geometry

chapters 1 - 8
includes nibs

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basic figures

point - “that which has no part”

line - “breadthless length”

plane - that which has no boundary

line segment - part of a line bound by two endpoints - 1 dimension

polygon - bound by line segments on one plane - 2 dimensions

polyhedron - bounded by polygons and exists in space - 3 dimensions

terms relating to points, lines, & planes

collinear points - all can fall on the same line

noncollinear - cannot all fall on the same line

coplanar points - can be on same plane

concurrent lines - contain the same point

perimeter - sum of lengths of a polygon’s sides

area - surface contained by lines

polygons

3 - triangle
4 - quadrilateral
5 - pentagon
6 - hexagon
7 - heptagon
8 - octagon
9 - enneagon
10 - decagon
11 - hendecagon
12 - dodecagon

angles

ray - part of a line extending endlessly in one direction

angle - a pair of rays with the same endpoint

constructions (all)

- To bisect a line segment
- To bisect an angle
- Copy a line segment
- Copy an angle
- Copy a triangle
- construct a line perpendicular to a given line through a given point
- construct a line parallel to a given line through a given point.

conditional statements

conditional statements - consist of if A, then B
Euler diagrams - have circle **a** contained by circle **b**

converse - if b, then a

    the converse of every definition is true

contrapositive - if not b, then not a

    switch and deny

inverse - if not a, then not b

conditional & contrapositive are logically equivalent

converse & inverse are logically equivalent.

proof

syllogism - $a \rightarrow b$; $b \rightarrow c$; therefore $a \rightarrow c$

this is a direct proof

$a \rightarrow b$; $b \rightarrow c$ are premises

$a \rightarrow c$ is conclusion

theorem - statement proved by reasoning deductively from accepted statements

indirect proof - assume the opposite of the desired conclusion

some postulates

postulate - assumed to be true without proof

Two points determine a line.

Three noncollinear points determine a plane.

some theorems

The Pythagorean Theorem the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

The sum of the angles of a triangle is 180°

If the diameter of a circle is d, its circumference is πd.

If the radius of a circle is r, its area is πr^2.

properties

The Reflexive Property a = a

The Substitution Property If a = b, a can be substituted for b.

The Addition Property If a = b, then a + c = b + c

The Subtraction Property If a = b, then a - c = b - c

The Multiplication Property If a = b, then ac = bc.

The Division Property If a = b and c /= 0, then a/c = b/c

symmetric property if a = b, then b = a

distance

The Ruler Postulate The points on a line can be numbered so that positive number differences measure distances.

A line segment has exactly one midpoint.

betweenness of points definition A point is between two other points on the same line iff its coordinate is between their coordinates.

Betweenness of points theorem If A - B - C, then AB + BC = AC

angles - 1

The Protractor Postulate The rays in a half-rotation can be numbered from 0 to 180 so that positive number distances measure angles.

An angle has exactly one ray that bisects it.

Betweenness of Rays definition A ray is between two other rays in the same half-rotation iff its coordinate is between their coordinates.

Betweenness of Rays theorem If OA - OB - OC, then $\angle AOB + \angle BOC = \angle AOC$

acute <90

right $=90$

all right angles are equal

obtuse $90 < x < 180$

straight 180

angles - 2

opposite rays - two rays are opposite rays if B-A-C

complementary angles sum is 90

complements of the same angle are equal

supplementary angles sum is 180

supplements of the same angle are equal

linear pair common side and other sides are opposite rays

the angles in a linear pair are supplementary

if the angles in a linear pair are equal, then their sides are perpendicular

vertical angles sides of one angle are opposite rays of sides of the other

vertical angles are equal

lines

perpendicular two lines form a right angle

perpendicular lines form four right angles

parallel lines that lie on the same plane and do not intersect

skew lines that lie on different planes and do not intersect

bisection

A point is the midpoint of a line segment iff it divides the line segment into two equal segments.

A line bisects an angle iff it divides the angle into two equal angles.

pre-congruence

distance formula: sqrt[(dif in x)^2 + (dif in y)^2]

polygon - connected set of at least three line segments in the same plane such that each segment intersects exactly two others, one at each endpoint

sides, vertices

congruence

congruent - same size and shape. Correspondence between vertices.

Two triangles are *congruent* iff there is a correspondence between their vertices such that all of their corresponding sides and angles are equal.

