Enhancing State Estimation and Outlier Rejection in Autonomous Navigation via Moving Horizon Estimation with Convex Measurement Fusion

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***Abstract*—State estimation plays a critical role in control system applications, especially when states are unobservable or corrupted by noise. Among various estimation techniques, Moving Horizon Estimation (MHE) is particularly notable for its ability to reconstruct states from noisy measurements using a mathematical model of the system. As an optimization-based technique, MHE can directly incorporate constraints on states and/or inputs while maintaining a low computational burden, enabling real-time implementation thanks to advances in numerical methods. Leveraging modern sensor technologies capable of delivering measurements at rates significantly faster than the system's dynamics, this work introduces an innovative MHE scheme that merges multiple sensor readings of the same state. This is achieved by exploiting either redundant sensor configurations or single sensors delivering measurements in bursts. From a practical perspective, the proposed formulation not only enhances state estimation, outperforming existing methods, but also effectively rejects outliers. Furthermore, from a theoretical standpoint, the cost associated with solving the optimization problem corresponds to an outlier-free cost, thereby tightening the upper bound on estimation error. The effectiveness of the proposed method is validated through comprehensive simulations and field experiments.**

***Index Terms*—Nonlinear Moving horizon estimation; Bounded disturbances; Outlier affected measurement; Control process.**

I. Introduction

When system states are not available for measurement or are affected by large noises, it becomes necessary to estimate them before making decisions. One of the most powerful estimation algorithms available nowadays is moving horizon estimation (MHE). This technique is optimisation-based, thus, it can handle constraints on states and/or inputs straightforwardly. It takes into account the system's output measurements within a moving window. When a new measurement is available, it is incorporated within the estimation window and the oldest is discarded, maintaining constant the computational burden, which gives room for real-time applications [1].

Several authors have addressed the problem of system state estimation using measurements of the system's output affected by noise and even the more challenging scenario where outliers are present too. One of the first work relying on MHE to mitigate measurements affected by outliers is [2], where the authors propose a moving horizon estimator for discrete-time linear time-invariant systems with measurements affected by outliers. The approach consists of minimising a set of least-squares cost functions in which each measure possibly contaminated by outliers is left out in turn. After solving $N_e + 2$ optimisation problems, where N_e is the length of the estimation window, the estimate that corresponds to the lowest cost is retained and propagated to the next time instant, where the procedure is repeated with the new information batch.

Following, the authors in [3] present an approach to improve the robustness of state estimation in linear systems when the measurement data may contain outliers. Instead of trying to detect and remove outliers, the method tries to select valid measurements within an optimisation framework. The method relies on selecting a subset of measurements per time step and the number of steps to reduce the probability of containing outliers within the moving horizon window. The authors extend the results for nonlinear systems in [4], where they include experimental results by estimating the state of a vehicle equipped with a Global Navigation Satellite System (GNSS) aiding an Inertial navigation System (INS). However, the outliers are generated artificially according to a uniform distribution, and the results are computed offline using Monte Carlo methods.

The authors of [5] enhance an Auto-Regressive-Moving-Average with exogenous input (ARMAX) model by considering additive error terms on the output. Moreover, they developed a moving horizon estimator that uses the enhanced ARMAX model, where the measurement errors are modelled as nuisance variables and estimated simultaneously with the states. Furthermore, by regularising the least-squared cost with the ℓ_2-norm of the nuisance variables, the optimisation problem has an analytical solution. In [6], the authors propose a horizon-based Maximum Likelihood (ML) state estimator for discrete-time linear systems robust against zero and non-zero mean outliers. The ML-based is used to find a batch solution for the filtering problem, and it uses an explicit outlier detection mechanism that enables its seamless working. Furthermore, the proposed filter switches between prediction and ML estimate accurately by detecting the occurrence of both types of outliers.

