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Color Image Analysis by Quaternion Zernike Moments

Beijing Chen[1,2], *Huazhong Shu*[1], *Hui Zhang*[1], *Gang Chen*[1], *Limin Luo*[1]
[1]School of Computer Science and Engineering, Southeast University, Nanjing, China
[2]School of Electronic and Information Engineering, Ningbo University of Technology, Ningbo, China
e-mail: nbutimage@126.com, shu.list@seu.edu.cn

***Abstract*—Moments and moment invariants are useful tool in pattern recognition and image analysis. Conventional methods to deal with color images are based on RGB decomposition or graying. In this paper, by using the theory of quaternions, we introduce a set of quaternion Zernike moments (QZMs) for color images in a holistic manner. It is shown that the QZMs can be obtained via the conventional Zernike moments of each channel. We also construct a set of combined invariants to rotation and translation (RT) using the modulus of central QZMs. Experimental results show that the proposed descriptors are more efficient than the existing ones.**

Keywords-color image; quaternions; Zernike moments

I. INTRODUCTION

Moment invariants have been extensively used in pattern recognition owing to their image description capability and invariance property [1]-[3]. Nowadays, almost all images acquired initially are chromatic. Two approaches have been used to deal with the color images. The first one transforms the color image to gray-scale one, which may loss some significant color features. The second approach decomposes the color image into three channels, and then the moment invariants of these images are calculated separately [4]-[6]. Among them, Mindru et al [4] proposed the generalized color moments for color images. Based on these moments, they constructed a set of invariants to geometric deformations and photometric changes. The affine geometric invariants for color images proposed by Suk and Flusser [5] are based on the product of moments defined for different color channels. Moreover, they also introduced the notion of common centroid to define the central moments for each channel. Both approaches are the generalization of traditional geometric moments, the difference between them is that the computation of Mindru's ones may refer to two or three color channels, and Suk's ones only relate to one channel. Because the kernel functions of geometric moments are not orthogonal, this leads to information redundancy and low noise robustness. The purpose of this paper is to extend the conventional orthogonal Zernike moments to color image in a holistic manner. To this end, we will use the theory of quaternions.

Quaternions have been successfully utilized for color image processing. A color image can be treated as a vector field with the quaternion-type color representation. Let $f(x, y)$ be an image in RGB color space, it can be represented by encoding three channels as a pure quaternion

$$f(x,y) = f_R(x,y)\boldsymbol{i} + f_G(x,y)\boldsymbol{j} + f_B(x,y)\boldsymbol{k}\ , \tag{1}$$

where $f_R(x,y), f_G(x,y)$ and $f_B(x,y)$ represent respectively the red, green and blue channel of the color image.

Ell and Sangwine have conducted a lot of researches in color image processing using quaternion theory [7]-[12]. Ell [8] first introduced the quaternion Fourier transform (QFT) in his Ph. D dissertation. Sangwine [9] applied it to color image. The QFTs have been used for color images registration [10], watermarking [13] and motions estimation [14]. Sangwine et al used the quaternion convolution for color image smoothing [11] and edge detection [12].

In this paper, we extend the conventional Zernike moments to color images using the quaternion theory, and construct the moment invariants with respect to rotation and translation (RT). Some experimental results are provided to validate the proposed descriptors.

II. QUATERNION ZERNIKE MOMENTS

A. Quaternions

Quaternions were first introduced by the mathematician Hamilton [15] in 1843, which are the generalization of complex numbers. The quaternion can be represented as a four-dimensional complex number with one real part and three imaginary parts as follows

$$q = a + b\boldsymbol{i} + c\boldsymbol{j} + d\boldsymbol{k}. \tag{2}$$

where a, b, c and d are real numbers, and $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ are three imaginary units obeying the following rules

$$\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = -1, \tag{3}$$

$$\boldsymbol{ij} = -\boldsymbol{ji} = \boldsymbol{k}, \boldsymbol{jk} = -\boldsymbol{kj} = \boldsymbol{i}, \boldsymbol{ki} = -\boldsymbol{ik} = \boldsymbol{j}. \tag{4}$$

