

Selection Procedures in Competitive Admission

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Abstract

Two identical firms compete to attract and hire from a pool of candidates of unknown productivity. Firms simultaneously post a selection procedure which consists of a test and an acceptance probability for each test outcome. After observing the firms' selection procedures, each candidate can apply to one of them. Firms can vary both the accuracy (Lehmann, 1988) and difficulty (Hancart, 2024) of their test. The firms face two key considerations when choosing their selection procedure: the statistical properties of their test and the selection into the procedure by the candidates. I show that there is a unique symmetric equilibrium where the test is maximally accurate but minimally difficult. Intuitively, competition leads to maximal but misguided learning: firms end up having precise knowledge that is not payoff relevant. I also consider the cases where firms face capacity constraints, have the possibility of making a wage offer and the existence of asymmetric equilibria where one firm is more selective than another.

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1 Introduction

An organisation's admission process, whether it is a firm hiring workers or a university admitting students, usually consists of two important parts: recruitment and selection. Recruitment is the process of attracting the most suitable pool of candidates while selection aims at identifying the best candidates from that pool. For candidates, applying to a position is a costly process in terms of time, effort and missed opportunities. Therefore, candidates will prioritise applications where the probability of being selected is the highest. From the organisation's perspective, it means that when choosing its selection procedure, it needs to take into account two elements: the statistical properties of the selection procedure and its impact on the pool of candidates it attracts. The goal of this paper is to study how these two elements interact in a competitive market for admission and to determine the properties of the selection procedures used in equilibrium.

There are various ways in which an organisation can vary the statistical properties of its selection procedures. One way is to vary how precise its testing is. The organisation could ask the candidates for more information, conduct longer interviews or require additional tests. Another possibility is to vary the direction of learning. The tests can be difficult, making them effective at identifying top candidates, or easy, making them effective at identifying poor candidates. These two dimensions of testing, one vertical, one horizontal, can be captured by the notions of accuracy (Lehmann, 1988) and difficulty (Hancart, 2024).

In this paper, I build a model of competition where firms compete by posting selection procedures. I characterise the properties of the test used in equilibrium and show how it depends on the characteristics of the admission procedure. In the baseline model, admission procedures are simple: firms only need to decide whether to accept candidates. Firms want to accept any candidates with positive productivity while all candidates want to be accepted to any firm. I show that in any symmetric equilibrium, the test used must be maximally accurate and minimally difficult. Intuitively, competition leads firms to use as much information as possible but learn too precisely about poor candidates compared to what would be optimal absent competition. I contrast this result with two modifications of the admission procedure. In the first one, firms face capacity constraints. In the second extension, firms also make a wage offer if they accept the candidate. I show that in both these extensions, firms use more difficult tests in equilibrium. A key mechanism for these results is how the candidates' selection into the selection procedures varies across environments.

Specifically, two identical firms post simultaneously a selection procedure that consists of a test and an acceptance rule. A test is a Blackwell experiment that outputs a binary signal. The firms can adjust both the accuracy and the difficulty of their test. The acceptance rule is an accept/reject decision based on the signal realisation. There is a continuum of candidates that differ in their privately observed productivity. Each candidate decides where to apply after having observed the selection procedures. Applying is costly for the candidates so after having observed the selection procedures, they apply to only one of the two firms.¹ I study symmetric subgame perfect equilibria of this game.

I first show that competition drives firms' profits to what they would be absent any information: firms accept until they make zero profits or they accept everyone. The intuition for this result is similar to price undercutting in a Bertrand competition model. If firms make positive profits, one of them can increase their acceptance probability by an arbitrarily small amount and attract all candidates.

I then turn to the characterisation of tests used in equilibrium, using two natural orders on experiments: accuracy (Lehmann, 1988) and difficulty (Hancart, 2024). Accuracy is a weaker order than Blackwell's (1953) informativeness order for environments satisfying monotonicity assumptions. It captures a notion of precision of tests. The difficulty order was introduced in Hancart (2024). This notion captures that varying the difficulty of a test changes which types are better identified: a more difficult test is informative after a high grade, as only high types are likely to produce a high grade but it is less informative after a low grade. I assume that firms can freely vary two parameters that determine the level of accuracy and difficulty: for a fixed accuracy level, increasing the difficulty level increases the difficulty of the test and vice-versa.

Theorem 1 shows that there is a unique symmetric equilibrium where firms use a maximally accurate but minimally difficult test. In other words, in equilibrium, the firms have precise information after a low signal but the high signal contains little information compared to what would be optimal in a decision problem. Competition creates a 'double' race to the bottom: not only does it incentivise a lower acceptance threshold for a fixed test, it also incentivises firms to use a test that outputs a low signal with higher probability.

The mechanism driving Theorem 1 is the selection of the candidates into the test. I say that selection into a test is positive if whenever one candidate prefers a test over another, then all

¹We can also interpret this constraint as a requirement that candidates must put some effort in the selection process and can direct that effort to at most one of the two firms.

candidates with higher productivity also prefer that test. If selection into a test is positive, any deviating firm can construct a deviation that attracts only positive types. Combined with the zero profits condition, this deviation is necessarily profitable.

In the case of tests ordered by accuracy, selection into a more accurate test is positive. Intuitively, higher types benefit more from a more precise test. To show that in any symmetric equilibrium, firms choose a minimally difficult test, I show that the selection into an *easier* test is positive. The reason is that if firms want to attract any candidate when offering a more difficult test, they must also have a more lenient acceptance rule, otherwise no type would ever want to deviate. Low productivity candidates benefit relatively more from a more lenient acceptance rule as they are more likely to produce a low signal, and even more so in a more difficult test. Therefore, the selection into the more difficult test is negative. These two observations imply that the existence and uniqueness of a symmetric equilibrium where firms use a maximally accurate test but minimally difficult test.

In Section 4, I consider two modifications to the admission environment and show how they can change the predictions of the baseline model. I first consider the case where firms face capacity constraints. I show that capacity constraints lead firms to use more difficult tests in equilibrium. One important factor behind this result is that when firms are at capacity, they cannot benefit from undercutting their competitor. Therefore, firms only accept after a high signal in equilibrium. In this case, higher productivity candidates benefit relatively more from a more difficult test as they are relatively more likely to receive a high signal, leading to positive selection into harder tests. At the extreme, when the capacity constraint is severe, the firms use the most difficult feasible test in equilibrium.

I also use the presence of capacity constraints to explore the possibility of asymmetric equilibria. In particular, I show conditions under which a *two-tier structure* can emerge in equilibrium, i.e., an equilibrium where a selective firm only attracts high types and a safe firm attracts lower types. I study this question assuming types are binary. I show that if firms do not face any capacity constraints, there cannot be any two-tier structure equilibria. On the other hand, when there are capacity constraints, it is possible to construct a two-tier structure equilibrium for some parameter values. In that equilibrium, the selective firm chooses a test that is either more accurate or more difficult than the safe firm. Therefore, ex-ante identical firms can become ex-post vertically differentiated endogenously through their choice of selection procedure.

In the second extension, firms can make a wage offer after having accepted the candidate and

do not face capacity constraints. In this case, firms compete both by using their acceptance rule and by making wage offers. I show that the test offered in any symmetric equilibrium is maximally difficult. This is because there is positive selection into harder tests under wage competition. When firms can make wage offers, competition is fierce not on the acceptance rule but on the wages firms offer: firms do not over-accept but they over-pay. Therefore, candidates only receive an offer after a high signal and selection into a more difficult test is positive.

Wage competition and capacity constraints show how selection into the selection procedure can vary depending on the type of admissions procedures firms conduct and how it affects the qualitative properties of tests used in equilibrium. When firms are not constrained by capacity and have fixed wages, the test used in a symmetric equilibrium is ‘lemon-dropping’, i.e., the test is good at identifying low types. If firms face capacity constraints or compete using wages, the model predicts that firms will use ‘cherry-picking’ tests, i.e., tests good at identifying high types.

Finally, in Section 5, I consider two other restrictions on feasible test. First, I show an equilibrium always exists when types are binary, provided the set of feasible test is closed and convex in an appropriate sense. This result obtains because the with binary types any two tests are comparable in difficulty or accuracy. In the second constraint, I assume that the set of feasible tests derives from a cost constraint: there is a continuous posterior separable cost function (Caplin et al., 2022) that determines the cost of each test and each firm can design any test less costly than some $\kappa > 0$. I show that in any symmetric equilibrium, the cost constraint binds but at the same time, the expected posterior productivity at the high signal is zero.

1.1 Relation to the literature

This paper introduces a model of competition where firms compete by posting selection procedures. When choosing their selection procedure, the firms must consider two key channels: the statistical properties of their test and the selection into their selection procedure. I show that natural orders related to the statistical properties of tests have strong implications on selection. Under the assumption of perfect competition, the selection effect determines the nature of tests in equilibrium. I also make predictions on the nature of the tests used depending on the primitive of the model. Finally, I show that ex-ante identical firms can be ex-post

differently productive by using different selection procedures.

This paper relates to the literature studying competitive markets with private information (e.g., Rothschild and Stiglitz, 1976; Peters, 1997; Guerrieri et al., 2010; Auster and Gottardi, 2019). This literature typically assumes that the firms can flexibly design a mechanism or a contract subject to incentive-compatibility constraints. Instead, in this paper, the firms have a limited set of feasible tests but they do not need to satisfy any incentive-compatibility constraints to reveal information. This approach allows me to study how the statistical properties of the tests interact with the strategic choice of the agents. It is also worth noting that absent any test and with the payoffs assumed, the firms could never elicit any information in an incentive-compatible way. Therefore, the firms need hard information through tests to inform their decision.²

There is a small literature that studies the design of selection procedures where strategic choices from applicants play a key role. Chade et al. (2014) study a competitive markets for admission procedures in the university context. They consider a fixed testing technology and analyse a game where universities and students make their decisions simultaneously. In this paper, I endogenise the testing technology and make it an additional instrument for competition. Another important difference is the fact that universities and candidate move simultaneously. This timing changes how selection operates, as students cannot respond to changes in university policies. Adda and Ottaviani (2024) and Alonso (2018) are two papers that study how changing statistical properties of tests changes candidates' application behaviour. Adda and Ottaviani (2024) study how changing the accuracy of scientific grant evaluation, in the sense of Lehmann (1988), affects application behaviour.³ Alonso (2018) examines the choice of selection procedure in a labour market setting with horizontally differentiated workers and wage bargaining. In his paper, workers differ in their fit for the firms and one firm's selection procedure is fixed while the other can adjust it. Both Adda and Ottaviani (2024) and Alonso (2018) only consider changes to the accuracy of the test and do not consider other statistical properties like difficulty. This paper illustrates the importance of going beyond accuracy. Here, firms endogenously choose maximally accurate tests. However, if they can also adjust the difficulty of their test, they mostly learn about non-payoff relevant information.

Finally, there is a literature on information intermediaries that study models where certifier(s)

²Here, communication cannot help the firm even in the presence of tests, see Hancart (2024), Silva (2024) or Weksler and Zik (2022).

³Adda and Ottaviani (2024) also study competition between fields as they can adjust the accuracy of grant evaluation.

can disclose the quality of an agent, e.g., Lizzeri (1999); Harbaugh and Rasmusen (2018); Asseyer and Weksler (2024). In a related context to this paper, there is also a literature that models education system as intermediaries (Ostrovsky and Schwarz, 2010; Boleslavsky and Cotton, 2015; Bizzotto and Vigier, 2024). In the case where the agent is privately informed (e.g., Lizzeri, 1999; Harbaugh and Rasmusen, 2018), selection into the certifier also play a key role in these papers. These information design problems are however different as the intermediaries are not trying to select candidates but to collect a fee or reveal something about them.

2 Model

There are two firms and a continuum of agents with mass normalised to one. Each agent has a private type which corresponds to his value to the firms $\theta \in \Theta = [\underline{\theta}, \bar{\theta}]$. Types are distributed according to a cdf F admitting a strictly positive density f .

