The solution for $t > 3$ is $v_c(t) = ke^{-2t}$. Evaluating at $t = 3$, we get

$$ke^{-6} = 3 + 3e^{-6}$$

$$k = 3e^6 + 3.$$

So $v_c(t) = (3e^6 + 3)e^{-2t}$. To check that $v_c(t)$ is a solution, we calculate

$$\frac{dv_c}{dt} = \frac{d}{dt}(3e^6 + 3)e^{-2t} = -2(3e^6 + 3)e^{-2t}$$

as well as

$$-2v_c = -2(3e^6 + 3)e^{-2t}.$$

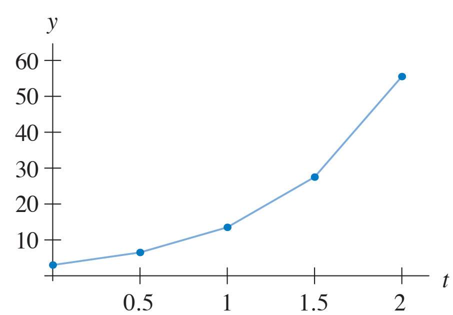
Since they agree, $v_c(t)$ is a solution.

## EXERCISES FOR SECTION 1.4

**1.**

Table 1.1
Results of Euler's method

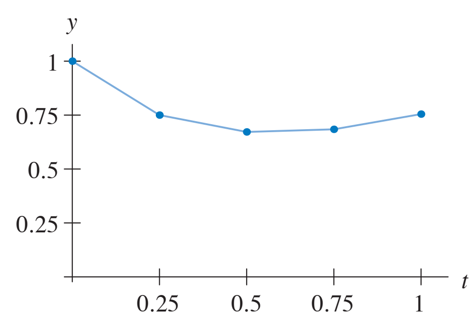
| $k$ | $t_k$ | $y_k$ | $m_k$ |
|---|---|---|---|
| 0 | 0 | 3 | 7 |
| 1 | 0.5 | 6.5 | 14 |
| 2 | 1.0 | 13.5 | 28 |
| 3 | 1.5 | 27.5 | 56 |
| 4 | 2.0 | 55.5 | |




**2.**

Table 1.2
Results of Euler's method ($y_k$ rounded to two decimal places)

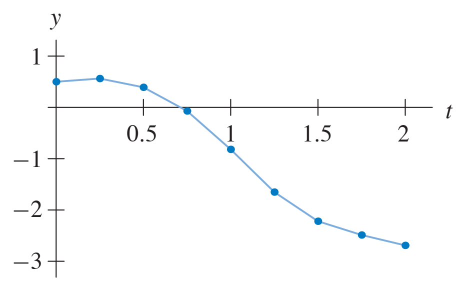
| $k$ | $t_k$ | $y_k$ | $m_k$ |
|---|---|---|---|
| 0 | 0 | 1 | -1 |
| 1 | 0.25 | 0.75 | -0.3125 |
| 2 | 0.5 | 0.67 | 0.0485 |
| 3 | 0.75 | 0.68 | 0.282 |
| 4 | 1.0 | 0.75 | |



**3.**

Table 1.3
Results of Euler's method (shown rounded to two decimal places)

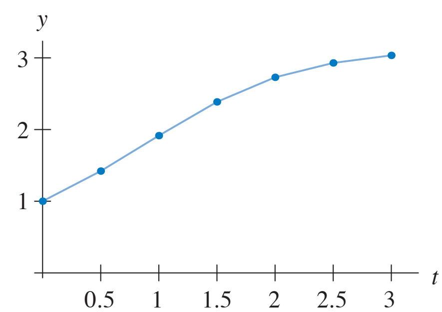
| $k$ | $t_k$ | $y_k$ | $m_k$ |
|---|---|---|---|
| 0 | 0 | 0.5 | 0.25 |
| 1 | 0.25 | 0.56 | −0.68 |
| 2 | 0.50 | 0.39 | −1.85 |
| 3 | 0.75 | −0.07 | −2.99 |
| 4 | 1.00 | −0.82 | −3.33 |
| 5 | 1.25 | −1.65 | −2.27 |
| 6 | 1.50 | −2.22 | −1.07 |
| 7 | 1.75 | −2.49 | −0.81 |
| 8 | 2.00 | −2.69 | |




**4.**

Table 1.4
Results of Euler's method (to two decimal places)

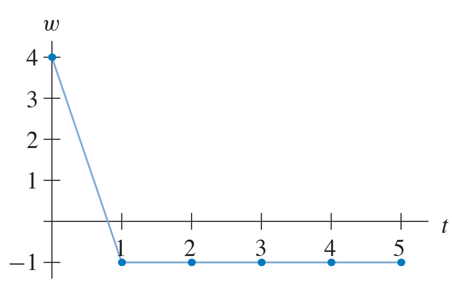
| $k$ | $t_k$ | $y_k$ | $m_k$ |
|---|---|---|---|
| 0 | 0 | 1 | 0.84 |
| 1 | 0.5 | 1.42 | 0.99 |
| 2 | 1.0 | 1.91 | 0.94 |
| 3 | 1.5 | 2.38 | 0.68 |
| 4 | 2.0 | 2.73 | 0.40 |
| 5 | 2.5 | 2.93 | 0.21 |
| 6 | 3.0 | 3.03 | |




