Assignment 1, Part 1: The Logic of Compound Statements

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January 14, 2024

Exercise Set 2.1

5 - b

"She is a mathematics major." is a proposition.

5 - c

$128 = 2^6$ is a proposition.

5 - d

$x = 2^6$ is not a proposition. The statement's truth value depends on the value of x. Since x may or may not be 128, the equation may be either true and false.

8 - c

$\sim h \wedge \sim w \wedge \sim q$

10 - e

$\sim p \vee (q \wedge r)$

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The dollar is not at an all-time high or the stock is not at a record low.

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$(x < -7) \vee (x \geq 0)$

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$(num\_orders \geq 50$ or $num\_instock \leq 300)$ and
$((num\_orders < 50$ or $num\_orders \geq 75)$ or $(num\_instock \leq 500)$

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p	q	r	$\sim p$	$\sim q$	$\sim p \wedge q$	$q \wedge r$	$((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$
T	T	T	F	F	F	T	F
T	T	F	F	F	F	F	F
T	F	T	F	T	F	F	F
T	F	F	F	T	F	F	F
F	T	T	T	F	T	T	F
F	T	F	T	F	T	F	F
F	F	T	T	T	F	F	F
F	F	F	T	T	F	F	F

The formula $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$ is a **contradiction**. This can also be understood considering the following proof:

$$\begin{aligned}((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q \\ (q \wedge \ \sim q) \wedge (\sim p \wedge q \wedge r) \\ \mathbf{c} \wedge (\sim p \wedge q \wedge r) \\ \mathbf{c}\end{aligned}$$

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a. Commutative Law

b. Distributive Law

c. Negation Law

d. Identity Law

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$$\begin{aligned} p &\equiv (p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q) && \\ &\equiv (p \wedge (p \wedge \sim q))) \vee (p \wedge q) && \text{De Morgan's Law} \\ &\equiv ((p \wedge p) \ \wedge \sim q) \vee (p \wedge q) && \text{Associative Law} \\ &\equiv (p \ \wedge \sim q) \vee (p \wedge q) && \text{Idempotent Law} \\ &\equiv p \wedge (\sim q \vee q) && \text{Distributive Law} \\ &\equiv p \wedge \mathbf{c} && \text{Negation Law} \\ &\equiv p && \text{Universal Bound Law} \end{aligned}$$

Canvas Problem

$$
\begin{aligned}
\sim p &\equiv ((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (\sim p \wedge q) & \\
&\equiv (\sim p \wedge \sim q) \vee ((\sim p \wedge q) \vee (\sim p \wedge q)) & \text{Commutative Law} \\
&\equiv ((\sim p \wedge q) \vee (\sim p \wedge q)) \vee (\sim p \wedge \sim q) & \text{Associative Law} \\
&\equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q) & \text{Idempotent Law} \\
&\equiv \sim p \wedge (q \vee \sim q) & \text{Distributive Law} \\
&\equiv \sim p \wedge \mathbf{t} & \text{Negation Law} \\
&\equiv \sim p & \text{Identity Law}
\end{aligned}
$$