Two triangles congruent to a third triangle are congruent to each other

triangle

scalene - no equal sides
isosceles - at least two equal sides
equilateral - all sides are equal

obtuse - an obtuse angle
right - a right angle
acute - all acute angles
equiangular - all angles equal

triangle congruence

If two sides of a triangle are equal, the angles opposite them are equal.

an equilateral triangle is equiangular

If two angles of a triangle are equal, the sides opposite them are equal.

an equiangular triangle is equilateral

ASA postulate; SAS postulate; SSS Theorem; AAS Theorem; HL Theorem

CPCTE

properties of inequality

“three possibilities” (trichotomy) - either $a > b$, $a = b$, $a < b$
transitive property* $a > b$ & $b > c$, then $a > c$

addition* $a > b$, then $a + c > b + c$
subtraction property $a > b$, then $a - c > b - c$
multiplication $a > b$ and $c > 0$, then $ac > bc$
division property $a > b$ and $c > 0$, then $a/c > b/c$

addition theorem of inequality* $a > b$ and $c > d$, then $a + c > b + d$

“whole greater than part” theorem if $a > 0$, $b > 0$, and $a + b = c$, then $c > a$ and $c > b$

*while says $>$, can also be $<$

more triangle properties

exterior angle of a triangle - angle forming linear pair with angle of triangle

Exterior Angle Theorem An exterior angle of a triangle is greater than either remote interior angle.

proof uses auxiliary lines - lines added to help prove a theorem

If two sides of a triangle are unequal, the angles opposite them are unequal in the same order.

If two angles of a triangle are unequal, the sides opposite them are unequal in the same order.

The Triangle Inequality Theorem The sum of any two sides of a triangle is greater than the third side.

The Midsegment Theorem The midsegment of a triangle is parallel to the third side and half as long.

hinge theorem

If **two sides** of one triangle are **equal** to two sides of a second triangle and if the included angle of the first triangle is **larger** than the included angle in the second triangle, then the third sides of the first triangle is **longer** than the third sides of the second triangle.

converse of hinge

If **two sides** of one triangle are **equal** to two sides of a second triangle and if the third side of the first triangle is **longer** than the third side of the second triangle, then the **angle** opposite the third side of the first triangle is **larger** than the angle opposite the third side of the second triangle.

line symmetry

Two points are *symmetric with respect to a line* iff the line is the perpendicular bisector of the line segment connecting the two points.

In a plane, two points each equidistant from the endpoints of a line segment determine the perpendicular bisector of the line segment.

proving lines parallel

parallel - two lines lie on the same plane and do not intersect

transversal - intersects two or more lines in different points.

Equal corresponding angles mean that lines are parallel

Equal alternate interior angles mean that lines are parallel.

Supplementary interior angles on the same side of a transversal mean that lines are parallel

In a plane, two lines perpendicular to a third line are parallel

parallel

Parallel Postulate (Euclid's Fifth Postulate) Through a point not on a line, there is exactly one line parallel to the given line.

In a plane, two lines parallel to a third line are parallel to each other.

Parallel lines form equal corresponding angles
form equal alternate interior angles
form supplementary interior angles on the same side of a transversal
In a plane, a line perpendicular to one of two parallel lines is also perpendicular to the other.

triangle angles

The sum of the angles of a triangle is 180.

If two angles of one triangle are equal to two angles of another triangle, the third angles are equal.

The acute angles of a right triangle are complementary.

Each angle of an equilateral triangle is 60.

An exterior angle of a triangle is equal to the sum of the remote interior angles.

quadrilateral terms

diagonal (of a polygon) - line segment connecting two nonconsecutive vertices

The sum of the angles of a quadrilateral is 360.

A quadrilateral is equiangular if it is a rectangle.

Two points are symmetric with respect to a point iff it is the midpoint of the line segment joining them.

regular polygon - equilateral + equiangular

median of a trapezoid - connects the midpoints of both legs of a trapezoid. parallel + average.

types of quadrilaterals

parallelogram - a quadrilateral whose opposite sides are parallel.

rectangle - a quadrilateral each of whose angles is a right angle.

square - quadrilateral all of whose sides and angles are equal.

rhombus - quadrilateral all of whose sides are equal.

trapezoid - quadrilateral that has exactly one pair of parallel sides.

isosceles trapezoid - trapezoid whose legs are equal.

parallelogram theorems

The opposite sides and angles of a parallelogram are equal.

The diagonals of a parallelogram bisect each other.

A quadrilateral is a parallelogram if its opposite sides are equal.

A quadrilateral is a parallelogram if its opposite angles are equal.

A quadrilateral is a parallelogram if two opposite sides are both parallel and equal.

A quadrilateral is a parallelogram if its diagonals bisect each other.

quadrilateral theorems

All rectangles are parallelograms.

All rhombuses are parallelograms.

The diagonals of a rectangle are equal.

The diagonals of a rhombus are perpendicular.

The diagonals of a rhombus bisect the rhombus angles and form four congruent right triangles.

The diagonals of a square form four congruent right isosceles triangles.

The base angles of an isosceles trapezoid are equal.

The diagonals of an isosceles trapezoid are equal.

kite

a quadrilateral having two disjoint pairs of consecutive sides that are equal.

The diagonals of a kite are perpendicular.