The above rules show that the multiplication of quaternions is not commutative. The conjugate and modulus of a quaternion are defined by

$$q^* = a - b\boldsymbol{i} - c\boldsymbol{j} - d\boldsymbol{k}, \tag{5}$$

$$|q| = \sqrt{a^2 + b^2 + c^2 + d^2}\ . \tag{6}$$

B. Quaternion Zernike Moments

The conventional Zernike moment of order n with repetition m of gray-scale image $g(r, \theta)$ is defined as

$$Z_{n,m}(g) = \frac{n+1}{\pi}\int_0^1\int_0^{2\pi} R_{n,m}(r)e^{-jm\theta}g(r,\theta)rdrd\theta\ ,$$
$$|m| \le n \text{ and } n - |m| \text{ being even}, \tag{7}$$

where $R_{n,m}(r)$ is the real-valued radial polynomial given by

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$$R_{n,m}(r)=\sum_{k=0}^{(n-|m|)/2}\frac{(-1)^k(n-k)!}{k!\left(\frac{n+|m|}{2}-k\right)!\left(\frac{n-|m|}{2}-k\right)!}r^{n-2k}. \quad (8)$$

The orthogonality property leads to the following approximated inverse transform

$$g(r,\theta)\approx\sum_{n=0}^{M}\sum_{m}Z_{n,m}(g)R_{n,m}(r)e^{jm\theta}, \quad (9)$$

where M is the maximum order of moments used in the reconstruction.

According to the above definition of Zernike moments for gray-scale image and the theory of quaternion, we introduce the right-side quaternion Zernike moments (QZMs) of order n with repetition m for color image $f(r,\theta)$ as

$$Z_{n,m}^{R}(f)=\frac{n+1}{\pi}\int_0^1\int_0^{2\pi}R_{n,m}(r)f(r,\theta)e^{-\boldsymbol{\mu}m\theta}rdrd\theta,$$

$$|m|\le n \text{ and } n-|m| \text{ being even}, \quad (10)$$

where $\boldsymbol{\mu}$ is a unit pure quaternion, which is chosen as $\boldsymbol{\mu}=(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k})/\sqrt{3}$ in this paper.

The color image can also be reconstructed as

$$f(r,\theta)\approx\sum_{n=0}^{M}\sum_{m}Z_{n,m}^{R}(f)R_{n,m}(r)e^{\boldsymbol{\mu}m\theta}. \quad (11)$$

If N is the number of pixels along each axis of the image, then (10) can be written in the discrete form as

$$Z_{n,m}^{R}(f)=\frac{n+1}{\pi(N-1)^2}\sum_{x=1}^{N}\sum_{y=1}^{N}R_{n,m}(r)f(x,y)e^{-\boldsymbol{\mu}m\theta}. \quad (12)$$

Note that by moving the exponential part $e^{-\boldsymbol{\mu}m\theta}$ in (10) to the left-side of $f(r,\theta)$, we can also define the left-side QZMs. In this paper, only the right-side QZMs will be used.

C. Relationship between Quaternion Zernike Moments and Conventional Zernike Moments for Single Channel of the RGB Color Image

Substitution of (1) into (10), we get

$$\begin{aligned}Z_{n,m}^{R}(f)&=\frac{n+1}{\pi}\int_0^1\int_0^{2\pi}R_{n,m}(r)r[f_R(r,\theta)\boldsymbol{i}+f_G(r,\theta)\boldsymbol{j}+f_B(r,\theta)\boldsymbol{k}]e^{-\boldsymbol{\mu}m\theta}drd\theta\\&=\boldsymbol{i}[\mathrm{Re}(Z_{n,m}(f_R))+\boldsymbol{\mu}\mathrm{Im}(Z_{n,m}(f_R))]+\boldsymbol{j}[\mathrm{Re}(Z_{n,m}(f_G))+\boldsymbol{\mu}\mathrm{Im}(Z_{n,m}(f_G))]\\&\quad+\boldsymbol{k}[\mathrm{Re}(Z_{n,m}(f_B))+\boldsymbol{\mu}\mathrm{Im}(Z_{n,m}(f_B))]\\&=A_{n,m}^{R}+\boldsymbol{i}B_{n,m}^{R}+\boldsymbol{j}C_{n,m}^{R}+\boldsymbol{k}D_{n,m}^{R},\end{aligned} \quad (13)$$

