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SHORT COMMUNICATIONS

Interpretation of Acceleration Measurements in Inertial Navigation

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Abstract—It is shown that, when the absolute and apparent accelerations are measured simultaneously, the problem of autonomous inertial navigation is reduced to the solvable inverse problem. Physical and geometrical (kinematic) conditions for solvability are formulated. *© 2004 MAIK "Nauka/Interperiodica".*

(1) Modern technology makes optical meters of absolute acceleration feasible [1]. It would be therefore of interest to test them in various applications, such as inertial navigation, where use of apparent-acceleration meters is common practice [2]. In this case, application of the inertial navigation method is essentially reduced to the solution of the direct problem using two (dynamic and kinematic) sets of equations. A solution to the direct problem thus stated is unstable, and straightforward use of the method faces certain difficulties [3].

The situation will change qualitatively if the absolute and apparent accelerations are measured simultaneously. Then, one can identify the inertial navigation method with the inverse problem provided that a model of gravitational field is known.

In this work, we state and discuss the inverse problem.

(2) While on the subject of f, gf, and g meters (measurements) of nongravitational specific forces (apparent acceleration), total specific forces (absolute acceleration), and gravitational specific forces or gravitational field strength (free-fall acceleration), we will consider 3D devices, bearing in mind that a g meter is a combination of f and gf meters [4]. It is reasonable to call these three devices f, gf, and g newtonmeters.

Let $oy = oy_1y_2y_3$ be an orthogonal coordinate rectangle uniquely related to a measuring platform whose constant (in the ideal case) orientation relative to the inertial frame of reference ($o\xi = o\xi_1\xi_2\xi_3$) is maintained by gyros.

Let oy physically simulate $o\xi$ (within a kinematic error) by a β vector of a small angular perturbation of the platform orientation, so that $y = (E + \hat{\beta})\xi$, where E is the unit matrix and $\hat{\beta}\xi = \beta \times \xi$.

Note that the case at hand differs substantially from that considered in [4], where oy physically simulated an attending geographically oriented coordinate trihedron. In essence, oy is an instrumental trihedron; that is, all vector measurements (including measurements by newtonmeters) are made relative to its coordinates. In view of the aforesaid, newtonmeter readings are representable in the form

$$J_{\mathrm{f}} = (E + \hat{\beta})f + \Delta_{\mathrm{f}},$$
$$J_{\mathrm{gf}} = (E + \hat{\beta})(g + f) + \Delta_{\mathrm{gf}}, \quad (1)$$

where g and f are the vectors of the gravitational field strengths and nongravitational specific forces projected onto the axes of the $o\xi$ coordinate system and Δ_{f} and Δ_{gf} are the instrumental errors of measurement.

In view of (1), the measurements of a g newtonmeter obviously take the form

$$J_{\mathrm{g}} = g + \hat{\beta}g + \Delta_{\mathrm{g}}, \quad (2)$$

where $\Delta_{\mathrm{g}} = \Delta_{\mathrm{gf}} - \Delta_{\mathrm{f}}$.

Above all, the gravimetric character of measurements (2) receives attention; however, no consideration will be given to this point, since it goes beyond the scope of this work.

(3) Integration of gf measurements on the axes of the trihedron oy makes it possible to determine the current values of the velocity, v, and position, r, of an object. However, these values involve errors arising from erroneous initial conditions of integration and the instrumental error of a gf newtonmeter.

Later on, we will proceed from the following assumptions (which do not limit the applied value of our considerations): (i) the form of the gravitational potential $U(r)$ is known: $g(r) = \partial U/\partial r = U'(r)$; (ii) gyros and newtonmeters do not introduce instrumental errors; that is, errors in input ($t = 0$) data are the only source of perturbations of the platform spatial orientation and at the integrator output. Then, $\beta = \mathrm{const}$, $\delta v = \delta v_0$, $\delta r = \delta r_0 + \delta v_0 t$, where $\delta v_0 = \delta v(0)$ and $\delta r_0 = \delta r(0)$.

