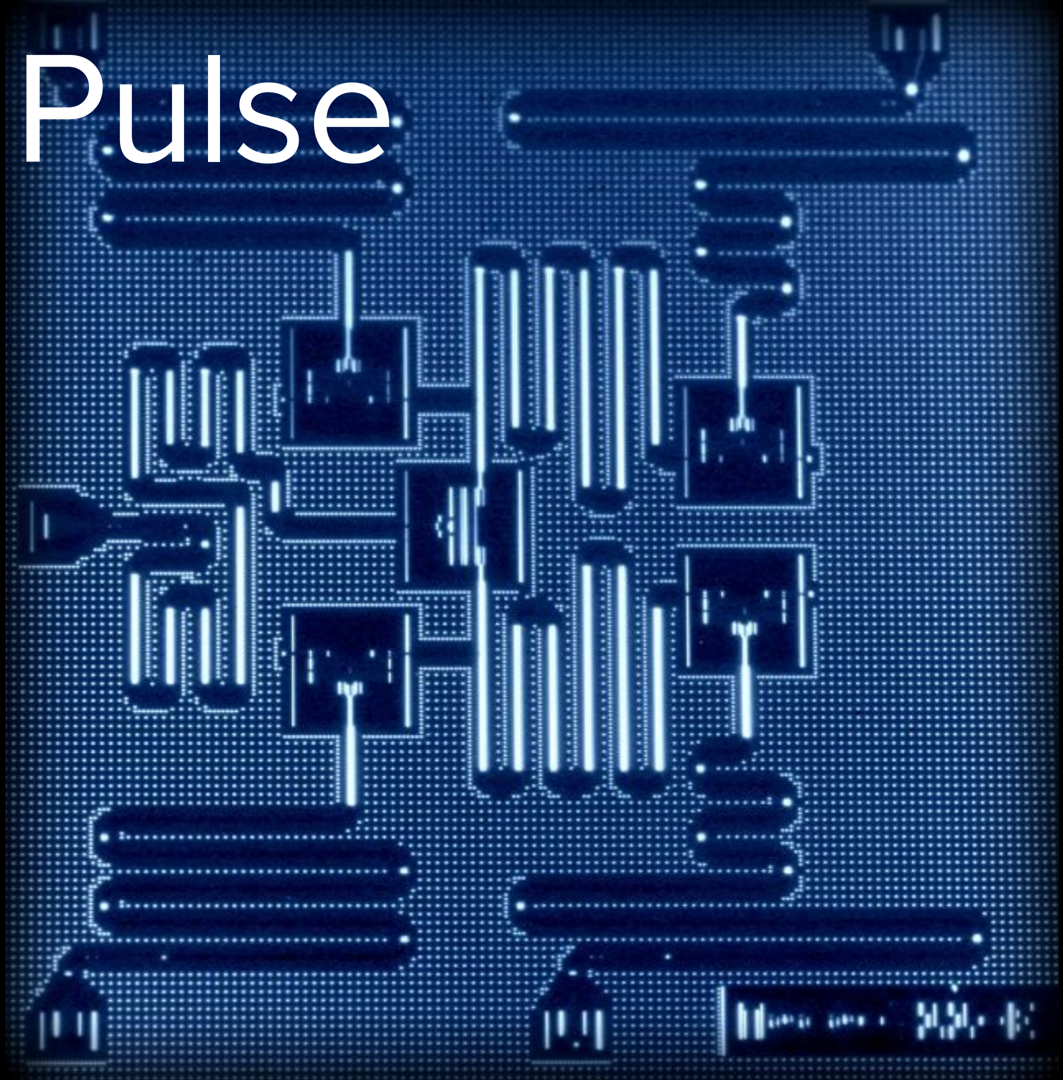
Qiskit Pulse



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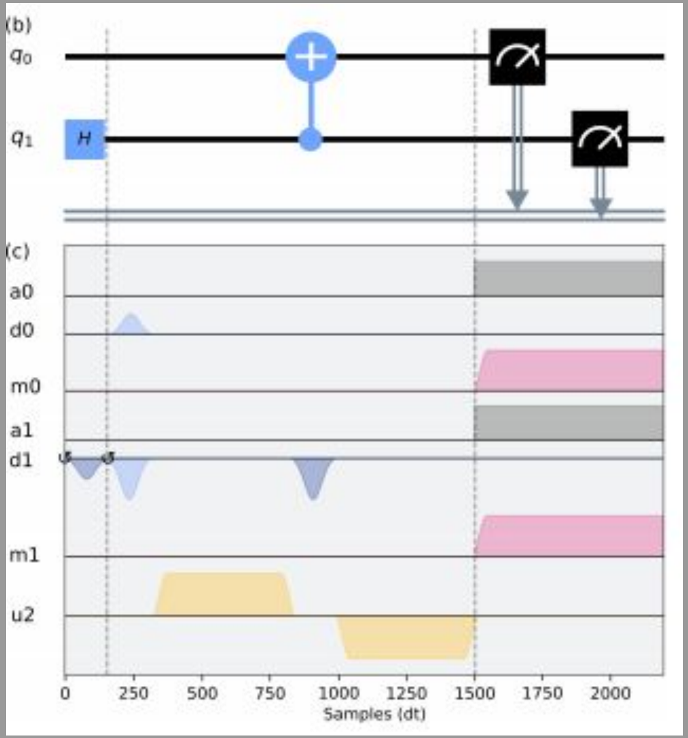
Outline

- Introduction
  - LC Circuit
  - Transmon Qubit
- Qiskit Pulse
  - Characterization of qubits
  - Rabi Experiment
  - Measurement of T_1 , T_2
  - Ramsey Experiment

Introduction: How low can we go?