The firms can decide whether to admit the agents, $a \in \{0, 1\}$, and an *accuracy* and *difficulty* level, $(\sigma, d) \in \Sigma \times D \subseteq \mathbb{R}^2$, that determine a binary test $\tau(\sigma, d) \in \Pi := \{\pi : \Theta \rightarrow \Delta\{l, h\}\}$. A generic test in Π is denoted by t . The conditional probabilities of test $t \in \Pi$ are denoted by $\pi_t(\cdot|\theta)$ and I interpret signal h as the high signal. To simplify notation, I denote by $\pi_t(\theta)$ the probability that type θ sends signal h . Denote by $\tau(\Sigma \times D)$ the set of feasible tests, i.e., the set of tests obtained by choosing some accuracy and difficulty level. The concepts of accuracy and difficulty are defined and explained in Section 2.1.

A selection procedure is a test $t \in \tau(\Sigma \times D)$, or equivalently an accuracy and difficulty level, $(\sigma, d) \in \Sigma \times D$, and a decision rule, $\alpha : \{h, l\} \rightarrow [0, 1]$, a mapping from the signal to a probability of accepting: $s = (t, \alpha)$. Let S be the set of selection procedure.

The firms post simultaneously a selection procedure $s \in S$. After observing the admission process, agents decides whether to apply to firm 1 or 2. Denote by $\phi : \Theta \times S \times S \rightarrow [0, 1]$ the probability agent θ chooses firm 1 given the selection procedures.

The agents have payoffs $u(a) = a$, i.e., they want to be accepted. Firm 1's payoffs are

$$v(s, s', \phi) = \int_{\Theta} \phi(s, s', \theta) \theta \left(\pi_t(\theta) \alpha(h) + (1 - \pi_t(\theta)) \alpha(l) \right) dF.$$

Firm 2's payoffs are defined analogously. The firm cares both about how many agents it attracts and their quality. It also assumes there is no capacity constraint for the firm. This captures the idea that the supply of agents is smaller than the total demand and therefore the two firms must compete to attract them. I introduce capacity constraints in Section 4.1.

Call $T_i = \{t \in \tau(\Sigma \times D) : \int_{\Theta} \theta (1 - \pi_t(\theta)) dF \leq 0 \leq \int_{\Theta} \theta \pi_t(\theta) dF\}$ the set of *minimally informative tests*. These are all the test that generate payoff relevant information for the firms.

I consider subgame-perfect equilibria of this game where agents break ties uniformly and firms use pure strategies.

2.1 Feasible tests

In this subsection, I describe in detail the set of feasible tests, $\tau(\Sigma \times D)$.

I maintain the following assumptions throughout.

Assumption 1. *For each test $t \in \tau(\Sigma \times D)$, $\pi_t(h|\theta)$ is increasing and $\pi_t(x|\theta) \in (0, 1)$ for $x = l, h$ and $\theta \in (\underline{\theta}, \bar{\theta})$.*

Assumption 1 guarantees that a high signal is good news about the type no set of types with positive measure can be identified or excluded from observing a signal.

I make use of two orders on tests, accuracy and difficulty, that capture vertical and horizontal properties of tests.

Definition 1 (Accuracy and Difficulty). *A test t is more accurate than a test t' , $t \succeq_a t'$, if for all $\theta, \theta' \in (\underline{\theta}, \bar{\theta})$ with $\theta' < \theta$,*

$$\frac{\pi_t(h|\theta)}{\pi_t(h|\theta')} \geq \frac{\pi_{t'}(x|\theta)}{\pi_{t'}(x|\theta')} \geq \frac{\pi_t(l|\theta)}{\pi_t(l|\theta')}, \text{ for } x = l, h.$$

A test t is more difficult than t' , $t \succeq_d t'$, if for all $\theta, \theta' \in (\underline{\theta}, \bar{\theta})$ with $\theta' < \theta$,

$$\frac{\pi_t(x|\theta)}{\pi_t(x|\theta')} \geq \frac{\pi_{t'}(x|\theta)}{\pi_{t'}(x|\theta')}, \text{ for } x = l, h.$$

Examples are provided in Section 2.1.1.

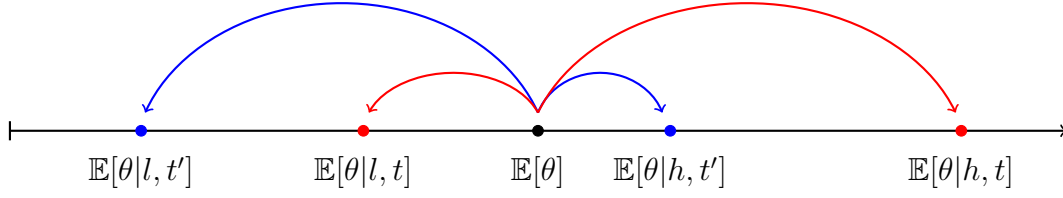


Figure 1: Illustration of posterior means for two tests, $t \succeq_d t'$. The good news signal h shifts the posterior towards a higher posterior mean in the more difficult test. The bad news signal l shifts the posterior towards lower posterior mean in the easier test.

Lehmann's (1988) accuracy order captures a notion of informativeness of a test. It allows for the comparison of more experiments than Blackwell's (1953) order.⁴ Accuracy is a concept defined for tests satisfying the monotone likelihood ratio property with arbitrary number of signals. However, given our focus on tests with binary signals, I give a definition that is equivalent to Lehmann's (1988) for binary signals. The equivalence is shown in Section A.1.

The difficulty order captures the following intuitive property of difficulty: when a test is more difficult than another, a high grade shifts beliefs more towards high type in the more difficult test. This is because it is harder to receive a high grade in a difficult test. On the other hand, after a low grade, beliefs are more pessimistic in the easier test. Indeed only bad candidates are expected to fail an easy test whereas it is expected to fail a difficult test. This intuition is formalised in the following proposition and illustrated in Figure 1.

Let $\mu(\cdot|t, x)$ denote the posterior beliefs in test t after signal x and \succeq_{FOSD} the first-order stochastic dominance order.

Proposition 1 (Hancart (2024)). *A test t is more difficult than t' if and only if $\mu(\cdot|t, x) \succeq_{FOSD} \mu(\cdot|t', x)$ for $x = h, l$ for any prior (including non full-support).*

We can now formalise the notion of accuracy and difficulty level as follows:

$$\begin{aligned} \text{for all } \sigma \in \Sigma, \tau(\sigma, d) \succeq_d \tau(\sigma, d') &\Leftrightarrow d \geq d', \\ \text{for all } d \in D, \tau(\sigma, d) \succeq_a \tau(\sigma', d) &\Leftrightarrow \sigma \geq \sigma'. \end{aligned}$$

⁴Lehmann (1988) showed that a decision maker with monotone decision preferences (Karlin and Rubin, 1956) always prefers a more accurate test. Quah and Strulovici (2009) have extended this result to preferences that form an interval dominance order family, a generalisation of both single-crossing preferences (Milgrom and Shannon, 1994) and monotone decision preferences. The term accuracy comes from Persico (2000).

All else equal, a higher σ indicates a higher accuracy and a higher d a higher difficulty.

Abusing notation, I write $\pi_{\sigma,d}(\theta)$ for the probability of sending a high signal in test $\tau(\sigma, d)$. I maintain the following technical assumption throughout.

Assumption 2. *The sets Σ and D are closed intervals.*

The probability $\pi_{\sigma,d}(\theta)$ is continuous in (σ, d) for all $\theta \in \Theta$.

If two tests $t, t' \in \tau(\Sigma \times D)$ are comparable in difficulty, they are not in accuracy.

The first two assumptions are technical assumptions to guarantee equilibrium existence. The last one simplifies the exposition.

2.1.1 Additional properties and examples

We first record the following observation.

Lemma 1. *Suppose test t is more difficult than t' . Then for all $\theta \in (\underline{\theta}, \bar{\theta})$, $\pi_t(\theta) \leq \pi_{t'}(\theta)$.*

All proofs are in Section A.

Lemma 1 shows that the difficulty order leads to the natural property that high signals are less likely in a more difficult test.

When two tests are comparable in terms of difficulty, they are not in terms of accuracy, except in knife-edge cases where the likelihood ratios are constant. When the state space is binary, all tests are comparable in difficulty or accuracy. In Section B, I also show that if we evaluate the cost of a test using continuous and posterior-separable cost function (Caplin et al., 2022), for any given test, we can find another test comparable in the difficulty order with equal cost.

I provide examples of tests comparable in terms of accuracy and difficulty below.

Examples. For simplicity, I normalise Θ to $\Theta = [0, 1]$. In each example, accuracy is increasing in σ and difficulty in d .

1. Let $\pi_{d,\sigma}(\theta) = \Pr[y \geq d|\theta]$ where $y = \sigma\theta + \epsilon$ and $\epsilon \sim N(0, 1)$.
2. Let $\pi_{d,\sigma}(\theta) = \sigma\pi(\theta) + (1 - \sigma)(1 - d)$ where $\pi : \Theta \rightarrow (0, 1)$ is increasing and $\sigma, d \in [0, 1]$.

3. Let $\pi_{d,\sigma}(\theta) = \left(\frac{1-\sigma}{2} + \sigma\theta\right)^d$ where $d > 0$ and $\sigma \in [0, 1]$.

In the first example, there is an underlying continuous signal y that is more sensitive to the type the higher σ is. The test outputs a high signal if the continuous signal y is above a threshold d . The higher the threshold, the more difficult the test. The second example can be interpreted as follows. With probability σ , the test is informative and the probability of a high signal depends on θ . With the complement probability, all types receive a high signal with probability $1 - d$. In the last example, the test is again more sensitive to the type and difficulty distorts downward the probability of receiving a high signal.

Intuitively, accuracy increases when the probability of receiving a high signal is more sensitive to the type. Difficulty increases when obtaining the higher signal is harder for all types, but more so for lower types.

3 Analysis

I first show that competition leads firms to ‘over-accept’ candidates in the sense that they reward the low signal even though it has negative posterior expected productivity. In any symmetric equilibrium, the payoffs of the firms are no different than if they could not observe any signals.

Lemma 2. *In any symmetric equilibrium $s = (t, \alpha)$, firms’ profits are $\max\{0, \frac{1}{2} \mathbb{E}[\theta]\}$.*

- If $\mathbb{E}[\theta] \geq 0$, $\alpha(h) = \alpha(l) = 1$.
- If $\mathbb{E}[\theta] < 0$ and $\int_{\Theta} \theta \pi_t(\theta) dF > 0$, $\alpha(h) = 1$, $\alpha(l) > 0$.

The intuition for Lemma 2 is the familiar Bertrand undercutting logic. If firms make positive profits, they can relax their acceptance rule and attract all candidates. If the increased probability of acceptance is small enough, the firms profits are larger than when sharing the market with the other firm. Therefore in equilibrium firms accept candidates until they make zero profits or they accept any candidates applying. This means that the firms make exactly the same profits as if they could not collect any information about candidates.

If $\mathbb{E}[\theta] \geq 0$, both firms accept all types and make weakly positive profits. Therefore, any test with $\alpha(h) = \alpha(l) = 1$ is an equilibrium. In the rest of the paper, I therefore focus on the case with $\mathbb{E}[\theta] < 0$.

I now turn to the characterisation of the tests used in equilibrium. I say an equilibrium is unique if in all equilibria payoffs and tests used are the same.

Theorem 1. *Suppose $\mathbb{E}[\theta] < 0$ and there is $t \in \tau(\Sigma \times D)$ such that $\int_{\Theta} \theta \pi_t(\theta) dF > 0$. Then there is an essentially unique symmetric equilibrium $s = (\tau(\sigma^*, d^*), \alpha)$. The accuracy and difficulty levels satisfy*

$$\sigma^* = \max\{\sigma \in \Sigma\},$$

$$d^* = \min\{d \in D : \int_{\Theta} \theta \pi_{\sigma^*, d}(\theta) dF \geq 0\}.$$

Theorem 1 shows that in the unique symmetric equilibrium, the test used is maximally informative but minimally difficult, as long as it reveals some payoff-relevant information. It is illustrated in Figure 2. There can be multiple equilibria if $\int_{\Theta} \theta \pi_{\sigma^*, d}(\theta) dF = 0$. In this case, the firms never accept after a low signal and accept after a high signal with any probability in $[0, 1]$.

