Ejercicio Individual - 2023

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a) Si se envía 0: $Z = N(-R, \sigma)$
Si se envía 1: $Z = N(R, \sigma)$

1º $P(\text{Querer transmitir } 0/\text{Transmitir } 1 \text{ por error}) = P(Q0/T1) = \overbrace{P(Z < \alpha / Z = N(R,\sigma))}^{\text{Dato}} =$

$= P(N(R,\sigma) < \alpha) \underset{\text{Tipificamos}}{=} P\left(N(0,1) < \frac{\alpha - R}{\sigma}\right) = 1 - \Phi\left(\frac{\alpha - R}{\sigma}\right)$

2º $P(Q1/T0) = \overbrace{P(Z > \alpha / Z = N(-R,\sigma))}^{\text{Dato}} = P(N(-R,\sigma) > \alpha) \overset{\text{Tipificamos}}{=} P\left(N(0,1) > \frac{\alpha - (-R)}{\sigma}\right)$

$= 1 - \Phi\left(\frac{\alpha + R}{\sigma}\right)$

→ Para que la probabilidad de error sea igual:

$1 - \Phi\left(\frac{\alpha - R}{\sigma}\right) = 1 - \Phi\left(\frac{\alpha + R}{\sigma}\right) \to \frac{\alpha - R}{\sigma} = \frac{\alpha + R}{\sigma} \to 0 = 2R$ → Pero queremos despejar α

→ Para poder despejar α: $P\left(N(0,1) < \frac{\alpha - R}{\sigma}\right) \to P\left(N(0,1) > \frac{-\alpha + R}{\sigma}\right)$ -

$\to 1 - \Phi\left(\frac{-\alpha + R}{\sigma}\right) = 1 - \Phi\left(\frac{\alpha + R}{\sigma}\right) \to 2\alpha = 0 \to \boxed{\alpha = 0}$

b) $SNR = R^2/\sigma^2$; $P(T0) = P(T1) = 0{,}5$; $\alpha = 0$

$\boxed{SNR = 1}$

$\to \frac{R^2}{\sigma^2} = 1 \to R = \sigma$

$P(Q0/T1) = 1 - \Phi\left(\frac{-\alpha + R}{\sigma}\right) \overset{\alpha = 0}{=} 1 - \Phi\left(\frac{R}{\sigma}\right)$

$P(Q1/T0) = 1 - \Phi\left(\frac{\alpha + R}{\sigma}\right) \overset{\alpha = 0}{=} 1 - \Phi\left(\frac{R}{\sigma}\right)$

$\to P(Q0/T1) = P(Q1/T0)$

$\to P(Q0/T1) = P(Q1/T0) = 1 - \Phi\left(\frac{R}{\sigma}\right) \overset{R=\sigma}{=} 1 - \Phi(1) = 1 - 0{,}8413 = 0{,}1587$

0,5 T0 — 0,8413 Q0
0,5 T0 — 0,1587 Q1 (Error)
0,5 T1 — 0,8413 Q1
0,5 T1 — 0,1587 Q0 (Error)

$\boxed{P(\text{Error})} = P(E) = P(T0) \cdot P(T0/Q1) + P(T1) \cdot P(T1/Q0)$

$= (0{,}5)(0{,}1587) + (0{,}5)(0{,}1587) = \boxed{0{,}1587}$

* Para los siguientes apartados:
$P(E) = 2[(P(T0) = P(T1) = 0{,}5) \cdot (P(T0/Q1) = P(T1/Q0))] =$
$= P(T0/Q1) = P(T1/Q0)$

$\boxed{SNR = 10}$ $\to \frac{R^2}{\sigma^2} = 10 \to \frac{R}{\sigma} = \sqrt{10} \to R = \sigma\sqrt{10}$

$\to P(Q0/T1) = P(Q1/T0) = 1 - \Phi\left(\frac{R}{\sigma}\right) \overset{R = \sigma\sqrt{10}}{=} 1 - \Phi(\sqrt{10}) = 1 - \Phi(3{,}16) = 0{,}0008$

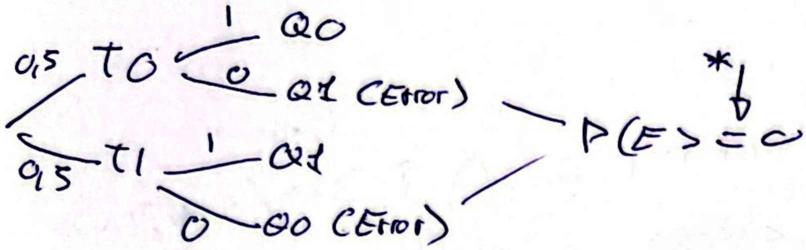
* usando la fórmula deducida anteriormente