“half-properties”

- one diagonal is the perpendicular bisector of the other
- one diagonal bisects a pair of opposite angles
- one pair of opposite angles are equal

polygon angles

sum of interior angles of a convex polygon is 180(n - 2)

sum of exterior angles of a convex polygon is 360

in a regular polygon:

interior angle measure: [180(n-2)]/n

exterior angle measure: 360/n

n = # of sides

transformations

transformation a one-to-one correspondence between two sets of points

reflection, glide reflection, rotation, translation, dilation

translation the composite of two successive reflections through parallel lines

rotation the composite of two successive reflections through intersecting lines

isometry preserves distance and angle measure

two figures are *congruent* if there is an isometry such that one figure is the image of the other

reflection

The *reflection* of point P through line *l* is P itself if P lies on *l*. Otherwise, it is the point P’ such that *l* is the perpendicular bisector of PP’.

construction - to reflect a point through a line (see page 306)

glide reflection composite of a translation and a reflection in a line parallel to the direction of the translation

symmetry

rotation symmetry iff coincide with rotation image through less than 360° about the point.

> figure has n-fold rotation symmetry iff smallest angle through which it can be turned to look exactly the same is 360°/n

reflection (line) symmetry coincide with reflection image through the line

translation symmetry coincides with translation image

Area - overview

polygonal region the union of a polygon and its interior

Area Postulate Every polygonal region has a positive number called its area such that congruent triangles have equal areas and the area of a polygonal region is equal to the sum of its non-overlapping parts.

area formulas

The area of a rectangle is the product of its base and altitude.

The area of a square is the square of its side.

The area of a right triangle is half the product of its legs.

The area of a triangle is half the product of any base and corresponding altitude.

Triangles with equal bases and equal altitudes have equal areas.

The area of a parallelogram is the product of any base and corresponding altitude.

The area of a rhombus (or kite) is half the product of the lengths of its diagonals.

The area of a trapezoid is half the product of its altitude and the sum of its bases.

or median times height

altitude

of a triangle the perpendicular line segment from a vertex of a triangle to the line containing the opposite side (its base).

acute - all inside; right - 1 inside, two on; obtuse - 1 inside, 2 outside

parallelograms perpendicular line segment that connects points on parallel sides

pythagorean theorem

$a^2 + b^2 > c^2$ | acute

$a^2 + b^2 = c^2$ | right

converse: If the square of one side of a triangle is equal to the sum of the square of the other two sides, the triangle is a right triangle.

$a^2 + b^2 < c^2$ | obtuse

—generator

in which m & n are positive & m < n

$n^2 - m^2$ is the shorter leg; 2mn is the longer leg; $n^2 + m^2$ is the hypotenuse

heron’s theorem

semiperimeter p / 2 = s

Heron’s Theorem: sqrt(s[s - a][s - b][s - c]) = a of a triangle

corollary: (a^2 * sqrt 3) / 4 = a of an equilateral triangle (a = side length)

ratio

the *ratio* of the number a to the number b is a/b (b =/ 0)

proportion an equality between two ratios

B is the *geometric mean* between numbers a and c if a, b, and c are positive and a/b = b/c

corresponding altitudes are drawn from corresponding vertices of two triangles

similarity

Figures are similar if there is a correspondence between their points such that corresponding segments are proportional are a similarity.

Two triangles are similar iff there is a correspondence between their vertices such that their corresponding sides are proportional and their corresponding angles are equal.

- center of dilation
- magnitude - gives relative size of image compared to original

some similarity theorems

SAS Similarity Theorem

SSS Similarity Theorem

AA Similarity Theorem two angles one triangle = two angle another triangle, the triangles are similar

two triangles similar to a third triangle are similar to each other

the side-splitter theorem

If a line parallel to one side of a triangle intersects the other two sides in different points, it divides the sides in the same ratio. a/b = c/d

corollary If a line parallel to one side of a triangle intersects the other two sides in different points, it cuts off segments proportional to the sides. a/a+b = c/c+d

more theorems on similarity

Corresponding altitudes of similar triangles have the same ratio as corresponding sides.

The ratio of the perimeters of two polygons is equal to the ratio of corresponding sides.

The ratio of the areas of two similar polygons is equal to the square of the ratio of corresponding sides.

Angle Bisector Theorem An angle bisector in a triangle divides the opposite side into segments that have the same ratio as the other two sides.

angle bisector theorem

An angle bisector in a triangle divides the opposite side into segments that have the same ratio as the other two sides.

Theorems w/ Projections

The altitude to the hypotenuse of a right triangle forms two triangles similar to it and each other.

The altitude to the hypotenuse of a right triangle is the geometric mean between the segments into which it divides the hypotenuse.

Each leg of a right triangle is the geometric mean between between the hypotenuse and its projection on the hypotenuse.

Special Right Triangles

The Isoscles Right Triangle Theorem In an isoscles right triangle, the hypotenuse is sqrt(2) times the length of a leg.

In a square, the diagonal is sqrt(2) times the length of a side.

The 30-60 Right Triangle Theorem In a 30-60 right triangle, the hypotenuse is twice the shorter leg and the longer leg is sqrt(3) times the shorter leg.