Following, in [7], the authors proposed a MHE-based multi-sensor fusion framework for localization of vehicle and mobile robots. Leveraging on the optimisation-based nature of MHE, the proposed scheme is capable of operating with different sensors' rates, missing measurements, and outliers, while to keep as low as possible the computational burden, a multi-threading architecture is used. Continuing with measurements affected by noises with unexpected uncertainties and noise distributions, like outliers, the authors in [8] presented an MHE framework that can explicitly accommodate unknown non-Gaussian distributions. The scheme approximates the unknown non-Gaussian distributions using an optimal Gaussian mixture model that is adapted online without increasing significantly the computational cost in comparison with the standard MHE formulation. In [9], the authors consider the state estimation problem for discrete-time linear systems suffering from dense measurement anomalies. Conventional moving horizon estimation algorithms can be used to solve cases containing sparse measurement anomalies, but their performance degrades dramatically as the number of outliers increases. To address this problem, they propose two outliers exclusion-moving horizon estimation strategies. That is, at each sampling instant, solving a set of least-squares cost functions aims to exclude all possible outliers. The state estimates corresponding to the optimal cost are retained and propagated to the next instant, and the procedure is repeated when new information arrives.

Following, in [10], the authors propose a generalised MHE (GMHE) approach that formulates MHE as a maximum posteriori estimation problem and extends the standard MHE with a generalised loss function. The proposed approach avoids the high computational complexity of existing methods and has no restriction on the system models. The authors demonstrate that the standard MHE is a special case of GMHE, where the loss function uses the Kullback-Leibler (KL) divergence between the empirical distribution of the observations and the assumed likelihood. Because KL divergence is sensitive to outliers, the authors replace it with a robust β-divergence and name the corresponding GMHE as β-MHE. Moreover, proves that for the case of linear Gaussian systems, the gross error sensitivity of the β-MHE remains bounded, which demonstrates its robustness against outliers under these conditions. Later, in [11], the authors formulate the MHE from a Bayesian perspective integrating a robust divergence measure to reduce the impact of outliers. The approach prioritises the fitting of uncontaminated data and lowers the weight of contaminated ones. The authors incorporated a tuning parameter into the framework to adjust the robustness degree to outliers.

Regarding sensor faults, the authors in [12] proposed the analysis of the statistical properties of decision variables of the unconstrained MHE to be used with fault diagnosis and identification. When a sensor fault is isolated, the fault magnitude information is used for on-line compensation of measurements sent to the controller. The proposed approach is able to isolate and compensate for multiple single faults occurring sequentially in time.

Several authors have addressed the problem of dealing with noisy measurements and outliers. However, available methods are focused mainly on linear systems and usually formulate combinatorial optimisation problems to tackle the challenge of measurements affected by outliers, which leads to computational demanding methods. Moreover, other available approaches rely on assumptions or prior knowledge about the noise distributions. By leveraging current sensor technology commonly used in robotics and autonomous navigation, such as Inertial Measurement Units (IMU), LiDARs and GNSS which can reach a sample frequency higher than the actually needed [13], we propose a Moving Horizon Estimation formulation for nonlinear systems that addresses noisy measurements possibly affected by outliers. In this approach, we include within the moving horizon formulation all available measurements at each sampling instant. Measurements are assumed to come from redundant sensors (measurements taken simultaneously) or by as single sensor capable of measuring in burst (consecutive measurements separated by a fraction of the sampling instant). Then, those states that are measured redundantly or by a single sensor capable of measuring in burst are estimated within the MHE taking into account all available measurements. The novelty of the proposed approach can be summarised as follows:

i) improved state estimation even in presence of outliers by incorporating all measurements available as a convex combination, ii) same computational burden as the standard MHE formulation allowing real-time implementation, and, iii) lower error upper bound in the presence of measurements corrupted by outliers.

The remainder of the article is organised as follows:

II. Problem Statement

Let us consider a continuous-time nonlinear system whose behaviour is given by:

$$\begin{aligned} \dot{x}_t = & \quad f\left(x_t, u_t\right) + w_t, \\ y_t = & \quad h\left(x_t\right) + \nu_t, \end{aligned} \tag{1}$$

where $x_t \in \mathcal{X} \subset \mathbb{R}^{n_x}$ is the system's state, $u_t \in \mathcal{U} \subset \mathbb{R}^{n_u}$ is the system's input and $w_t \in \mathcal{W} \subset \mathbb{R}^{n_w}$ is the unmeasured additive process disturbance. The output of the system is $y_t \in \mathcal{Y} \subset \mathbb{R}^{n_y}$ and $\nu_t \in \mathcal{V} \subset \mathbb{R}^{n_v}$ is the measurement noise. The function $f(\cdot,\cdot)$ is assumed to be at least locally Lipschitz in its arguments, and the function $h(\cdot)$ is also assumed to be a Lipschitz function. The sets $\mathcal{X}$, $\mathcal{U}$, $\mathcal{W}$, $\mathcal{V}$ and $\mathcal{Y}$ are assumed to be convex, containing the origin in its interior. Then, the estimation problem aims to reconstruct x_t as accurately as possible from measurements y_t and the model f. The standard moving horizon estimator minimises at each sampling time the following cost function:

