The Solutions

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Problem 1

Question

Divide. Check your answer by multiplication.

$$\frac{x^2 - 5x - 10}{x - 8}$$

Answer

$$\frac{x^2 - 5x - 10}{x - 8}$$

$$\frac{(x - 8)(x + 3) + 14}{x - 8}$$

$$\frac{(x - 8)(x + 3)}{x - 8} + \frac{14}{x - 8}$$

$$x + 3 + \frac{14}{x - 8}$$

Now, you're probably wondering how I miraculously pulled the expression $(x - 8)(x + 3) + 14$ out of my ass. Here's how can do this too:

Let's start by focusing on the numerator: $x^2 - 5x - 10$. When you typically factor out these kinds of expressions, you're probably doing something like this in your head:

$$x^2 - 5x - 10 = (x + a)(x + b)$$

Where,

$$\begin{aligned}(a+b)x &= -5x \\ ab &= -10\end{aligned}$$

Or, more simply:

$$\begin{aligned}a+b &= -5 \\ ab &= -10\end{aligned}$$

In other words, you're probably thinking 'What two numbers (a and b) multiply to get -10 and add to get -5?' But now you're at a dead end, there's no simple solution here.

Here's a better approach:

$$x^2 - 5x - 10 = (x+a)(x+b) + c$$

Where,

$$\begin{aligned}a+b &= -5 \\ ab+c &= -10\end{aligned}$$

Compare this with the old approach. The only difference is that we have a bonus $+c$ to get -10. This makes things a lot easier.

Let's continue solving with this approach and you should see what I mean:

$$x^2 - 5x - 10 = (x+a)(x+b) + c$$

Since we want to cancel out the $x-8$ in the denominator, we should set $a = -8$:

$$x^2 - 5x - 10 = (x-8)(x+b) + c$$

(Now I know that this doesn't *fully* cancel out the $x-8$ in the denominator. But frankly, that $+c$ can be pulled out on its own later on).

Next, we still want a and b to summate to be -5. Or rather, we want to satisfy our equation from before:

$$a+b = -5$$

Plugging in -8 for a, and solving:

$$\begin{aligned}a+b &= -5 \\ -8+b &= -5 \\ b &= 3\end{aligned}$$

Now that we've determined a and b, let's solve for c using the other equation from before:

$$\begin{aligned} ab + c &= -10 \\ (-8)(3) + c &= -10 \\ -24 + c &= -10 \\ c &= 14 \end{aligned}$$

Great! Let's put this all back into the equation:

$$\begin{aligned} x^2 - 5x - 10 &= (x + a)(x + b) + c \\ x^2 - 5x - 10 &= (x - 8)(x + 3) + 14 \end{aligned}$$

And there you have it. You factored out the expression. You can continue solving the division problem.

Of course, you're going to end up with $\frac{14}{x-8}$ in your result. But I'd like to think the final answer is nonetheless better.

Problem 2

Question

Divide.

$$\frac{2t^3 - t + 11}{t + 3}$$

Answer

I'm going to take a slightly more advanced approach from Problem 1 and factor out the numerator like so:

$$2t^3 - t + 11 = (t + 3)(at^2 + bt + c) + d$$

We obviously want the numerator to have some factor of $(t + 3)$ in it. But since the expression has a highest exponent of 3, we are expecting to factor out some high order expression. Hence, we use $(at^2 + bt + c)$.

Let's continue:

$$2t^3 - t + 11 = (t + 3)(at^2 + bt + c) + d$$

This tells us that:

$$\begin{aligned} 3c + d &= 11 \\ (3b + c)t &= -t \\ (3a + b)t^2 &= 0t^2 \\ at^3 &= -2t^3 \end{aligned}$$

Or, more simply:

$$\begin{aligned} 3c + d &= 11 \\ 3b + c &= -1 \\ 3a + b &= 0 \\ a &= -2 \end{aligned}$$

Notice we have four equations with four unknowns. I'm not going to show the simple algebra but the result of these four equations gives us:

$$\begin{aligned} a &= -2 \\ b &= -6 \\ c &= 17 \\ d &= -40 \end{aligned}$$

Plugging it back in:

$$2t^3 - t + 11 = (t + 3)(-2t^2 - 6t + 17) - 40$$

Going back to the original problem:

$$\begin{aligned} \frac{2t^3 - t + 11}{t + 3} &= \frac{(t + 3)(-2t^2 - 6t + 17) - 40}{t + 3} \\ &= -2t^2 - 6t + 17 - \frac{40}{t + 3} \end{aligned}$$