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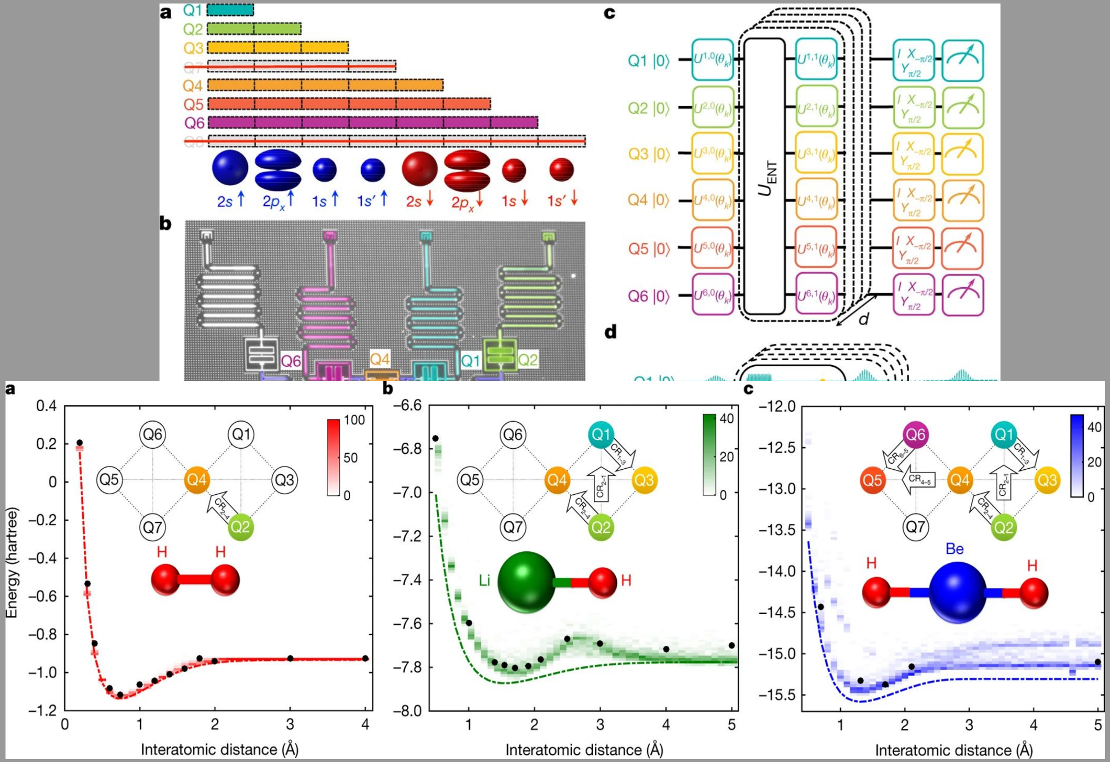
