Symmetric (secret-key) cryptography

What a crypto-system is (the three algorithms) You can describe DES or AES - but you can also just give a high-level description of what a block cipher is. Definitions of PRF and CPA security. Specification of CBC or CTR modes (or both), proofs of CPA security for CBC or CTR mode (or both). Perhaps a brief talk about stream ciphers and how to make one from a block cipher.

Core Vocabulary & Syntax (Total Recall)

Cryptosystem

> A cryptosystem is a triple (G, E, D) of algorithms, for key generation, encryption and decryption. The definition says nothing about security.

> For a cryptosystem, we always require that for any key K output by G, correct decryption is possible, i.e. it holds that for any $x \in P$, $x = D_K(E_K(x))$.
> The simplest cryptosystem system ”encrypts” by sending x itself.

> For symmetric cryptosystems, the information you need to encrypt a message is the same as what you need to decrypt it.

G (Key Generation)

> This algorithm is probabilistic, takes no input and always outputs a key $K \in \mathscr{K}$, usually G simply outputs a key chosen uniformly from $\mathscr{K}$.

E (Encryption)

> This algorithm takes as input K and $x \in \mathscr{P}$ and produces as output $E_K(x) \in \mathscr{C}$.

> Note that E **may be probabilistic**; even for a given x_0 and K_0, many different ciphertexts may be produced as output from E as a result of random choices made during the encryption process. In other words, the ciphertext will have a probability distribution that is determined from x and K, typically uniform in some subset of the ciphertexts.

D (Decryption)

> This algorithm takes as input K, $y \in C$ and produces as output $D_K(y) \in \mathscr{P}$. It is (not strictly, but) most often deterministic.

Block cipher

> Cryptosystem, where the key is a bit string of fixed length and key generation just chooses a uniform key. Furthermore, it takes as input a bitstring of fixed length and outputs a ciphertext

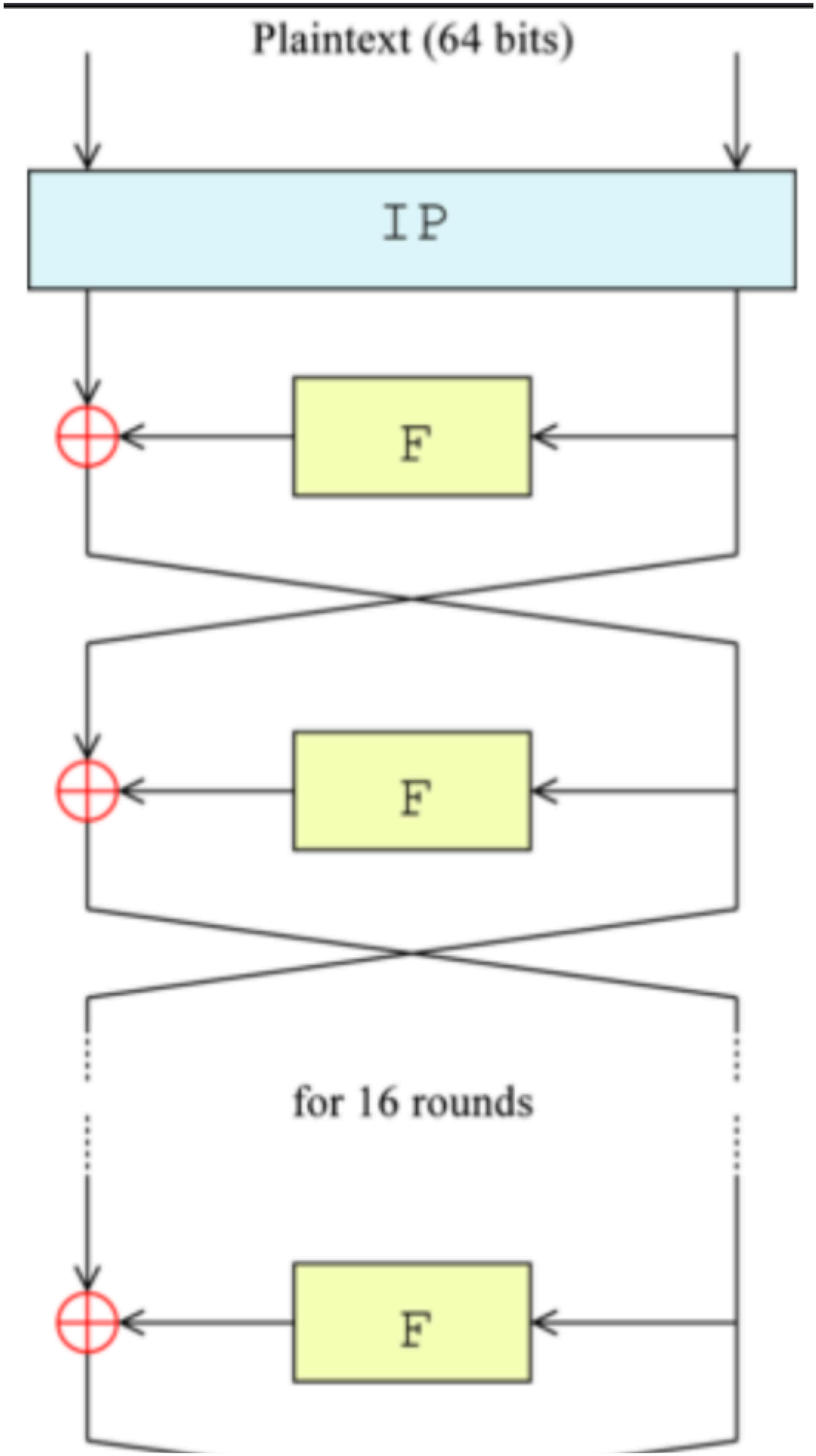
> of the same length.

