CS-225: Discrete Structures in CS
Homework 4, Part 2

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February 6, 2024

Exercise Set 6.2

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Suppositions:

Suppose sets A, B, and C are arbitrarily chosen sets.

Goal:

Prove $(A - B) \cap (C - B) = (A \cap C) - B$.

Deductions:

By set equality, $(A - B) \cap (C - B) = (A \cap C) - B$ is true if and only if each side of the equation is a subset of the other. Hence, the following set relations must be proved:

$$(A - B) \cap (C - B) \subseteq (A \cap C) - B \tag{i}$$

and

$$(A \cap C) - B \subseteq (A - B) \cap (C - B) \tag{ii}$$

Proving (i) is to prove that $\forall x$, if $x \in (A - B) \cap (C - B)$ then $x \in (A \cap C) - B$.
Proving (ii) is to prove that $\forall x$, if $x \in (A \cap C) - B$ then $x \in (A - B) \cap (C - B)$.

(i) $\forall x$, if $x \in (A - B) \cap (C - B)$ then $x \in (A \cap C) - B$.

- Suppose $x \in (A - B) \cap (C - B)$.
- By definition of intersection, $x \in A - B$ and $x \in C - B$.
- By definition of difference; $x \in A$, $x \in C$, and $x \notin B$.

- By definition of intersection, $x \in A \cap C$.
- By definition of difference, $x \in (A \cap C) - B$.
- Hence, $x \in (A \cap C) - B$.

(ii) $\forall x$, if $x \in (A \cap C) - B$ then $x \in (A - B) \cap (C - B)$.

- Suppose $x \in (A \cap C) - B$.
- By definition of difference, $x \in A \cap C$ and $x \notin B$.
- By definition of intersection, $x \in A$ and $x \in C$.
- By definition of difference, $x \in (A - B)$ and $x \in (C - B)$.
- By definition of intersection, $x \in (A - B) \cap (C - B)$.
- Hence, $x \in (A - B) \cap (C - B)$.

Since (i) and (ii) are true, then by set equality the original equation $(A - B) \cap (C - B) = (A \cap C) - B$ is also true.

Conclusion:

Therefore, $(A - B) \cap (C - B) = (A \cap C) - B$.

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Suppositions:

Suppose sets A, B, and C are arbitrarily chosen sets. Also presume $A \subseteq B$.

Goal:

Prove $A \cup C \subseteq B \cup C$.

Deductions:

The supposition $A \subseteq B$ is equivalent to the statement that $\forall x$, if $x \in A$ then $x \in B$.

Proving $A \cup C \subseteq B \cup C$ is to prove that $\forall x$, if $x \in A \cup C$ then $x \in B \cup C$.

$\forall x$, if $x \in A \cup C$ then $x \in B \cup C$.

- Let's take looking at the following cases.

- Case 1: $x \in C$.
- By definition of union, $x \in B \cup C$ because $x \in C$.
- Case 2: $x \in A$.
- By our original supposition, $x \in B$ because $x \in A$.
- Looking at these cases, if $x \in A$ or $x \in C$ then $x \in B$ or $x \in B \cup C$ respectively.
- Through definition of union, this can be rewritten as if $x \in A \cup C$ then $x \in B \cup C$.
- Hence, $\forall x$, if $x \in A \cup C$ then $x \in B \cup C$.

Since it is true that $\forall x$, if $x \in A \cup C$ then $x \in B \cup C$; it must also be true that $A \cup C \subseteq B \cup C$.

Conclusion:

Therefore, $A \cup C \subseteq B \cup C$.

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Suppositions:

Suppose A, B, and C are arbitrarily chosen sets. Also presume $A \subseteq C$ and $B \subseteq C$.

Goal:

Prove $A \cup B \subseteq C$.

Deductions:

The supposition $A \subseteq C$ is equivalent to the statement that $\forall x$, if $x \in A$ then $x \in C$.
The supposition $B \subseteq C$ is equivalent to the statement that $\forall x$, if $x \in B$ then $x \in C$.

Proving $A \cup B \subseteq C$ is to prove that $\forall x$, if $x \in A \cup B$ then $x \in C$.

$\forall x$, if $x \in A \cup B$ then $x \in C$.

- Let's take looking at the following cases.
- Case 1: $x \in A$.
- By definition of union, $x \in C$ because $x \in A$.
- Case 2: $x \in B$.
- By our original supposition, $x \in C$ because $x \in B$.

- Looking at these cases, if $x \in A$ or $x \in B$ then $x \in C$.
- Through definition of union, this can be rewritten as if $x \in A \cup B$ then $x \in C$.

Since it is true that $\forall x$, if $x \in A \cup B$ then $x \in C$; it must also be true that $A \cup B \subseteq C$.

Conclusion:

Therefore, $A \cup B \subseteq C$.

Exercise Set 6.3

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$$\begin{aligned} (B^c \cup (B^c - A))^c &= B \cap (B^c - A)^c && \text{De Morgan's Law} \\ &= B \cap (B^c \cap A)^c && \text{Set Difference Law} \\ &= B \cap (B \cup A^c) && \text{De Morgan's Law} \\ &= B && \text{Absorption Law} \end{aligned}$$

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$$\begin{aligned} (A \cap B)^c \cap A &= (A^c \cup B^c) \cap A && \text{De Morgan's Law} \\ &= (A^c \cap A) \cup (B^c \cap A) && \text{Distributive Law} \\ &= \varnothing \cup (B^c \cap A) && \text{Complement Law} \\ &= B^c \cap A && \text{Identity Law} \\ &= A \cap B^c && \text{Communative Law} \\ &= A - B && \text{Set Difference Law} \end{aligned}$$