

Note: Partial credit can not be awarded unless there is legible work to assess.

1. Suppose we have the differential equation

$$\frac{dy}{dt} = f(t, y).$$

Assume the function f satisfies the hypotheses of the Existence and Uniqueness Theorem in the entire ty -plane. Also assume that $y_1(t) = t + 2$ and $y_2(t) = -t^2$ are solutions for all t . What can you conclude about a solution $y(t)$ defined for all t with the initial condition $y(0) = 1$? Specifically, give any bounds on the values of $y(t)$ and discuss its limiting behavior when t approaches negative infinity and when t approaches positive infinity.

2. Give the phase line of

$$\frac{dy}{dt} = 3y^3 - 12y^2$$

and sketch possible solutions in the ty -plane relative to the equilibrium solutions.

Bonus: (1 point) Identify any equilibrium points as sinks, sources, or nodes.