$$\mathcal{J}_e := \Gamma_\chi(\hat{\chi}) + \int_{t-T_e}^{t} \left(\ell_w(\hat{w}_\tau) + \ell_\nu\left(\hat{\nu}_\tau\right)\right) d\tau \tag{2}$$

where $\hat{\chi} = \hat{x}_{t-T_e|t} - \bar{x}_{t-T_e}$ is the so-called arrival-cost, with corresponding penalisation function $\Gamma_\chi(\hat{\chi})$, and $\bar{x}_{t-T_e}$ represents the best knowledge of the system's state at the beginning of the estimation window. Variables $\hat{w}_\tau$ and $\hat{\nu}_\tau$ are the sequences of estimated process disturbances affecting the systems and residuals of the estimation, respectively. The function ℓ_w and ℓ_ν are definite positive functions, and penalise large values of $\hat{w}_\tau$ and $\hat{\nu}_\tau$, respectively. Moreover, it is assumed that they are lower and upper bounded by $\mathcal{K}$

functions such that $\underline{\ell}_w \leq \ell_w \leq \overline{\ell}_w$ and $\underline{\ell}_\nu \leq \ell_\nu \leq \overline{\ell}_\nu$, as usual in the MHE literature [14]. The optimisation problem to be solved at each sampling time is the following:

$$\begin{aligned} &\min_{\hat{\chi}} \quad \mathcal{J}_e \\ &\text{s.t.} \begin{cases} \hat{\chi} = & \hat{x}_{t-T_e|t} - \bar{x}_{t-T_e} \\ \dot{\hat{x}}_\tau = & f\left(\hat{x}_\tau, \hat{u}_\tau\right) + \hat{w}_\tau \\ y_\tau = & h\left(\hat{x}_\tau\right) + \hat{\nu}_\tau \end{cases} \end{aligned} \tag{3}$$

Note that the main optimisation variables are $\hat{\chi}$ and sequence $\hat{w}_\tau$ since $\hat{\nu}_\tau$ can be determined as $y_\tau - h(\hat{x}_\tau)$. In addition, since measurements are usually taken at discrete instants, it is convenient to formulate problem (3) in the discrete-time domain. The time interval T_e is divided into N_e+1 nodes with time stamps $t_k - T_e + k\,T_s$, with $k \in \mathbb{Z}_{[0,N_e]}$, such that $T_e = N_e T_s$. Each node has associated the measurement taken at time $t_k - T_e + k\,T_s$. Subsequently, the continuous-time system presented in Eq. (1) is evaluated and integrated at each node using a proper numerical integrator Φ. Then, the discrete-time version of the cost function (2) is the following:

$$\mathbb{J}_e := \Gamma_\chi(\hat{\chi}) + \sum_{k=0}^{N_e} (\ell_w(\hat{w}_{j|t_k}) + \ell_\nu(\hat{\nu}_{j|t_k})) \tag{4}$$

where $j = t_k - T_e + k\,T_s$ for simplicity. A slightly abuse of natation is introduced to denote $j+1 = t_k - T_e + (k+1)\,T_s$. Then, the estimation problem that is solved at each sampling time is the following:

$$\begin{aligned} &\min_{\hat{\chi}} \quad \mathbb{J}_e \\ &\text{s.t.} \begin{cases} \hat{\chi} = & \hat{x}_{t_k-T_e|t_k} - \bar{x}_{t_k-T_e} \\ \hat{x}_{j+1|t_k} = & \Phi\left(\hat{x}_{j|t_k}, \hat{u}_{j|t_k}\right) + \hat{w}_{j|t_k} \\ y_j = & h\left(\hat{x}_{j|t_k}\right) + \hat{\nu}_{j|t_k} \end{cases} \end{aligned} \tag{5}$$

When measurements are corrupted by noise, the accuracy of the estimates reduces. The scenario is more challenging when noises do not follow a distributions, such as in the case of outliers. Then, more information, i.e., a larger estimation window is usually required to improve the estimation which increases the computation burden. Furthermore, when measurements are corrupted with outliers, enlarging the estimation window is not enough to obtain an accurate estimation of the system's current state [10]. We aim to formulate a moving horizon scheme able to handle noisy measurements and outliers without increasing significantly the computational burden in comparison with a standard MHE formulation. To achieve that, we assume that several measurements of the measurable states are available at each time node within the interval $[t_k - N_e,\, t_k]$. It is further expanded following.

