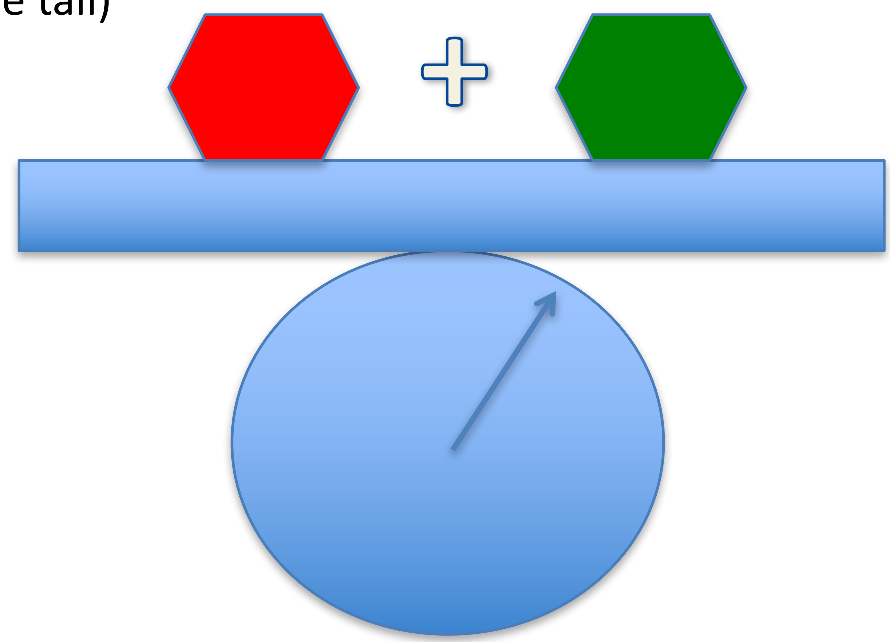
# Radiative corrections: A*X + B vs. A'*X

Sebastian

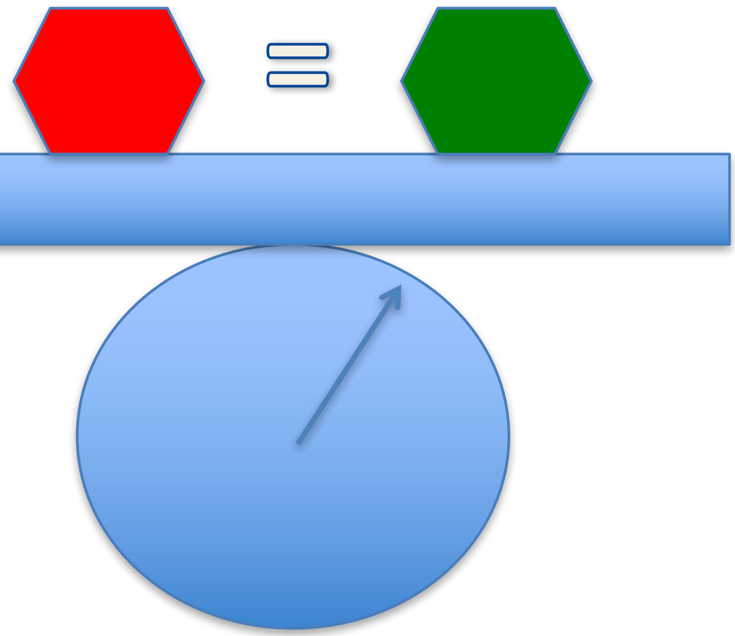
**Consider the following analogy:**

Assume 2 weights, “red” and “green”. We want to measure the weight of “red”, but it always comes tethered with “green”, so we have to put BOTH on the scale together. (E.g., “red” = true inel. events, “green” = excl. radiative tail)



I believe it’s fair to say that most people would measure “red” by measuring the combined weight and then subtracting the weight of “green”. This would correspond to an equation like $W_{\mathrm{Red}} = W_{Meas} - W_{Green}$ where $W_{\mathrm{Red}}$ plays the role of $N_{\mathrm{True}}$ and $W_{\mathrm{Green}}$ is the background (“*b*” in the title). So, if we ignore any scale factor (“acceptance”) for now, this is the extreme case of the formula $N_{meas} = a \cdot N_{true} + b$ with $a = 1$ and $b = W_{\mathrm{Green}}$.

