Math 2110-003 Quiz 9 **Name:** ______________________________ **Due:** 12/6/17

This is a take-home quiz. It is due completed at the beginning of class on Wednesday 12/6/17. This quiz will be graded with partial credit. Feel free to use the back of this page if you need extra room. Please indicate if you do so.

1. (3 points) Give a parametrization, including the appropriate u- and v-bounds, for the portion of the sphere $x^2 + y^2 + z^2 = 16$ where $x \geq 0$ and $z \geq 0$.

Solution: $\mathbf{r}(\phi, \theta) = \langle 4\sin\phi\cos\theta, 4\sin\phi\sin\theta, 4\cos\phi\rangle$ with $(\phi, \theta) \in [0, \pi/2] \times [-\pi/2, \pi/2]$.

2. (7 points) Let S be the surface given by the cone $z = \sqrt{4x^2 + 4y^2}$ below $z = 6$ including its top, oriented outward.
   (a) (1 point) Sketch S including it's orientation.
   (b) (6 points) Let $\mathbf{F} = \langle 2x, 2y, xyz\rangle$. Compute $\iint_S \mathbf{F} \bullet d\mathbf{S}$.

Solution: We compute the desired integral by computing two constituent integrals

$$\iint_S \mathbf{F} \bullet d\mathbf{S} = \iint_{S_1} \mathbf{F} \bullet d\mathbf{S} + \iint_{S_2} \mathbf{F} \bullet d\mathbf{S}$$

So, we need parametrize both surfaces:

$$\begin{aligned} S_1 : &\mathbf{r}(u, v) = \langle u\cos v, u\sin v, 2u\rangle \\ &(u, v) \in [0, 3] \times [0, 2\pi] \\ &\mathbf{r}_u \times \mathbf{r}_v = \langle -2u\cos v, -2u\sin v, u\rangle \text{ is the wrong orientation!} \\ &\mathbf{r}_v \times \mathbf{r}_u = \langle 2u\cos v, 2u\sin v, -u\rangle \text{ is the right orientation!} \end{aligned}$$

$$\begin{aligned} S_2 &\mathbf{r}(x, y) = \langle x, y, 6\rangle \\ &(x, y) \in \{(x, y) : x^2 + y^2 \leq 9\} \\ &\mathbf{r}_x \times \mathbf{r}_y = \langle 0, 0, 1\rangle \end{aligned}$$

With that we simply compute both integrals. First,

$$\begin{aligned} \iint_{S_1} \mathbf{F} \bullet d\mathbf{S} &= \int_0^{2\pi}\int_0^3 \mathbf{F}(\mathbf{r}(u, v)) \bullet (\mathbf{r}_u \times \mathbf{r}_v)\, du\, dv \\ &= \int_0^{2\pi}\int_0^3 \langle 2u\cos v, 2u\sin v, 2u^3\cos v\sin v\rangle \bullet \langle 2u\cos v, 2u\sin v, -u\rangle\, du\, dv \\ &= \int_0^{2\pi}\int_0^3 4u^2 - 2u^4\cos v\sin v\, du\, dv \\ &= 72\pi \end{aligned}$$

and second,

$$\begin{aligned} \iint_{S_2} \mathbf{F} \bullet d\mathbf{S} &= \iint_D \langle 2x, 2y, xy6\rangle \bullet \langle 0, 0, 1\rangle\, du\, dv \\ &= \int_0^{2\pi}\int_0^3 6r^3\cos\theta\sin\theta\, dr\, d\theta \\ &= 0 \end{aligned}$$

$$\boxed{\text{Thus, we can conclude } \iint_S \mathbf{F} \bullet d\mathbf{S} = 72\pi.}$$