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In view of assumption (i), the inverse problem mentioned above is stated in general form (2); in view of both assumptions, in the form

$$J_g = g(r) - \hat{g}(r)\beta = g(r) - G(r)\hat{\beta}, \qquad (3)$$

where $G(r) = \hat{g}(r)$.

Our aim is to find a phase vector $s = (\beta^T, r^T, v^T)^T$ for both (kinematic and dynamic) sets of equations that constitute the inertial navigation method. Accordingly, we must answer the question as to whether the problem is solvable for the vector s.

(4) Let us return to integration of *gf* measurements, i.e., to the solution of the direct problem. Taking into account the results of integration, we may localize the initially nonlinear problem by constructing a residual vector and passing to the problem in the small according to the following linear model:

$$\delta J_g = g'(r)\delta r + G(r)\beta, \qquad (4)$$

where δJ_g is the residual vector of measurements and $g'(r) = U''(r)$.

Before proceeding further, we will consider a hypothetical case that is similar to the case under discussion and is of independent interest (as will be seen from the following). Let $g(r) = \text{const}$ or $g'(r) = 0$. Then, from (4), we have

$$\delta\tilde{J}_g = G(r)|g|^{-1}\beta = \hat{\tau}\beta = \hat{\tau}\beta^+, \qquad (5)$$

where $\tau = g/|g|$, $\delta\tilde{J}_g = \delta J_g/|g|$, and β^+ is the component of the vector β that is orthogonal to the unit vector τ or, which is the same, to the field line.

As follows from (5), a *g* newtonmeter shows the properties of a “telescope”; that is, it locates the platform up to rotation about the unit vector τ of the “sighting” of a specific “star.”

It is known that, in the case of real telescopes, this problem is completely solved by sighting two or more stars or by sighting one fixed (at $\tau \neq \text{const}$) object whose angular coordinates are known.

In the case of a *g* newtonmeter as a telescope, sighting of several stars is impossible because field lines outside sources do not intersect. However, the case $\tau \neq \text{const}$ seems quite realistic but requires that model (4) be invoked.

In the general case of an arbitrary field, when $g'(r) = U''(r) \neq 0$, model (4) is representable (with regard to assumption (ii)) in the form

$$\delta J_g = W\delta s_0, \qquad (6)$$

where $\delta s_0 = (\beta^T, \delta r_0^T, \delta v_0^T)^T$ and $W = \|G \vdots U'' \vdots U''t\|$.

In the final interval of measurements, the set of equations that is generated by model (6) can be solved for the constant vector δs_0. This statement follows from the fact that the columns of the matrix W as functions of time (paths) are generally linearly independent unless the Hessian $U''(r)$ of the field is nondegenerate. Exceptions are cases when paths for which $\tau = g/|g| = \text{const}$ ($G = \text{const}$) are realized in the time intervals of measurements. In this case, the vector β is a “weak point,” since its component $\beta^- = (\tau^T\beta)\tau$ becomes nonidentifiable.

If, by way of example, we pass from the general case to the specific case of a central field, by which the exterior field of terrestrial gravitation is frequently simulated, the Hessian $U''(r)$ of such a field is nondegenerate (its singular numbers relate as 2 : 1 : 1 [5]) and singular (for the matrix W) trajectories (on which $\tau = \text{const}$) are realized on central straight lines.

Thus, the inverse problem is solvable if two conditions are fulfilled simultaneously. The first one, $\det U''(r) \neq 0$, has the purely physical meaning; the other, $\tau(r) \neq \text{const}$, is of geometric or kinematic (if it is kept in mind that “kinematics is the geometry of motion” [2]) character. The former condition is universal; the latter leaves room for selection.

(5) To conclude, the basic result of this study and its applied value are as follows. Combined use of *f* and *gf* newtonmeters allows one to reduce the inertial navigation method to the inverse problem, which is fundamentally solvable for the phase vector of a set of dynamic and kinematic equations that simulate the evolution of the trajectory and the frame of reference. This offers scope for designing autonomous asymptotically stable operating inertial navigation systems (unlike conventional systems [3]).

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