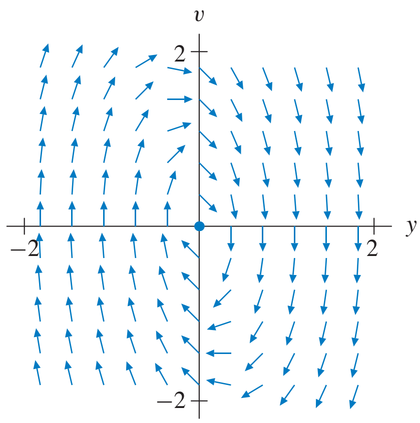
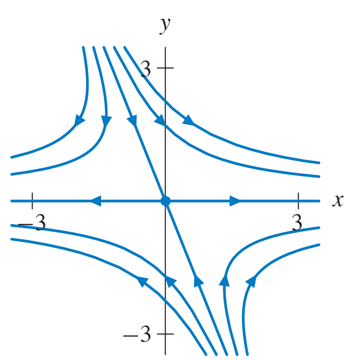
The characteristic polynomial is $\lambda^2 + \lambda + 4$, and its roots are the complex numbers $(-1 \pm \sqrt{15}\, i)/2$. Therefore there are no straight-line solutions. According to the direction field, the solution curves seem to spiral around the origin.

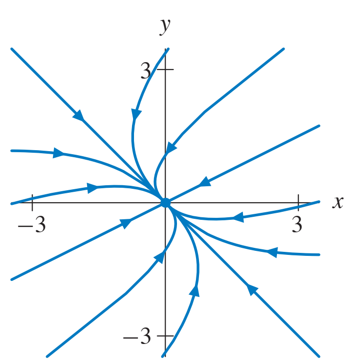


EXERCISES FOR SECTION 3.3

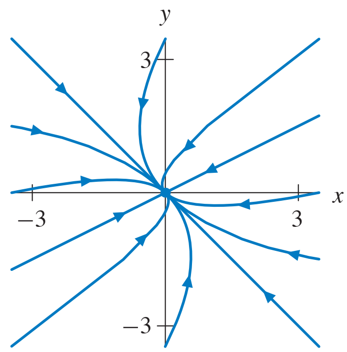
1. As we computed in Exercise 1 of Section 3.2, the eigenvalues are $\lambda_1 = -2$ and $\lambda_2 = 3$. The eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1 = -2$ satisfy $5x_1 = -2y_1$, and the eigenvectors (x_2, y_2) for $\lambda_2 = 3$ satisfy the equation $y_2 = 0$. The equilibrium point at the origin is a saddle.



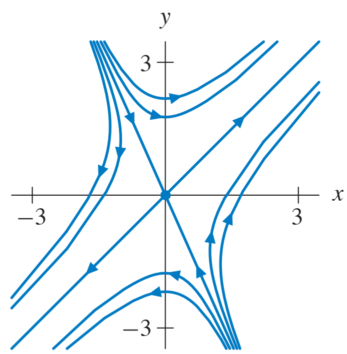
2. As we computed in Exercise 2 of Section 3.2, the eigenvalues are $\lambda_1 = -2$ and $\lambda_2 = -5$. The eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1 = -2$ satisfy $y_1 = -x_1$, and the eigenvectors (x_2, y_2) for $\lambda_2 = -5$ satisfy $x_2 = 2y_2$. The equilibrium point at the origin is a sink.



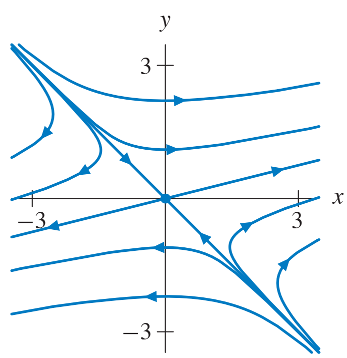
3. As we computed in Exercise 3 of Section 3.2, the eigenvalues are $\lambda_1 = -3$ and $\lambda_2 = -6$. The eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1 = -3$ satisfy $y_1 = -x_1$, and the eigenvectors for $\lambda_2 = -6$ satisfy $x_2 = 2y_2$. The equilibrium point at the origin is a sink.



