

Arccos Eternity Function

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Introduction:

Inverse Trigonometric functions used to determine an angle from any of the ratios. The prefix "arc" is used to represent the inverse function. In general all the trigonometric functions when measured in radians is of Θ radian. However the inverse function lies under the radian of $(r\Theta)$. Whereas the Inverse function is used to find the value of an unknown length of right angle.

Domain and Range:

The mapping of the inverse function is one to many which lead to the need of restricting the range of the inverse function. The ranges of the arcs functions are usually the proper subsets of domains of original trigonometric functions.

Domain of the function:

$$[-1,1]$$

Range Of the function:

$$[0,\pi]$$

Cos and \cos^{-1} :

The relation between the Inverse and the original Cosine function can be better explained using the following example:

If we have to calculate the value of

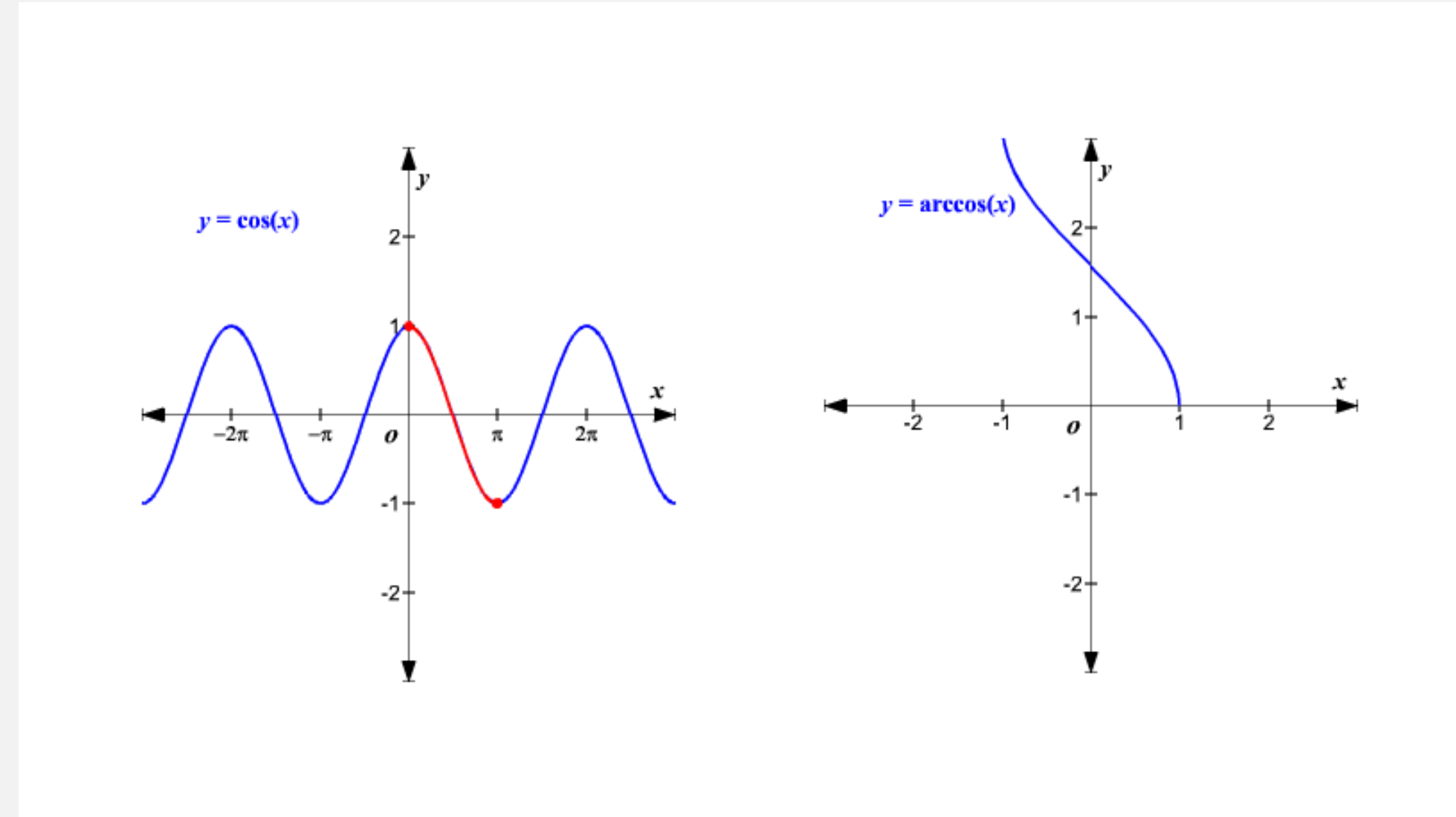
$$\cos^{-1}(-1/2)$$

The above statement will be equivalent to:

