Economics 1016 Gregory A. Bruich, Ph.D.
Summer 2020 Harvard University

Elasticities and Income vs. Subsitution Effects

I. Review of elasticity concept

An elasticity $e_{x,p}$ tells you that when p changes by 1 percent, x changes by $e_{x,p}$ percent.

$$e_{x,p} = \frac{\%\text{ change in } x}{\%\text{ change in } p} = \frac{dx/x}{dp/p} = \frac{dx}{dp}\frac{p}{x} = \frac{d\log x}{d\log p}$$

If we change p from p_0 to p_1, then x changes by $[e_{x,p} \cdot (p_1 - p_0)/p_0]\%$

Inelastic vs. elastic demand / supply:

- Small elasticity (quantity changes little in response to price changes) → inelastic demand / supply. Corresponds to a steep demand / supply curve.
- Large elasticity (quantity changes a lot in response to price changes) → elastic demand / supply. Corresponds to a demand / supply curve that is not steep.
- Vertical demand / supply is perfectly inelastic (an elasticity of zero).
- Horizontal demand / supply is perfectly elastic (infinite elasticity).

Two concepts: compensated elasticity $\varepsilon^c_{x,p}$ and and uncompensated elasticity $\varepsilon_{x,p}$.

- Compensated elasticity calculated from compensated demand curve ($x^c = f(p, u_0)$); holds *utility* constant, only includes substitution effects and only depends on the slope of the budget constraint.
- Uncompensated elasticity calculated from market demand curve ($x = f(p, R)$ where $\$R$ is unearned income); includes both income and substitution effects, and depends on both the slope (controlled by p) and height (controlled by R) of the budget constraint.

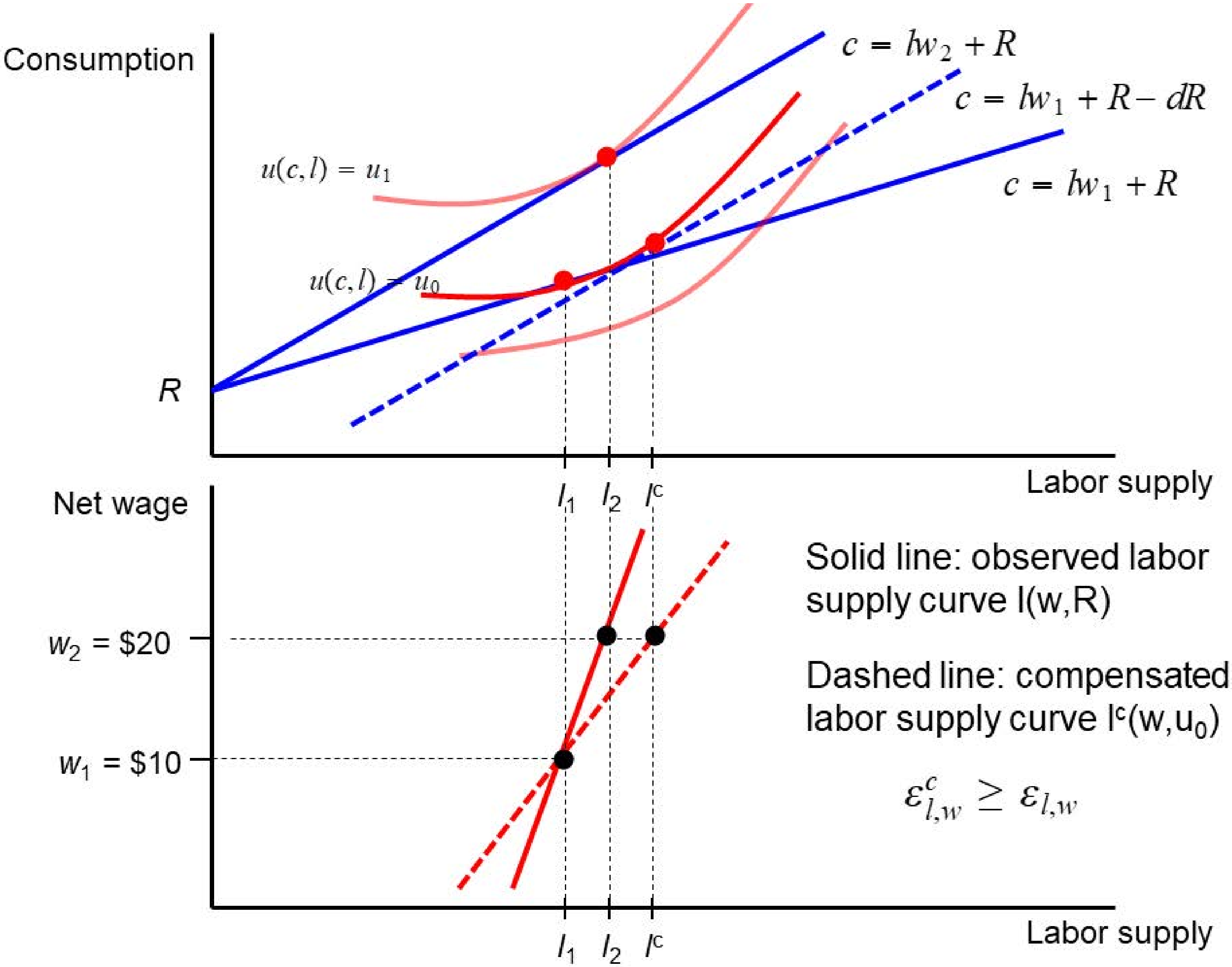
Relationship between the two: $\varepsilon_{x,p} = \varepsilon^c_{x,p} + \sigma$ where σ is the income effect

