

Homework 2 : Advanced Robot Motion Planning

(a)  $h_m()$  is an admissible heuristic for  $G_1$ .

Let's prove with Contradiction.

Let  $G$  be the goal

$\therefore h(G) = 0$  ; Manhattan distance of Goal from Goal is 0.

Let us assume, for some initial state  $i$ , optimal cost  $[v^*(i) < h(i)]$  — (1)

↳ Since only four actions are allowed, each action can reduce the manhattan distance  $h$  by at most 1.

↳ Since optimal cost of the path from  $i$  to  $G$  is  $v^*$ , goal can be reached in  $v^*$  steps.

$\therefore h(G)$  can be greater or (in the best case) equal to  $h(i) - v^*$

$$\therefore [h(G) \geq h(i) - v^*] \text{ — (2)}$$

$$\text{But from (1) ; } [h(i) - v^* > 0] \text{ — (3)}$$

From (3), we can rewrite (2) as

$$[h(G) \geq h(i) - v^* > 0]$$

But  $h(G) = 0$  ; hence Contradiction on assumption.

$\therefore$  Manhattan distance is the shortest path from any  $i$  to  $G$ . Other paths are either equal to  $h_m()$  or suboptimal.  $\therefore h_m()$  is ADMISSIBLE. ✓



## Planning

$G_2$

from

initial

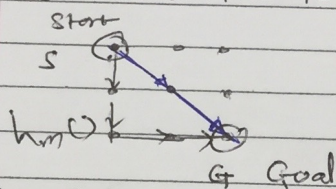
action

at

is  $V^*$ ,

equal

- (b) The Heuristic  $h_m()$  in case of  $G_8$ , overestimates the diagonal cost.  
 $\therefore h_m()$  in case of  $G_8$  is NOT ADMISSIBLE.  
 $\rightarrow$  consider the following case  $\rightarrow$



$$\begin{aligned} * h_m(s) &= |x_G - x_s| + |y_G - y_s| \\ &= 2 + 2 = 4 \end{aligned}$$

- \*  $V^*(s)$ , optimal path is shown in Blue.  
 $\therefore V^*(s) = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

clearly,  $h_m(s) > V^*(s)$

$\therefore$  we can see that,  $h_m()$  in case of  $G_8$  is NOT ADMISSIBLE.

(3)

$h_m()$

Output costs:

30.142135	30.142135	30.627416	31.798989	31.
798989	31.798989			
28.727926	29.798996	34.041638	32.041638	31.
455852	32.041638			
26.727926	32.21321	40.384782	43.091889	43.
091889	43.091889			

Number of Iterations:

99	63	31	27	28	27
379	198	195	156	197	199
1560		1178		923	853
				794	770