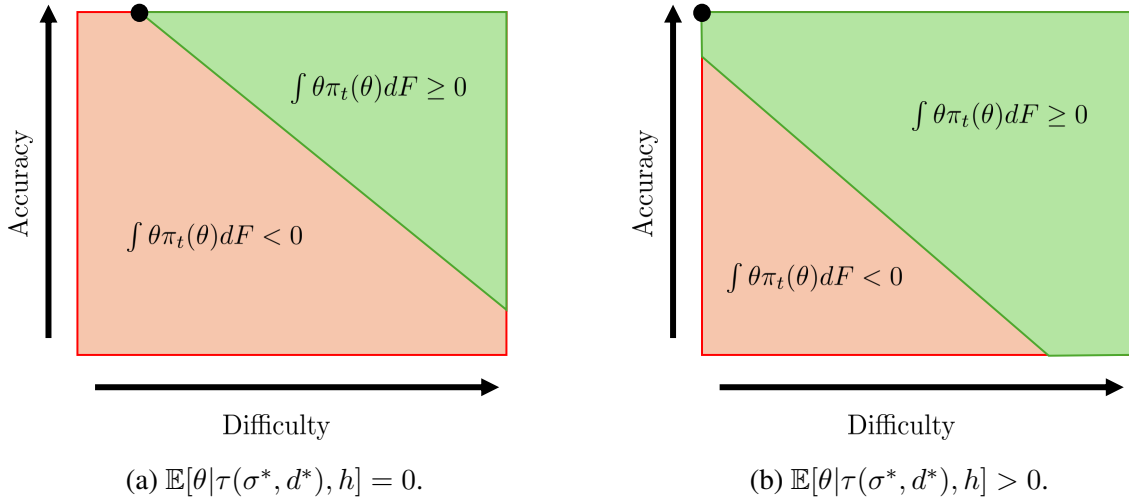


Figure 2: Each point in the rectangle represents a test. A test is minimally informative if it is accurate or difficult enough. The black dot indicates the test used in symmetric equilibrium.

If the set of feasible tests is sufficiently rich, at the equilibrium test, $\int_{\Theta} \theta \pi_{\sigma^*, d^*}(\theta) dF = 0$. Therefore, in equilibrium, we have $\alpha(h) \geq 0$ and $\alpha(l) = 0$. Under that strategy, the firms best-reply to their signals in equilibrium. In that case, the equilibrium we have found is also an equilibrium of a game with an alternative timing where firms do not commit to an acceptance rule but decide whether to accept only after having seen the test result.

Theorem 1 follows from the selection into tests. I first show that there is positive selection into a more accurate test. Intuitively, higher productivity agents benefit relatively more from a more accurate test no matter what strategy is employed. Because any candidate equilibrium has zero profits, any deviation that attracts only positive types is profitable.

I also show that there is positive selection into an *easier* test. Consider the case where both firms use the easiest minimally informative test in equilibrium and consider a deviation to a harder test. To make this deviation successful, firms must make their acceptance rule more lenient as no candidate would want to choose a more difficult with harsher acceptance rule. But when raising the acceptance probability at the low signal in the more difficult test, the agents most likely to benefit from that selection procedure are low productivity agents. Indeed, they are the agents most likely to generate low signals. Therefore, the selection into the more difficult *and* more lenient acceptance rule is negative.

When the set of feasible test is rich enough, the equilibrium test has $\mathbb{E}[\theta|\tau(\sigma^*, d^*), h] = 0$, i.e., the posterior expectation at the high signal is zero. In equilibrium, the firms set $\alpha(l) = 0$, i.e., a low signal is no longer rewarded. If a firm deviates to an easier, it would need to use a harsher decision rule: if not, all types would choose the deviating firm but in the easier test, the expected payoffs are negative. Therefore, the acceptance rule from a profitable deviation must only reward the high signal. But in this case, the selection into an easier test when only the high signal is rewarded is negative as high productivity candidate are more likely to generate a high signal and are therefore more inclined to choose the harder test.

4 Extensions

4.1 Capacity constraint

As in the analysis of markets with Bertrand competition, capacity constraints can radically change the predictions of our model. The reason is that capacity constraints shift the focus of firms from both the quality and the quantity of applicants to the quality only. This gives incentives to use more difficult tests in equilibrium. To focus the analysis, I only consider the case where the firms can adjust the difficulty of their tests but not the accuracy, i.e., $\Sigma = \{\sigma\}$.

I model capacity constraints as follows. Each firm has a capacity $k > 0$, i.e., in equilibrium we must have $\int_{\Theta} \phi(s, s', \theta) (\alpha(l)(1 - \pi_t(\theta)) + \alpha(h)\pi_t(\theta)) dF \leq k$.

The timing of the game is as follows:

1. Firms post selection procedures simultaneously
2. Candidates choose where to apply
3. Candidates applying to firm i form a queue whose order is random. Firm i treats applications sequentially until it exhausts the pool of applicants or hits its capacity constraint.

The solution concept is still symmetric subgame equilibrium where firms use pure strategies and agents break ties symmetrically.

Given the strategy of the agents ϕ , let $p_1 = \min\{1, \frac{k}{\int_{\Theta} \phi(s, s', \theta) (\alpha(l)(1 - \pi_t(\theta)) + \alpha(h)\pi_t(\theta)) dF}\}$ be the probability of having a candidate's application considered in firm 1. The payoffs of firm 1 in equilibrium are

$$p_1 \cdot \int_{\Theta} \phi(s, s', \theta) \theta (\alpha(l)(1 - \pi_t(\theta)) + \alpha(h)\pi_t(\theta)) dF.$$

The payoffs of firm 2 are defined analogously. I illustrate the effect of capacity constraints in the extreme case where firms do not even have the capacity to accept all types that get a high signal in the most difficult test. Recall that in this section the firms cannot adjust the accuracy level and therefore I identify a test with its difficulty level.

Proposition 2. *Let $\bar{d} = \max D$ and assume that $k < \frac{1}{2} \int_{\Theta} \pi_{\bar{d}}(\theta) dF$.*

There is a symmetric equilibrium $s = (\tau(\bar{d}), \alpha)$, i.e., both firms offer the most difficult test.

We get the inverse prediction to the case with no capacity constraints. The reason is that capacity constraints limit the scope for undercutting the competitor: once the firms are at capacity, there is no benefit to increasing the acceptance probability to attract all the candidates. Given the assumption on the capacity constraint, there is no incentive to accept after a low signal. When the low signal is not rewarded, the selection into the easier test is negative. Moreover, only the expected productivity of the accepted candidates matter for the payoffs as the firms are at capacity. From Proposition 1, a more difficult test always has a higher posterior expected value. Therefore a deviation to an easier test results both in negative selection and lower expected value, even absent any selection effect.

Proposition 2 gives us the following (informal) comparative statics result:

Observation 1. *Firms facing a capacity constraint use more difficult tests in equilibrium than firms not facing a capacity constraint.*

This result predicts that whether the firms face capacity constraints or not will affect the qualitative nature of the tests they use. Loosely speaking, a firm facing a tight capacity constraint will use a difficult selection procedure, a ‘cherry-picking’ type of test whereas a firm without capacity constraint will use easier tests, a ‘lemon-dropping’ type of test.

4.2 Asymmetric equilibria

A natural question in this context is whether different selection procedures can generate differentiation amongst firms despite being ex-ante identical. In particular, I will look at whether a two-tier structure can emerge in equilibrium where one firm attracts all candidates above a threshold and the other firm attracts candidates below the threshold. To answer this question, I specialise the setting to a binary type model. I show that a two-tier structure never emerges when there are no capacity constraints but that it can emerge when there are.

A *two-tier equilibrium* is an equilibrium where only types above a threshold apply to a firm. I call the firm where only types above the threshold apply the *selective firm* and the firm where types below the threshold apply the *safe firm*.

The set $\tau(\Sigma \times D)$ induce binary types if there is θ^* such that for all $t \in \tau(\Sigma \times D)$, $\pi_t(\theta)$ is constant above and below θ^* .

Whenever $\tau(\Sigma \times D)$ induces binary types, then abusing notation, I write $\underline{\theta} = \mathbb{E}[\theta | \theta \leq \theta^*]$ and $\bar{\theta} = \mathbb{E}[\theta | \theta > \theta^*]$. I denote by μ the mass of $\theta > \theta^*$.

As the equilibrium construction in Proposition 3 below will use mixed strategies, I no longer require that agents break ties uniformly. I do require however that any types generating the same distribution over signals use the same strategy. If there are capacity constraints, the timing and rationing is as defined in Section 4.1.

Proposition 3. *Suppose $\tau(\Sigma \times D)$ induces binary types. Let $\bar{t} \in \arg \max_{t \in \tau(\Sigma \times D)} \frac{\pi_t(\bar{\theta})}{\pi_t(\underline{\theta})}$.*

If there are no capacity constraints, then there are no two-tier equilibria.

If there are capacity constraints, there is a two-tier equilibrium where the selective firm chooses \bar{t} and the safe firm chooses some other test $t \in \tau(\Sigma \times D)$ if

$$\begin{aligned} \underline{\theta} &\geq 0, \\ (1 - \mu)\pi_{\bar{t}}(\underline{\theta}) &\geq \mu\pi_{\bar{t}}(\bar{\theta}), \\ \frac{\mu\pi_{\bar{t}}(\bar{\theta}) + (1 - \mu)\pi_{\bar{t}}(\underline{\theta})}{2} &\geq k, \\ \text{and } \frac{\pi_t(\underline{\theta})}{2} &\geq k. \end{aligned}$$

In equilibrium, $\bar{\theta}$ chooses the selective firm and $\underline{\theta}$ chooses the safe firm with probability $\frac{\mu\pi_{\bar{t}}(\bar{\theta}) + (1 - \mu)\pi_{\bar{t}}(\underline{\theta})}{2(1 - \mu)\pi_{\bar{t}}(\underline{\theta})}$.

The reason two-tier equilibria cannot exist without capacity constraints is that as long as the safe firm makes positive profits, the selective firm has an incentive to lower its standards to attract more candidates. Here decreasing standards corresponds to offering a selection procedure with a lower ratio of likelihood of acceptance between high and low types. On the other hand, if the selective firm is at capacity, the benefits from decreasing standards could be limited.

With capacity constraints, the selective firm uses the test with the highest likelihood ratio at the high signal. That means that the test is either more accurate or more difficult than the other feasible tests. In equilibrium, the selective firm only accepts after a high signal. Under this strategy, the selection into the most difficult or accurate test is positive. Moreover, because in equilibrium firms are at capacity, they cannot improve payoffs by simply lowering their standards and attracting more types. For example, when the selective firm decreases its standards, it increases the share of lower quality students applying to it, thereby decreasing its payoffs.

The equilibrium I construct has high types choosing the competitive firm and low types mixing between the competitive and the safe firm. The sufficient conditions in Proposition 3 reflect the equilibrium conditions to maintain that equilibrium. The first one, $\underline{\theta} \geq 0$ is necessary to make sure that the safe firm makes profits in equilibrium. The second condition ensures that the mixed strategy is feasible. The last two conditions guarantee that the capacity constraints of both firms are binding.

4.3 Wage competition

In this subsection, I consider the consequences of wage setting for the choice of equilibrium tests. Formally, a firm can offer a positive transfer to the agent based on the signal it received: $m : \{h, l\} \rightarrow \mathbb{R}_+$. An admission procedure is a test, a decision rule and a transfer rule, $s = (t, \alpha, m)$. An agent's payoff is the transfer $a \cdot m$. Firm 1's payoffs are

$$v(s, s', \phi) = \int_{\Theta} \phi(s, s', \theta) \left(\pi_t(\theta) \alpha(h) (\theta - m(h)) + (1 - \pi_t(\theta)) \alpha(l) (\theta - m(l)) \right) dF.$$

As in Section 4.1, I focus the analysis on the case where the firms can only adjust their difficulty level, i.e., $\Sigma = \{\sigma\}$. Therefore, I identify a test with its difficulty level.

Proposition 4. *Let $\bar{d} = \max D$ and suppose there is d such that $\int_{\Theta} \theta \pi_d(\theta) dF > 0$. In any symmetric equilibrium $s = (\tau(\bar{d}), \alpha)$ and $\alpha(l) = 0$.*

The main consequence of wage setting is that competition moves from the acceptance rule to the wage offered. Instead of over-accepting, i.e., set $\alpha(l) > 0$, firms compete on the wage offered, conditional on receiving a high signal. This changes the selection effect of offering a more difficult test. Now, high productivity agents benefit relatively more from a more difficult test as they are more likely to get a high signal. Because of this positive selection into a more difficult test, firms can always deviate if there is a more difficult test available.

