Math 2410-10/15 # Quiz #4 **February 27^{th}, 2017**

Note: Partial credit can not be awarded unless there is legible work to assess.

1. Find all equilibrium points of the following predator-prey model and explain the significance of these points in terms of the predator and prey populations.

$$\frac{dR}{dt} = 10R\left(1 - \frac{R}{10}\right) - 2RF$$
$$\frac{dF}{dt} = -8F + 4RF$$

Solution: Recall equilibrium points are values for R and F such that:

$$10R\left(1 - \frac{R}{10}\right) - 2RF = R(10 - R - 2F) = 0$$
$$-8F + 4RF = F(-8 + 4R) = 0$$

It is easy to see that this happens if R and F both equal zero. If $R \neq 0$ then either $F = 0$ or $(-8 + 4R) = 0$. If $F = 0$, then we see $(10 - R) = 0$ so R must be 10. If $F \neq 0$, then $-8 + 4R = 0$ so R must be 2. This in turn implies that $10 - 2 - 2F = 0$ so F must be 4. Hence, the equilibrium points for this system are (0,0) in which both populations are extinct; (10,0) in which the predator population is extinct and the prey population is at the carrying capacity of the environment; and (2,4) in which the prey and predator populations are both present (of size 2 units and 4 units respectively) and are interacting, reproducing and dying off at rates which correspond to a perfect balance in the growth of both populations.

2. Draw a 9 point vector field on the unit square and sketch the phase portrait for the following system.

$$\frac{dx}{dt} = x$$
$$\frac{dy}{dt} = -y$$

Solution: The vector field I wish to draw on the unit square is $\mathbf{F}(x, y) = (x, -y)$. At each point (x, y), I draw the corresponding vector $\mathbf{F}(x, y)$ (see figure 1, below).

To sketch the phase portrait, I first plot any equilibrium points: here that is only the origin. Next, I consider solutions that lie along the x and y-axes. Notice that on the x-axis (i.e. when $y = 0$), the dy/dt-component is 0 while the dx/dt-component is positive. Hence solution curves on this axis tend away from the origin in a straight line. Similarly, along the y-axis, the dx/dt-component is 0 and the dy/dt-component is negative. Hence solution curves on this axis tend toward the origin in a straight line. Finally, I consider curves lying in one of the four quardrents: if we consider a solution curve in the first quadrant we see its dx/dt-component is positive and its

dy/dt-component is negative. Thus, solutions tend away from the y-axis and towards the x-axis. Similar analysis allows us to conclude the shapes of solution curves in the other three quadrants. See figure 2, below, for my sketch of the phase portrait. Figure 3 presents both the vector field and the phase portrait together in the phase plane.

(x, y)	$\mathbf{F}(x, y)$
$(-1, 1)$	$(-1, -1)$
$(0, 1)$	$(0, -1)$
$(1, 1)$	$(1, -1)$
$(-1, 0)$	$(-1, 0)$
$(0, 0)$	$(0, 0)$
$(1, 0)$	$(1, 0)$
$(-1, -1)$	$(-1, 1)$
$(0, -1)$	$(0, 1)$
$(1, -1)$	$(1, 1)$

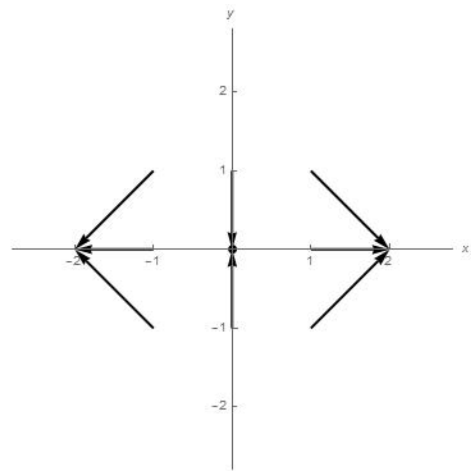

Figure 1: The 9 point vector field on the unit square

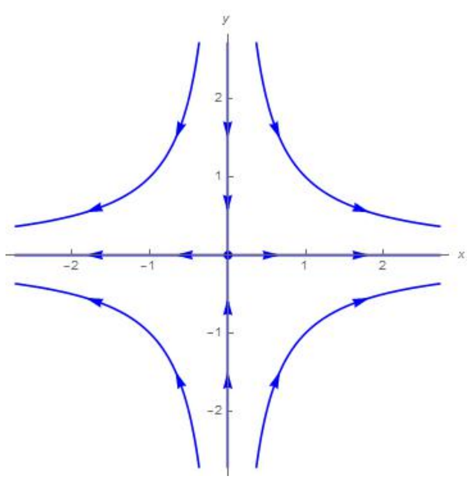

Figure 2: The phase portrait of this system

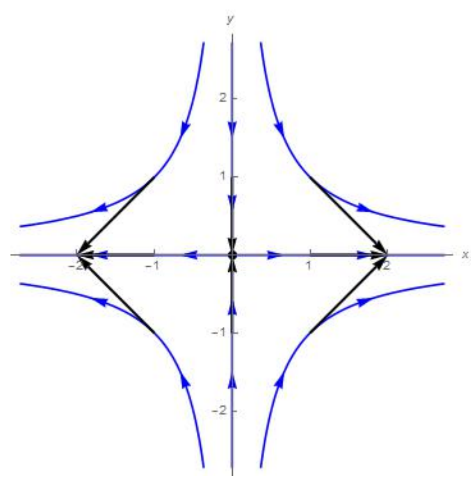

Figure 3: The phase portrait with the desired vector field overlain