**5.**

Table 1.5
Results of Euler's method

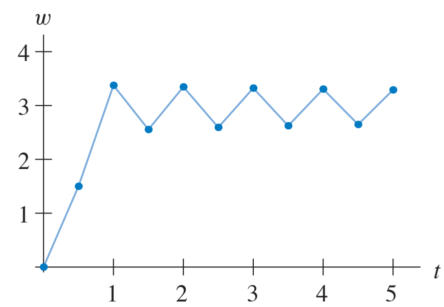
| $k$ | $t_k$ | $w_k$ | $m_k$ |
|---|---|---|---|
| 0 | 0 | 4 | −5 |
| 1 | 1 | −1 | 0 |
| 2 | 2 | −1 | 0 |
| 3 | 3 | −1 | 0 |
| 4 | 4 | −1 | 0 |
| 5 | 5 | −1 | |



**6.**

Table 1.6
Results of Euler's method (shown rounded to two decimal places)

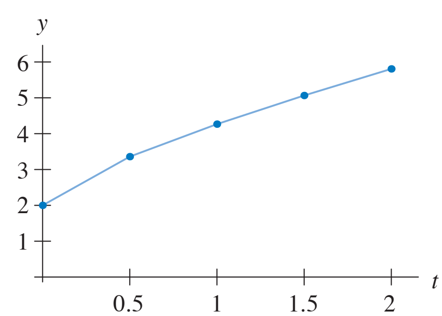
| $k$ | $t_k$ | $w_k$ | $m_k$ |
|---|---|---|---|
| 0 | 0 | 0 | 3 |
| 1 | 0.5 | 1.5 | 3.75 |
| 2 | 1.0 | 3.38 | −1.64 |
| 3 | 1.5 | 2.55 | 1.58 |
| 4 | 2.0 | 3.35 | −1.50 |
| 5 | 2.5 | 2.59 | 1.46 |
| 6 | 3.0 | 3.32 | −1.40 |
| 7 | 3.5 | 2.62 | 1.36 |
| 8 | 4.0 | 3.31 | −1.31 |
| 9 | 4.5 | 2.65 | 1.28 |
| 10 | 5.0 | 3.29 | |




**7.**

Table 1.7
Results of Euler's method (shown rounded to two decimal places)

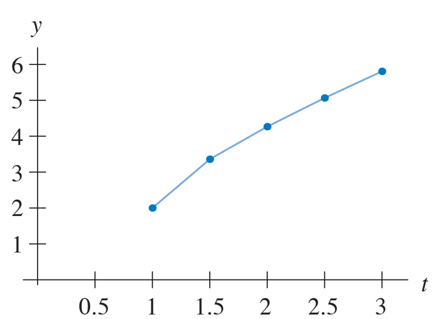
| $k$ | $t_k$ | $y_k$ | $m_k$ |
|---|---|---|---|
| 0 | 0 | 2 | 2.72 |
| 1 | 0.5 | 3.36 | 1.81 |
| 2 | 1.0 | 4.27 | 1.60 |
| 3 | 1.5 | 5.06 | 1.48 |
| 4 | 2.0 | 5.81 | |




**8.**

Table 1.8
Results of Euler's method (shown rounded to two decimal places)

| $k$ | $t_k$ | $y_k$ | $m_k$ |
|---|---|---|---|
| 0 | 1.0 | 2 | 2.72 |
| 1 | 1.5 | 3.36 | 1.81 |
| 2 | 2.0 | 4.27 | 1.60 |
| 3 | 2.5 | 5.06 | 1.48 |
| 4 | 3.0 | 5.81 | |

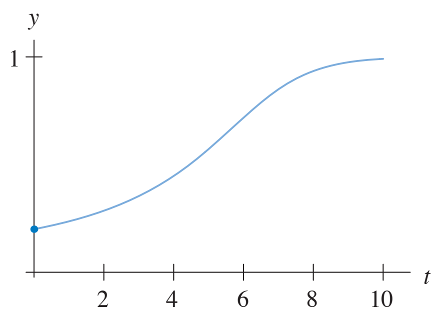


**9.**

Table 1.9

Results of Euler's method (shown rounded to three decimal places)

| $k$ | $t_k$ | $y_k$ | $m_k$ |
|---|---|---|---|
| 0 | 0.0 | 0.2 | 0.032 |
| 1 | 0.1 | 0.203 | 0.033 |
| 2 | 0.2 | 0.206 | 0.034 |
| 3 | 0.3 | 0.210 | 0.035 |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 99 | 9.9 | 0.990 | 0.010 |
| 100 | 10.0 | 0.991 | |