$$\begin{aligned}\arccos(-1/2) &= \theta \\ \cos \theta &= -1/2\end{aligned}$$

After solving the angles on the unit ellipse. The value of $\arccos(-1/2)=120^\circ$ as per the range defined in the previous section.

Graph: Arccos and Cos



The above image represents both the Inverse function and the original Cosine function.

Characteristics of Arccos Function:

1. The inverse function $\cos^{-1} x$ has a 3 branch points: $x=\pm 1$, $x=\infty$
2. It is a single value function and continuous on the interval $(-\infty,1]$ and from below on the interval $[1,\infty]$.
3. Function is neither even nor odd.
4. It is an analytic function of x and can be defined over complex numbers.

Algorithm for Implementation:

There can be possible solutions for solving the Arccos function. We used Taylor's Series for the implementation.

Taylor's series for Arccos function:

$$\sin^{-1}x = x + \frac{1x^3}{2 \cdot 3} + \frac{13x^5}{24 \cdot 5} + \frac{135x^7}{246 \cdot 7} + \dots \quad |x| < 1$$

$$\begin{aligned}\cos^{-1}x &= \frac{\pi}{2} - \sin^{-1}x \\ &= \frac{\pi}{2} - x + \frac{1x^3}{2 \cdot 3} + \frac{13x^5}{24 \cdot 5} + \frac{135x^7}{246 \cdot 7} + \dots\end{aligned}$$

Taylor's series was preferred due to its simplicity.

This series can be used to solve even the most complex arguments and As in case of Arccos function there can be many possible solutions.

Algorithm for Arccos Function:

```
Algorithm 1 Calculate arccos Function
1: procedure FACT(c)
2:   factorial ← 1
3:   for i ← 1, exponent do
4:     factorial ← factorial * i
5:   return factorial
6: end procedure

7: procedure CALCULATRARCOS(x, n)
8:   pi ← 3.14
9:   i ← (value - 1)/(value + 1) for i ← 1, ∞ do
10:    factorial ← (factorial * i)
11:    sum ← pow(x, i)FACT(i)()
12:    approx ← 1 + value
13:    z ← (pi/2) - approx
14:  end procedure

16: a ← CALCULATRARCOS(x)           ▷ Calculates Arccosx
17: b ← CALCULATRARCOS(b)           ▷ Calculates Arccosb
18: result ← a/b                    ▷ Final result of Arccosx
```

Advantages Of Taylor's Series:

- The biggest advantage of using Taylor's series is its **Simplicity**.
- This series can be very convenient for solving the most complex problems.
- Taylor's series is helpful in making basis of many methods and functions.
- Taylor's series also helps in getting Theoretical error bounds.

Disadvantages Of Taylor's Series:

- The main disadvantage of using Taylor's Series is the time it takes to solve the equations.
- Taylor's series is not compatible for successive terms. The derivation becomes very complex.
- The truncation error may grow rapidly if Taylor's series is used.
- It is not as efficient when comes to direct approximation.

Working of the Function:

```
Console
<terminated> ArccosF [Java Application] C:\Program Files\Java\jdk\bin\javaw.exe (Aug
Enter the number to calculate the Inverse:
5
Valid values for the function should be between -1 and 1
```

Language used for Implementation: Java.

Tool Used: Eclipse IDE.

Type of Interface: TUI.

Conclusion:

- There can be other methods or series as well for the implementation of the function. However in our case the use of Taylor's series helped in getting efficient and accurate output.
- Few tools were used for the code to be more efficient, readable and maintainable like: **PMD** and **CheckStyle**. The two plugins were easy to use and made the code more understandable.
- For checking the correctness testcases were written in Junit.

References

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