CS-225: Discrete Structures in CS
Assignment 8 Part 1

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Exercise Set 9.5

8 - b

i.

$$\binom{6}{4}\binom{8}{6} = 420$$

ii. All possible choices of 10 must contain at least one:

$$\binom{14}{10} = 1001$$

iii.

$$\binom{6}{2}\binom{8}{8} + \binom{6}{3}\binom{8}{8}$$

8 - c

Q1 only:

$$Q_1 = \binom{12}{9}$$

Q2 only:

$$Q_2 = \binom{12}{9}$$

Nether Q1 or Q2:

$$Q_{neither} = \binom{12}{10}$$

Total:

$$Q1 + Q2 + Q_{neither} = \binom{12}{9} + \binom{12}{9} + \binom{12}{10}$$

12

An even sum occurs when either two even or two odd numbers are summated. There are 50 even numbers and 51 odd numbers in this list.

Summation of two even numbers:

$$\binom{50}{2}$$

Summation of two odd numbers:

$$\binom{51}{2}$$

<u>Total</u>:

$$\binom{50}{2} + \binom{51}{2}$$

17 - c

$$\binom{10}{3}$$

17 - d

$$\binom{9}{3}$$

Exercise Set 9.6

14

Every value must be greater than or equal to 10. Hence, we're distributing 450 elementsacross 5 locations:

$$\binom{450+5-1}{450}$$

19 - a

Let T represent the set of all possible selections of 20 pastries where there are 6 pastry types.
Let E_i represent the set of all selections within T where each selection contains i eclairs.

Then the total number of ways to select 20 of 6 types of pastries:

$$N(T) = \binom{20+6-1}{20}$$

Number of ways to select 20 of 6 types of pastries where 11 or more are eclairs. This is equivocal to selecting 9 of 6 types of pastries:

$$N(E_{\geq 11}) = \binom{9+6-1}{9}$$

Answer: Now we can calculate the number of ways to select 20 of 6 types of pastries where at most 10 are eclairs:

$$\begin{aligned}
E_{\leq 10} &= T \cap E^c_{\geq 11} \\
&= T - E_{\geq 11} \\
N(E_{\leq 10}) &= N(T - E_{\geq 11}) \\
&= N(T) - N(E_{\geq 11}) \\
&= \binom{20+6-1}{20} - \binom{9+6-1}{9}
\end{aligned}$$

19 - b

Let S_i represent the set of all possible selections of pastries where each selection contains i napolean slices.

Next, let's calculate the number of ways to select 20 of 6 types of pastries where 9 or more are napolean slices. This is equivocal to selecting 11 of 6 types of pastries:

$$N(S_{\geq 9}) = \binom{11+6-1}{11}$$

Answer: Number of ways to select 20 of 6 types of pastries where at most 8 are napolean slices and at most 10 are eclairs.

$$\begin{aligned}
E_{\leq 10} \cap S_{\leq 8} &= E_{\leq 10} - S_{\geq 9} \\
N(E_{\leq 10} - S_{\leq 9}) &= N(E_{\leq 10}) - N(S_{\geq 9}) \\
&= \binom{20+6-1}{20} - \binom{9+6-1}{9} - \binom{11+6-1}{11}
\end{aligned}$$

20 - a

Let T represent the set of all possible selections of 30 batteries where there are 8 battery types.

Let A_i represent the set of all selectrions within T where each selection contains i A76 batteries.
The total number of ways to select 30 of 8 types of batteries:

$$N(T) = \binom{30+8-1}{30}$$

Now we can calculate the number of ways to select 30 of 8 types of batteries where at 11 or more are A76 batteries. This is equivocal to selecting 19 of 8 types of batteries:

$$N(A_{\geq 11}) = \binom{19+8-1}{19}$$

<u>Answer</u>: Finally, let's calculate the number of ways to select 30 of 8 types of batteries where at most 10 are A76 batteries:

$$\begin{aligned} A_{\leq 10} &= T \cap A^c_{\geq 11} \\ &= T - A_{\geq 11} \\ N(A_{\leq 10}) &= N(T - A_{\geq 11}) \\ &= N(T) - N(A_{\geq 11}) \\ &= \binom{30+8-1}{30} - \binom{19+8-1}{19} \end{aligned}$$

20 - b

Let D_i represent the set of all possible selections of batteries where there are i D303 batteries.

Number of ways to select 30 of 8 types of batteries where 7 or more are D303 batteries. This is equivocal to selecting 23 of 8 types of batteries:

$$N(D_{\geq 7}) = \binom{23+8-1}{23}$$

<u>Answer</u>: Number of ways to select 30 of 8 types of batteries where at most 6 are D303 batteries and at most 10 are A76 batteries.

$$\begin{aligned} A_{\leq 10} \cap D_{\leq 6} &= A_{\leq 10} - D_{\geq 7} \\ N(A_{\leq 10} - D_{\leq 6}) &= N(A_{\leq 10}) - N(D_{\geq 7}) \\ &= \binom{30+8-1}{30} - \binom{19+8-1}{19} - \binom{23+8-1}{23} \end{aligned}$$