4. As we computed in Exercise 6 of Section 3.2, the eigenvalues are $\lambda_1 = -4$ and $\lambda_2 = 9$. The eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1 = -4$ satisfy $9x_1 = -4y_1$, and the eigenvectors (x_2, y_2) for $\lambda_2 = 9$ satisfy the equation $y_2 = x_2$. The equilibrium point at the origin is a saddle.



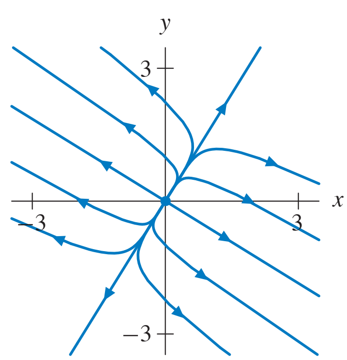
5. As we computed in Exercise 7 of Section 3.2, the eigenvalues are $\lambda_1 = -1$ and $\lambda_2 = 4$. The eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1 = -1$ satisfy $y_1 = -x_1$, and the eigenvectors (x_2, y_2) for $\lambda_2 = 4$ satisfy $x_2 = 4y_2$. The equilibrium point at the origin is a saddle.



6. As we computed in Exercise 8 of Section 3.2, the eigenvalues are

$$\lambda_1 = \frac{3 + \sqrt{5}}{2} \quad \text{and} \quad \lambda_2 = \frac{3 - \sqrt{5}}{2}.$$

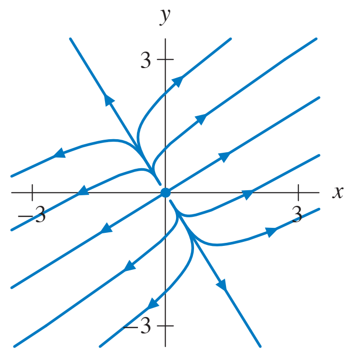
The eigenvectors (x_1, y_1) for the eigenvalue λ_1 satisfy $y_1 = (1 - \sqrt{5})x_1/2$, and the eigenvectors (x_2, y_2) for the eigenvalue λ_2 satisfy $y_2 = (1 + \sqrt{5})x_2/2$. The equilibrium point at the origin is a source.



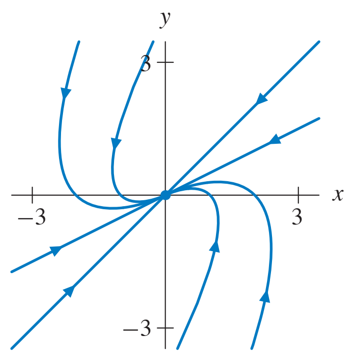
7. As we computed in Exercise 9 of Section 3.2, the eigenvalues are

$$\lambda_1 = \frac{3+\sqrt{5}}{2} \quad \text{and} \quad \lambda_2 = \frac{3-\sqrt{5}}{2}.$$

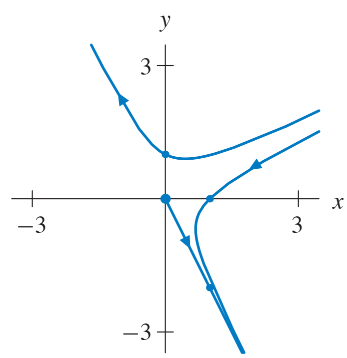
The eigenvectors (x_1, y_1) for the eigenvalue λ_1 satisfy $y_1 = (-1+\sqrt{5})x_1/2$, and the eigenvectors (x_2, y_2) for λ_2 satisfy $y_2 = (-1-\sqrt{5})x_2/2$. The equilibrium point at the origin is a source.