Here we cannot establish generally that there is an equilibrium where the most difficult test is chosen. A deviating firm to an easier test faces negative selection but can compensate by offering lower wages. Therefore whether a deviation is profitable depends on the specification of the feasible tests and the prior. For example, one can show that if $\theta \sim U[0, 1]$ and $\tau(\Sigma \times D) = \{t : \pi_t(\theta) = \sigma\theta + (1 - \sigma)(1 - d), d \in [0, 1]\}$, then an equilibrium exists.

This second extension gives us a second informal comparative statics result:

Observation 2. *If firms compete using wages, they use more difficult tests in equilibrium than when they can only compete using admission probability. Moreover, the hiring probability is lower for almost all types.*

As in the case of capacity constraints, firms offer a more difficult test in equilibrium when they can compete using wage offers. Because in equilibrium they only accept after a high

signal, it also implies that the probability of accepting any given type decreases.⁵

5 Other structures on the feasible tests

In this section, I show how the difficulty and accuracy order can also be useful for other specification of the set of feasible tests. First, I show that when types are binary, there is always an equilibrium, provided the set of feasible tests is suitably closed and convex. In the second case, I show how to characterise equilibria when the restriction on the feasible tests comes from a cost function in the spirit of rational inattention (Sims, 2003).

5.1 Binary types

Denote by T the set of feasible tests, i.e., $T \subseteq \Pi$ and all $t \in T$ respect Assumption 1. In the case where T induces binary types,⁶ each test is comparable in terms of either accuracy or difficulty. Therefore, by using a similar reasoning as in Theorem 1, an equilibrium exists provided there exists a most accurate or easiest test. One sufficient condition is the following. When T induces binary types, each test can be described as a pair of likelihood ratios: $(\frac{\pi(\bar{\theta})}{\pi(\underline{\theta})}, \frac{1-\pi(\bar{\theta})}{1-\pi(\underline{\theta})}) \in [1, \infty) \times (0, 1]$ with $\pi(\bar{\theta}) \geq \pi(\underline{\theta})$. I say that T is closed and convex if the set of tests interpreted as a subset $[1, \infty) \times (0, 1]$ is closed and convex. Let T_i be the set of minimally informative tests, i.e., $T_i = \{t \in T : \mathbb{E}[\theta|t, l] \leq 0 \leq \mathbb{E}[\theta|t, h]\}$.

Corollary 1. *Suppose $\mathbb{E}[\theta] < 0$ and T induces binary types and is closed and convex. Then there is a unique symmetric equilibrium. The test used in the symmetric equilibrium is*

$$\arg \max_{t \in T} \left\{ \frac{\pi_t(\bar{\theta})}{\pi_t(\underline{\theta})} : t \in \arg \min_{t' \in T_i} \frac{1 - \pi_{t'}(\bar{\theta})}{1 - \pi_{t'}(\underline{\theta})} \right\}.$$

The test used in equilibrium is the one with smallest likelihood ratio at the low signal amongst the minimally informative tests: $\arg \min_{t' \in T_i} \frac{1 - \pi_{t'}(\bar{\theta})}{1 - \pi_{t'}(\underline{\theta})}$. If there are multiple, then we select the one that is maximally accurate amongst them, i.e., with the largest likelihood ratio at the high

⁵Under capacity constraints, the probability of being accepted decreased as well but that was exogenously imposed by the capacity constraint.

⁶Recall that, as defined in Section 4.2, the set T induce binary types if there is θ^* such that for all $t \in T$, $\pi_t(\theta)$ is constant above and below θ^* .

signal. The test selected will be either more informative or easier than any other test amongst the minimally informative ones.

5.2 Cost constraint

For a given test t , let $\bar{\pi}_t = \int_{\Theta} \pi_t(\theta) dF$. We define the cost as follows let $f_{th}(\theta) = f(\theta) \frac{\pi_t(\theta)}{\bar{\pi}_t}$ and $f_{tl}(\theta) = f(\theta) \frac{1-\pi_t(\theta)}{1-\bar{\pi}_t}$. I assume that the cost associated with test t is posterior separable (Caplin et al., 2022):

$$C(t) = \bar{\pi}_t c(f_{th}) + (1 - \bar{\pi}_t) c(f_{tl}),$$

where $c : \Delta\Theta \rightarrow \mathbb{R}$ is a strictly convex and continuous function.⁷ This class of cost function includes many commonly used cost functions including mutual information cost or log-likelihood ratio cost (Sims, 2003; Pomatto et al., 2023). The representation in terms of posterior beliefs will also be convenient in the proofs. I make the further assumption that the test cannot rule out any state: if $\pi_t(\theta) \in \{0, 1\}$ for some θ then $\pi_t(\theta') = \pi_t(\theta)$ for all θ' . Or put differently, if $\pi_t(\theta) \in \{0, 1\}$ for some θ and the test is informative then the cost is infinite.

For any $\kappa > 0$, let $T = \{\pi : \Delta\Theta \rightarrow \Delta\{h, l\} : \pi \text{ satisfies Assumption 1 and } C(\pi) \leq \kappa\}$. Posterior separable cost functions depend implicitly on the prior belief over types. Here I define the costs at the prior belief f and I will keep that fixed, even though the firm might have additional information through selection prior to administrating the test. The interpretation of this constraint is that the firms have to pay a cost to design the test, captured by C . But once the test is designed, there are no further costs.

Proposition 5. *Suppose $\mathbb{E}[\theta] < 0$. Suppose there is $t \in T$ with $\int_{\Theta} \theta \pi_t(\theta) dF > 0$. In any symmetric equilibrium, $s = (t, \alpha)$, we have*

$$\int_{\Theta} \theta \pi_t(\theta) dF = 0.$$

and $C(t) = \kappa$.

The proof works by showing that for any test with $\int_{\Theta} \theta \pi_t(\theta) dF > 0$, it is possible to find an easier test that still has positive posterior expected productivity at the high signal and is less costly. Therefore by the positive selection into easier tests when $\alpha(l) > 0$, t cannot be

⁷I endow the set $\Delta\Theta$ with the weak* topology. In this topology, a sequence $(F_n)_n$ converges to F if for any continuous $\phi : \Theta \rightarrow \mathbb{R}$, $\int \phi(\theta) dF_n(\theta) \rightarrow \int \phi(\theta) dF$.

part of a symmetric equilibrium. The budget constraint is binding because C is increasing in the Blackwell order. Therefore, if the budget constraint is not binding, there is a test that is more informative (and thus more accurate) and feasible. Not also that Proposition 5 does not establish that an equilibrium exists.

Given the flexibility in the design of the test with these constraints, the proof sketched above is not the only one. However, it does illustrate how the results derived above can be useful in a more flexible environment. Moreover, it shows that the results do not rely on the extreme flexibility of the set of feasible tests but rather on the monotonicity in the Blackwell order and the possibility of finding isocost or cheaper tests comparable in difficulty.

6 Discussion

Cost of applying In this model, the constraint on the application strategy of the agents is that they can apply to only one firm. This constraint can be interpreted in different ways and is natural in a number of markets. If applying requires effort to tailor the application package or prepare for firm-specific tests, candidates need to prioritise effort towards a subset of firms.⁸ There can also be institutional constraints. For example, in the UK, applicants for undergraduate programmes can apply to up to five different programmes (UCAS, 2025). Finally, the constraint can come from the nature of the test. If the test is a task the agent needs to perform during a probation period, the test can be performed at at most one firm.

The results would be qualitatively the same if there was a fixed cost of applying to each firm and the cost is high enough so that no agent applies to two firms. The key assumption is that agents cannot apply to all firms in the market.

Binary tests Binary signals are a simplification on the testing technology. An important implication of binary signals is that all signals have an unambiguous interpretation as either a high or a low signal. This would not be the case with more signals.

This is especially important when comparing selection procedure where the tests differ in their difficulty. In this case, the selection into a more difficult test varies depending on

⁸For example, PrepLounge, an interview preparation platform for jobs in consulting and the financial sector, argues that “with your limited time, you might be better off focusing on just a handful to maximize the quality of your applications.” (PrepLounge, 2025).

whether the low signal is rewarded or not. If only the high signal is rewarded, then higher types tend to prefer a more difficult test as they are more likely to generate a high signal. On the other hand, when the low signal is also rewarded, low types tend to prefer a more difficult test as they are more likely to generate a low signal. The clean distinction between high and low signals allows for a transparent interpretation of the selection effects into more difficult tests. To make progress beyond binary signals, one would need to develop a theory of difficult tests beyond binary signals. For accuracy, the analysis extends to arbitrarily many signals.

7 Conclusion

I have introduced a new model of competition where firms compete by posting selection procedures. The key channel I explored is how statistical properties of the tests imply different strategic choices from the tested agents. In particular, I showed that two natural orders on tests, accuracy and difficulty, create single-crossing utility differences for the agent. This led to positive or negative selection into a test that in turn determined the equilibrium.

The model makes some predictions the qualitative nature of the tests used in equilibrium depending on the primitives of the game. In the absence of capacity constraints, the firms use maximally accurate but the easiest test that is minimally informative. We can interpret this as maximal but misguided learning. In equilibrium, firms are very confident the type is of low quality after a low signal but their posterior expectation is barely high enough to make them accept the agent. On the other hand, when firms face capacity constraints or can compete using wages, they use more difficult tests in equilibrium.

I see this model as a first step towards studying the effect of competition on the choice of tests. There are many natural extensions one would want to consider such as differentiated firms, both horizontally and vertically. Another interesting extension would be introducing peer effects which is particularly relevant in a university admission context.

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A Proofs

A.1 Preliminary results

Lehmann (1988) defined his notion of accuracy as follows. Take a compact signal space $\tilde{X} \subset \mathbb{R}$ and let $F_t(\cdot|\theta)$ the conditional cdf of test t . Lehmann (1988) shows how information structures with discrete signal spaces can be rewritten as information structures with continuous signal spaces.

Definition 2 (Lehmann (1988)). *A test t is more accurate than a test t' if*

$$x^*(\theta, x) \equiv F_t(x^*|\theta) = F_{t'}(x|\theta)$$

is weakly increasing in θ for each $x \in \tilde{X}$.

The following result shows the equivalence between Lehmann's (1988) definition and the one given in Section 2.1.

Proposition 6. *Suppose the signal space is binary. A test t is more accurate than t' if and only if for all $\theta > \theta'$,*

$$\frac{\pi_t(h|\theta)}{\pi_t(h|\theta')} \geq \frac{\pi_{t'}(h|\theta)}{\pi_{t'}(h|\theta')} \quad \text{and} \quad \frac{\pi_{t'}(l|\theta)}{\pi_{t'}(l|\theta')} \geq \frac{\pi_t(l|\theta)}{\pi_t(l|\theta')}.$$

Proof. Adda and Ottaviani (2024) show that t more accurate than t' is equivalent to having for all $\theta > \theta'$,

$$F_t(F_t^{-1}(q|\theta')|\theta) \leq F_{t'}(F_{t'}^{-1}(q|\theta')|\theta),$$

for all $q \in [0, 1]$.

