1. Indices for measuring network balance

Let $G = (V, E, \sigma)$ be an undirected signed network, where V and E are the set of verices and edges, and σ is the sign function $\sigma : E \to \{-1, +1\}$. The *signed adjacency matrix* and *unsigned adjacency matrix* are defined as

$$A_{uv} = \tag{1}$$

$$|A|_{uv} = \tag{2}$$

The sign of a cycle is the product of the sign of its edges. A cycle is *balanced* if its sign is positive, and *unbalanced* if its sign is negative. The total number of positive (negative) cycles is denoted by O^+ (O^-). Similarly, the total number of positive (negative) walks is denoted by Q^+ (Q^-). Thus, the total number of cycles (walks) in the graph is $O = O^+ + O^-$ ($Q = Q^+ + Q^-$). Finally, the signed Laplacian of the graph is defined as $L = D - A$, where $D_ij = \sum_j |a_ij|$ is the diagonal matrix of degrees.

There are several ways of measuring balance in signed networks (for a review on the topic, we refer the reader to [**?**]).

The triangle index $T(G)$ is defined as

$$T(G) = \frac{O_3^+}{O_3} = \frac{Tr(A^3) + Tr(|A^3|)}{2Tr(|A^3|)}. \tag{3}$$

The walk-based index of balance is defined as

$$W(G) = \frac{K(G) + 1}{2}, \quad K(G) = \frac{\sum_k \frac{Q_k^+ - Q_k^-}{K!}}{\sum_k \frac{Q_k^+ + Q_k^-}{K!}} = \frac{Tr(e^A)}{Tr(e^{|A|})}. \tag{4}$$

The algebraic conflict $\lambda(G)$ is defined as the smallest eigenvalue of the sign Laplacian matrix. This eigenvalue equals zero if and only if the networks is balanced. The normalized algebraic conflict is defined as

$$A(G) = 1 - \frac{\lambda(G)}{\bar{d}_{\max} - 1}, \quad \bar{d}_{\max} = \max_{(u,v) \in E)} (d_u + d_v)/2, \tag{5}$$

and is bounded between 0 and 1 (see [**?**]).

2. Results

We built $N_s = 5000$ interaction matrices with $N_a = 10$ agents.

Let $C_a^{(n)}$ be the complexity of the time series asociated with agent a for the simulation performed using the interaction matrix n. For each simulation, we consider as representative values the mean and maximum complexity

$$\bar{C}^{(n)} = \frac{1}{N_a} \sum_a C_a^{(n)} \tag{6}$$

$$C_{\max}^{(n)} = \max_a C_a^{(n)}. \tag{7}$$

In Figure 1 (left), we show the histogram for the number of matrices with a given value of the triangle index T. [TODO: prove that the histogram can be expressed in terms of the Binomial distribution with parameters $q = 0{,}5$ and $N =$ number of cycles]. For each value of T, we plot in the center and right panel of the figure, the maximum and the mean complexity, averaged over all the matrices having the same T. It can be observed that there is not clear dependence of the complexity on this balance index.

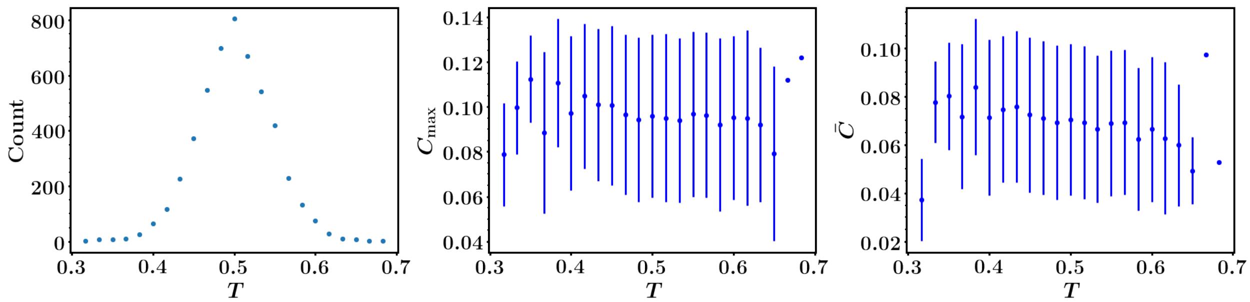

Figura 1: (Left) Histogram for the triangle index ditribution in the 5000 interaction matrices generated. Values for the max (center) and mean (right) complexities as a function of the triangle index. Dots and bars represent the mean and standard deviation computed over all the matrices with the same value of T

