Math 2410-10/15 **Quiz #1** **January 23rd, 2017**

Note: Partial credit can not be awarded unless there is legible work to assess.

1. Consider the population model

$$\frac{dP}{dt} = 0.7P\left(1 - \frac{P}{115}\right),$$

where $P(t)$ is the population at time t.

(i) For what values of P is the population in equilibrium?

(ii) For what values of P is the population increasing?

(iii) For what values of P is the population decreasing?

Solution: To solve all three questions we simply need to identify what values of P make $dP/dt = 0$, positive, or negative respectively for parts (i), (ii), and (iii). So, for (i) we see $P = 0$ and $P = 115$ make $dP/dt = 0$ and thus are the values for which P is in equilibrium. For (ii) we see if $P > 0$ but below 115 then $dP/dt > 0$ and thus for values of P in $(0, 115)$ we see that P is increasing. And finally, for (iii), if $P < 0$ or $P > 115$ then $dP/dt < 0$ and thus for values of P in $(-\infty, 0)$ or $(115, \infty)$ we have that P is decreasing.

2. Find the general solution of $\dfrac{dy}{dt} = \dfrac{t}{t^2y + y}$.

Solution: First, we see that $y(t)$ cannot be 0 for any value of t. Second, we see that this diff. eq. is seperable:

$$\frac{dy}{dt} = \frac{t}{t^2y + y} = \frac{1}{y} \cdot \frac{t}{t^2 + 1}.$$

Thus, we solve the following equation for y:

$$\int y\,dy = \int \frac{t}{t^2+1}\,dt \Longrightarrow \frac{y^2}{2} = \frac{1}{2}\ln|t^2+1| + C$$
$$\Longrightarrow y(t) = \pm\sqrt{\ln(t^2+1) + C}.$$

Thus $y(t) = \pm\sqrt{\ln(t^2+1) + C}$ is our general solution where sign is determined by an initial condition.