Let $\tilde{X} = [0, 1]$. We can rewrite an information structure with binary signals where the probability of a high signal is $\pi_t(\theta)$ as

$$F_t(x|\theta) = \begin{cases} 2(1 - \pi_t(\theta))x & \text{if } x < 1/2, \\ 1 + 2\pi_t(\theta)(x - 1) & \text{if } x \geq 1/2. \end{cases}$$

The inverse is

$$F_t^{-1}(q|\theta) = \begin{cases} \frac{q}{2(1-\pi_t(\theta))} & \text{if } q < 1 - \pi_t(\theta), \\ \frac{q}{2\pi_t(\theta)} + \frac{2\pi_t(\theta)-1}{2\pi_t(\theta)} & \text{if } q \geq 1 - \pi_t(\theta). \end{cases}$$

For any $\theta > \theta'$, we have

$$F_t(F_t^{-1}(q|\theta')|\theta) = \begin{cases} q \frac{1-\pi_t(\theta)}{1-\pi_t(\theta')} & \text{if } q < 1 - \pi_t(\theta'), \\ q \frac{\pi_t(\theta)}{\pi_t(\theta')} + 1 - \frac{\pi_t(\theta)}{\pi_t(\theta')} & \text{if } q \geq 1 - \pi_t(\theta'). \end{cases}$$

Given that $F_t(F_t^{-1}(q|\theta')|\theta) = F_{t'}(F_{t'}^{-1}(q|\theta')|\theta)$ for $q = 0, 1$, to have $F_t(F_t^{-1}(q|\theta')|\theta) \leq F_{t'}(F_{t'}^{-1}(q|\theta')|\theta)$ for all q , we must have

$$\frac{\pi_t(\theta)}{\pi_t(\theta')} \geq \frac{\pi_{t'}(\theta)}{\pi_{t'}(\theta')} \text{ and } \frac{1 - \pi_{t'}(\theta)}{1 - \pi_{t'}(\theta')} \geq \frac{1 - \pi_t(\theta)}{1 - \pi_t(\theta')}.$$

□

A.2 Proof of Lemma 1

Proof. Suppose there is $\theta' \in (\underline{\theta}, \bar{\theta})$ such that $\pi_{t'}(\theta') < \pi_t(\theta')$. If $t \succeq_d t'$, then for all $\theta > \theta'$,

$$\pi_t(\theta)\pi_{t'}(\theta') \geq \pi_t(\theta')\pi_{t'}(\theta).$$

Adding $\pi_{t'}(\theta)\pi_{t'}(\theta')$ on both sides, we obtain

$$\pi_{t'}(\theta')(\pi_t(\theta) - \pi_{t'}(\theta)) > \pi_{t'}(\theta)(\pi_t(\theta') - \pi_{t'}(\theta')) \Leftrightarrow \frac{\pi_t(\theta) - \pi_{t'}(\theta)}{\pi_t(\theta') - \pi_{t'}(\theta')} > \frac{\pi_{t'}(\theta)}{\pi_{t'}(\theta')}, \quad (1)$$

where we have used that $\pi_t(\theta) - \pi_{t'}(\theta') > 0$.

Test t more difficult than t' also implies

$$(1 - \pi_t(\theta))(1 - \pi_{t'}(\theta')) \geq (1 - \pi_t(\theta'))(1 - \pi_{t'}(\theta)).$$

Rearranging and adding $\pi_{t'}(\theta)\pi_{t'}(\theta')$ on both sides again, we obtain

$$(1 - \pi_{t'}(\theta))(\pi_t(\theta') - \pi_{t'}(\theta')) > (1 - \pi_{t'}(\theta'))(\pi_t(\theta) - \pi_{t'}(\theta)) \Leftrightarrow \frac{1 - \pi_{t'}(\theta)}{1 - \pi_{t'}(\theta')} > \frac{\pi_t(\theta) - \pi_{t'}(\theta)}{\pi_t(\theta') - \pi_{t'}(\theta')}.$$

Together with inequality (1), we can get

$$\frac{1 - \pi_{t'}(\theta)}{1 - \pi_{t'}(\theta')} > \frac{\pi_t(\theta) - \pi_{t'}(\theta)}{\pi_t(\theta') - \pi_{t'}(\theta')} > \frac{\pi_{t'}(\theta)}{\pi_{t'}(\theta')}.$$

This implies $\pi_{t'}(\theta') > \pi_{t'}(\theta)$, a contradiction. \square

A.3 Proof of Lemma 2

Proof. In any equilibrium, profits must be weakly positive for otherwise the firm can just set $\alpha(x) = 0$ for $x = h, l$ and increase profits.

Suppose first that $\mathbb{E}[\theta] \geq 0$ and suppose that $\alpha(h) < 1$. In any symmetric equilibrium $s = (t, \alpha)$, all agents choose either firm with probability 1/2 and we have

$$v(s, s, \phi) = \frac{1}{2} \int_{\Theta} \theta \left(\pi_t(\theta) \alpha(h) + (1 - \pi_t(\theta)) \alpha(l) \right) dF > 0.$$

If one first sets $s' = (t, \alpha')$ with $\alpha'(h) = \alpha(h) + \epsilon$ and leave the test unchanged, almost all types prefer s' to s . The resulting profits are

$$\int_{\Theta} \theta \left(\pi_t(\theta) (\alpha(h) + \epsilon) + (1 - \pi_t(\theta)) \alpha(l) \right) dF > \frac{1}{2} \int_{\Theta} \theta \left(\pi_t(\theta) \alpha(h) + (1 - \pi_t(\theta)) \alpha(l) \right) dF,$$

for any ϵ . Therefore $\alpha(h) = 1$.

If $\alpha(l) < 1$, we can set $\alpha'(l) = \alpha(l) + \epsilon$ and leave the test unchanged. Almost all types prefer s' to s . The resulting profits are

$$\int_{\Theta} \theta \left(\pi_t(\theta) + (1 - \pi_t(\theta)) (\alpha(l) + \epsilon) \right) dF > \frac{1}{2} \int_{\Theta} \theta \left(\pi_t(\theta) + (1 - \pi_t(\theta)) \alpha(l) \right) dF,$$

for ϵ small enough. We thus get that equilibrium profits are $\frac{1}{2} \mathbb{E}[\theta]$.

If $\mathbb{E}[\theta] < 0$, the same argument holds: as long as profits are strictly positive any firm can increase $\alpha(x)$ and have a strictly profitable deviation. As long as $\int_{\Theta} \theta \pi_t(\theta) dF > 0$, there will also always be an incentive to increase $\alpha(h)$. If $\int_{\Theta} \theta \pi_t(\theta) dF = 0$ then we must have $\alpha(l) = 0$ and we could have $\alpha(h) \in [0, 1]$ in equilibrium. \square

A.4 Proof of Theorem 1

The proof of Theorem 1 is in three steps. First, I show that in any equilibrium, the accuracy level must be σ^* . Second, I show that any equilibrium must have difficulty level satisfying $\min\{d \in D : \int_{\Theta} \theta \pi_{\sigma,d}(\theta) dF \geq 0\}$. And finally that the equilibrium as described in Theorem 1 is indeed an equilibrium.

I start with the following preliminary lemma on the acceptance rule.

Lemma 3. *In any symmetric equilibrium, $s = (t, \alpha)$, the acceptance rule takes a cutoff form: $\alpha(l) > 0 \Rightarrow \alpha(h) = 1$ and $\alpha(h) < 1 \Rightarrow \alpha(l) = 0$.*

Moreover, it is without loss of optimality to consider deviations from symmetric strategy profiles where the acceptance rule takes a cutoff form.

Proof. Take any selection procedure. Let α^* be the cutoff strategy that solves

$$\alpha(l)\pi_t(l|0) + \alpha(h)\pi_t(h|0) = \alpha^*(l)\pi_t(l|0) + \alpha^*(h)\pi_t(h|0).$$

Such strategy always exists. It is easy to verify that

$$\begin{aligned} \text{for } \theta < 0, \quad & \alpha(l)\pi_t(l|\theta) + \alpha(h)\pi_t(h|\theta) \geq \alpha^*(l)\pi_t(l|\theta) + \alpha^*(h)\pi_t(h|\theta), \\ \text{for } \theta > 0, \quad & \alpha(l)\pi_t(l|\theta) + \alpha(h)\pi_t(h|\theta) \leq \alpha^*(l)\pi_t(l|\theta) + \alpha^*(h)\pi_t(h|\theta). \end{aligned}$$

These inequalities are strict if α is not a cutoff strategy and $\pi_t(h|\theta) \neq \pi_t(h|0)$.

Therefore, all symmetric equilibria must have a cutoff acceptance rule, otherwise the cutoff rule as defined above would constitute a profitable deviation. Moreover, when testing for profitable deviation, the cutoff rule as defined here always makes the deviator at least as well off as the original rule. \square

Lemma 4. *In any symmetric equilibrium, the accuracy level is $\sigma^* = \max \Sigma$.*

Proof. The proof of this lemma uses the original notion of accuracy from Lehmann (1988) as defined in Section A.1. Using this definition shortens the argument and shows that its logic does not depend on the binary signal setting.

For each test t , let

$$\tilde{F}_t(x|\theta) = \begin{cases} 2(1 - \pi_t(\theta))x & \text{if } x \in [0, 1/2), \\ 1 + 2\pi_t(\theta)(x - 1) & \text{if } x \in [1/2, 1]. \end{cases}$$

First note that for each strategy cutoff strategy α and test t , there is a corresponding cutoff $x \in [0, 1]$ such that $\alpha(h)\pi_t(\theta) + \alpha(l)(1 - \pi_t(\theta)) = 1 - \tilde{F}_t(x|\theta)$. Because each test is monotonic in types, i.e., they have the monotone likelihood ratio property, we have $\tilde{F}_t(x|\theta) \leq \tilde{F}_t(x|\theta')$ for any $\theta' < \theta$ and $x \in [0, 1]$.

Suppose there is a symmetric equilibrium $s' = (\tau(\sigma, d), \alpha)$ with $\sigma < \sigma^*$. Let $t' = \tau(\sigma, d)$ and x' be the corresponding cutoff in the modified test $\tilde{F}_{t'}$.

From Lemma 2, it must be that firms' profits are zero. Take test $t = \tau(\sigma^*, d)$ and for $\theta = 0$, find the cutoff strategy α_θ^* such that

$$\alpha(h)\pi_{t'}(\theta) + \alpha(l)(1 - \pi_{t'}(\theta)) = \alpha_\theta^*(h)\pi_t(\theta) + \alpha_\theta^*(l)(1 - \pi_t(\theta)).$$

and let $x^*(\theta)$ be the corresponding cutoff in the modified test. Observe that

$$1 - \tilde{F}_t(x^*(0)|\theta) \geq 1 - \tilde{F}_t(x^*(\theta)|\theta) = 1 - \tilde{F}_{t'}(x'|\theta), \text{ for all } \theta \in (0, \bar{\theta}),^9$$

where the inequality comes from the definition of accuracy (in Section A.1) and the equality from the definition of $x^*(\theta)$. The inequality is reversed for $\theta < 0$. Moreover, there must be some types for whom the inequality is strict as $t' \prec_a t$. This implies that if one of the firms deviates to $s = (t, \alpha_0^*)$, they achieve strictly positive profits, making them better off than in the candidate equilibrium. To see this formally, let $\Theta_s, \Theta_i, \Theta_{s'}$ be the set of types strictly

⁹Here, I use Lehmann's (1988) original definition, see Section A.1.

preferring s , indifferent and strictly preferring s' . The profits following the deviation to s are

$$\begin{aligned}
& \frac{1}{2} \int_{\Theta_i} (1 - \tilde{F}_t(x^*(0)|\theta))\theta dF + \int_{\Theta_s} (1 - \tilde{F}_t(x^*(0)|\theta))\theta dF \\
& > \frac{1}{2} \int_{\Theta_i} (1 - \tilde{F}_{t'}(x^*(0)|\theta))\theta dF + \int_{\Theta_s} (1 - \tilde{F}_{t'}(x^*(0)|\theta))\theta dF \\
& > \frac{1}{2} \int_{\Theta_i} (1 - \tilde{F}_{t'}(x^*(0)|\theta))\theta dF + \int_{\Theta_s} (1 - \tilde{F}_{t'}(x^*(0)|\theta))\theta dF \\
& \quad + \frac{1}{2} \int_{\Theta_{s'}} (1 - \tilde{F}_{t'}(x^*(0)|\theta))\theta dF - \frac{1}{2} \int_{\Theta_s} (1 - \tilde{F}_{t'}(x^*(0)|\theta))\theta dF = 0,
\end{aligned}$$

where the first inequality holds because only positive types strictly prefer s and the second by adding negative terms. \square

Lemma 5. *If $s = (\tau(\sigma, d), \alpha)$ is a symmetric equilibrium, then $d = \min\{d' \in D : \int_{\Theta} \theta \pi_{\sigma, d'}(\theta) dF\}$.*

Proof. Suppose there is a symmetric equilibrium $s' = (\tau(\sigma, d'), \alpha')$ with $d' > d = \min\{d' \in D : \int_{\Theta} \theta \pi_{\sigma, d'}(\theta) dF\}$. Take $\tilde{d} \in (d, d')$ and let $t = \tau(\sigma, \tilde{d})$ and $t' = \tau(\sigma, d')$. By Assumption 2, t and t' are not comparable in accuracy and therefore $\frac{\pi_t(\theta)}{\pi_{t'}(\theta)}$ and $\frac{\pi_{t'}(\theta)}{\pi_{\sigma, d}(\theta)}$ are not constant and thus $\mathbb{E}[\theta|t', h] > \mathbb{E}[\theta|t, h] > \mathbb{E}[\theta|\tau(\sigma, d), h] \geq 0$. Consider a deviation to $s = (t, \alpha)$ with $\alpha(h) = 1$ and $\alpha(l) \geq 0$. To simplify notation, I write α for $\alpha(l)$ as it will be the only parameter varying in the proof.