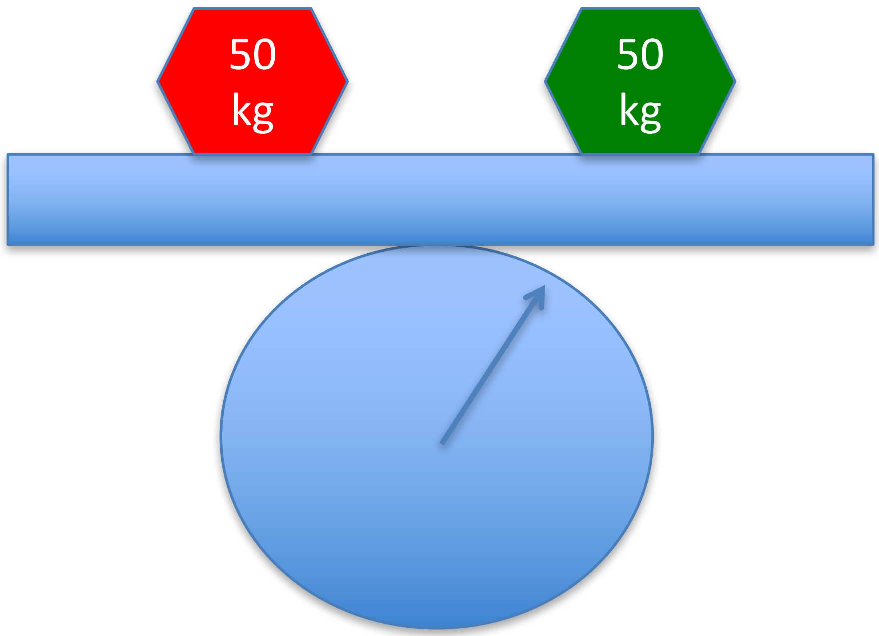
Now, you **may** have a model that predicts, on a fundamental basis, that the two weights **MUST** always be the same (e.g., we KNOW that they are produced in pairs of identical weights). In that case, you can conclude that



$W_{\mathrm{Red}} = \frac{1}{2} W_{Meas}$ or $N_{meas} = a \cdot N_{true} + b$ with *a* = 2 and *b* = 0,

which is the other extreme of the general (generic) equation and corresponds to the often-used "factor only" method. In general, the truth may be somewhere in between these 2 extremes (SOME correlation between the 2 weights, but no strict proportionality) – hence the general form of the equation with both *a* and *b*.

What I claim you should NOT do is use a model that predicts BOTH $W_{\mathrm{Red}}$ and $W_{Green}$ and then take the ratio predicted by the model to get away with a "factor only" correction:




