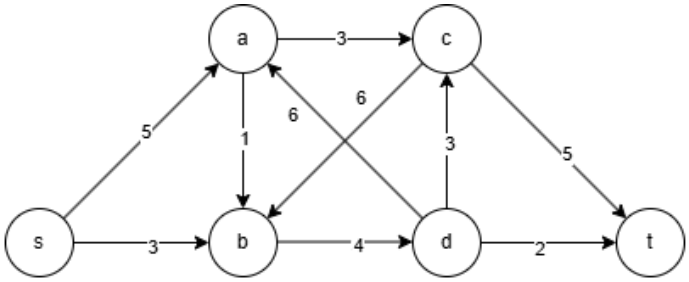
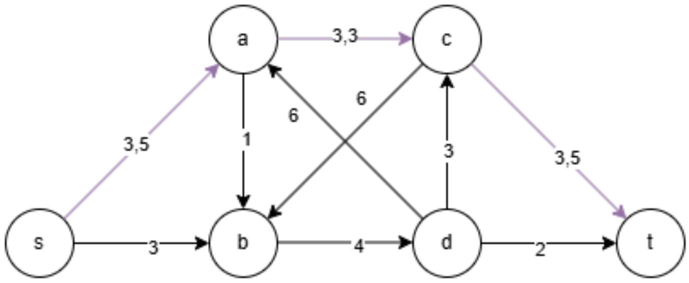
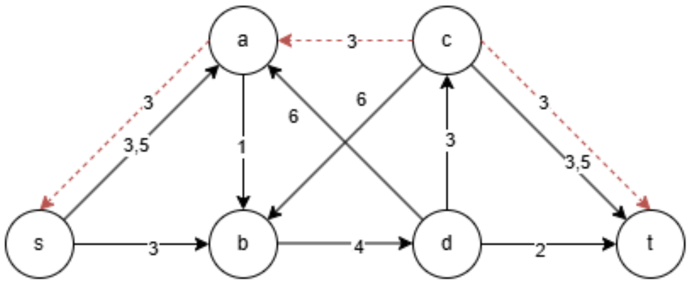
Design and Analysis of Algorithms - Naiara Alonso Montes