where

$$\begin{aligned}A_{n,m}^{R}&=-\frac{1}{\sqrt{3}}[\mathrm{Im}(Z_{n,m}(f_R))+\mathrm{Im}(Z_{n,m}(f_G))+\mathrm{Im}(Z_{n,m}(f_B))],\\B_{n,m}^{R}&=\mathrm{Re}(Z_{n,m}(f_R))+\frac{1}{\sqrt{3}}[\mathrm{Im}(Z_{n,m}(f_G))-\mathrm{Im}(Z_{n,m}(f_B))],\\C_{n,m}^{R}&=\mathrm{Re}(Z_{n,m}(f_G))+\frac{1}{\sqrt{3}}[\mathrm{Im}(Z_{n,m}(f_B))-\mathrm{Im}(Z_{n,m}(f_R))],\\D_{n,m}^{R}&=\mathrm{Re}(Z_{n,m}(f_B))+\frac{1}{\sqrt{3}}[\mathrm{Im}(Z_{n,m}(f_R))-\mathrm{Im}(Z_{n,m}(f_G))].\end{aligned} \quad (14)$$

Here $Z_{n,m}(f_R)$, $Z_{n,m}(f_G)$ and $Z_{n,m}(f_B)$ are respectively the conventional Zernike moments for the red channel, green channel and blue channel, Re(x) represents the real part of complex number x, and Im(x) the imaginary part.

From (13) and (14), the QZMs can be obtained from the conventional Zernike moments for single channel.

III. ROTATION AND TRANSLATION INVARIANTS OF QUATERNION ZERNIKE MOMENTS

A. Translation Invariants

Suk and Flusser [5] introduced the notion of common centroid (x_c, y_c) of all three channels, which is defined by

$$\begin{aligned}x_c&=(m_{1,0}^{(R)}+m_{1,0}^{(G)}+m_{1,0}^{(B)})/m_{0,0},\\y_c&=(m_{0,1}^{(R)}+m_{0,1}^{(G)}+m_{0,1}^{(B)})/m_{0,0},\\m_{0,0}&=m_{0,0}^{(R)}+m_{0,0}^{(G)}+m_{0,0}^{(B)}.\end{aligned} \quad (15)$$

where $m_{0,0}^{(R)}$, $m_{1,0}^{(R)}$, $m_{0,1}^{(R)}$ are the zero-order and first-order geometric moments for R channel, $m_{0,0}^{(G)}$, $m_{1,0}^{(G)}$, $m_{0,1}^{(G)}$ for G channel, and $m_{0,0}^{(B)}$, $m_{1,0}^{(B)}$, $m_{0,1}^{(B)}$ for B channel. Let the origin of coordinate system be located at (x_c, y_c), the central QZMs are given by

$$\bar{Z}_{n,m}^{R}=\frac{n+1}{\pi}\int_0^1\int_0^{2\pi}R_{n,m}(\bar{r})\bar{r}f(\bar{r},\bar{\theta})e^{-\boldsymbol{\mu}m\bar{\theta}}d\bar{r}d\bar{\theta}, \quad (16)$$

where

$$\bar{r}=\sqrt{(x-x_c)^2+(y-y_c)^2},\quad \bar{\theta}=\tan^{-1}\left(\frac{y-y_c}{x-x_c}\right). \quad (17)$$

The central QZMs are invariant to translation

B. Rotation Invariants and Combined Invariants

Let f' be a rotated version of f, i.e., $f'(r,\theta)=f(r,\theta-\alpha)$, where α denotes the rotation angle, then we have

$$\begin{aligned}Z_{n,m}^{R}(f')&=\frac{n+1}{\pi}\int_0^1\int_0^{2\pi}R_{n,m}(r)f'(r,\theta)e^{-\boldsymbol{\mu}m\theta}rdrd\theta\\&=\frac{n+1}{\pi}\int_0^1\int_0^{2\pi}R_{n,m}(r)f(r,\theta-\alpha)e^{-\boldsymbol{\mu}m\theta}rdrd\theta\\&=\frac{n+1}{\pi}\int_0^1\int_0^{2\pi}R_{n,m}(r)f(r,\theta)e^{-\boldsymbol{\mu}m\theta}rdrd\theta\cdot e^{-\boldsymbol{\mu}m\alpha}\\&=Z_{n,m}^{R}(f)e^{-\boldsymbol{\mu}m\alpha}.\end{aligned} \quad (18)$$

