CS-225: Discrete Structures in CS

Homework 2, Part 2

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Canvas Problems

1

For the sequence,

$$\frac{1}{4}, \frac{3}{12}, \frac{5}{36}, \frac{7}{108}, \frac{9}{324}, \ldots$$

The corresponding sequence formula is,

$$a_n = \frac{2n+1}{4 \cdot 3^n} \quad \text{for every integer } n \geq 0$$

For the sequence,

$$0, -\frac{1}{3}, \frac{2}{4}, -\frac{3}{5}, \frac{4}{6}, -\frac{5}{7}, \ldots$$

The corresponding sequence formula is,

$$a_n = \frac{(-1)^n \cdot n}{n+2} \quad \text{for every integer } n \geq 0$$

2

Given,

$$\sum_{k=1}^{n} \frac{2k+1}{k^2(k+1)^2}$$

We can use partial fractions to simplify the inside expression. Lets start by assuming,

$$\frac{2k+1}{k^2(k+1)^2} = \frac{A}{k^2} + \frac{B}{(k+1)^2}$$

Now solving for A and B,

$$\frac{2k+1}{k^2(k+1)^2} = \frac{A}{k^2} + \frac{B}{(k+1)^2}$$

$$\frac{2k+1}{k^2(k+1)^2} = \frac{A(k+1)^2}{k^2(k+1)^2} + \frac{Bk^2}{k^2(k+1)^2}$$

$$\begin{aligned}
2k+1 &= A(k+1)^2 + Bk^2 \\
2k+1 &= A(k^2+2k+1) + Bk^2 \\
2k+1 &= Ak^2 + 2Ak + A + Bk^2 \\
2k+1 &= (A+B)k^2 + A(2k+1)
\end{aligned}$$

Solving for A,

$$\begin{aligned}
2k+1 &= A(2k+1) \\
A &= 1
\end{aligned}$$

Solving for B when $A = 1$,

$$\begin{aligned}
0 &= (A+B)k^2 \\
0 &= (1+B)k^2 \\
0 &= 1+B \\
B &= -1
\end{aligned}$$

Substituting $A = 1$ and $B = -1$ into the original expression,

$$\frac{2k+1}{k^2(k+1)^2} = \frac{A}{k^2} + \frac{B}{(k+1)^2}$$

$$\frac{2k+1}{k^2(k+1)^2} = \frac{1}{k^2} - \frac{1}{(k+1)^2}$$

Substituting this new expression into the original summation formula,

$$\sum_{k=1}^{n} \frac{2k+1}{k^2(k+1)^2} = \sum_{k=1}^{n} \left(\frac{1}{k^2} - \frac{1}{(k+1)^2} \right)$$

$$= \left(\frac{1}{1} - \frac{1}{4} \right) + \left(\frac{1}{4} - \frac{1}{9} \right) + \left(\frac{1}{9} - \frac{1}{16} \right) + ... + \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right)$$

$$= \frac{1}{1} + \left(\frac{1}{4} - \frac{1}{4} \right) + \left(\frac{1}{9} - \frac{1}{9} \right) + ... + \left(\frac{1}{n^2} - \frac{1}{n^2} \right) - \frac{1}{(n+1)^2}$$

$$= 1 - \frac{1}{(n+1)^2}$$

3

$$\sum_{i=30}^{500} \left(10i - \frac{5}{2} \right) = \sum_{i=30}^{500} 10i - \sum_{i=30}^{500} \frac{5}{2}$$

$$= 10 \sum_{i=30}^{500} i - \frac{5}{2}(500 - 30 + 1)$$

$$= 10 \sum_{i=30}^{500} i - \frac{5}{2}(500 - 30 + 1)$$

$$= 10 \left(\sum_{i=1}^{500} i - \sum_{i=1}^{29} i \right) - \frac{5}{2}(500 - 30 + 1)$$

$$= 10 \left(\frac{500(500+1)}{2} - \frac{29(29+1)}{2} \right) - \frac{5}{2}(500 - 30 + 1)$$

4

$$\sum_{j=0}^{200}(200j^2 - (-20)^j) = \sum_{j=0}^{200} 200j^2 - \sum_{j=0}^{200}(-20)^j$$

$$= 200\sum_{j=0}^{200} j^2 - \sum_{j=0}^{200}(-20)^j$$

$$= 200\left(\frac{200(200+1)(2\cdot 200+1)}{6}\right) - \left(\frac{(-20)^{200+1}}{-20-1}\right)$$

5

$$4\sum_{k=1}^{15}(14k^2+9) + 6\sum_{k=1}^{15}(15k^2-7) = \sum_{k=1}^{15}(56k^2-36) + \sum_{k=1}^{15}(90k^2-42)$$

$$= \sum_{k=1}^{15}(56k^2 - 36 + 90k^2 - 42)$$

$$= \sum_{k=1}^{15}(146k^2 - 78)$$

$$= \sum_{k=1}^{15} 146k^2 - \sum_{k=1}^{15} 78$$

$$= 146\sum_{k=1}^{15} k^2 - 78\sum_{k=1}^{15} 1$$

$$= 146\left(\frac{15(15+1)(2\cdot 15+1)}{6}\right) - 78(15-1+1)$$