Summer 2020 Gregory A. Bruich, Ph.D.
Economics 1016 Harvard University

Cumulative Effects as Estimated in Dube, Lester, and Reich (2010)

Consider the following regression:

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 X_{it-1} + \beta_3 X_{it-2} + u_{it} \tag{1}$$

The sum $\beta_1 + \beta_2 + \beta_3$ is the cumulative effect of a unit change in X_{it} on Y_{it} over three periods.

The cumulative dynamic multiplier can be estimated directly as:

$$Y_{it} = \delta_0 + \delta_1 \Delta X_{it} + \delta_2 \Delta X_{it-1} + \delta_3 X_{it-2} + u_{it} \tag{2}$$

where $\delta_0 = \beta_0$, $\delta_1 = \beta_1$, $\delta_2 = \beta_1 + \beta_2$, and $\delta_3 = \beta_1 + \beta_2 + \beta_3$ (the long run cumulative dynamic multiplier); $\Delta X_{it} = X_{it} - X_{it-1}$ and $\Delta X_{it-1} = X_{it-1} - X_{it-2}$. Notice that X_{it-2} is not differenced.

We can derive this expression by re-writing Equation (1). Start by adding $0 = \beta_1 X_{i-1} - \beta_1 X_{it-1}$:

$$\begin{aligned} Y_{it} &= \beta_0 + \beta_1 X_{it} + \beta_2 X_{it-1} + \beta_3 X_{it-2} + \{\beta_1 X_{it-1} - \beta_1 X_{it-1}\} + u_{it} \\ &= \beta_0 + \beta_1 \{X_{it} - X_{it-1}\} + (\beta_1 + \beta_2) X_{it-1} + \beta_3 X_{it-2} + u_{it} \end{aligned}$$

To finish, we add $0 = (\beta_1 + \beta_2) X_{it-2} - (\beta_1 + \beta_2) X_{it-2}$

$$\begin{aligned} Y_{it} &= \beta_0 + \beta_1 \{X_{it} - X_{it-1}\} + (\beta_1 + \beta_2) X_{it-1} + \beta_3 X_{it-2} \\ &\quad + \{(\beta_1 + \beta_2) X_{it-2} - (\beta_1 + \beta_2) X_{it-2}\} + u_{it} \\ &= \beta_0 + \beta_1 \{X_{it} - X_{it-1}\} + (\beta_1 + \beta_2) \{X_{it-1} - X_{it-2}\} \\ &\quad + (\beta_1 + \beta_2 + \beta_3) X_{it-2} + u_{it} \\ &= \beta_0 + \beta_1 \Delta X_{it} + (\beta_1 + \beta_2) \Delta X_{it-1} + (\beta_1 + \beta_2 + \beta_3) X_{it-2} + u_{it} \end{aligned}$$

We have shown that we can estimate cumulative effects in a single regression. The graphs and Appendix Table in Dube, Lester, and Reich (2010) that check for pre-trends are from models like this one.