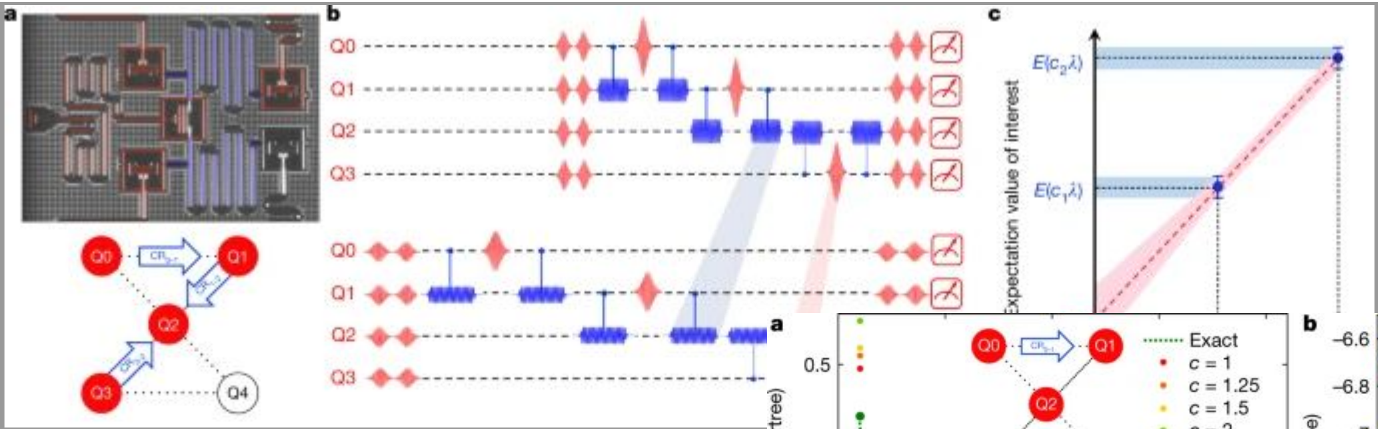
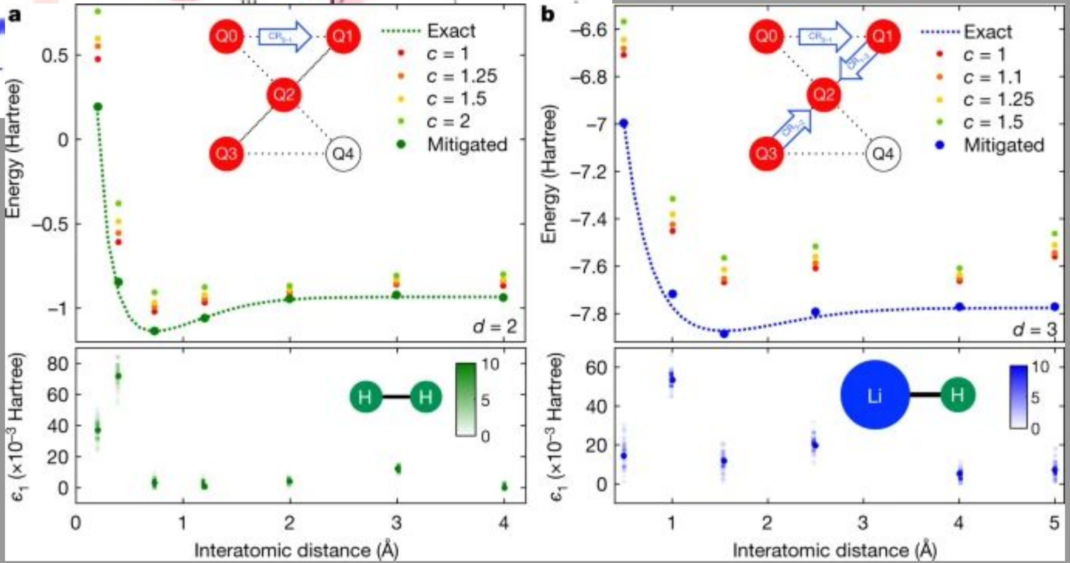