**10.**

Table 1.10

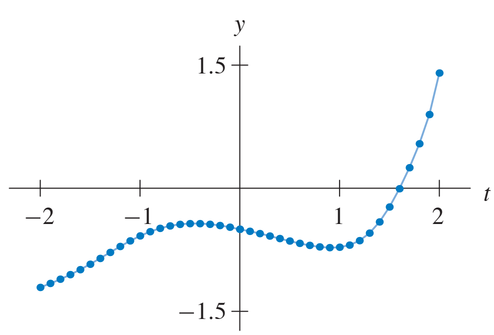
Results of Euler's method with $\Delta t$ negative (shown rounded to three decimal places)

| $k$ | $t_k$ | $y_k$ | $m_k$ |
|---|---|---|---|
| 0 | 0 | −0.5 | −0.25 |
| 1 | −0.1 | −0.475 | −0.204 |
| 2 | −0.2 | −0.455 | −0.147 |
| 3 | −0.3 | −0.440 | −0.080 |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 19 | −1.9 | −1.160 | 0.488 |
| 20 | −2.0 | −1.209 | 0.467 |

Table 1.11

Results of Euler's method with $\Delta t$ positive (shown rounded to three decimal places)

| $k$ | $t_k$ | $y_k$ | $m_k$ |
|---|---|---|---|
| 0 | 0 | −0.5 | −0.25 |
| 1 | 0.1 | −0.525 | −0.279 |
| 2 | 0.2 | −0.553 | −0.298 |
| 3 | 0.3 | −0.583 | −0.306 |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 19 | 1.9 | 0.898 | 5.058 |
| 20 | 2.0 | 1.404 | 9.532 |




**11.** As the solution approaches the equilibrium solution corresponding to $w = 3$, its slope decreases. We do not expect the solution to "jump over" an equilibrium solution (see the Existence and Uniqueness Theorem in Section 1.5).

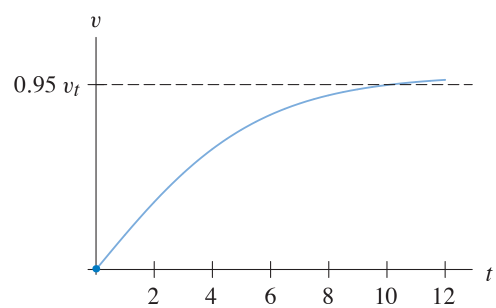
**12.** According to the formula derived in part (b) of Exercise 12 of Section 1.1, the terminal velocity ($v_t$) of the freefalling skydiver is

$$v_t = \sqrt{\frac{mg}{k}} = \sqrt{\frac{(54)(9.8)}{0.18}} = \sqrt{2940} \approx 54.22 \text{ m/s}.$$

Therefore, 95% of her terminal velocity is $0.95v_t = 0.95\sqrt{2940} \approx 51.51$ m/s. At the moment she jumps from the plane, $v(0) = 0$. We choose $\Delta t = 0.01$ to obtain a good approximation of when the skydiver reaches 95% of her terminal velocity. Using Euler's method with $\Delta t = 0.01$, we see that the skydiver reaches 95% of her terminal velocity when $t \approx 10.12$ seconds.

Table 1.12
Results of Euler's method (shown rounded to three decimal places)

| $k$ | $t_k$ | $v_k$ | $m_k$ |
|---|---|---|---|
| 0 | 0.0 | 0.0 | 9.8 |
| 1 | 0.01 | 0.098 | 9.800 |
| 2 | 0.02 | 0.196 | 9.800 |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 1011 | 10.11 | 51.498 | 0.960 |
| 1012 | 10.12 | 51.508 | 0.956 |
| ⋮ | ⋮ | ⋮ | ⋮ |




**13.** Because the differential equation is autonomous, the computation that determines $y_{k+1}$ from $y_k$ depends only on $y_k$ and $\Delta t$ and not on the actual value of $t_k$. Hence the approximate $y$-values that are obtained in both exercises are the same. It is useful to think about this fact in terms of the slope field of an autonomous equation.

**14.** Euler's method is not accurate in either case because the step size is too large. In Exercise 5, the approximate solution "jumps onto" an equilibrium solution. In Exercise 6, the approximate solution "crisscrosses" a different equilibrium solution. Approximate solutions generated with smaller values of $\Delta t$ indicate that the actual solutions do not exhibit this behavior (see the Existence and Uniqueness Theorem of Section 1.5).

**15.**

Table 1.13
Results of Euler's method with $\Delta t = 1.0$ (shown to two decimal places)

| $k$ | $t_k$ | $y_k$ | $m_k$ |
|---|---|---|---|
| 0 | 0 | 1 | 1 |
| 1 | 1 | 2 | 1.41 |
| 2 | 2 | 3.41 | 1.85 |
| 3 | 3 | 5.26 | 2.29 |
| 4 | 4 | 7.56 | |