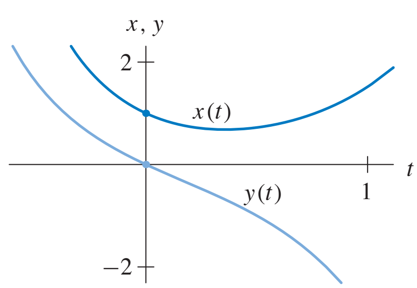
8. As we computed in Exercise 10 of Section 3.2, the eigenvalues are $\lambda_1 = -2$ and $\lambda_2 = -3$. The eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1 = -2$ satisfy $x_1 = 2y_1$, and the eigenvectors (x_2, y_2) for $\lambda_2 = -3$ satisfy $x_2 = y_2$. The equilibrium point at the origin is a sink.



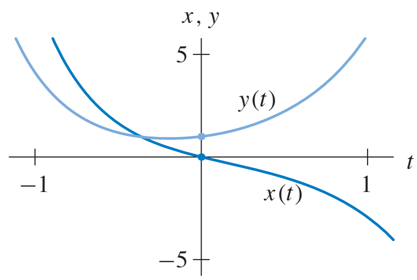
9. As we computed in Exercise 11 of Section 3.2, the eigenvalues are $\lambda_1 = 2$ and $\lambda_2 = -3$. The eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1 = 2$ satisfy $y_1 = -2x_1$, and the eigenvectors for the eigenvalue $\lambda_2 = -3$ satisfy $x_1 = 2y_1$. The equilibrium point at the origin is a saddle. The solution curves in the phase plane for the initial conditions $(1, 0)$, $(0, 1)$, and $(1, -2)$ are shown in the figure on the right.



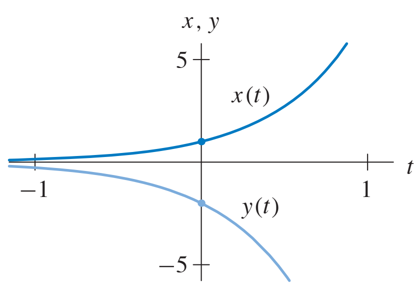
(a) The solution with initial condition $(1, 0)$ is asymptotic to the line $y = -2x$ in the fourth quadrant as $t \to \infty$ and to the line $x = 2y$ in the first quadrant as $t \to -\infty$.



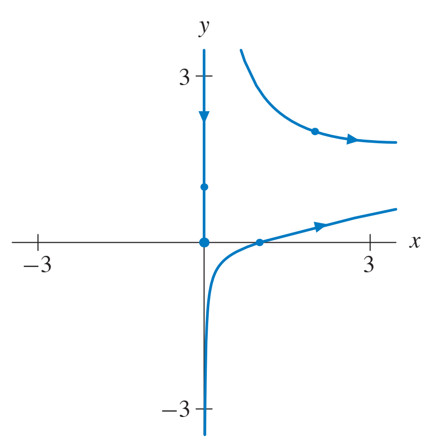
(b) The solution curve with initial condition $(1, 0)$ is asymptotic to the line $y = -2x$ in the second quadrant as $t \to \infty$ and to the line $x = 2y$ in the first quadrant as $t \to -\infty$.



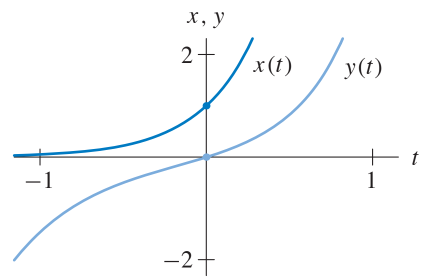
(c) The solution curve with initial condition $(1, -2)$ is on the line of eigenvectors for the eigenvalue $\lambda_1 = 2$. Hence, this solution curve stays on the line $y = -2x$. It approaches the origin as $t \to -\infty$, and it tends to ∞ in the fourth quadrant as $t \to \infty$.



