

CS-225: Discrete Structures in CS

Initial- Post Wk 5 & 6 Problem Sets #6 and #15

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Question

Prove the following statement by mathematical induction:

$$\forall n \in \mathbb{N}, \quad \sum_{i=1}^n i!(i^2 + 1) = (n + 1)!n$$

Answer

Proof:

$P(n) \equiv \sum_{i=1}^n i!(i^2 + 1) = (n + 1)!n$ for every integer $n \geq 1$

Basis Step:

For the case $P(1)$:

$$\begin{aligned} \sum_{i=1}^1 i!(i^2 + 1) &= (1 + 1)1! \\ 1!(1^2 + 1) &= (2)(1) \\ (1)(2) &= 2 \\ 2 &= 2 \end{aligned}$$

The base case $P(1)$ holds true.

Inductive Hypothesis:

Suppose that for an arbitrary but particular integer k ,

$$P(k) \equiv \sum_{i=1}^k i!(i^2 + 1) = (k+1)!k \text{ for every integer } k \geq 1$$

is true.

Inductive Steps:

We must show that $P(k+1)$ is true. Hence we must demonstrate,

$$P(k+1) \equiv \sum_{i=1}^{k+1} i!(i^2 + 1) = (k+2)!(k+1) \text{ for every integer } k \geq 1$$

- Let the expression $\sum_{i=1}^{k+1} i!(i^2 + 1)$ be referred to the left-hand side (LHS) of the equation.
- Let the expression $(k+2)!(k+1)$ be referred to the right-hand side (RHS) of the equation.
- To show that $P(k+1)$ is true, we must show that the LHS is equal to the RHS.

The LHS simplifies to the RHS

- Recall the LHS is $\sum_{i=1}^{k+1} i!(i^2 + 1)$
- By algebra and supposition of $P(k)$, the LHS can be simplified:

$$\begin{aligned} \sum_{i=1}^{k+1} i!(i^2 + 1) &= \sum_{i=1}^k i!(i^2 + 1) + (k+1)!((k+1)^2 + 1) \\ &= (k+1)!k + (k+1)!(k^2 + 2k + 1 + 1) \\ &= (k+1)!(k^2 + 3k + 2) \\ &= (k+1)!(k+1)(k+2) \\ &= (k+2)!(k+1) \end{aligned}$$

- Recall the RHS is $(k+2)!(k+1)$
- Thus, the LHS is equal to the RHS.

Thus, $P(k+1) \equiv \sum_{i=1}^{k+1} i!(i^2 + 1) = (k+2)!(k+1)$ for every integer $k \geq 1$ was to be shown.

Conclusion:

Since both the basis step and the inductive step have been proved, the original expression,

$$P(n) \equiv \sum_{i=1}^n i!(i^2 + 1) = (n + 1)!n \text{ for every integer } n \geq 1$$

is true.

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Question

- i. Give a recursive definition for the set of all strings of 0's and 1's that have more 0's than 1's.
- ii. Give a recursive definition for the set of all strings of a 's and b 's that begins with an a and ends in a b .
- iii. Give a recursive definition for the set of all strings of 0's and 1's that are odd palindromes

Answer

i.

I - Base: The string 0 is in set S .

II - Recursion: New strings are formed according to the following rules:

- a. If u is any string in S , then

$0u$ and $u0$ are strings in S

where $0u$ and $u0$ are the concatenations of u and 0. Each is obtained by appending 0 to the left side of u and the right side of u respectively.

- b. If u is any string in S , and u contains at least two more 0's than 1's, then

$1u$ and $u1$ are strings in S

where $1u$ and $u1$ are the concatenations of u and 1. Each is obtained by appending 1 to the left side of u and the right side of u respectively.

III - Restriction: Every string in S is obtainable from the base and the recursion.

ii.

I - Base: The string ab is in set S .

II - Recursion: New strings are formed according to the following rules:

a. If u is any string in S , then

au and ub are strings in S

where au and ub are the concatenations of u with a and b respectively.

b. If u and v are any strings in S , then

uv is a string in S

where uv is the concatenation of u and v .

III - Restriction: Every string in S is obtainable from the base and the recursion.

iii.

I - Base: The strings 0 and 1 are in set S .

II - Recursion: New strings are formed according to the following rules:

a. If u is any string in S , then

$0u0$ is a string in S

where $0u0$ is the concatenation of u and 0. $0u0$ is obtained by appending 0 to the left side of u and the right side of u .

b. If u is any string in S , then

$1u1$ is a string in S

where $1u1$ is the concatenation of u and 1. $1u1$ is obtained by appending 1 to the left side of u and the right side of u .

III - Restriction: Every string in S is obtainable from the base and the recursion.