A. Proposed moving horizon estimation scheme

To improve the performance of the MHE estimator in the presence of disturbances noisy measurements and outliers, we slightly modify the constraints to include all measurements taken simultaneously from those measurable states. Let assume that N_s simultaneous measurements for each measurable state are available. They enter as a convex weighted sum within the constraints of the MHE problem, as follows:

$$\begin{aligned} &\min_{\hat{\chi}} \quad \mathbb{J}_e \\ &\text{s.t.} \begin{cases} \hat{\chi} = & \hat{x}_{t_k-T_e|t_k} - \bar{x}_{t_k-T_e} \\ \hat{x}_{j+1|t_k} = & \Phi\left(\hat{x}_{j|t_k}, \hat{u}_{j|t_k}\right) + \hat{w}_{j|t_k} \\ \sum_{i=1}^{N_s} y_{j,i}\,\alpha_{j,i} = & h\left(\hat{x}_{j|t_k}\right) + \hat{\nu}_{j|t_k} \\ \sum_{i=1}^{N_s} \alpha_{j,i} = & 1,\ \alpha_{j,i} \geq 0 \end{cases} \end{aligned} \tag{6}$$

where the constraints $\sum_{i=1}^{N_s} \alpha_{j,i} = 1$ and $\alpha_{j,i} \geq 0$ ensure the convexity of parameter $\alpha_{j,i}$ for each j. Note that $\alpha_{j,i} = 1/Ns$ averages the measurements taken at instant j, and $\alpha_{j,1} = 1$ corresponds to a standard MHE. Moreover, each measurement obeys the system's output equation $y_{j,i} = h(x_j) + \nu_{j,i}$ with corresponding measurement noise $\nu_{j,i}$.

B. Analysis of the proposed formulation

The problem (6) is solved at each sampling time, delivering the optimal sequence of states and disturbances, in the sense that they minimise the cost function given by Eq. (4). Let us denote $\mathbb{J}^*(\hat{\chi}^*, \hat{w}^*_{j|t_k})$ as the optimal cost, where $\hat{\chi}^*$ and $\hat{w}^*_{j|t_k}$ are the optimal solutions. Then, the following holds:

$$\mathbb{J}^*(\hat{\chi}^*, \hat{w}^*_{j|t_k}) = \quad \Gamma_\chi(\hat{\chi}^*) + \sum_{k=0}^{N_e} (\ell_w(\hat{w}^*_{j|t_k}) + \ell_\nu(\hat{\nu}^*_{j|t_k})) \tag{7}$$

and due to optimality, the following is always verified:

$$\mathbb{J}^*(\hat{\chi}^*, \hat{w}^*_{j|t_k}) \leq \quad \mathbb{J}(\chi, w_j) \tag{8}$$

where $\chi = x_{t_k-T_e} - \bar{x}_{t_k-T_e}$ with $x_{t_k-T_e}$ being the true state at the beginning of the estimation window, and w_j is the true sequence of disturbances acting on the system. Then, the cost incurred solving the problem given by Eq. (6) can be bounded as follows:

$$\begin{aligned} &\Gamma_\chi(\hat{\chi}^*) + \sum_{k=0}^{N_e} (\ell_w(\hat{w}^*_{j|t_k}) + \ell_\nu(\hat{\nu}^*_{j|t_k})) \leq \\ &\quad \Gamma_\chi(\chi) + \sum_{k=0}^{N_e} (\ell_w(w_j) + \ell_\nu(\underline{\nu}_j)) \end{aligned} \tag{9}$$

where $\underline{\nu}_j := \min_i\{\ell_\nu(\nu_{j,i})\}$ with $i \in \mathbb{Z}_{[1,N_s]}$ for each j. Then, assuming that at least one of the N_s measurements is free of outliers, then, the cost incurred in solving the estimation problem given by Eq. (6) correspond to a free of outliers estimation cost. Note that this assumption was already made in the literature [15]. However, its validity depends on the quality of the sensors and if the environments is well controlled. In harsh environments (e.g., underwater, space, or high-temperature industrial settings), sensors are more prone to failures or outliers, making the assumption less realistic. However, our work is in the context of robotics and autonomous navigation of grounded vehicles, where the assumption can be considered to hold.

