1 Model

Parameters:

- N: Network size
- N_c: Number of nodes per block
- ℓ: Number of blocks
- $\langle k \rangle$: Average degree of the overall network
- $\langle k \rangle^{\text{in}}$: Average degree counting only intrablock links
- $\langle k \rangle^{\text{out}}$: Average degree counting only interblock links
- p^{in}: Probability of existence of intrablock links
- p^{out}: Probability of existence of interblock links

Relation between parameters:

$$N = \ell N_c \tag{1}$$
$$\langle k \rangle = \langle k \rangle^{\text{in}} + \langle k \rangle^{\text{out}} = p^{\text{in}}(N_c - 1) + p^{\text{out}}(N - N_c) \tag{2}$$
$$N_r = N/N_c \tag{3}$$
$$P_{\text{out}} = p^{\text{out}} N_c^2 \tag{4}$$

2 Results

It seems that the effective size of the system is not the number of nodes N but the ratio between N and the size of the blocks. Thus, we define the variable $N_r = N/N_c$ and perform the analysis in terms of this variable.

For constant N_c and $\langle k \rangle$ large enough ($\langle k \rangle \gtrsim 4$), the transition occurs when

$$N_r P_{\text{out}} = 1, \tag{5}$$

where $P_{\text{out}} = p^{\text{out}} N_c^2$ is the average number of links connecting two given blocks. For smaller values of $\langle k \rangle$, the percolation point moves to the right, as it can be seen in Figures 1 and 2. The critical exponents do not seem to vary.

Hipothesis: the (finite-size) percolation threshold and the peak of N_2 and $\langle s \rangle$, for a given value of N, satisfy a scaling relation of the type

$$q_c(N, \langle k \rangle) - q_c(N, \langle k \rangle_{\max}) \sim \langle k \rangle^a \tag{6}$$
$$\frac{1}{N_2^{\max}(N, \langle k \rangle)} - \text{const} \sim \langle k \rangle^b \tag{7}$$
$$\frac{1}{\langle s \rangle^{\max}(N, \langle k \rangle)} - \text{const} \sim \langle k \rangle^c \tag{8}$$

