dx = \frac{\frac{\pi}{6}}{3}\left[f(x_0)+f(x_n)+4(f(x_1)+f(x_3)+f(x_5)\right]+2(f(x_2)+f(x_4))$$

$$= \frac{\frac{\pi}{6}}{3}\left[0+(-\pi)+4\left[\frac{\sqrt{3}\pi}{12}+0+\frac{(-2\pi)}{12}+2\left(\frac{\pi}{6}+\frac{(-2\pi)}{6}\right)\right.\right. \cong -1{,}99735$$