Source: Nature volume 549, pages242–246(2017)

Introduction: How low can we go?



Source: Nature volume 567, pages 491–495 (2019)



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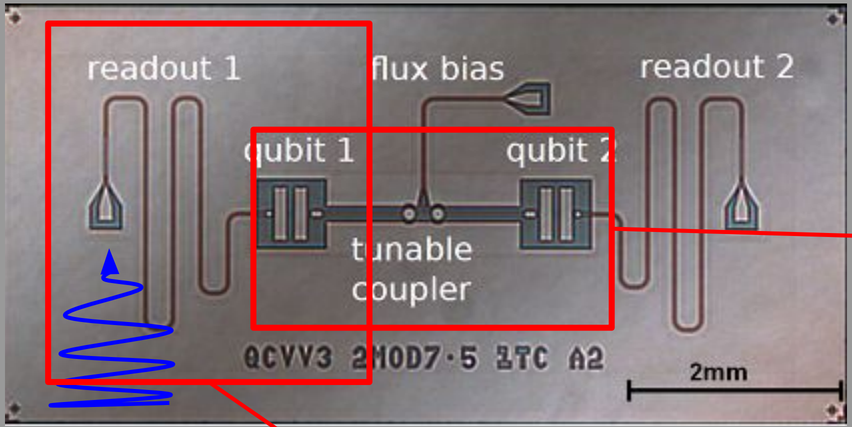
Introduction: Superconducting Qubits



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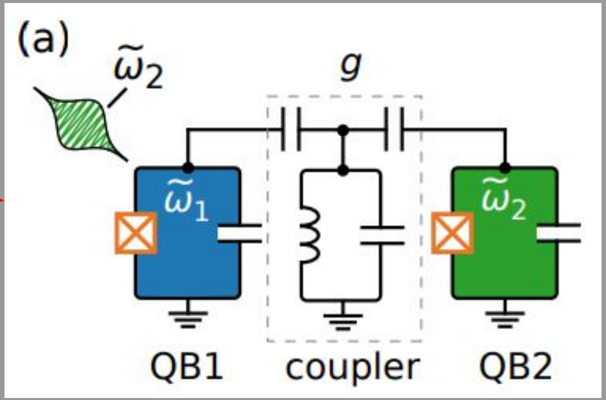
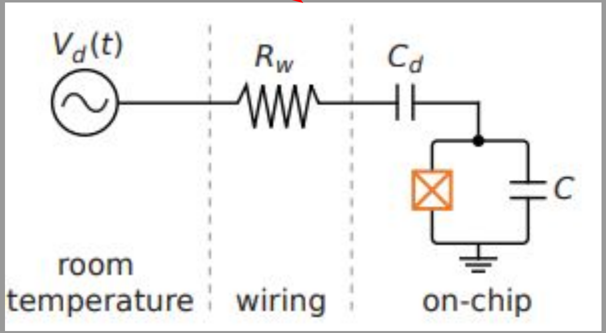
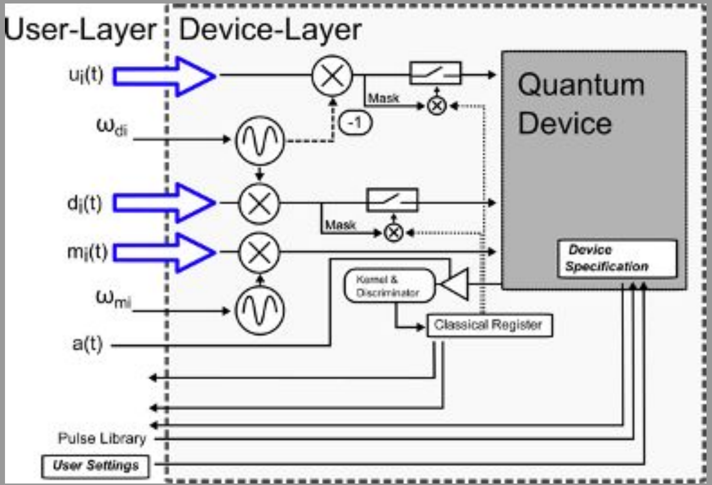