Equation (18) shows that the modulus of $Z_{n,m}^{R}$ is invariant to rotation. Combining (16) and (18), we have

Proposition 1. Let

$$\varphi_{n,m}^{R}=\left|\bar{Z}_{n,m}^{R}\right|, |m|\le n \text{ and } n-|m| \text{ being even}. \quad (19)$$

Then, $\varphi_{n,m}^{R}$ is invariant to both rotation and translation.

IV. EXPERIMENTAL RESULTS

In this section, two experiments are carried out to validate the proposed QZMs and their invariance.

In order to compare our proposed QZM invariants (QZMIs) defined in (19) with the affine geometric deformations and diagonal photometric transformations invariants (GPDIs) proposed by Mindru et al [4] and the affine color moment invariants (ACMIs) introduced by Suk et al [5], the same number of invariants is used in three approaches (12 invariants for this experiment). The 12 ACMIs include I_1, I_2, I_3 and I_4 from each channel given in [5]. The 12 GPDIs are the first four invariants from each channel described in [4]. The 12 QZMIs of order up to 5 are listed in the first column of Table I. Note that the Zernike moments are calculated with the modified Kintner's method [16]. All the three sets of invariants are normalized as follows [5]

$$\hat{I}_p = sign(I_p)\left|I_p\right|^{1/s}, p = 1, 2, \dots, 12. \tag{20}$$

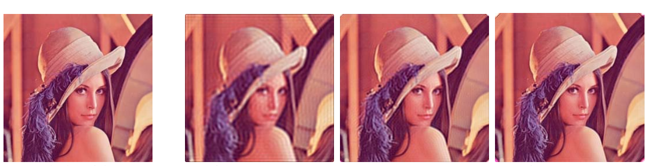
where s is the number of moments in one term. We define the feature vector $V = (\hat{I}_1, \hat{I}_2, \cdots, \hat{I}_{12})$.

A. *Test of Image Reconstruction Capability*

For this experiment, a standard Lena image (Fig. 1(a)) of size 256 × 256 pixels was used. The reconstructed images using (11) with different values of M are shown in Fig. 1. The results show that the reconstructed images are close to the original image.

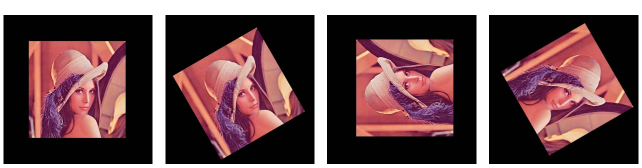
B. *Test of Invariance to Rotation and Translation*

This experiment is carried out to test the performance of the proposed QZMIs with respect to rotation and translation. The Lena image (Fig. 2(a)) was undergone different RT transformations (Fig. 2(b)-(d)). The proposed QZMIs of order n up to 5 were calculated for each image. Table I shows the invariant values and that of σ/μ, where μ denotes the mean of invariants and σ the standard deviation. The average value of σ/μ is 0.7293E-4. From this table, it can be seen that excellent results have been obtained whatever the RT transformations. GPDIs and ACMIs were also computed for these images, the average value of σ/μ is 2.7080E-3 for GPDIs and 3.6219E-4 for ACMIs.



(a)Original image (b) M = 150 (c) M = 250 (d) M = 350

Figure 1. Reconstruction Lena images with different maximum order M of moments used

