# CS-225: Discrete Structures in CS
Initial- Post Wk 5 & 6 Problem Sets #6 and #15

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## 6

### Question

Prove the following statement by mathematical induction:

$$\forall n \in \mathbb{N}, \quad \sum_{i=1}^{n} i!(i^2+1) = (n+1)!n$$

### Answer

Proof:

$P(n) \equiv \sum_{i=1}^{n} i!(i^2+1) = (n+1)!n$ for every integer $n \geq 1$

Basis Step:

For the case $P(1)$:

$$\begin{aligned}\sum_{i=1}^{1} i!(i^2+1) &= (1+1)1! \\ 1!(1^2+1) &= (2)(1) \\ (1)(2) &= 2 \\ 2 &= 2\end{aligned}$$

The base case $P(1)$ holds true.

Inductive Hypothesis:

Suppose that for an arbitrary but particular integer $k$,

$$P(k) \equiv \sum_{i=1}^{k} i!(i^2+1) = (k+1)!k \text{ for every integer } k \geq 1$$

is true.

Inductive Steps:

We must show that $P(k+1)$ is true. Hence we must demonstrate,

$$P(k+1) \equiv \sum_{i=1}^{k+1} i!(i^2+1) = (k+2)!(k+1) \text{ for every integer } k \geq 1$$

- Let the expression $\sum_{i=1}^{k+1} i!(i^2+1)$ be referred to the left-hand side (LHS) of the equation.
- Let the expression $(k+2)!(k+1)$ be referred to the right-hand side (RHS) of the equation.
- To show that $P(k+1)$ is true, we must show that the LHS is equal to the RHS.

The LHS simplifies to the RHS

- Recall the LHS is $\sum_{i=1}^{k+1} i!(i^2+1)$
- By algebra and supposition of $P(k)$, the LHS can be simplified:

$$\sum_{i=1}^{k+1} i!(i^2+1) = \sum_{i=1}^{k} i!(i^2+1) + (k+1)!((k+1)^2+1)$$

By inductive hypothesis, $\sum_{i=1}^{k} i!(i^2+1) \Rightarrow (k+1)!k$. Continuing,

$$\begin{aligned}
&= \sum_{i=1}^{k} i!(i^2+1) + (k+1)!((k+1)^2+1) \\
&= (k+1)!k + (k+1)!(k^2+2k+1+1) \\
&= (k+1)!(k^2+3k+2) \\
&= (k+1)!(k+1)(k+2) \\
&= (k+2)!(k+1)
\end{aligned}$$

- Recall the RHS is $(k+2)!(k+1)$

- Thus, the LHS is equal to the RHS.

Thus, $P(k+1) \equiv \sum_{i=1}^{k+1} i!(i^2+1) = (k+2)!(k+1)$ for every integer $k \geq 1$ was to be shown.

Conclusion:

Since both the basis step and the inductive step have been proved, the original expression,

$$P(n) \equiv \sum_{i=1}^{n} i!(i^2+1) = (n+1)!n \text{ for every integer } n \geq 1$$

is true.

# 15

### Question

i. Give a recursive definition for the set of all strings of 0's and 1's that have more 0's than 1's.

ii. Give a recursive definition for the set of all strings of $a$'s and $b$'s that begins with an $a$ and ends in a $b$.

iii. Give a recursive definition for the set of all strings of 0's and 1's that are odd palindromes

### Answer

**i.**

I - Base: The string 0 is in set $S$.

II - Recursion: New strings are formed according to the following rules:

a. If $u$ is any string in $S$, then

$$0u \text{ and } u0 \text{ are strings in } S$$

where $0u$ and $u0$ are the concatenations of $u$ and 0. Each is obtained by appending 0 to the let side of $u$ and the right side of $u$ respectively.

b. If $u$ is any string in $S$, then

$$01u,\ 10u,\ u01,\ u10,\ 0u1, \text{ and } 1u0 \text{ are strings in } S$$

where all six of these are the concatenations of $u$, 0, and 1. Each is obtained by appending 0 and 1 to the let side of $u$ and the right side of $u$.

III - Restriction: Every string in $S$ is obtainable from the base and the recursion.

**ii.**

I - Base: The string $ab$ is in set $S$.

II - Recursion: New strings are formed according to the following rules:

a. If $u$ is any string in $S$, then

$$au \text{ and } ub \text{ are strings in } S$$

where $au$ and $ub$ are the concatenations of $u$ with $a$ and $b$ respectively.

b. If $u$ and $v$ are any strings in $S$, then

$$uv \text{ is a string in } S$$

where $uv$ is the concatenation of $u$ and $v$.

III - Restriction: Every string in $S$ is obtainable from the base and the recursion.

**iii.**

I - Base: The strings 0 and 1 are in set $S$.

II - Recursion: New strings are formed according to the following rules:

a. If $u$ is any string in $S$, then

$$0u0 \text{ is a string in } S$$

where $0u0$ is the concatenation of $u$ and 0. $0u0$ is obtained by appending 0 to the left side of $u$ and the right side of $u$.

b. If $u$ is any string in $S$, then

$$1u1 \text{ is a string in } S$$

where $1u1$ is the concatenation of $u$ and 1. $1u1$ is obtained by appending 1 to the left side of $u$ and the right side of $u$.

III - Restriction: Every string in $S$ is obtainable from the base and the recursion.