

① Pseudo-code for insertion-sort algorithm

input: array \rightarrow arr

def insertionsort(arr)

for i in range(0, length of arr)

put $j = i$

while $j > 0$ and $arr[j-1] > arr[j]$

~~swap~~ swap $arr[j-1]$ and $arr[j]$

$j = j - 1$

Following the above algorithm we get

Unsatisfied condition
 $j = 0$

- pass 1: $\langle 31, 41, 59, 26, 41, 58 \rangle$
- pass 2: $\langle 31, 41, 59, 26, 41, 58 \rangle$ $arr[j-1] > arr[j]$
- pass 3: $\langle 31, 41, 59, 26, 41, 58 \rangle$ $arr[j-1] > arr[j]$
- pass 4: $\langle 26, 31, 41, 59, 41, 58 \rangle$
- pass 5: $\langle 26, 31, 41, 41, 59, 58 \rangle$
- pass 6: $\langle 26, 31, 41, 41, 58, 59 \rangle$

In pass 4 we shift: $\langle 31, 41, 59, 26, 41, 58 \rangle$

In pass 5 we shift: $\langle 26, 31, 41, 59, 41, 58 \rangle$

In pass 6 we shift: $\langle 26, 31, 41, 41, 59, 58 \rangle$

Final output: $\langle 26, 31, 41, 41, 58, 59 \rangle$

(3) Given algorithm:-

Input A of N numbers:-

Unknown (N)

for $j = 1$ to $N-1$

if $(A[N] < A[j])$

swap $A[j]$ and $A[N]$

output $A[N]$

$\left. \begin{array}{l} \{ O(1) \} \end{array} \right\} O(n)$

(1) The output of this algorithm is always going to be ~~to~~ ^a sorted array so $A[N]$ will always give the largest element in the array. Since the largest element is always swapped

(2) to the right of the array, leaving the last element as the largest element.

(2) The time complexity of this algorithm is $O(n)$

③ let there be k arrays each having N elements

Input:- array 1 = $[1 \dots N]$

array 2 = $[1 \dots N]$

;

array k = $[1 \dots N]$

```
define function count-func (array)  
count  
for i in Range (array length)  $\rightarrow$  count = 0 }  $O(N)$   
    count ++;  
print count
```

count_func (array 1), count_func (array 2) ... count_func (array k)

Total time complexity = $O(kN)$. } $O(k)$

The above algorithm can also be written as:

```
for i in range(0 to k):  
    for j in range(0 to n) }  $O(N)$  }  $O(k)$   
        count ++;  
    print (count)
```

Total time complexity = $O(k \times N)$
= $O(kN)$

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Consider the following Algorithm.

input : array \leftarrow arr , step = 0 , counter = 0

```
for i 0 to N-2
{
    if (arr[i] - i == step)
    {
        // do nothing
    }
    else
    {
        print ("Missing : " + i);
        step = counter++;
        step = step + arr[i] - i;
        if (counter == 2)
        {
            exit();
        }
    }
}
```

}
O(N)
time
complexity

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⑤ Pseudo-code for finding inversions

```
for i in range(array-length)
  for j in range(i, array-length)
    if  $i < j$  and  $arr[i] > arr[j]$ 
      print (i, j)
```

(1) let's follow the above algorithm and get the inversions of $\{2, 3, 8, 6\}$ ↗

pass 1: (0, 4)

pass 2: (1, 4)

pass 3: (2, 3)

pass 4: (2, 4)

pass 5: (3, 4)

} They all satisfy the inversion conditions.

(2) An array will ~~also~~ always have the most inversions when they are sorted in a descending order i.e. starting from maximum and going to minimum so the set $\{n, n-1, n-2, \dots, 1\}$ will have the most inversions.

(3) The more inversions more is the runtime of selection sort algorithm as it will have to get each element from n to 1; $n-1$ to 2, $n-2$ to 3 etc. This will become the worst case run-time of algorithm $O(n^2)$. The number of operations for shifting each element will be ' n ' for the ^{first} iteration. The total would be $n + n-1 + n-2 + \dots$ till 1. This adds up to $O(n^2)$.

⑥ Consider the following pseudo-code for the given problem:-

Consider the following Algorithm:-

input: ~~array~~ 2 sorted arrays arr1, arr2

define small(start1, end1, start2, end2, k)

{

if (~~arr1~~^{start1} == end1) return arr2[k]

if (~~arr2~~^{start2} == end2) return arr1[k]

else

{

mid1 = ~~arr~~(start1 + end1) / 2

mid2 = (start2 + end2) / 2

if (mid1 + mid2 < k)

{

if (arr[mid1] > arr2[mid2])

{ return

small(start1, end1,

start2 + mid2 + 1, end2, k - mid2 - 1)

}

else

{ ~~small~~

return (start1 + mid1 + 1, end1, start2,

end2, k - mid1 - 1)

}

}

else {

if (arr1[mid1] > arr2[mid2]) {

return ^{small}(start1, start1 + mid1, start2, end2, k)

}

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else

{

Return small (start1, end1, start2, ~~end~~^{start2 + mid2}, k)

}

}

}

Total time complexity $\Rightarrow O(\log M + \log n)$

where $M \Rightarrow$ total No. of elements in sorted array arr1

$n \Rightarrow$ total No. of elements in sorted array arr2

⑦ Consider the following pseudo-code for the given problem statement:-

input: array \rightarrow arr of length 'n'
def function-permute(arr, n)

for i in range (1, n)

j = pick any random no. between 0 and i

swap(arr[i], arr[j])

return arr

- Here the probability of any element going to any other position would be:-

= probability that element was not picked previously \times
probability that element is - picked now

$$= \left(1 - \frac{1}{n}\right) \times \left(\frac{1}{n-1}\right)$$

$$= \frac{n-1}{n} \times \frac{1}{n-1} = \frac{1}{n} \quad \text{--- this holds true for all the positions}$$

hence its of equal probability.