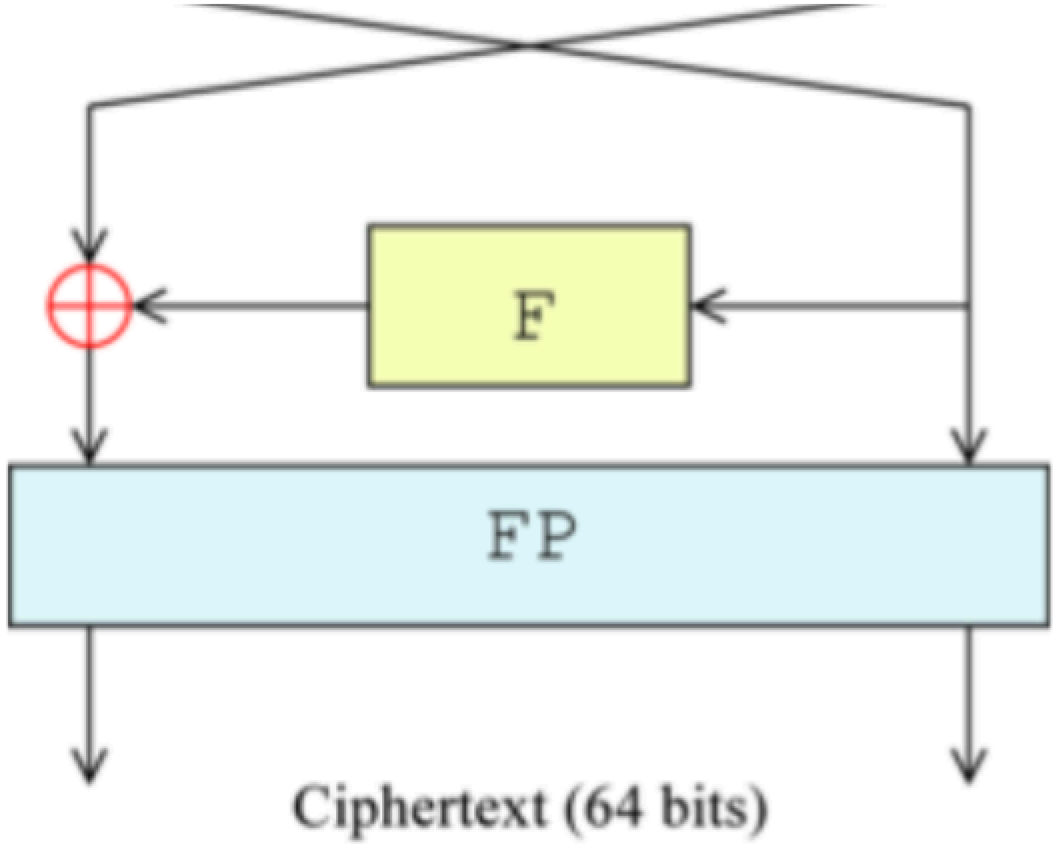
Feistel ciphers

> Named after Horst Feistel, who was on the IBM team that designed DES, a Feistel cipher is a symmetric block cipher design model that processes plaintext through multiple rounds of substitution and permutation operations.

> They have the following structure: encryption consists of repeating some computation a number of times, one such computation is called a round, and the number of rounds is denoted n. There is an algorithm called the Key Schedule that takes the key as input and outputs the rounds keys $K_1, ..., K_n$, where each round uses its own round key. More concretely, each round does the following:
> The input is a bitstring P that we **split in two halves**, called L_0, R_0.
> Then for $i = 1 \ldots n-1$ we, for a given **round function** f do: $L_i = R_{i-1}, R_i = L_{i-1} \oplus f(R_{i-1}, K_i)$.
> The last round is slightly different: we set $R_n = R_{n-1}, L_n = L_{n-1} \oplus f(R_{n-1}, K_n)$.
> The output ciphertext C is now defined to be $C = (L_n, R_n)$.
> It is not hard to see that if you know the round key K_i, you can invert the i'th round: given L_i, R_i, you can compute $L_{i-1} = R_i \oplus f(L_i, K_i)$, $R_{i-1} = L_i$, and so given the key, one can always decrypt a Feistel cipher.
> Note that this works no matter what the function f does, so this means that one can design f to get the most secure cipher possible, without having other constraints in mind. For instance f doesn't have to be invertible.