Introduction: Superconducting Qubits

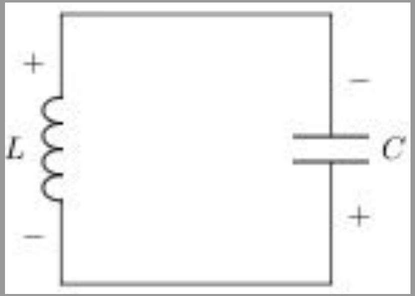


Channel	Description	Alias
drive-channel (DriveChannel)	Transmit channel connected to qubit i, with signals typically modulated at a frequency in resonance with qubit i.	d_i
control-channel (ControlChannel)	Transmit channel with signals typically modulated at a frequency off resonant from its associated qubit.	u_i
measure-channel (PulseChannel)	Transmit channel connected to the readout resonator of qubit i.	m_i
acquire-channel (AcquireChannel)	Receive channel connected to the readout resonator of qubit and store result into a register slot	a_i
Instruction	Operands	Description
Play (Pulse)	*pulse, pulse-channel*	Output the waveform described by pulse on the pulse-channel.
Delay (Delay)	*duration, pulse-channel*	Idle the pulse-channel for the given duration.
shift-phase (Framechange)	*phase, pulse-channel*	Increase the phase of the pulse-channel by phase radians.
Acquire (Acquire)	*duration, acquire-channel, register*	Trigger the acquire-channel to collect data for the given duration, and store the measurement result in a register.

LC Circuit

Quantizing a LC Circuit

We define the flux and charge as follows:



$$\Phi(t) = \int_{-\infty}^{t} V(t')dt' \qquad Q(t) = \int_{-\infty}^{t} I(t')dt'$$

Using Kirchoff's law we get the following equation of motion (EoM):

$$\ddot{\Phi} = \omega^2 \Phi \qquad \omega = \frac{1}{\sqrt{LC}}$$

From this EoM, we can "guess" the Lagrangean and derive the Hamiltonian by doing a Legendre Transform:

$$\mathcal{L} = \frac{1}{2L}\Phi^2 - \frac{1}{2}C\dot{\Phi}^2 \xrightarrow{Q \equiv \frac{\partial \mathcal{L}}{\partial \dot{\Phi}} = C\dot{\Phi} = CV} \mathcal{H} = Q\Phi - \mathcal{L} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

Quantizing a LC Circuit

Now to quantize our Hamiltonian, we put hat on our conjugate variables.

$$\mathcal{H} = Q\Phi - \mathcal{L} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L} \longrightarrow \hat{\mathcal{H}} = \frac{\hat{Q}}{2C} + \frac{\hat{\Phi}}{2L}$$

By matching Poisson Brackets and Commutation relations for our conjulgate variables, we have that:

$$\{\Phi, Q\} = 1 \Rightarrow [\hat{\Phi}, \hat{Q}] = i\hbar$$

We can rewrite the Hamiltonian using the reduced charge $\hat{n} = \frac{\hat{Q}}{2e}$ and reduced flux $\hat{\phi} = \frac{2\pi\Phi}{\Phi_0}$ $\Phi_0 = \frac{h}{2e}$

$$\hat{H} = 4E_C\hat{n}^2 + \frac{1}{2}E_L\hat{\phi}^2$$

$E_C = \frac{e^2}{2C}$ Charge energy

$E_L = \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{L}$ Inductive energy