By Lemma 2, profits are zero and $\alpha'(h) = 1$ and $\alpha'(l) > 0$. To simplify notation, I write α' for $\alpha'(l)$ and α for $\alpha(l)$.

Type θ chooses selection procedure s if

$$\pi_t(\theta) + \alpha(1 - \pi_t(\theta)) \geq \pi_{t'}(\theta) + \alpha'(1 - \pi_{t'}(\theta)) \Leftrightarrow (1 - \alpha)(1 - \pi_t(\theta)) \leq (1 - \alpha')(1 - \pi_{t'}(\theta)). \quad (2)$$

Because $\frac{1 - \pi_{t'}(\theta)}{1 - \pi_t(\theta)}$ is increasing, if the inequality is satisfied for θ , it is satisfied for all $\theta' > \theta$, i.e., there is positive selection into s .

We now show that there is an α such that s is a profitable deviation.

Consider first the case where $\alpha = 0$. If inequality (2) is satisfied for some types, then by the positive selection and $\mathbb{E}[\theta|t, h] > 0$, s is a profitable deviation.

If inequality (2) is not satisfied for all types, then in particular, it is not satisfied for $\theta = 0$,

i.e.,

$$(1 - \alpha)(1 - \pi_t(0)) > (1 - \alpha')(1 - \pi_{t'}(0)), \text{ at } \alpha = 0.$$

At the same time, inequality (2) holds for $\theta = 0$ when $\alpha = \alpha'(l)$ as $\pi_t(\theta) \geq \pi_{t'}(\theta)$ (Lemma 1). Therefore, by the intermediate value theorem, there is $\alpha \in (0, \alpha'(l))$ such that

$$(1 - \alpha)(1 - \pi_t(0)) = (1 - \alpha')(1 - \pi_{t'}(0)),$$

By the positive selection, s is a profitable deviation. \square

Lemma 6. *The strategy $s = (\tau(\sigma^*, d^*), \alpha)$ with $\sigma^* = \max\{\sigma \in \Sigma\}$ and $d^* = \min\{d \in D : \int_{\Theta} \theta \pi_{\sigma^*, d}(\theta) dF \geq 0\}$ is a symmetric equilibrium.*

Proof. Take a symmetric equilibrium $s = (\tau(\sigma^*, d^*), \alpha)$ and let $t = \tau(\sigma^*, d^*)$. Set $\alpha(l) \geq 0$ and $\alpha(h) = 1$. To simplify notation, write $\alpha(l) = \alpha$.

Take a deviation to $t' = \tau(\sigma, d)$ with $\sigma \leq \sigma^*$.

First, suppose that selection procedure $(\tau(\sigma, d), \alpha')$ with $\sigma < \sigma^*$ is a profitable deviation. In that case, $(\tau(\sigma^*, d), \alpha'')$ where α'' is such that positive types have a higher probability of being accepted and negative types a lower one is also a profitable deviation. Such α'' exists using the same reasoning as in the proof of Lemma 5.

Take test $t' = \tau(\sigma^*, d)$ with $d > d^*$. We have $\tau(\sigma^*, d) \succeq_d \tau(\sigma^*, d^*)$ and therefore for all $\theta, \theta' \in (\underline{\theta}, \bar{\theta})$ with $\theta > \theta'$,

$$\frac{1 - \pi_{\sigma^*, d^*}(\theta)}{1 - \pi_{\sigma^*, d^*}(\theta')} \leq \frac{1 - \pi_{\sigma^*, d}(\theta)}{1 - \pi_{\sigma^*, d}(\theta')}.$$

Note that any deviation to $s' = (\tau(\sigma^*, d), \alpha')$ must have $\alpha'(l) > \alpha(l) \geq 0$ as $\pi_{\sigma^*, d^*}(\theta) \geq \pi_{\sigma^*, d}(\theta)$ for all θ . Otherwise, no type would choose s' .

Let $\Delta u(\theta)$ denote the difference in probability of being accepted of type θ between s' and s :

$$\begin{aligned} \Delta u(\theta) &= \pi_{t'}(\theta) + \alpha'(1 - \pi_{t'}(\theta)) - \pi_t(\theta) - \alpha(1 - \pi_t(\theta)) \\ &= (1 - \pi_{t'}(\theta))(\alpha' - 1 + (1 - \alpha) \frac{1 - \pi_t(\theta)}{1 - \pi_{t'}(\theta)}). \end{aligned}$$

Both $1 - \pi_{t'}(\theta)$ and $\alpha' - 1 + (1 - \alpha) \frac{1 - \pi_t(\theta)}{1 - \pi_{t'}(\theta)}$ are decreasing function of θ . Therefore, whenever $\alpha' - 1 + (1 - \alpha) \frac{1 - \pi_t(\theta)}{1 - \pi_{t'}(\theta)} \geq 0$, and thus $\Delta u(\theta) \geq 0$, $\Delta u(\theta)$ is decreasing as the product of two

positive decreasing function is decreasing. Moreover, the function $\Delta u(\theta)$ is single crossing from above.

Let $\Theta_{s'}$, Θ_i and Θ_s be the set of types for which $\Delta u(\theta) > 0$, $= 0$ and < 0 .

The profits from s' are

$$\begin{aligned}
& \int_{\Theta_{s'}} (\pi_{t'}(\theta) + \alpha'(1 - \pi_{t'}(\theta))\theta) dF + \frac{1}{2} \int_{\Theta_i} (\pi_{t'}(\theta) + \alpha'(1 - \pi_{t'}(\theta))\theta) dF \\
& - \int_{\Theta} (\pi_t(\theta) + \alpha(1 - \pi_t(\theta))\theta) dF \\
& = \int_{\Theta_{s'}} \theta \Delta u(\theta) dF + \frac{1}{2} \int_{\Theta_i} \theta \Delta u(\theta) dF \\
& - \frac{1}{2} \left(\int_{\Theta_i} (\pi_t(\theta) + \alpha(1 - \pi_t(\theta))\theta) dF + \int_{\Theta_s} (\pi_t(\theta) + \alpha(1 - \pi_t(\theta))\theta) dF \right) \\
& - \frac{1}{2} \int_{\Theta_s} (\pi_t(\theta) + \alpha'(1 - \pi_t(\theta))\theta) dF,
\end{aligned}$$

where we have added the profits s which are zero on the first line and rearranged to obtain the inequality. Now note that if s' is a profitable deviation, it must be that $\theta = 0 \in \Theta_{s'}$ or $\in \Theta_i$, i.e., some positive types must be choosing s' . This implies that all types in Θ_s must be positive. We can now prove that the profits from s' are negative.

The last term is negative as all types in Θ_s are positive. The second last term is negative as it is the profits from equilibrium after having removed types at the bottom. The term $\int_{\Theta_i} \theta \Delta u(\theta) dF = 0$ by definition of Θ_i . Finally, because on $\Theta_{s'}$, $\Delta u(\theta)$ is decreasing, we have

$$\int_{\Theta_{s'}} \theta \Delta u(\theta) dF \leq \int_{\Theta_{s'}} \theta \Delta u(0) dF \leq 0,$$

using that $\mathbb{E}[\theta] < 0$.

Now consider a deviation to $s' = (\tau(\sigma^*, d), \alpha')$ with $d < d^*$. If such test is feasible, it must be that $\int_{\Theta} \pi_{\sigma^*, d^*}(\theta) \theta dF = 0$ (by continuity of $\pi_{\sigma, d}$) and $\int_{\Theta} \pi_{\sigma^*, d}(\theta) \theta dF < 0$.

The candidate equilibrium $s = (\tau(\sigma^*, d^*), \alpha)$ has $\alpha(l) = 0$

Let $t' = \tau(\sigma^*, d)$. If $\alpha'(l) > 0$, then all types choose s' as $\pi_{d, \sigma^*}(\theta) \geq \pi_{d^*, \sigma^*}$. But $\mathbb{E}[\theta|t', h] < 0$ and therefore payoffs from s' are negative. Therefore $\alpha'(l) = 0$.

Let $\Delta u(\theta)$ the difference in probability of acceptance between s' and s for type θ :

$$\Delta u(\theta) = \alpha'(h)\pi_{t'}(\theta) - \pi_t(\theta) = \pi_{t'}(\theta)(\alpha'(h) - \frac{\pi_t(\theta)}{\pi_{t'}(\theta)}).$$

The function $\Delta u(\theta)$ is single-crossing from above and therefore there is negative selection into s' . Combined with the fact that $\mathbb{E}[\theta|t', h] < 0$, the deviation is not profitable. \square

A.5 Proof of Proposition 2

Proof. We show that the strategy $s = (\tau(\bar{d}), \alpha)$ with $\alpha(h) = 1$, $\alpha(l) = 0$ is an equilibrium. For simplicity, let $t = \tau(\bar{d})$. Equilibrium payoffs for both firms are

$$k \cdot \mathbb{E}[\theta|t, h].$$

Consider a deviation of firm 1 to $s' = (t' = \tau(d'), \alpha')$ with $d' < \bar{d}$. Let p_i denote the probability of a given type to have its application considered by firm i . Suppose first that $\alpha'(l) = 0$. For simplicity let $\alpha'(h) = \alpha'$. I will first show that selection into firm 1 is negative. The agent's utility difference between firm 1 and 2 is

$$\Delta u(\theta) = p_1 \alpha' \pi_{t'}(\theta) - p_2 \pi_t(\theta).$$

Using that $\frac{\pi_t(\theta)}{\pi_{t'}(\theta)}$ is increasing, we can establish that selection is negative. Let $\Theta_{t'}$ and Θ_i be the set of types for which $\Delta u(\theta) > 0$ and $\Delta u(\theta) = 0$. If the capacity constraint is binding, then

$$k \cdot \mathbb{E}[\theta|t', h, \theta \text{ chooses 1}] \leq k \cdot \mathbb{E}[\theta|t', h] \leq k \cdot \mathbb{E}[\theta|t, h].$$

If the capacity constraint is binding, equilibrium profits are

$$\begin{aligned} \int_{\Theta_d} \theta \alpha' \pi_{t'}(\theta) dF + \frac{1}{2} \int_{\Theta_i} \theta \alpha' \pi_{t'}(\theta) dF &\leq \frac{k}{p_1} \int_{\Theta_{t'}} \theta \alpha' \pi_{t'}(\theta) dF + \frac{1}{2} \int_{\Theta_i} \theta \alpha' \pi_{t'}(\theta) dF \\ &= k \cdot \mathbb{E}[\theta|t', h, \theta \text{ chooses 1}] \\ &\leq k \cdot \mathbb{E}[\theta|t', h] \leq k \cdot \mathbb{E}[\theta|t, h]. \end{aligned}$$

Now suppose that $\alpha'(l) > 0$. For simplicity let $\alpha'(l) = \alpha'$. First observe that to have a profitable deviation, it must be $p_2/p_1 > 1$ for otherwise no type would choose firm 2 as

$\pi_{t'}(\theta) \geq \pi_t(\theta)$. But in that case firm 1's capacity constraint binds and its profits are lower than under the equilibrium profits as $\mathbb{E}[\theta|t, h] \geq \mathbb{E}[\theta|t', h]$. Let $p = p_2/p_1$ and

$$\Delta u(\theta) = p_1(\pi_{t'}(\theta) + \alpha'(1 - \pi_{t'}(\theta))) - p_2\pi_t(\theta) = \pi_t(\theta) \left(p_1(1 - \alpha') \frac{\pi_{t'}(\theta)}{\pi_t(\theta)} + \frac{p_1\alpha'}{\pi_t(\theta)} - p_2 \right).$$

The term in parenthesis is negative whenever $\Delta u \leq 0$ and it is decreasing while $\pi_t(\theta)$ is increasing and positive. Therefore whenever $\Delta u(\theta) \leq 0$, $\Delta u(\theta)$ is decreasing. This shows there is negative selection into s' and the acceptance probability satisfies decreasing differences for the types choosing s' . By a similar argument as above, the deviation cannot be profitable. \square

A.6 Proof of Proposition 3

I show that there is an equilibrium where firm 1 chooses selection procedure $(\bar{t}, \alpha(h) = 1, \alpha(l) = 0)$ and firm 2 chooses $(t, \alpha(h) = 1, \alpha(l) = 0)$. To simplify notation, I denote by $\bar{\pi}_i$ and $\underline{\pi}_i$ the probability of $\bar{\theta}$ and $\underline{\theta}$ to generate the high signal in the test chosen by firm i .