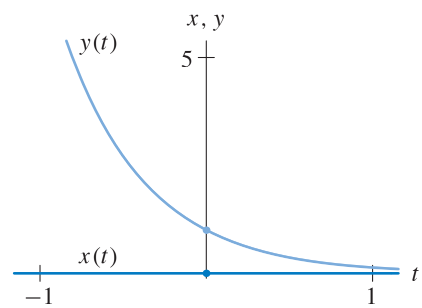
10. As we computed in Exercise 12 of Section 3.2, the eigenvalues are $\lambda_1 = 3$ and $\lambda_2 = -2$. The eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1 = 3$ satisfy $y_1 = x_1/5$, and the eigenvectors (x_2, y_2) for the eigenvalue $\lambda_2 = -2$ satisfy $x_2 = 0$. The equilibrium point at the origin is a saddle. Therefore, the solution curves in the phase plane for the initial conditions $(1, 0)$, $(0, 1)$, and $(2, 2)$ are shown in the figure on the right.



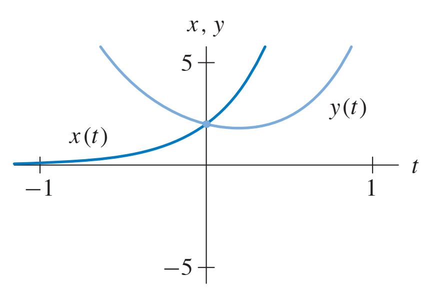
(a) The solution curve with initial condition $(1, 0)$ is asymptotic to the negative y-axis as $t \to -\infty$ and is asymptotic to the line $y = x/5$ in the first quadrant as $t \to \infty$.



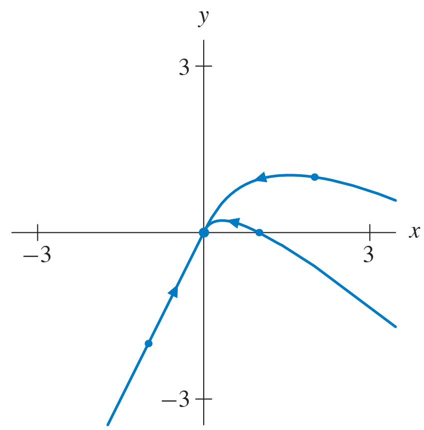
(b) The solution curve with initial condition $(0, 1)$ lies entirely on the positive y-axis, and $y(t) \to 0$ in an exponential fashion as $t \to \infty$.



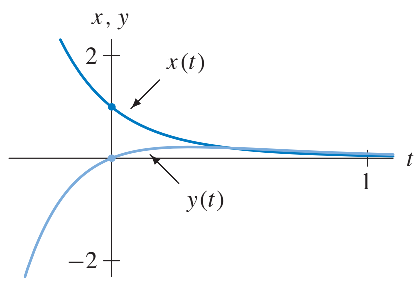
(c) The solution curve with initial condition $(2, 2)$ lies entirely in the first quadrant. It is asymptotic to the positive y-axis as $t \to -\infty$ and asymptotic to the line $y = x/5$ as $t \to \infty$.



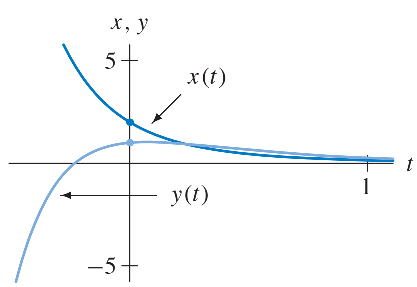
11. As we computed in Exercise 13 of Section 3.2, the eigenvalues are $\lambda_1 = -5$ and $\lambda_2 = -2$. The eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1 = -5$ satisfy $y_1 = -x_1$, and the eigenvectors (x_2, y_2) for the eigenvalue $\lambda_2 = -2$ satisfy $y_2 = 2x_2$. The equilibrium point at the origin is a sink. The solution curves in the phase plane for the initial conditions $(1, 0)$, $(2, 1)$, and $(-1, -2)$ are shown in the following figure.



(a) The solution curve with initial condition $(1, 0)$ approaches the origin tangent to the line $y = 2x$.



(b) The solution curve with initial condition $(2, 1)$ approaches the origin tangent to the line $y = 2x$.



(c) The initial condition $(-1, -2)$ is an eigenvector associated to the eigenvalue $\lambda_2 = -2$. The corresponding solution curve approaches the origin along the line $y = 2x$ as $t \to \infty$.