

1 Introduction

F5: $\Gamma(x)$ which is named as gamma function, is a commonly used extension of the factorial function to complex numbers

Lets define f be the Gamma Function from A to B , therefore A is the domain and B is the co-domain of the Gamma Function.

(A) Domain of function: includes all complex numbers and the positive integer.

(B) Co-domain of function:

When a in A is a positive integer, then the gamma function is related to the factorial function $\Gamma(a) = (a-1)!$

When a in A for complex numbers with a positive real part, then the $\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx$.

2 Characteristics

(1) when $a \rightarrow 0^+$, $\Gamma(a) \rightarrow +\infty$

(2) Extreme property: For any real number a , $a \in \mathbf{R}$, $\lim_{n \rightarrow \infty} \frac{\Gamma(n+a)}{\Gamma(n)n^a} = 1$,

(3) Assisting computation of probability density function, $\Gamma\left(n + \frac{1}{2}\right) = \frac{(2n)! \sqrt{\pi}}{n! 4^n}$

(4) Satisfies the recursive property: $\Gamma(a) = (a-1) * \Gamma(a-1)$

3 Special Number

$$\Gamma(1) = 0! = 1$$

$$\Gamma(2) = 1! = 1$$

$$\Gamma(3) = 2! = 2$$

$$\Gamma(4) = 3! = 6$$

Results can be worked out by Characteristics(4)—recursive property, use $\Gamma(5)$ as an example
 $\Gamma(5) = 4 * \Gamma(4) = 4 * 3 * \Gamma(3) = 4 * 3 * 2 * \Gamma(2) = 4 * 3 * 2 * 1 * \Gamma(1) = 4! = 24$

4 References

[En.wikipedia.org] Gamma function https://en.wikipedia.org/wiki/Gamma_function

[Course Resource] Function <http://users.encs.concordia.ca/kamthan/courses/soen-6011/functions.pdf>

[Jekyll.math.byuh.edu] Properties of the Gamma function
<http://www.jekyll.math.byuh.edu/courses/m321/handouts/gammaproperties.pdf>