0,5 T0 — 0,9992 Q0
0,5 T0 — 0,0008 Q1 (Error)
0,5 T1 — 0,9992 Q1
0,5 T1 — Q0 (Error)

$P(E) = 0{,}0008$

$\boxed{SNR=\infty} \rightarrow \frac{K^2}{\sigma^2}=\infty \rightarrow \frac{K}{\sigma}=\infty \rightarrow \eta=\infty\cdot\sigma$

$\rightarrow P(Q0/T1) = P(Q1/T0) = 1 - G\left(\frac{\eta}{\sigma}\right) = 1 - G(\infty) \simeq 1-1=0$



c) v.a. $X \sim B(\underbrace{n=1000, p=0{,}0008}_{SNR=\sqrt{10}})$

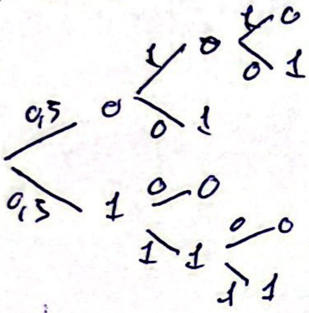
$X \sim \text{Poisson}(\lambda=np)$: $P(X=x) = e^{-\lambda}\frac{\lambda^x}{x!}$

v.a. $X \sim B(n=1000, p=0{,}0008) \xrightarrow{aprox} \text{Poisson}(\lambda=np=1000\cdot 0{,}0008) = \text{Poisson}(\lambda=0$

$\rightarrow \boxed{P(X\leq 2)} = P(X=0)+P(X=1)+P(X=2) = e^{-0,8}\frac{0{,}8^0}{0!} + e^{-0,8}\frac{0{,}8^1}{1!} + e^{-0,8}\frac{0{,}8^2}{2!} =$

$= 0{,}4493 + 0{,}3594 + 0{,}1437 = \boxed{0{,}9524}$

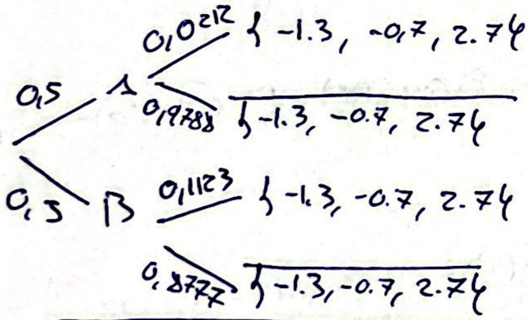
d) $SNR=\infty \rightarrow P(E)=0 \approx$ "No hay fallo al recibir el símbolo enviado"



$\rightarrow P(0)\cdot P(0/0)\cdot P(0/(0\cap 0)) \overset{n \text{ veces}}{\cdots} + P(1)\cdot P(1/1)\cdot P(1/(1\cap 1))$

$= \frac{1}{2}\underbrace{(1\cdot 1\cdot 1)}_{n \text{ veces}} + \frac{1}{2}\underbrace{(1\cdot 1\cdot 1)}_{n \text{ veces}} = 2\underbrace{\left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right)}_{n \text{ veces}} = \boxed{2\left(\frac{1}{2}\right)^n}$

e) $A=010$; $B=101$; $SNR=1$



$\boxed{P(\{-1.3, -0.7, 2.7/010) = P((-1.3/0)\cap(-0.7/1)\cap(2.7/0}$

Si $x\geq\alpha \rightarrow 1$; $x>0 \rightarrow 1$; $\rightarrow x<0 \rightarrow 0$ (Dato)

*Cogemos las probabilidades del b)

$\rightarrow P((0/0)\cap(0/1)\cap(1/0)) = (0{,}8413)(0{,}1587)(0{,}158$

$= \boxed{0{,}0212}$

$\rightarrow \boxed{P(\{-1.3,-0.7,2.7/101)} = P((0/1)\cap(0/0)\cap(1/1)) = 0{,}1587\cdot 0{,}8413\cdot 0{,}8413 = \boxed{0{,}1123}$

$\boxed{P(A/\{-1.3,-0.7, 2.74} = \frac{P(A)\cdot P(\{-1.3,-0.7,2.7/A)}{P(\{-1.3,-0.7,2.7)} = \frac{\frac{1}{2}0{,}0212}{0{,}06675} = \boxed{0{,}1588}$

$\rightarrow \frac{1}{2}\cdot 0{,}0212 + \frac{1}{2}0{,}1123 = 0.06675$

$\boxed{P(B/\{-1.3,-0.7,2.74} = \frac{P(B)\cdot P(\{-1.3,-0.7,2.7/B)}{P(\{-1.3,-0.7,2.74)} = \frac{\frac{1}{2}0{,}1123}{0.06675} = \boxed{0.8412}$

$0.8412 > 0.1588 \rightarrow$ La palabra más probable es la B

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