In Figure 2 we show the maximum and mean complexity plotted against the normalized algebraic conflict. No clear correlation can be seen for this balance measure.

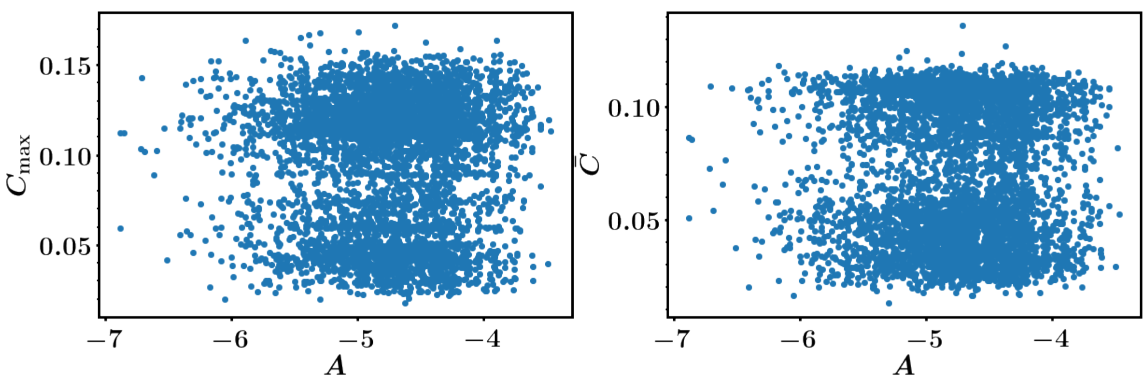

Figura 2: Max (left) and mean (right) complexities of each realization plotted against the normalized algebraic conflict.

In Figure 3 we show the maximum and mean complexity plotted against the walk-based index of balance. Again, there is not a good correlation using this measure.

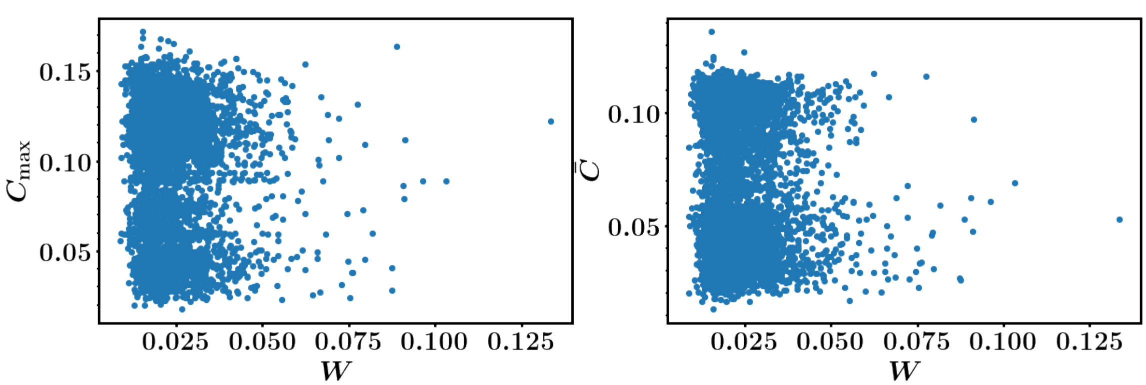

Figura 3: Max (left) and mean (right) complexities of each realization plotted against the walk-based measure of balance.