C. Measurements read in burst

The estimation problem given by Eq. 6 can be solved when the measurements $y_{j,i}$ are not taken simultaneously but in burst by the same sensor. Let us assume that N_s measurements are taken within a temporal window of length δ centred at the time instant j. For the sake of simplify the analysis, let us assume that momentarily the measurements are free of noise. Then, for the first and last measurements, $y_{j-\delta/2,1}$ and $y_{j+\delta/2,N_s}$, respectively, the following holds:

$$\begin{aligned} |y_{j-\delta/2,1} - y_{j+\delta/2,N_s}| &= |h(x_{j-\delta/2}) - h(x_{j+\delta/2})| \\ &\leq \mathcal{L}_h |x_{j-\delta/2} - x_{j+\delta/2}| \\ &= \mathcal{L}_h \frac{|x_{j-\delta/2} - x_{j+\delta/2}|}{\delta}\delta \end{aligned} \tag{10}$$

Note that $|x_{j-\delta/2} - x_{j+\delta/2}|/\delta$ can be identified as a first order backward approximation of $\dot{x}$ at $t = j + \delta/2$, such that the following holds $|\dot{x} - (x_{j-\delta/2} - x_{j+\delta/2})/\delta| = O(\delta)$. Then, the following holds:

$$\begin{aligned} |y_{j-\delta/2} - y_{j+\delta/2}| &\approx \mathcal{L}_h |\dot{x}|\,\delta \\ &\leq \mathcal{L}_h |\dot{X}|\,\delta \end{aligned} \tag{11}$$

where $|\dot{X}| := \max\{|\dot{x}|\}\ \forall x \in \mathcal{X}$. Now let us introduce the effect of the measurement noises:

$$\begin{aligned} |y_{j-\delta/2} - y_{j+\delta/2}| = &\ |h(x_{j-\delta/2}) + \nu_{j-\delta/2} - h(x_{j+\delta/2}) - \\ &\ \nu_{j+\delta/2}| \\ \leq &\ |h(x_{j-\delta/2}) - h(x_{j+\delta/2})| + \\ &\ |\nu_{j-\delta/2} - \nu_{j+\delta/2}| \\ \leq &\ \mathcal{L}_h |\dot{X}|\,\delta + |\nu_{j-\delta/2} - \nu_{j+\delta/2}| \end{aligned} \tag{12}$$

Thus, taking measurements in a burst is equivalent to reading them simultaneously with higher noise, whose amplitude depends on the output function's Lipschitz constant, the system's dynamic, and, the delay between the first and the last measurements.

D. Robust stability Analysis

The robust stability of the moving horizon estimator given by Eq. 6 with multiple simultaneous or burst measurements can be established whenever the inequality given by Eq. (9) holds. Under this consideration, the procedures needed to prove the robust stability are the same as reported in [14]. It is worth noting that the error bound for the estimator given by Eq. (6) is the same as the one in [14], which does not consider outliers but regular process and measurement noises.

In the context of our estimator, we require that inequality (9) holds with $\underline{\nu}_{j,i}$ corresponding to an inlier measurement noise. However, even in the case that $\underline{\nu}_{j,i}$ corresponds to an outlier noise, inequality (9) holds due to optimality. In this case, the robust stability can be guaranteed for a larger estimation window N_e and increased error bound. For a detailed explanation, the reader is referred to [14]. □

III. Experiments and Results

The proposed framework given by Eq. (6) was implemented with CasADI [16] on MATLAB. The algorithms run on a PC ASUS TUF F15 with 16 GB of RAM and a 13-th Gen Intel Core i7 with Ubuntu 22.04.

A. Simulated experiments

B. Field experiments

IV. Conclusions

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