The structure illustrated specifically for DES


Plaintext (64 bits)
IP
F
F
for 16 rounds
F

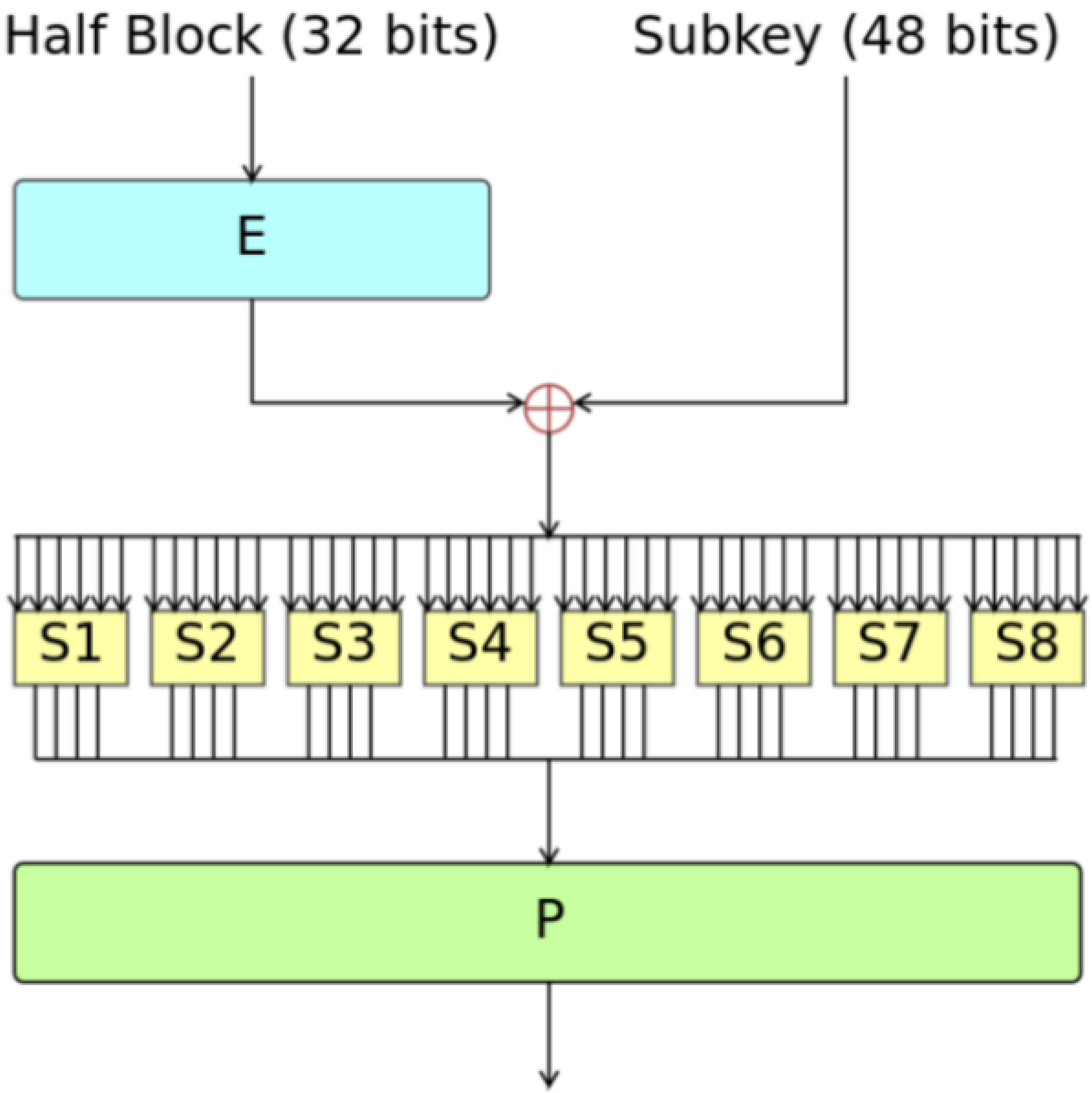


DES

> Data Encryption Standard (DES) is a symmetric block cipher that encrypts data in fixed-size blocks using the Feistel structure. It has been deprecated and AES took its place in 2001 It takes a 64-bit input block and a 56-bit secret key to produce a 64-bit ciphertext.

> At a high level, DES works as follows:
>
> - The 64-bit plaintext is permuted and then split into two 32-bit halves, L_0 and R_0.
> - DES applies 16 Feistel rounds, where the round function $f(R, K) = P(S(K \oplus E(R)))$, using functions E, S, P.
>   - $E(R)$ expands the 32-bit right half to 48 bits
>   - $K \oplus \ldots$ mixes it with the subkey
>   - $S(\ldots)$ passes it through nonlinear S-boxes reducing the length back to 32 bits [LUT that for each 6 bits gives a 4 bit chunk which are then concatenated]
>   - and $P(\ldots)$ permutes the result according to the specification.
> - After the 16th round, the two halves are swapped and a final permutation produces the 64-bit ciphertext block.

> Decryption runs the **same** Feistel network but uses the subkeys in reverse order, so DES encryption and decryption share the same core structure. Intuitively, DES is “just” a 16-round Feistel cipher with carefully chosen bit permutations and S-boxes to resist classical attacks, though its 56-bit key is small by modern standards.



PRF Security

> We imagine that for a good cryptosystem, an adversary couldn't really differentiate between the encryption of a given message and completely random output of the same format. This would mean our cryptosystem works like a pseudorandom function.

