CS-225: Discrete Structures in CS
Assignment 9 Part 1

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Canvas Problem

Graph 1

i. e_2, e_3, e_4, e_5, and e_7

ii. v_1 and v_2

iii. e_2, e_6, and e_7

iv e_3 and e_6

v. $e_3 \parallel e_4$

vi. 6

Graph 2

i. e_2, e_3, e_5, and e_7

ii. v_2, v_3, and v_5

iii. e_8, e_9, and e_{10}

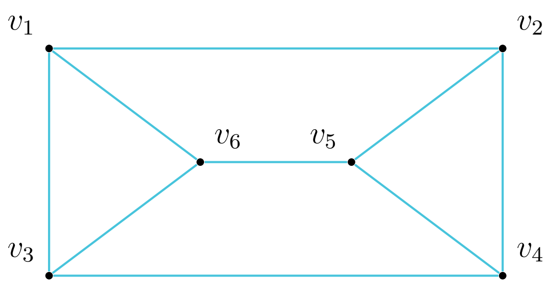
iv. e_{10}

v. None

vi. 4

Exercise Set 4.9

13

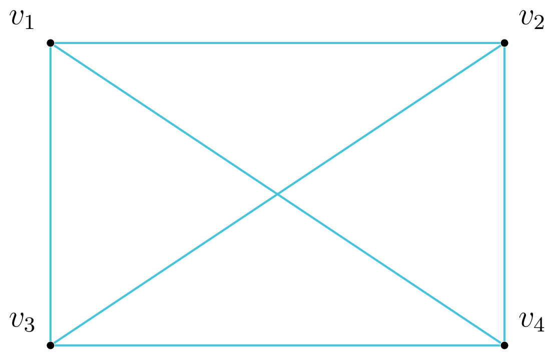


16 - b

Yes.

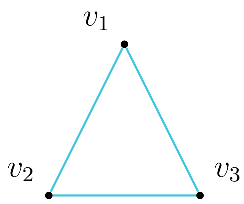
Everyone can be a friend of each other. Since there are 4 total people, each person could have a maximum of 3 friends. In this case where everyone has the maximum, then each person would have exactly 3 friends. In other words, every person of a friend of each other.

Let's visualize this. The vertices a, b, c, and d will represent the 4 people. The edges will represent the friendship between each person. Then in the case where each of the 4 people has exactly 3 friends:



18

Yes, see the following graph:



21 - c

[PROOF BY CONTRADICTION]

Suppositions:

Suppose not. Let n be the number of v_i vertices in the simple graph K, and $n \geq 5$. Then it must be true that all vertices to each have a different degree.

Goal:

Arrive at contradiction.

Deductions:

The maximum degree of every vertex $n - 1$.

. By defintion of simple, each vertex may only to connect to any other vertex once; a vertex may not connect to itself and there may not be multiple connections between any two vertices.

. To maximize the degree of each vertex, each vertex must connect to *all other* vertices once.

. Therefore, each vertex must have a maximum degree of $n - 1$.

Contradiction: It is not possible for all vertices to each have a different degree.

. Let's define the vertices such that the first vertex v_1 has the highest degree, v_2 has the second highest degree, and so on.

. By the previous proof, the higest degree for v_1 would be the maximum value $n - 1$. Then stepping down by 1 for each vertex; v_2 would have a degree of $n-2$, v_3 would have a degree of $n-3$, and so on. Hence, v_n would then have a degree of $n-(n) = 0$.

. However, $deg(v_n) \neq 0$ because the graph is simple.

. Therefore we have arrived at contradiction.

Therefore, it is contradictory to state that all vertices each have a different degree.

Conclusion:

For any integer $n \geq 5$, it is impossible for all vertices to each have a different degree.

23 - e

Presume the following defintion from the textbook for a **complete bipartite graph**:

Let m and n be positive integers. A **complete bipartite graph** on (m, n) vertices, denoted $K_{m,n}$ is a simple graph whose vertices are divided into two distinct subsets, V with m vertices and W with n vertices, in such a way that:

i. Every vertex of $K_{m,n}$ belongs to one of V or W, but no vertex belongs to both V and W.

ii. There is exactly one edge from each vertex of V to each vertex of W.

iii. There is no edge from any one vertex of V to any other vertex of V.

iv. There is no edge from any one vertex of W to any other vertex of W.

Let's now count the total number of degrees in $K_{m,n}$.

. By (ii) and (iii), each vertex in V has n connections since each vertex in V is connected to every vertex in W.

. Since there are m vertices in V and n vertices in W, the total number of degrees in V is mn.

. By (ii) and (iv), each vertex in W has m connections since each vertex in W is connected to every vertex in n.

. Again, since there are m vertices in V and n vertices in W, the total number of degrees in W is mn.

. Hence, the total number of degrees in $K_{m,n}$ is $deg(V) + deg(W) = mn + mn = 2mn$.

Therefore, the total number of degrees in $K_{m,n}$ is $2mn$.

23 - f

Since the total number of degrees is $2mn$, and every two degrees comprise its own edge, then the total number of edges is $\frac{2mn}{2} = mn$.

24 - d

This graph is not bipartite.

Suppositions:

Suppose not. Suppose the graph from Exercise 4.9.24, denoted as F. Then F must be bipartite.

Goal:

Arrive at contradiction.

Deductions:

Denote the defintion of a **bipartite graph** as follows:

Let m and n be positive integers. A **bipartite graph** on (m, n) vertices, denoted $K_{m,n}$ is a simple graph whose vertices are divided into two distinct subsets, V with m vertices and W with n vertices, in such a way that:

i. Every vertex of $K_{m,n}$ belongs to one of V or W, but no vertex belongs to both V and W.

ii. There is no edge from any one vertex of V to any other vertex of V.

iii. There is no edge from any one vertex of W to any other vertex of W.

Contradiction: The vertices v_2, v_3, and v_4 cannot be properly partitioned in the bipartite graph F.

. In the graph; v_2, v_3, and v_4 are each connected to each other. In other words, each of these three vertices would have a degree of 2 if we werer to exclude all other vertices.

. By (i), the vertices v_2, v_3, and v_4 must be between V and W.

. Yet by (ii) and (iii), there cannot be two vertices in either V or W that are connected to each other.

. Since there must be any two of v_2, v_3, and v_4 in the same set, then in either V or W there must be two vertices connected to each other.

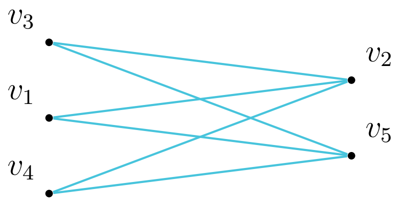
. Hence, there is a contradiction.

Therefore, it is contradictory to state that the vertices v_2, v_3, and v_4 can be properly partitioned in the bipartite graph F.

Conclusion:

The graph is not bipartite.

24 - e



The nodes on the left column ($v3$, $v1$, and $v4$) are part of V and the nodes on the right column ($v2$ and $v5$) are part of W.