CS-225: Discrete Structures in CS
Homework 4, Part 1

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Exercise Set 6.1

7-a

<u>Suppositions</u>:

(None)

<u>Goal</u>:

Prove $A \nsubseteq B$.

<u>Deductions</u>:

$4 \in A$

- Let $x \in A$.
- By definition of A, $x \in \mathbb{Z} | x = 6a + 4$.
- Then for $a = 0$, $x = 6(0) + 4 = 4$.
- Hence $4 \in A$.

$4 \notin B$.

- By defintion of B, if $4 \in B$ then $18b - 2 = 4$ for some integer b.
- By algebra, $4 = 18b - 2 \Rightarrow b = \frac{1}{3}$.
- Yet b cannot equal $\frac{1}{3}$ because b is an integer.
- Therefore, $4 \notin B$.

Since $4 \in A$ and $4 \notin B$, $A \nsubseteq B$.

Conclusion:

$A \nsubseteq B$.

7-b

Suppositions:

Let t be a particular but arbitrarily chosen element of B.

Goal:

Prove $B \subseteq A$.

Deductions:

$t \in \mathbb{Z} | t = 18b - 2$ for some integer b.

- By supposition, $t \in B$.
- Then by defintion of B, $t \in \mathbb{Z} | t = 18b - 2$ for some integer b.

$t \in A$.

- Let $a = 3b - 1$.
- By closure, a is an integer because it is the product and summation of integers.
- By definition of A, $6a + 4 \in A$
- By algebra, $t = 6a + 4$:

$$\begin{aligned} 6a + 4 &= 6(3b - 1) + 4 \\ &= 18b - 6 + 4 \\ &= 18b - 2 \\ &= t \end{aligned}$$

- By equality, $t \in A$ because $6a + 4 \in A$.
- Hence $t \in A$.

By definition of subset, $B \subseteq A$ because $t \in A \rightarrow t \in B$.

Conclusion:

Therefore, $B \subseteq A$.

7-c

By definition of set equality, $B = C$ if and only if $B \subseteq C$ and $C \subseteq B$.

Proof: $B \subseteq C$

Suppositions:

Let t be a particular but arbitrarily chosen element of B.

Goal:

Prove $B \subseteq C$.

Deductions:

$t \in \mathbb{Z} | t = 18b - 2$ for some integer b (See problem 7-b for proof).

$t \in C$.

- Let $c = b - 1$.
- By closure, c is an integer because it is the product and summation of integers.
- By definition of C, $18c + 16 \in C$
- By algebra, $t = 18c + 16$:

$$\begin{aligned} 18c + 16 &= 18(b-1) + 16) \\ &= 18b - 18 + 16 \\ &= 18b - 2 \\ &= t \end{aligned}$$

- By equality, $t \in C$ because $18c + 16 \in C$.
- Hence $t \in C$.

By definition of subset, $B \subseteq C$ because $t \in B \rightarrow t \in C$.

Conclusion:

Therefore, $B \subseteq C$.

Seeing that this first proof is complete, do not carry any established variables into the next proof. Looking ahead, the only known variable definitions should be that of B and C, and their respective formulas, from the initial prompt.

Proof: $C \subseteq B$

Suppositions:

Let t be a particular but arbitrarily chosen element of C.

Goal:

Prove $t \in B$.

Deductions:

$t \in \mathbb{Z} | t = 18c + 16$ for some integer c.

- By supposition, $t \in C$.
- Then by defintion of C, $t \in \mathbb{Z} | t = 18c + 16$ for some integer c.

$t \in B$.

- Let $b = c + 1$.
- By closure, b is an integer because it is the product and summation of integers.
- By definition of B, $18b - 2 \in B$.
- By algebra, $t = 18b - 2$:

$$\begin{aligned} 18b - 2 &= 18(c+1) - 2 \\ &= 18c + 18 - 2 \\ &= 18c + 16 \\ &= t \end{aligned}$$

- By equality, $t \in B$ because $18b - 2 \in B$.
- Hence $t \in B$.

By definition of subset, $C \subseteq B$ because $t \in C \to t \in B$.

<u>Conclusion</u>:

Therefore, $C \subseteq B$.

By definition of set equality, $B = C$ because $B \subseteq C$ and $C \subseteq B$.

26 - a

$$\begin{aligned}
\bigcup_{i=1}^{4} R_i &= \bigcup_{i=1}^{4} \left[1, 1 + \frac{1}{i}\right] \\
&= \left[1, 1 + \frac{1}{1}\right] \cup \left[1, 1 + \frac{1}{2}\right] \cup \left[1, 1 + \frac{1}{3}\right] \cup \left[1, 1 + \frac{1}{4}\right] \\
&= [1, 2] \cup \left[1, \frac{3}{2}\right] \cup \left[1, \frac{4}{3}\right] \cup \left[1, \frac{5}{4}\right] \\
&= [1, 2]
\end{aligned}$$

26 - b

$$\begin{aligned}
\bigcap_{i=1}^{4} R_i &= \bigcap_{i=1}^{4} \left[1, 1 + \frac{1}{i}\right] \\
&= \left[1, 1 + \frac{1}{1}\right] \cap \left[1, 1 + \frac{1}{2}\right] \cap \left[1, 1 + \frac{1}{3}\right] \cap \left[1, 1 + \frac{1}{4}\right] \\
&= [1, 2] \cap \left[1, \frac{3}{2}\right] \cap \left[1, \frac{4}{3}\right] \cap \left[1, \frac{5}{4}\right] \\
&= \left[1, \frac{5}{4}\right]
\end{aligned}$$

26 - c

No, the sets R_1, R_2, R_3,... are not mutually disjoint. This is because all sets share the element, 1. In fact, for $i \neq j$, no set R_i is mutually disjoint from R_j.

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