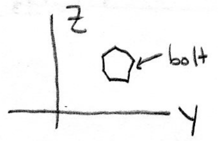
Position Algorithm

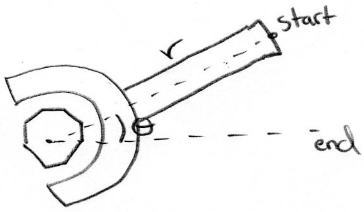
From current accelerometer and gyroscope fusion, we have accurate degrees of rotation.

We will use the degrees of rotation along with length of wrench to estimate displacement.

Scenario 1:

- assuming on YZ plane:

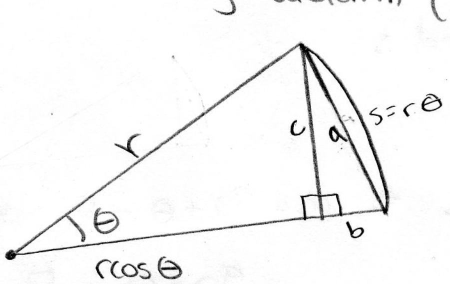




r = length of wrench (cm)

θ = degrees of rotation

- we will be using arclenth ($s = r\theta$) and it very small integrals this is accurate



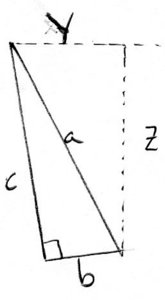
major assumption → $s = r\theta$ is equal to a

$a = r\theta$

Length of a $= r\theta$ holds if θ is small which it will be because processor will run fast.

$c = r\sin\theta$ (known Value)

- Now let's zoom in to abc triangle:



true displacement is Y and Z not C and B

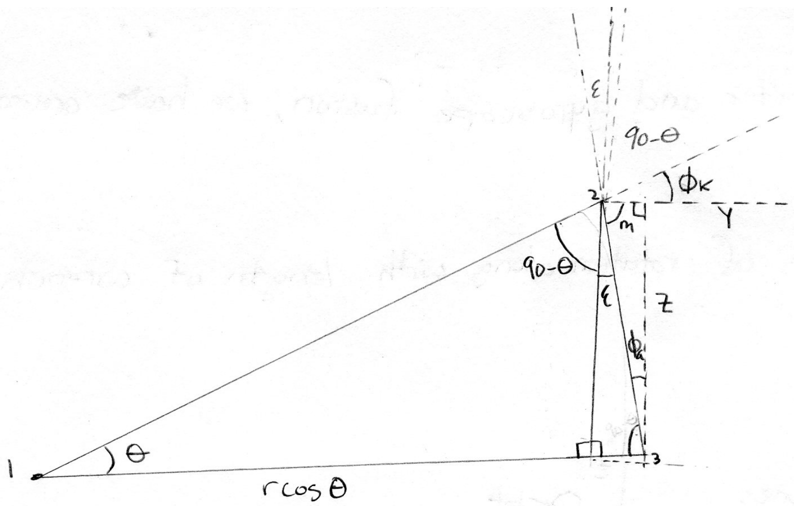
To get X and Z, we have to do Triangulation.

known:

$b = r - r\cos\theta$

$a = r\theta$

$c = r\sin\theta$



ϕ_K is neglected because the smaller the θ the smaller that angle is.

Find:

ϕ_a, Y, Z

Triangle 123 creates an isocelles triangle so

$$180 = 2(90-\theta+\varepsilon)+\theta \quad \text{solve for } \varepsilon$$

$$\varepsilon = \frac{\theta}{2}$$

to get <u>m</u> we will assume k is very small and:

$$180 = m + \varepsilon + 90 - \theta$$

$$90 = m - \theta + \varepsilon \Rightarrow -m = -90 - \theta + \varepsilon \Rightarrow m = 90 + \theta - \varepsilon \quad \text{where } \varepsilon = \frac{\theta}{2}$$

$$= 90 + \theta - \frac{\theta}{2}$$

$$m = 90 - \frac{\theta}{2}$$

Now we know: $Z = r\theta \sin m$ and $Y = r\theta \cos m$

$$Z = r\theta \sin\left(90 - \frac{\theta}{2}\right) \quad \text{and} \quad Y = r\theta \cos\left(90 - \frac{\theta}{2}\right)$$

Now we can do some examples:

ex) assume $r = 25$ cm (10 inch wrench) and angle is θ

$\theta = 2°$ and in radians it is $\frac{2\pi}{180}$ =

$$z = r\theta \sin\left(\frac{\pi}{2} - \frac{10\pi}{180}\right) = (25)\left(\frac{10\pi}{180}\right)\sin\left(\frac{\pi}{2} - \frac{10\pi}{180}\right)$$

$$z = 4.3467 \text{ cm} \downarrow$$

$$Y = r\theta \cos\left(\frac{\pi}{2} - \frac{10\pi}{180}\right)$$

$$Y = 0.3803 \text{ cm} \rightarrow$$

To check our measurements we did a live measurent with p

$Z_{actual} = 4.2$ cm. (live)

$Y_{actual} = 0.30$ cm (rough live estimate)

Error calculation

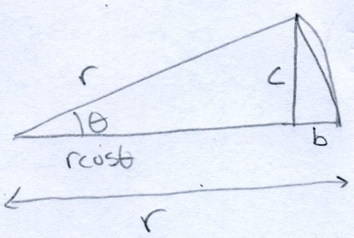
$$P_z(\%) = \left|\frac{4.3467 - 4.2}{4.2}\right| \cdot 100 = 3.49\%$$

We have a 3.49% error, which is perfect!

Now for more simplification →

more testing and more equations, we derived it down:

same example #'s:



$c = r\sin\theta$

$= (25)\sin\left(\frac{10\pi}{180}\right)$

$c = 4.3412\ cm$

, $b = r - r\cos\theta$

$= r(1-\cos\theta)$

$= 25(1-\cos(\frac{1}{1}$

$= 0.3798$

you can see the approximations are very precise so:

$$Y = r\theta\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = r\sin\theta$$

$$Z = r\theta\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = r(1-\cos\theta)$$

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have succesfully determined position of wrench within 0.01 cm

Final equations (YZ plane)

$$\boxed{Y = r\sin\theta}$$

$$\boxed{Z = r(1-\cos\theta)}$$

will be repeated for XZ and XY planes!