In the suggested equilibrium, type $\bar{\theta}$ chooses firm 1 and type $\underline{\theta}$ mixes between firm 1 and firm 2. Denote by ϕ the probability that type $\underline{\theta}$ chooses firm 2. Both firms' capacities are binding.

Type $\underline{\theta}$ is willing to mix between firm 1 and firm 2 if

$$\frac{k}{(1 - \mu)\phi\underline{\pi}_2} \bar{\pi}_2 = \frac{k}{\mu\bar{\pi}_1 + (1 - \mu)(1 - \phi)\underline{\pi}_1} \bar{\pi}_1.$$

Solving for ϕ , we get $\phi = \frac{\mu\bar{\pi}_1 + (1 - \mu)\underline{\pi}_1}{2(1 - \mu)\underline{\pi}_1}$. We have $\phi \leq 1 \Leftrightarrow \mu\bar{\pi}_1 \leq (1 - \mu)\underline{\pi}_1$. This inequality corresponds to the second condition in Proposition 3. Note also that we have $\phi \geq \frac{1}{2(1 - \mu)}$.

Type $\bar{\theta}$ prefers firm 1 if

$$\frac{k}{\mu\bar{\pi}_1 + (1 - \mu)(1 - \phi)\underline{\pi}_1} \bar{\pi}_1 \geq \frac{k}{(1 - \mu)\phi\underline{\pi}_2} \bar{\pi}_2 \Leftrightarrow \frac{\bar{\pi}_1}{\underline{\pi}_1} \geq \frac{\bar{\pi}_2}{\underline{\pi}_2}. \quad (3)$$

This inequality is always satisfied by definition of \bar{t} , the test chosen by firm 1.

The capacity constraints bind if

$$\text{Firm 1: } \mu\bar{\pi}_1 + (1 - \mu)(1 - \phi)\underline{\pi}_1 \geq k,$$

$$\text{Firm 2: } (1 - \mu)\phi\underline{\pi}_2 \geq k.$$

Plugging in the value of ϕ for the first inequality, we obtain the third condition in Proposition 3. For the second inequality, we obtain the fourth inequality of Proposition 3 using the minimal value of ϕ .

Let's now consider deviations of firms. First, firm 2 must make positive profits in equilibrium. This is the case only if $\underline{\theta} \geq 0$ (first condition in Proposition 3).

Consider a deviation to $s' = (t', \alpha')$. If firm 2 deviates, there are two possibilities. Either its capacity constraint binds following the deviation or it does not.

If firm 2's capacity constraint still binds in the continuation equilibrium, then we still have that $\underline{\theta}$ mixes between firm 1 and 2 using the same strategy (that only depended on the strategy of firm 1). Type $\bar{\theta}$ would want to deviate to firm 2 if

$$\frac{\bar{\pi}_1}{\underline{\pi}_1} < \frac{\alpha'(h)\pi_{t'}(\bar{\theta}) + \alpha'(l)(1 - \pi_{t'}(\bar{\theta}))}{\alpha'(h)\pi_{t'}(\underline{\theta}) + \alpha'(l)(1 - \pi_{t'}(\underline{\theta}))},$$

using the same calculations as (3). The inequality above is never satisfied because

$$\frac{\bar{\pi}_1}{\underline{\pi}_1} \geq \frac{\pi_{t'}(\bar{\theta})}{\pi_{t'}(\underline{\theta})} \geq \frac{\alpha'(h)\pi_{t'}(\bar{\theta}) + \alpha'(l)(1 - \pi_{t'}(\bar{\theta}))}{\alpha'(h)\pi_{t'}(\underline{\theta}) + \alpha'(l)(1 - \pi_{t'}(\underline{\theta}))},$$

where the first inequality follows from the definition of \bar{t} , the test used by firm 1. The inequality above also shows that there is negative selection into firm 2's selection procedure. Therefore, firm 2's profits following such deviation are still $k \cdot \underline{\theta}$, the same as the equilibrium profits.

If capacity constraint of firm 2 does not bind following the deviation, we consider continuation equilibria where if indifferent, type $\bar{\theta}$ chooses firm 1.¹⁰ Therefore, firm 2 benefits from a deviation only if both types strictly prefer firm 2. This is because there is negative selection

¹⁰This assumption on the continuation equilibrium is not necessary to check for profitable deviations but makes the proof shorter.

into firm 2's strategy. In this case, no one is applying to firm 1 and the IC constraints are

$$\begin{aligned}\alpha'(h)\pi_{t'}(\bar{\theta}) + \alpha'(l)(1 - \pi_{t'}(\bar{\theta})) &\geq \bar{\pi}_1, \\ \alpha'(h)\pi_{t'}(\underline{\theta}) + \alpha'(l)(1 - \pi_{t'}(\underline{\theta})) &\geq \underline{\pi}_2.\end{aligned}$$

Combining these inequalities, we have

$$\begin{aligned}\mu[\alpha'(h)\pi_{t'}(\bar{\theta}) + \alpha'(l)(1 - \pi_{t'}(\bar{\theta}))] + (1 - \mu)[\alpha'(h)\pi_{t'}(\underline{\theta}) + \alpha'(l)(1 - \pi_{t'}(\underline{\theta}))] \\ \geq \mu\bar{\pi}_1 + (1 - \mu)\underline{\pi}_1.\end{aligned}$$

The RHS of this equation is larger than k by the last condition in Proposition 3, contradicting that the capacity constraint is not binding.

We now need to verify that firm 1 does not have any profitable deviation. Again, we need to distinguish the cases where the capacity constraint is binding or not in the continuation equilibrium.

Let's first consider the case where the capacity constraint binds in equilibrium. Consider a deviation to $s' = (t', \alpha')$. Let $\bar{q} = \alpha'(h)\pi_{t'}(\bar{\theta}) + \alpha'(l)(1 - \pi_{t'}(\bar{\theta}))$, the acceptance probability under s' of $\bar{\theta}$ and define \underline{q} similarly.

Suppose it is still true that

$$\frac{\mu\bar{q} + (1 - \mu)\underline{q}}{2} \geq k.$$

Then the same equilibrium as above holds as long as $\frac{\bar{q}}{\underline{q}} \geq \frac{\bar{\pi}_2}{\underline{\pi}_2}$. Moreover, we necessarily have that

$$\frac{\bar{\pi}_1}{\underline{\pi}_1} \geq \frac{\bar{q}}{\underline{q}}.$$

Let ϕ_q be the probability of type $\underline{\theta}$ to choose firm 2. The strategy ϕ_q is decreasing in the likelihood ratio $\frac{\bar{q}}{\underline{q}}$ and therefore this deviation is not profitable for firm 1 as it attracts more $\underline{\theta}$. If $\frac{\bar{q}}{\underline{q}} < \frac{\bar{\pi}_2}{\underline{\pi}_2}$ we can construct an equilibrium where the roles of firms are reversed and firm 1 does not benefit from a deviation either.

Suppose now that

$$\mu\bar{q} + (1 - \mu)(1 - \phi_q)\underline{q} = \frac{\mu\bar{q} + (1 - \mu)\underline{q}}{2} < k.$$

In that case, the equilibrium we constructed is not feasible. The best case scenario for firm 1 is that it attracts only high types. Given the inequality above, the capacity constraint cannot

bind. The deviation is profitable if

$$\mu\bar{q}\bar{\theta} > \frac{k}{\mu\bar{\pi}_1 + (1-\mu)(1-\phi)\underline{\pi}_1} (\mu\bar{\pi}_1\bar{\theta} + (1-\mu)(1-\phi)\underline{\pi}_1\underline{\theta}).$$

We get,

$$\begin{aligned} & \frac{k}{\mu\bar{\pi}_1 + (1-\mu)(1-\phi)\underline{\pi}_1} (\mu\bar{\pi}_1\bar{\theta} + (1-\mu)(1-\phi)\underline{\pi}_1\underline{\theta}) \\ & < \mu\bar{q}\bar{\theta} \\ & \leq \mu\bar{q}\bar{\theta} + (1-\mu)(1-\phi_q)q\underline{\theta} \\ & < \frac{k}{\mu\bar{q} + (1-\mu)(1-\phi_q)\underline{q}} (\mu\bar{q}\bar{\theta} + (1-\mu)(1-\phi_q)\underline{q}\underline{\theta}). \end{aligned}$$

It is easy to verify that this chain of inequality cannot hold.

A.7 Proof of Proposition 4

Proof. In any symmetric equilibrium, both firms get zero profits. If it is not the case and firm gets strictly positive profits, then there is at least one signal where firms get positive profits. Then one of them can raise the transfer by ϵ and attract all agents for an arbitrarily small increase.

First, I show that there is no ‘cross-subsidisation’ in equilibrium, i.e., $\alpha(l) = 0$.

Suppose it is not the case. Let $s = (t, \alpha, m)$ be the selection procedure in equilibrium. Let $m(h) = m_h$ and $m(l) = m_l$. First note that if $m_l = 0$, then any firm can decrease $\alpha(l)$ and increase its profits. Because $m_l = 0$, this does not change the payoffs of the agents. So $m_l > 0$. We have

$$\int_{\Theta} \pi_t(\theta) \alpha_h(\theta - m_h) + (1 - \pi_t(\theta))(\theta - m_l) dF = 0.$$

Consider the following deviation s' that leaves all aspects of the selection procedure unchanged but $m'(h) = m_h + \epsilon$ and $m'(l) = m_l - \delta$ with $\epsilon, \delta > 0$ such that

$$\int_{\Theta} \pi_t(\theta) \alpha_h(\theta - m_h - \epsilon) + (1 - \pi_t(\theta)) \alpha_l(\theta - m_l + \delta) dF = 0.$$

We choose ϵ, δ small enough such that $m'(l) > 0$. I will show that some types will choose the deviating firm and that the deviation will exhibit positive selection. This will imply that the deviation is profitable.

Type θ chooses s' if

$$\begin{aligned} \pi_t(\theta)\alpha_h(\theta - m_h - \epsilon) + (1 - \pi_t(\theta))\alpha_l(\theta - m_l + \delta) &\geq \pi_t(\theta)\alpha_h(\theta - m_h) + (1 - \pi_t(\theta))\alpha_l(\theta - m_l) \\ \Leftrightarrow \pi_t(\theta)\alpha_h\epsilon &\leq (1 - \pi_t(\theta))\alpha_l\delta \end{aligned}$$

Plugging in the solution of δ as a function of ϵ and the fact that the original profits are zero, we get that type θ chooses s' if

$$\pi_t(\theta) \int_{\Theta} (1 - \pi_t(\theta)) dF \geq (1 - \pi_t(\theta)) \int_{\Theta} \pi_t(\theta) dF.$$

This has to hold for some types as otherwise we have $\pi_t(\theta) \int_{\Theta} (1 - \pi_t(\theta)) dF < (1 - \pi_t(\theta)) \int_{\Theta} \pi_t(\theta) dF$ for all θ which implies $\int_{\Theta} \pi_t(\theta) dF \cdot \int_{\Theta} (1 - \pi_t(\theta)) dF < \int_{\Theta} (1 - \pi_t(\theta)) dF \cdot \int_{\Theta} \pi_t(\theta) dF$, a contradiction. Using the same argument, some types must prefer the original selection procedure s . Therefore, we get positive selection into the new selection procedure. Since absent positive selection, the profits are zero, this must be a strictly profitable deviation.

