EXERCISES FOR SECTION 2.5

1. **(a)** We compute

$$\frac{dx}{dt} = \frac{d(\cos t)}{dt} = -\sin t = -y \quad \text{and} \quad \frac{dy}{dt} = \frac{d(\sin t)}{dt} = \cos t = x,$$

so $(\cos t, \sin t)$ is a solution.

(b)

Table 2.1

t	Euler's approx.	actual	distance
0	$(1, 0)$	$(1, 0)$	
4	$(-2.06, -1.31)$	$(-0.65, -0.76)$	1.51
6	$(2.87, -2.51)$	$(0.96, -0.28)$	2.94
10	$(-9.21, 1.41)$	$(-0.84, -0.54)$	8.59

(c)

Table 2.2

t	Euler's approx.	actual	distance
0	$(1, 0)$	$(1, 0)$	
4	$(-.81, -.91)$	$(-0.65, -0.76)$	0.22
6	$(1.29, -.40)$	$(0.96, -0.28)$	0.35
10	$(-1.41, -.85)$	$(-0.84, -.54)$	0.65

(d) The solution curves for this system are all circles centered at the origin. Since Euler's method uses tangent lines to approximate the solution curve and the tangent line to any point on a circle is entirely outside the circle (except at the point of tangency), each step of the Euler approximation takes the approximate solution farther from the origin. So the Euler approximations always spiral away from the origin for this system.

2. **(a)** We compute

$$\frac{dx}{dt} = \frac{d(e^{2t})}{dt} = 2e^{2t} = 2x \quad \text{and} \quad \frac{dy}{dt} = \frac{d(3e^{t})}{dt} = 3e^{t} = y,$$

so $(e^{2t}, 3e^{t})$ is a solution.

(b)

Table 2.3

t	Euler's approx.	actual	distance
0	$(1, 3)$	$(1, 3)$	
2	$(16, 15.1875)$	$(54.59, 22.17)$	39.22
4	$(256, 76.88)$	$(2981, 164)$	2726
6	$(4096, 389)$	$(162755, 1210)$	158661