II. Note on income and substitution effects

In the labor market, the income effect $\sigma = d[wl]/dR$ and substitution effects have opposite signs. This implies that $\varepsilon^c \geq \varepsilon$. Recall that supply elasticities should be positive.

In a market for a normal good, the income $\sigma = -p_x dx/dR$ and substitution effects have the same sign. If the price of a good increases, the substitution effect leads me to consume less of it and I am effectively poorer now, which also leads me to consume less of it. This implies that $|\varepsilon^c| \leq |\varepsilon|$. Demand elasticities should be negative.

III. Derivation of the compensated and uncompensated labor supply curves



IV. Average vs. Marginal tax rates

Let's start with a simple proportional tax t on earnings $z = wl$. The average and marginal tax rates are equal:

$$ATR_0 = MTR_0 = t$$

Suppose now we add in an exclusion: the first $\$E$ of earnings are untaxed. This reduces the average tax rate:

$$ATR_1 = t\frac{z-E}{z} < t = MTR_1 = MTR_0 = ATR_0$$

The budget constraint for after-tax consumption before the exclusion is:

$$c = z(1-t) + R_0$$

When we add in the exclusion we have:

$$\begin{aligned} c &= z - t(z-E) + R_0 && \text{(if } z > E\text{)} \\ &= z(1 - t\frac{z-E}{z}) + R_0 \\ &= z(1 - ATR_1) + R_0 \\ c &= z + R_0 && \text{(if } z < E\text{)} \end{aligned}$$

Therefore, the average tax rate controls the height or intercept of our budget constraint. Alternatively, we could write:

$$\begin{aligned} c &= z - t(z-E) + R_0 && \text{(if } z > E\text{)} \\ &= z(1-t) + tE + R_0 \\ c &= z + R_0 && \text{(if } z < E\text{)} \end{aligned}$$

so that the budget constraint with the exclusion is simply a vertical shift of the initial no-exclusion budget constraint for earnings $\$z > \$E$. Therefore, for individuals with earnings above $\$E$, the change in the average tax rate only affects earnings through an income effect (which depends on the income effect parameter $\sigma = dz/dR$) because the slope has remained the same at $1-t$. It is exactly as if unearned income has increased to $R_1 = tE + R_0$.

How would this tax change affect the compensated labor supply curve and the uncompensated labor supply curve? Recall that $z^c = f(1 - MTR, u_0)$ and $z = f(1 - MTR, R)$.