Quantizing a LC Circuit

Now we can define creation and annihilation operators in terms of zero-point fluctuations of the charge and phase:

$$\hat{n} = i\, n_{ZPF}(\hat{a} + \hat{a}^\dagger)$$

$$\hat{\phi} = \phi_{ZPF}(\hat{a} - \hat{a}^\dagger)$$

$$n_{ZPF} = \left(\frac{E_L}{32E_C}\right)^{\frac{1}{4}}$$

$$\phi_{ZPF} = \left(\frac{2E_C}{E_L}\right)^{\frac{1}{4}}$$

Then our Hamiltonian becomes: (Dropping the hats)

$$\mathcal{H} = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$$

$$\omega = \frac{\sqrt{8E_L E_C}}{\hbar} = \frac{1}{\sqrt{LC}}$$

Transmon Qubits

Transmon Hamiltonian

All the previous work was done for the quantization of LC circuits, for the Transmon circuit we need to change the inductor for a josephson junction which has the following current-flux relation:

$$I = I_0 \sin\left(\frac{2\pi\Phi}{\Phi_0}\right)$$

Using Kirchoff's law, just as before, we have the following equation of motion:

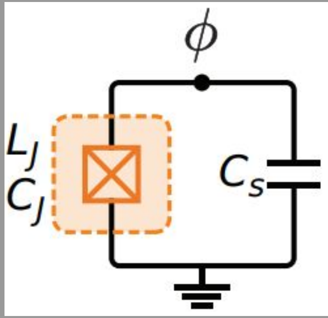
$$I_0 \sin\left(\frac{2\pi\Phi}{\Phi_0}\right) + C\ddot{\Phi} = 0$$

Now we need to convert this equation of motion into a Lagrangian, which is:

$$\mathcal{L} = \frac{I_0\Phi_0}{2\pi}\cos\left(\frac{2\pi\Phi}{\Phi_0}\right) + \frac{C}{2}\dot{\Phi}^2$$

We have the same conjugate variable, thus the Hamiltonian is:

$$\mathcal{H}_T = \frac{Q^2}{2C} - \frac{I_0\Phi_0}{2\pi}\cos\left(\frac{2\pi\Phi}{\Phi_0}\right)$$



Transmon Hamiltonian

Joseph Junction Energy

Now we can quantize the Transmon Hamiltonian: $$\mathcal{H}_T = 4\,E_C\,\hat{n}^2 - E_J\cos\hat{\phi} \quad , \quad E_J = \frac{I_0\Phi_0}{2\pi}$$

Just as the LC circuit, we define annihilation and creation operators:

$$\hat{n} = i\,n_{ZPF}(\hat{c} + \hat{c}^\dagger) \quad n_{ZPF} = \left(\frac{E_J}{32E_C}\right) \qquad\qquad \hat{\phi} = \phi_{ZPF}(\hat{c} - \hat{c}^\dagger) \quad \phi_{ZPF} = \left(\frac{2E_C}{E_J}\right)$$

In the transmon regime, we have that Φ << 1 , then $\frac{E_J}{E_C} >> 1$ and we take the Taylor Expansion:

$$\mathcal{H}_T \approx \sqrt{8E_C E_J}\left[c^\dagger c + \tfrac{1}{2}\right] - E_J - \tfrac{E_C}{12}(c^\dagger + c)^4$$

Expanding on c, c† and dropping fast rotating terms:

$$\mathcal{H}_T = \left(\omega_0 + \frac{\delta}{2}\right)c^\dagger c + \frac{\delta}{2}(c^\dagger c)^2$$

Which is a Hamiltonian of a Duffing Oscillator.