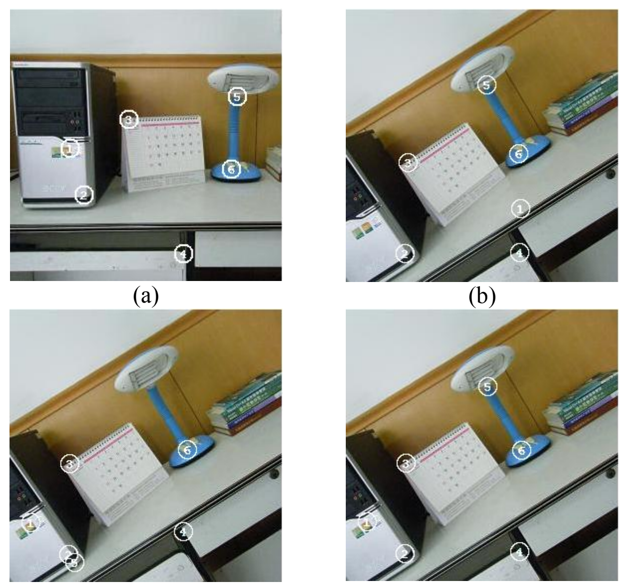


(a) Original image (b) Δx = -6, Δy =8, θ =30^0 (c) Δx = 6, Δy =8, θ =90^0 (d) Δx = 6, Δy =-8, θ=300^0

Figure 2. Lena images with different rotation and translation

TABLE I. THE QZMIS OF IMAGES SHOWED IN FIG. 2

	Fig. 2(a)	Fig. 2(b)	Fig.2(c)	Fig. 2(d)	σ/μ (E-4)
$\varphi^R_{0,0}$	62.5183	62.5183	62.5183	62.5191	0.0640
$\varphi^R_{1,1}$	1.8763	1.8762	1.8763	1.8762	0.3077
$\varphi^R_{2,0}$	134.9488	134.9469	134.9488	134.9495	0.0827
$\varphi^R_{2,2}$	1.4740	1.4739	1.4740	1.4740	0.3392
$\varphi^R_{3,1}$	5.6789	5.6786	5.6789	5.6785	0.3630
$\varphi^R_{3,3}$	0.3242	0.3244	0.3242	0.3241	3.8810
$\varphi^R_{4,0}$	101.2417	101.2364	101.2417	101.2416	0.2601
$\varphi^R_{4,2}$	5.4245	5.4240	5.4245	5.4247	0.5505
$\varphi^R_{4,4}$	3.3263	3.3264	3.3263	3.3264	0.1736
$\varphi^R_{5,1}$	10.5006	10.5005	10.5006	10.4984	1.0327
$\varphi^R_{5,3}$	1.9241	1.9245	1.9241	1.9237	1.6974
$\varphi^R_{5,5}$	0.6537	0.6537	0.6537	0.6537	0.0000



(a) (b) (c) (d)

Figure 3. Images of the indoor scene: (a) The original templates, (b) The matched templates using GPDIs, (c) The matched templates using ACMIs, (d) The matched templates using QZMIs

C. *Template Matching*

The objective of this experiment is to verify the performance of our descriptors for the real indoor scene images. The template matching test was performed. For that purpose, two pictures (Fig. 3) were taken by digital camera (FUJIFILM FinePix Z200fd) with different position by rotating the camera. Fig. 3(a) serves as the original image, while Fig. 3(b) is used as the test image. Six circular areas (numbered from 1 to 6) in Fig. 3(a) with radius r = 9 were extracted as templates.

The template was shifted across the other image (Fig. 3(b)) and in each position the 12 invariants were calculated and compared with the invariants of the original six templates. Then, these invariants were normalized through (20). The 'matching position' corresponds to the location where the Euclidean distance $d(V_f, V_g)$ reaches the minimum value, where V_f and V_g represent the feature vector of the original and test template. The matching results obtained with different moment invariants are shown in Fig. 3(b)-(d), respectively. It can be seen that the proposed QZMIs matches correctly for all six templates, but two templates (1 and 6) are mismatched for the GPDIs, and two templates (4 and 5) are matched incorrectly for the ACMIs. The computational time of shifting across the entire image is 411s for GPDIs, 423s for ACMIs and 491s QZMIs. Note that these methods were implemented in Matlab R2006b on a PC Dual Core 2.33 GHz, 2GB RAM.

V. CONCLUSION

In this paper, we have extended the conventional Zernike moments defined for gray-scale image to color image using the theory of quaternions. The invariance property with respect to image rotation and translation has been investigated. The advantages of the proposed QZMs over the existing color moments are as follows: (1) The proposed moments are based on the orthogonal Zernike polynomials, thus they have better discriminative power than the existing two methods; (2) The quaternion-type color representation, dealing with a color image as a vector field, is employed in the definition of moments.

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