$$W_{\mathrm{Red}} = \frac{50}{50+50} W_{Meas}$$

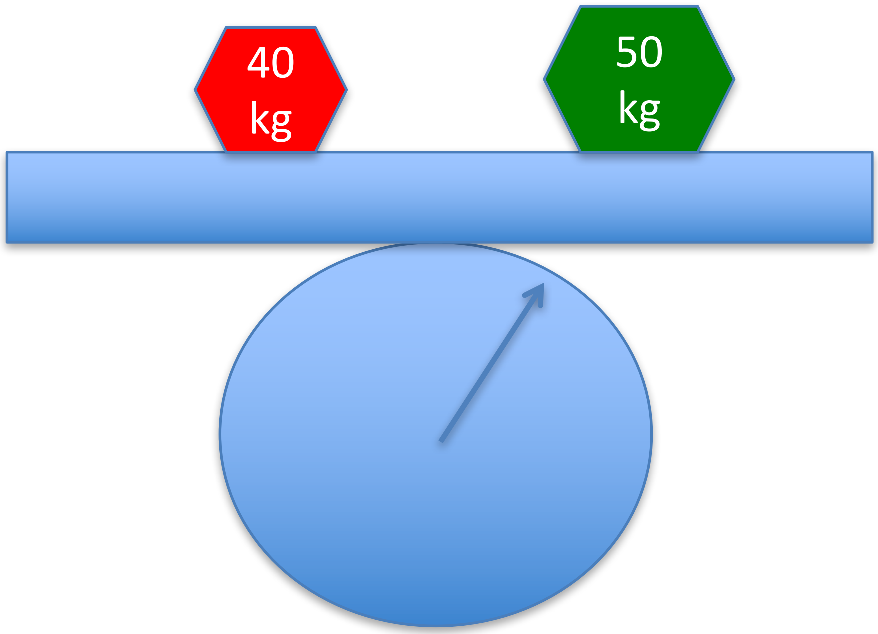
based on

$$N_{meas} = a \cdot N_{true} + b$$

with $a$ = 2 and $b$ = 0

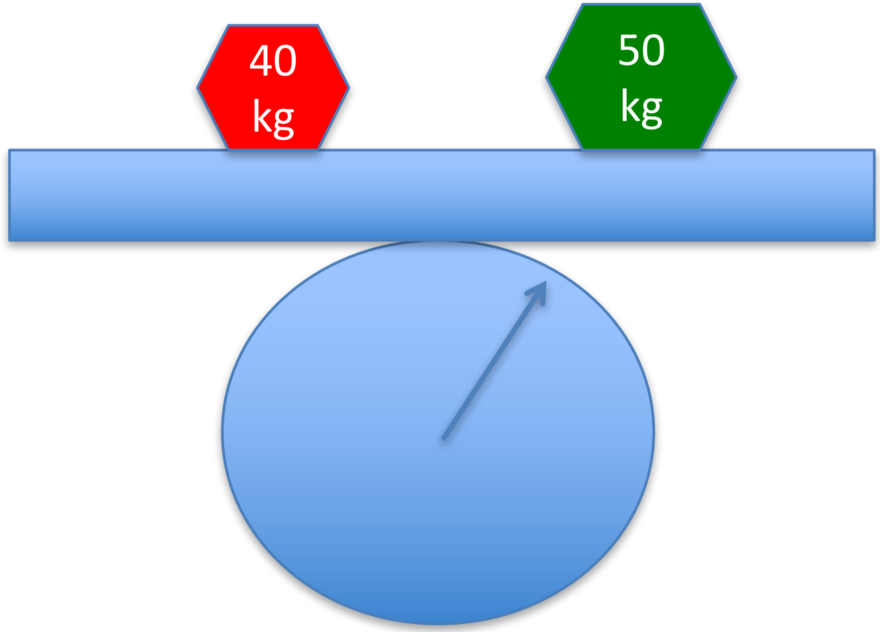
(Here and in the following I assume that the weight of $W_{\mathrm{Green}}$ = 50 kg is known with absolute precision).
Either your model is absolutely correct (and you are certain of that) – in that case, you didn't need to do a measurement in the first place. Or there is some model uncertainty on $W_{\mathrm{Red}}$ and your measurement should yield an unbiased estimator (with equally unbiased experimental uncertainties). As the example on the next 2 pages shows, this is NOT generally the case if you use the "factor only" correction.

**Example:** Assume that the "true" $W_{red}$ = 40 kg but the model predicts 50 kg, so the factor method would still use $W_{Red} = 0.5 \cdot W_{Meas}$ . Let's also assume that the scale has a statistical uncertainty of 10 kg (bad, I know).




Therefore, the expected distribution of $W_{meas}$ is a Gaussian with mean of 90 kg and a standard deviation of 10 kg. Using the "factor only" conversion, we would get a distribution of predicted weights $W_{red}$ with mean 45 kg and standard deviation of 5 kg. Using the formula with
a = 1 and b = 50, i.e. subtracting 50 from the measured weight, we would instead get a distribution with mean 40 and standard deviation of 10. Clearly, the factor method leads to a bias in the mean while the subtraction method is unbiased; however this could be fixed by iterating the model several times until it gets close to the true value. But how about the standard deviation?

**Example continued:**




$$W_{\mathrm{Red}} = 0.5 \cdot W_{Meas}$$

Intuitively, I think it is clear that if the scale has an uncertainty of 10 kg, you cannot claim that you "measured" $W_{red}$ with an uncertainty of only 5 kg. The statistical argument is based on the significance of hypothesis testing. Here is an example: Assume a hypothesis that $W_{red}$ = 60 kg. This corresponds to an expected $W_{meas}$ of 110 kg, given the 50 kg offset. Therefore, if we measure 90 kg +/- 10 kg, the hypothesis would be rejected by 2 standard deviations.

If we convert our measurement to the extracted $W_{red}$ using the subtraction method, we get $W_{red}$ = 40 +/-10, meaning we still reject the hypothesis by 2 standard deviations. However, if we use the factor method, we have $W_{red}$ = 45 +/-5 so the hypothesis would be rejected by THREE standard deviations! If we iterate the model to get closer to the true factor of 0.44 $(W_{\mathrm{Red}} = 40/90 \cdot W_{Meas} = 0.444 \cdot W_{Meas})$ the extracted value gets closer to 40 but the significance gets even more unrealistic – (60-40)/4.44 = 4.5 standard deviations!

# Summary

- In the case where the absolute magnitude of some background (radiative tails, bin in-migration,…) is known, it should be subtracted, not corrected for with a multiplicative factor (only).
- If the background is known to have a fixed ratio to the signal, the “factor method” alone may be sufficient or even more appropriate.
- Having a priori knowledge of the ratio is not the same as having a (“good”) model of both the signal and the background and then correcting with the ratio of signal over the sum (“factor method”).
- While it is true that an imperfect model can be improved iteratively so that the “factor method” eventually gives the same result as the subtraction method, this is not true for the statistical significance of the result which can be severely misinterpreted using simple error propagation following the “factor method”.
- In the most general case, backgrounds should be treated by a combination of multiplicative and additive corrections.