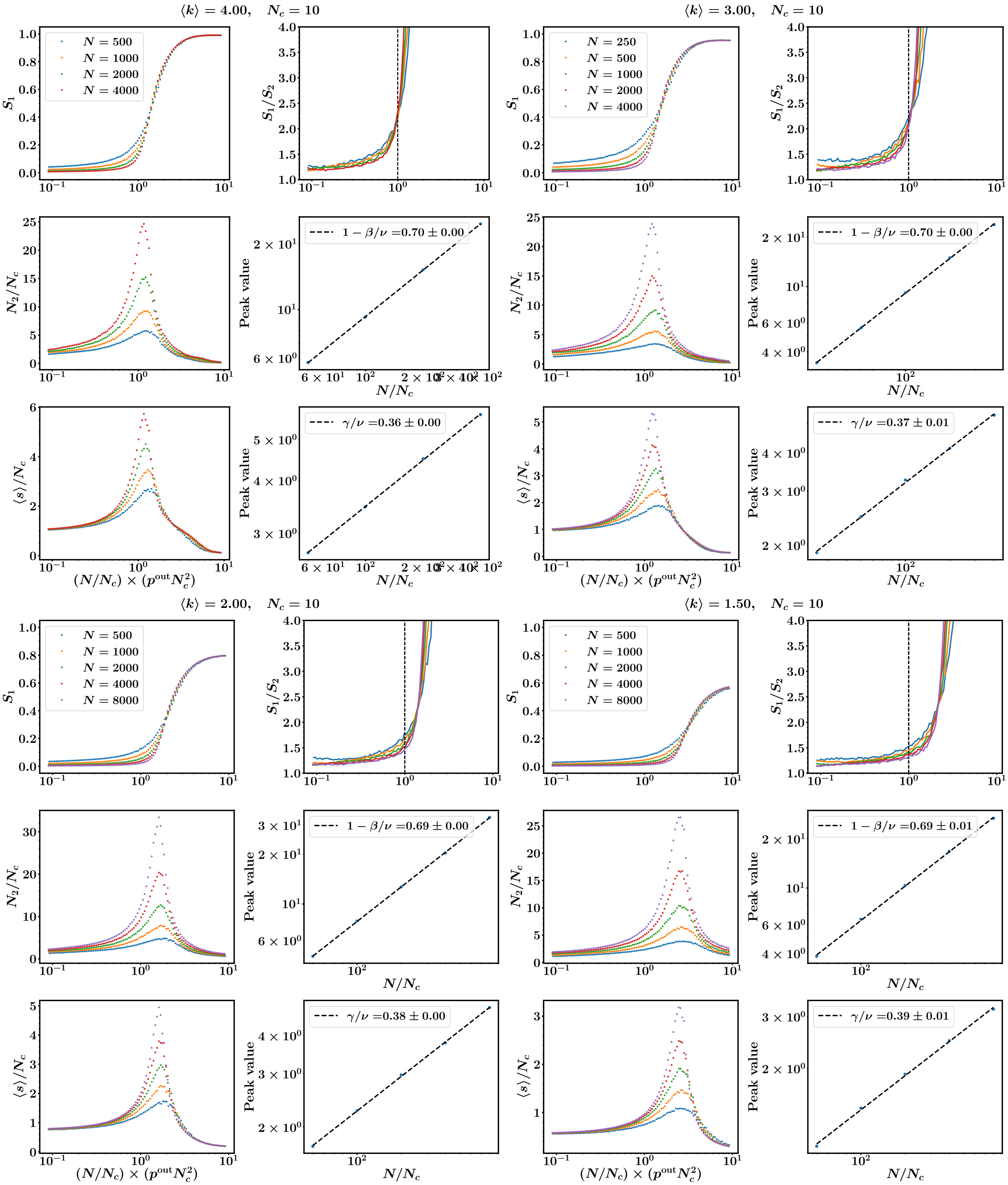

Figure 1: Scaling of the percolation transition for different values of $\langle k \rangle$.

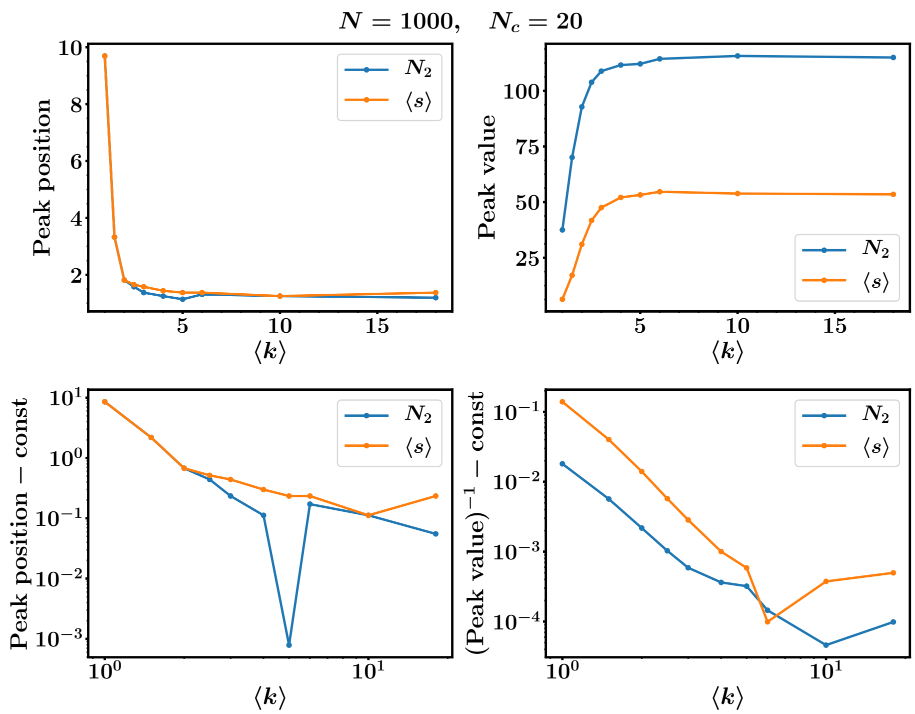

Figure 2: Variation of the (finite-size) percolation threshold and size of the susceptibility peak with $\langle k \rangle$.