> Let's put this into numbers for a function f_K, where $K \in \{0,1\}^k$, $f_K : \{0,1\}^n \to \{0,1\}^m$.
> Let's say an adversary (A, a probabilistic function) is blindly presented with one of 2 oracles (~black-box functions):
> **The ideal world**: A gets access to an oracle O_{Ideal} which initially chooses a random mapping R from $\{0,1\}^n$ to $\{0,1\}^m$ (uniformly among all such mappings), and then, when A sends an input x, it answers with $R(x)$.
> **The real world**: The adversary gets access to an oracle O_{Real} which initially chooses K at random from $\{0,1\}^k$, and fixes K for the duration of the game. After this, on input x, it answers with $f_K(x)$.

> The adversary needs to then guess which oracle it encountered by outputting a bit. Define $p(A,0) := \text{ guessed ideal}$ anf $p(A,1) := \text{ guessed real}$. The advantage of A is then $Adv_A(O_{ideal}, O_{real}) = |p(A,0) - p(A,1)|$.

> We say that f_K is a (t,q,ε) PRF-secure, if any adversary A that runs in time at most t and makes at most q calls to the oracle, satisfies $Adv_A(O_{Real}, O_{Ideal}) \leq \varepsilon$.

CPA Security

> Deterministic block cyphers have a very clear problem: if I encrypt m today and tomorrow, I'll get the same result. This is giving away too much information to an attacker.

> PRF security is nice and dandy, but it doesn't take this weakness into account - let's define something stronger. Let's require the adversary that it cannot tell the difference between a real encryption of **a message x it chooses**, and a completely random ciphertext chosen with no relation to x! Note that this doesn't hold for DES; A could submit x multiple times and could be reasonably sure that if they got back the same thing they are dealing with the real deal, otherwise they are in the ideal world.

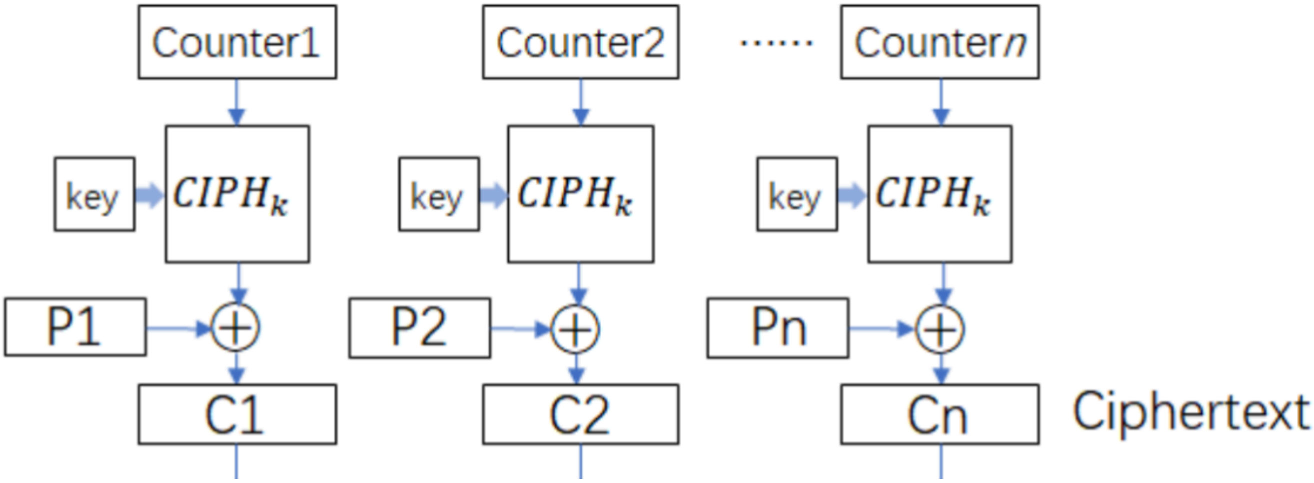
> **The ideal world**: A gets access to an oracle O_{Ideal} which on input a plaintext x answers with $E_K(r)$, where r is a randomly chosen message with the same length as x, and K is produced by G, but fixed in the entire attack.
> **The real world**: A gets a access to normal chosen message attack: an oracle O_{Real} which on input a plaintext x answers with $E_K(x)$, where K is produced by G, but fixed in the entire attack.