Let $\bar{t} = \tau(\bar{d})$. Suppose there is symmetric equilibrium with $t \prec_d \bar{t}$ where $t = \tau(d)$ for some $d < \bar{d}$.

In equilibrium, it must be that $m(l) = 0$ and from the zero profits condition, $m(h) = m = \frac{\int_{\Theta} \theta \pi_t(\theta) dF}{\int_{\Theta} \pi_t(\theta) dF}$.

Take some $\epsilon \in (0, \frac{\pi_t(0) - \pi_{\bar{t}}(0)}{\pi_{\bar{t}}(0)})$. We want to find t' with $t \prec_d t' \prec_d \bar{t}$ such that

$$m\pi_t(0) = m(1 + \epsilon)\pi_{t'}(0).$$

For $t' = t$, we have $m\pi_t(0) < m(1 + \epsilon)\pi_{t'}(0)$ and for $t' = \bar{t}$ we have $m\pi_t(0) > m(1 + \epsilon)\pi_{t'}(0)$, using the bound on ϵ . Because D is an interval and the continuity assumption (Assumption 2), by the intermediate value theorem, there is d' and $t' = \tau(d')$ with $t \prec_d t' \prec_d \bar{t}$ such that $m\pi_t(0) = m(1 + \epsilon)\pi_{t'}(0)$.

We want to show that for ϵ small enough, this constitutes a profitable deviation, i.e.,

$$\int_0^{\bar{\theta}} \pi_{t'}(\theta)(\theta - m(1 + \epsilon))dF > 0.$$

Because t' is more difficult than t , we have $\mathbb{E}[\theta|t, \theta \in [0, \bar{\theta}]] < \mathbb{E}[\theta|t', \theta \in [0, \bar{\theta}]]$. Moreover, we have that for ϵ small enough, $\mathbb{E}[\theta|t, \theta \in [0, \bar{\theta}]] > \mathbb{E}[\theta|t](1 + \epsilon)$. Combining these facts, we get

$$\frac{\int_0^{\bar{\theta}} \pi_{t'}(\theta)\theta dF}{\int_0^{\bar{\theta}} \pi_{t'}(\theta)dF} > \frac{\int_0^{\bar{\theta}} \pi_t(\theta)\theta dF}{\int_0^{\bar{\theta}} \pi_t(\theta)dF} > \frac{\int_{\underline{\theta}}^{\bar{\theta}} \pi_t(\theta)\theta dF}{\int_{\underline{\theta}}^{\bar{\theta}} \pi_t(\theta)dF}(1 + \epsilon).$$

Recalling that $m = \frac{\int_{\underline{\theta}}^{\bar{\theta}} \pi_t(\theta)\theta dF}{\int_{\underline{\theta}}^{\bar{\theta}} \pi_t(\theta)dF}$, this is what we needed to show. \square

A.8 Proof of Corollary 1

Take $t \succeq_d t'$. Then $t_\beta = \beta t + (1 - \beta)t'$ has $t \preceq_d t_\beta \preceq_d t'$ where $\beta \in [0, 1]$ and the convex combination is taken with respect to the likelihood ratios:

$$\frac{\pi_{t'}(x|\bar{\theta})}{\pi_{t'}(x|\underline{\theta})} \geq \beta \frac{\pi_t(x|\bar{\theta})}{\pi_t(x|\underline{\theta})} + (1 - \beta) \frac{\pi_{t'}(x|\bar{\theta})}{\pi_{t'}(x|\underline{\theta})} \geq \frac{\pi_t(x|\bar{\theta})}{\pi_t(x|\underline{\theta})}, \text{ for } x = l, h.$$

Let $\pi_\beta(\theta)$ be the induced probability of sending the high signal for type θ . It is easy to verify that $\pi_\beta(\theta)$ is continuous in β .

If T induces binary types and is closed, then $t = \arg \max \left\{ \frac{\pi_t(\bar{\theta})}{\pi_t(\underline{\theta})} : t \in \arg \min_{t' \in T_i} \frac{1 - \pi_{t'}(\bar{\theta})}{1 - \pi_{t'}(\underline{\theta})} \right\}$ exists. If $t \in \arg \min_{t' \in T_i} \frac{1 - \pi_{t'}(\bar{\theta})}{1 - \pi_{t'}(\underline{\theta})}$, then either $t \succeq_a t'$ or $t \preceq_d t'$ for all $t' \in T_i$. Then using a similar argument as in Lemma 6 in Theorem 1, there is no deviation to any $s' = (t', \alpha')$.

Any test in $t' \in T \setminus T_i$ has negative posterior expectation at the high signal. Therefore it is necessarily either less accurate or easier than t . Again, using a similar argument as in Lemma 6 in Theorem 1, there is no deviation to any $s' = (t', \alpha')$.

A similar argument to Lemma 5 and Lemma 4 in Theorem 1 establishes that this must be the unique symmetric equilibrium.¹¹

¹¹Recall that we have used the continuity in d assumption to prove Lemma 6 in Theorem 1.

A.9 Proof of Proposition 5

Proof. Suppose there is a symmetric equilibrium $s = (t, \alpha)$ with $\int_{\Theta} \theta \pi_t(\theta) dF > 0$. Let $\mu \in [0, \frac{1-\pi_t(\bar{\theta})}{1-\bar{\pi}_t})$ (such a μ exists because $\pi_t(\bar{\theta}) < 1$). Define the test t' as follows:

$$\pi_{t'}(\theta) = \frac{\lambda \bar{\pi}_t + (1-\lambda)\pi_t(\theta)}{(1-\mu)(1-\lambda(1-\bar{\pi}_t)) + \mu \bar{\pi}_t}.$$

One can check that with the assumption on μ , we always have $\pi_{t'}(\theta) \leq 1$.

Monotonicity of t' follows from the definition.

Let $\bar{\pi}_{t'} = \int_{\Theta} \pi_{t'}(\theta) dF$. Under our assumption on μ , we get $\bar{\pi}_{t'} = \frac{\bar{\pi}_t}{(1-\mu)(1-\lambda(1-\bar{\pi}_t)) + \mu \bar{\pi}_t}$. Note as well that for all $\theta \in \Theta$, we have

$$\begin{aligned} \lambda f(\theta) + (1-\lambda)f_{th}(\theta) &= f_{t'h}(\theta), \\ \mu f(\theta) + (1-\mu)f_{t'l}(\theta) &= f_{tl}(\theta). \end{aligned}$$

These expression can be easily verified plugging in the values of $\pi_{t'}$ and $\bar{\pi}_{t'}$.

This implies that

$$\begin{aligned} \lambda + (1-\lambda) \frac{\pi_t(\theta)}{\bar{\pi}_t} &= \frac{\pi_{t'}(\theta)}{\bar{\pi}_{t'}} \\ \text{and } \mu + (1-\mu) \frac{1-\pi_t(\theta)}{1-\bar{\pi}_t} &= \frac{1-\pi_{t'}(\theta)}{1-\bar{\pi}_{t'}}. \end{aligned}$$

If $\pi_{t'}(\theta) \in (0, 1)$, one can check from these expressions that $\frac{\pi_t(\theta)}{\pi_{t'}(\theta)}$ and $\frac{1-\pi_t(\theta)}{1-\pi_{t'}(\theta)}$ are increasing using that $\pi_{t'}$ is increasing. Therefore, $t \succeq_d t'$. Whenever $\pi_{t'}(\theta) \in \{0, 1\}$, the condition for difficulty is satisfied.

Note also that $\frac{1-\pi_t(\theta)}{1-\pi_{t'}(\theta)}$ is not constant whenever $\mu > 0$.

Set $\mu = 0$. We want to show that for any $\lambda \in [0, 1]$, this test is well-defined, easier than t and $C(t') < C(t)$, i.e.,

$$\bar{\pi}_{t'} c(f_{t'h}) + (1-\bar{\pi}_{t'}) c(f_{t'l}) \leq \bar{\pi}_t c(f_{th}) + (1-\bar{\pi}_t) c(f_{tl}).$$

We can take the LHS and using the strict convexity of c obtain

$$\bar{\pi}_{t'}c(f_{t'h}) + (1 - \bar{\pi}_{t'})c(f_{t'l}) < \bar{\pi}_{t'}(\lambda c(f) + (1 - \lambda)c(f_{th})) (1 - \bar{\pi}_{t'})c(f_{t'l}).$$

Using that $f_{tl} = f_{th}$ if $\mu = 0$, it is enough to verify that

$$\bar{\pi}_{t'}(\lambda c(f) + (1 - \lambda)c(f_{th})) (1 - \bar{\pi}_{t'})c(f_{t'l}) \leq \bar{\pi}_t c(f_{th}) + (1 - \bar{\pi}_t)c(f_{tl}).$$

Using that $c(f) = 0$, this is equivalent to

$$0 < \left(1 - \frac{\bar{\pi}_t}{1 - \lambda + \lambda \bar{\pi}_t}\right) (\bar{\pi}_t c(f_{th}) + (1 - \bar{\pi}_t)c(f_{tl})),$$

which is satisfied.

Now observe that because $\int_{\Theta} \theta \pi_t(\theta) dF > 0$, we can always pick λ such that $\int_{\Theta} \theta \pi_{t'}(\theta) dF > 0$. We can also increase μ by an arbitrarily small amount and by continuity of the integral and the cost $C(t') < C(t)$ and $\int_{\Theta} \theta \pi_{t'}(\theta) dF > 0$. By Lemma 5, this contradicts s is an equilibrium.

Budget binding: The budget constraint must bind for otherwise we can find a Blackwell more informative test that is still affordable. By Lemma 4, this contradicts we are in a symmetric equilibrium. \square

B Isocost tests with difficulty comparison

Proposition 7. *Let $\pi_t(\theta) \in (0, 1)$ for all $\theta \in \Theta$. Then there exists a test t' with $t \succ_d t'$ and $C(t) = C(t')$.*

Proof. Let $\mu \in (0, \frac{1 - \pi_t(\bar{\theta})}{1 - \bar{\pi}_t})$, $\lambda \in [0, 1]$ and $\pi_{t'}(\theta) = \frac{\lambda \bar{\pi}_t + (1 - \lambda) \pi_t(\theta)}{(1 - \mu)(1 - \lambda(1 - \bar{\pi}_t)) + \mu \bar{\pi}_t}$. We have already shown in the proof of Proposition 5 that this test is well-defined.

We want to show there exists $\lambda \in (0, 1)$ such that

$$\bar{\pi}_{t'}c(f_{t'h}) + (1 - \bar{\pi}_{t'})c(f_{t'l}) = \bar{\pi}_t c(f_{th}) + (1 - \bar{\pi}_t)c(f_{tl}).$$

Note that if $\lambda = 1$, then $\pi_{t'}(\theta) = 1$ for all θ and $f_{t'h} = f$. Therefore, t' is uninformative and

$C(t') < C(t)$. If on the other hand, $\lambda = 0$, we obtain $\bar{\pi}_{t'} = \frac{\bar{\pi}_t}{1-\mu+\mu\bar{\pi}_t}$ and $f_{t'h} = f_{th}$. To apply the intermediate value theorem and prove our claim, we want to show that

$$\frac{\bar{\pi}_t}{1-\mu+\mu\bar{\pi}_t}c(f_{th}) + (1 - \frac{\bar{\pi}_t}{1-\mu+\mu\bar{\pi}_t})c(f_{t'l}) > \bar{\pi}_t c(f_{th}) + (1 - \bar{\pi}_t)c(f_{tl}).$$

We can use the fact c is convex and $f_{tl} = \mu f + (1 - \mu)f_{t'l}$, to strengthen this inequality to

$$\frac{\bar{\pi}_t}{1-\mu+\mu\bar{\pi}_t}c(f_{th}) + (1 - \frac{\bar{\pi}_t}{1-\mu+\mu\bar{\pi}_t})c(f_{t'l}) > \bar{\pi}_t c(f_{th}) + (1 - \bar{\pi}_t)(\mu c(f) + (1 - \mu)c(f_{t'l})).$$

Using that $C(f) = 0$ and rearranging, we obtain

$$\bar{\pi}_t c(f_{th}) + (1 - \mu)(1 - \bar{\pi}_t)c(f_{t'l}) > 0,$$

which is satisfied. Therefore, by continuity of c , there is $\lambda \in (0, 1)$ that delivers $C(t') = C(t)$. \square