CS-225: Discrete Structures in CS

Homework 2, Part 1

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January 23, 2024

Exercise Set 3.1

29 - b

There exists a geometric figure that is a rectangle and is a square. **True**, all squares are rectangles.

29 - c

There exists a geometric figure that is a rectangle and is a not square. **True**, a *long* rectangle is not a square since not all side lengths are equal.

30 - a

There exists an integer such that it is prime and it is not odd. **True**, the number 2 is prime and not odd.

30 - c

There exists an integer such that it is odd and is is a perfect square. **True**, the number 25 is odd and is a perfect square.

Exercise Set 3.2

8***

Informal Negation: There exists a simple solution to life's problems.

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Informal Negation: For all real numbers x_1 and x_2, if $x_1 = x_2$ then $x_1^2 = x_2^2$.

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Original Statement: $\forall$ integer n, if n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

Let n be a variable and $\mathbb{Z}$ be the set of all integers.
Let $P(n)$ be the statement "if n is divisible by 6."
Let $Q(n)$ be the statement "if n is divisible by 2."
Let $R(n)$ be the statement "if and n is divisible by 3."

The Original Statement can then be formulated as $\forall n \in \mathbb{Z}, P(n) \to (Q(n) \wedge R(n))$

Converse: $\forall n \in \mathbb{Z}, (Q(n) \wedge R(n)) \to P(n)$
Inverse: $\forall n \in \mathbb{Z}, \sim P(n) \to \ (\sim Q(n) \vee \sim R(n))$
Contrapositive: $\forall n \in \mathbb{Z}, (\sim Q(n) \vee \sim R(n)) \to \sim P(n)$

Rewriting these verbally,

Converse: $\forall$ integer n, if n is divisible by 2 and n is divisible by 3, then n is divisible by 6. This statement is **true**.

Inverse: $\forall$ integer n, if n is not divisible by 6, then neither n is divisible by 2 nor n is divisible by 3. This statement is **false** for the counterexample $n = 2$.

Contrapositive: $\forall$ integer n, if neither n is divisible by 2 nor n is divisible by 3, then n is not divisible by 6. This statement is **true**.

44

If a polygon is a square, then it has four sides.

48

Original Statement: Being a polynomial is not a sufficient condition for a function to have a real root.

Let f be a variable and F be the set of all functions.
Let $P(f)$ be the statement "the function is a polynomial."
Let $Q(f)$ be the statement "the function has a real root."

The Original Statement can then be formulated as $\sim (\forall f \in F, P(f) \to Q(f))$.

Reformulating to not use the confitional symbology,

$$\sim (\forall f \in F, P(f) \to Q(f)) \equiv \exists f \in F, P(f) \wedge \sim Q(f)$$

Rewriting this verbally,

Answer: There exists a function such that the function is a polynomial and the function does not have a real root.

49

Original Statement: The absence of error messages during translation of a computer program is only a necessary and not a sufficient condition for reasonable [program] correctness.

Let x be a variable and C be the set of all computer programs.
Let $P(x)$ be the statement "there is an absence of error messages during translation"
The $Q(x)$ be the statement "there is reasonable correctness."

The Original Statement is the conjunction of the both following,
Necessary: $\forall x \in C, \sim P(x) \to \sim Q(x) \equiv \forall x \in C, P(x) \vee \sim Q(x)$
Not Sufficient: $\sim (\forall x \in C, P(x) \to Q(x)) \equiv \exists x \in C, P(x) \wedge \sim Q(x)$
The Original Statement can therefore be formulated as,

$$(\forall x \in C, P(x) \vee \sim Q(x)) \wedge (\exists x \in C, P(x) \wedge \sim Q(x))$$

Rewriting this verbally,

Answer: For all computer programs, there is an absence of error messages during translation or there is no reasonable correctness. And, there exists a computer program such that there is an absence of error messages during translation and there is no reasonable correctness.

Canvas Problem

a

$\forall x \in D, C(x) \to A(x)$

b

$\forall x \in D, C(x) \to (F(x) \wedge H(x))$

c

$\forall x \in D, \sim (A(x) \to L(x))$

d

$\exists x \in D, C(x) \to (\sim H(x) \vee \sim F(x))$

e

$\forall x \in D, C(x) \to A(x)$