> The cryptosystem (G,E,D) is (t,q,μ,ε) CPA-secure, if for any adversary A that runs in time at most t, and makes at most q calls to the oracle, with plaintexts consisting of a total of μ bits, it holds that $Adv_A(O_{Real}, O_{Ideal}) \leq \varepsilon$.

CTR mode and proof for its CPA security

> PRF-secure block cyphers can be converted into probabilistic, CPA-secure cryptosystems that as a bonus benefit can handle any length. These are often called *modes of use*, and one of these is the CTR (Counter) mode.

> The trick is to create a random initialization vector IV or nonce of length n (block size). Then split the message up into n bit chunks. Encrypt IV with the key K, then XOR the result to the first block. Then increase IV by one, encrypt it and XOR the next block. The result is $[IV, m_1 \oplus E_K(IV), m_2 \oplus E_K(IV+1)\ldots]$ (where the square brackets denote the concatenation of bits). Decryption is rather trivial.



> Suppose $\{E'_K \mid K \in \mathscr{K}\}$ is (t', q', ε') PRF-secure. Then CTR encryption based on this system is (t, q, μ, ε) CPA-secure for any q, and for
>
> $$t \leq t' \quad \frac{\mu}{n} \leq q' \quad \varepsilon = \varepsilon' + \left(\frac{\mu}{n}\right)^2 \cdot \frac{1}{2^n}$$

> TODO proof of CPA sec

Stream Cipher from CTR block cipher

> Trivial

Logic Flow / Mechanism (Process)

Construct a symmetric cryptosystem (G, E, D). Fill in the requirements and correctness condition.

1. Define finite sets: **K**, **P**, **C**.
2. [____________________]
3. [____________________]
4. Require: For any K from **G**, and any $x \in P$, $x = D_K(E_K(x))$.

Encrypt a message with the Shift Cipher. Derive the steps.

1. Choose K uniformly from $K = \mathbb{Z}_n$.
2. [____________________]
3. [____________________]

The "Exam Trap" (Distinctions)

Distinguish between attack models based on oracle access. Complete the matrix:

Attack Type	Oracle Input	Oracle Output	Adversary Knowledge
Ciphertext Only	[Student fills]	[Student fills]	[Student fills]

Attack Type	Oracle Input	Oracle Output	Adversary Knowledge
Known Plaintext	[Student fills]	[Student fills]	[Student fills]
Chosen Plaintext (CPA)	[Student fills]	[Student fills]	[Student fills]
Chosen Ciphertext	[Student fills]	[Student fills]	[Student fills]

Exam Simulation

Oral Exam Question 1: "Walk me through the definition of a symmetric cryptosystem, including the three algorithms and their inputs/outputs. Specify the sets involved and the correctness requirement."

Oral Exam Question 2: "Describe DES and AES as modern symmetric cryptosystems. What historical context led to DES, and why was a successor like AES needed? Reference their structure (e.g., Feistel for DES)."

Oral Exam Question 3: "Source data missing for PRF definition, CPA security definitions, CBC/CTR mode specifications, and their CPA security proofs. Explain from course material: Define PRF and CPA security. Specify CBC or CTR mode and prove its CPA security assuming a secure block cipher."

Source Map

- CryptographyV6.pdf | Page 27-28 | Covers: Symmetric cryptosystem definition (G, E, D), sets K/P/C, correctness.
- CryptographyV6.pdf | Page 28 | Covers: Shift Cipher example.
- CryptographyV6.pdf | Page 29 | Covers: Attack models (ciphertext-only, known/chosen plaintext/ciphertext).
- CryptographyV6.pdf | Page 60-61 | Covers: DES blockcipher, Feistel structure, history.
- CryptographyV6.pdf | Page 58,63 | Covers: DES and AES blockciphers (high-level).