Qubit Drive

We know about Transmon qubits and all, but we want to control them! This is done by applying a Electric Field which induces a dipole interaction between the transmon and microwave field. Now we have the qubit Hamiltonian and a drive Hamiltonian:

$$H = H_0 + H_d \quad , \quad H_0 = -\frac{1}{2}\hbar\omega_q\sigma^Z$$

$$c \longrightarrow \sigma^+|0\rangle = |1\rangle$$

$$c^\dagger \longrightarrow \sigma^-|1\rangle = |0\rangle$$

Since the electric field will excite and de-excite the qubit, we define the dipole operator as $\vec{d} = \vec{d}_0\sigma^+\vec{d}_0^*\sigma^-$
The drive Hamiltonian is:

$$H_d = -\vec{d}\cdot\vec{E}(t) \qquad \vec{E}(t) = \vec{E}_0 e^{-i\omega_d t} + \vec{E}_0^* e^{i\omega_d t}$$

$$H_d = -\hbar\left[\Omega e^{-i\omega_d t} + \tilde{\Omega} e^{i\omega_d t}\right]\sigma^+ - \hbar\left[\tilde{\Omega}^* e^{-i\omega_d t} + \Omega^* e^{i\omega_d t}\right]\sigma^+$$

$$\Omega = \vec{d}_0\cdot\vec{E}_0$$

$$\tilde{\Omega} = \vec{d}_0\cdot\vec{E}_0^*$$

Qubit Drive

Moving to the interaction picture:

$$H_{d,I} = U H_d U^\dagger \qquad U = e^{i\frac{H_0 t}{\hbar}} = 1\cos\left(\frac{\omega_q t}{2}\right) - i\,\sigma^Z \sin\left(\frac{\omega_q t}{2}\right)$$

$$\Delta_q = \omega_q - \omega_d$$

$$H_{d,I} = -\hbar\left[\Omega e^{-i\Delta_q t} + \tilde{\Omega} e^{i(\omega_d+\omega_q)t}\right] e^{i\omega_q t}\sigma^+ - \hbar\left[\tilde{\Omega}^* e^{-i(\omega_d+\omega_q)t} + \Omega^* e^{i\omega_d t}\right] e^{-i\Delta_q t}\sigma^+$$

Now, we apply the Rotating Wave Approximation: $\omega_q + \omega_d >> \Delta_q$

$$H_{d,I}^{\mathrm{RWA}} = -\hbar\Omega e^{-i\Delta_q t}\sigma^+ - \hbar\Omega^* e^{i\Delta_q t}\sigma^-$$

$$H_d^{\mathrm{RWA}} = -\hbar\Omega e^{-i\omega_d t}\sigma^+ - \hbar\Omega^* e^{i\omega_d t}\sigma^-$$

$$H^{\mathrm{RWA}} = -\frac{1}{2}\hbar\omega_q\sigma^Z - \hbar\Omega e^{-i\omega_d t}\sigma^+ - \hbar\Omega^* e^{i\omega_d t}\sigma^-$$

Qubit Drive

We can go to the frame of the drive by doing the following transformation $U_d = e^{-i\omega_d t \frac{\sigma}{2}}$, then in the frame of the drive using the Schrödinger Equation, the effective Hamiltonian is:

$$H_{\text{eff}} = U_d H^{\text{RWA}} U_d^\dagger - i\hbar U_d \dot{U_d}^\dagger$$

$$H_{\text{eff}} = -\frac{1}{2}\hbar\omega_q\sigma^Z - \hbar\Omega\sigma^+ - \hbar\Omega^*\sigma^- + \frac{1}{2}\hbar\Omega_d\sigma^Z = -\frac{1}{2}\hbar\Delta_q\sigma^Z - \hbar\Omega\sigma^+ - \hbar\Omega^*\sigma^-$$

Assuming that the drive is real, we have the desired Hamiltonian:

$$H_{\text{eff}} = -\frac{1}{2}\hbar\Delta_q\sigma^Z - \hbar\Omega\sigma^X$$

This shows that when the drive is resonant with the qubit ($\Delta_q = 0$) the drive causes an X rotation in the Bloch sphere. On the other hand, an off-resonant qubit drive has additional Z rotations generated by the σ^Z contribution, and those manifest themselves as oscillations in a Ramsey experiment.

Qiskit Pulse Demo

References

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