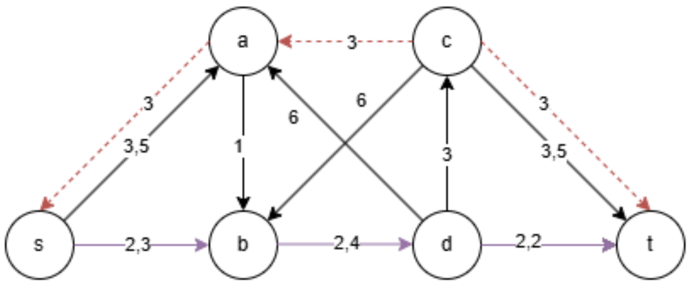
Exercise 1

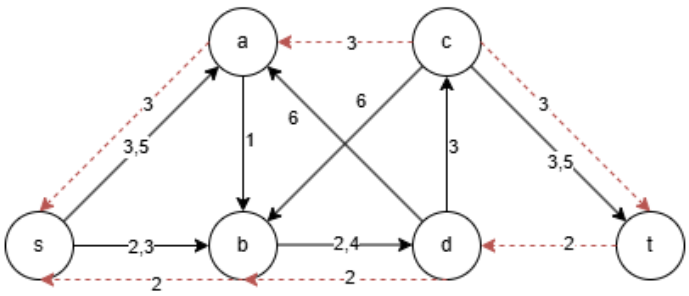




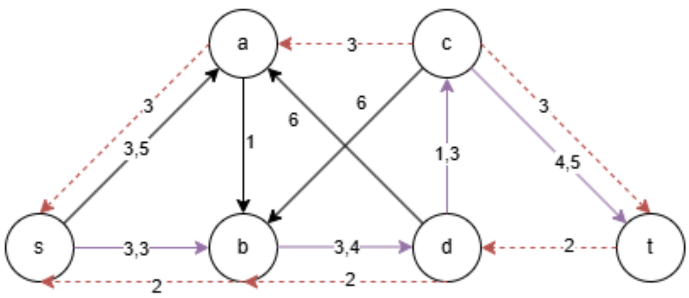


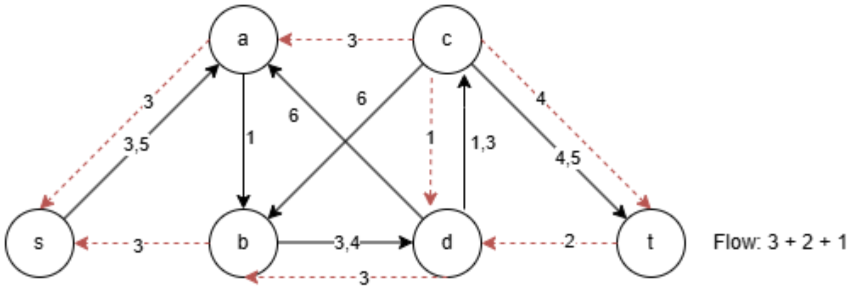
Flow: 3

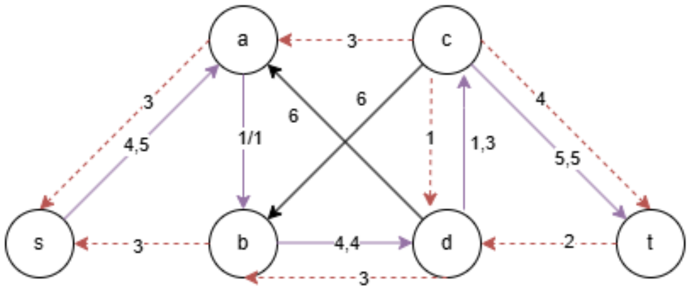


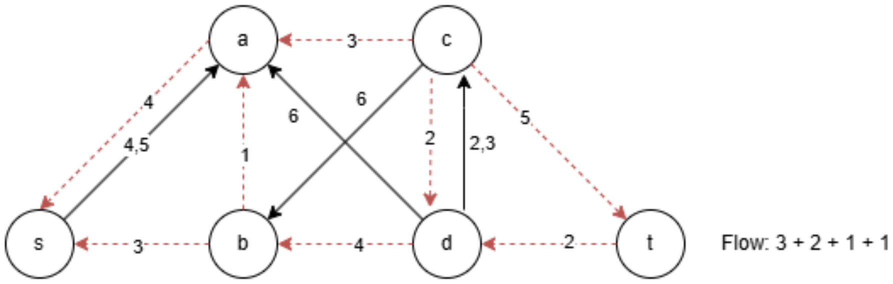


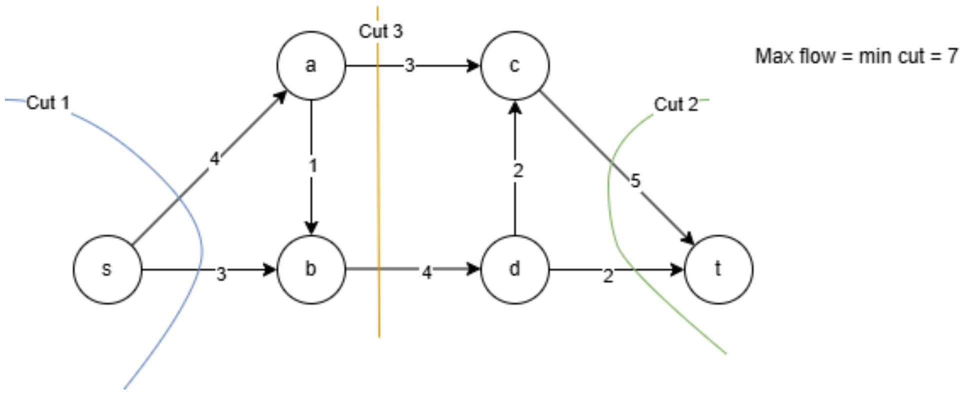
Flow: 3 + 2











The maximal flow is equal to 7, same to the value of the minimum cut. I found 3 minimum cuts.

Exercise 2

Assume P_1 as the first augmented path found by Ford-Fulkerson algorithm, with V_1 vertices and E_1 edges.

$$E_1 = e_0, e_1, \ldots, e_n$$

$$|E_1| \geq 4$$

$$d_1(t) = |E_1|$$

Assume P_2 as the second augmented path found, with V_2 vertices and E_2 edges such that:

$$E_2 = E_1 \notin e_1$$

$$|E_2| = |E_1| - 1$$

$$d_2(t) = |E_2|$$

After second iteration we found that:

$$d_1(t) = |E_1| > |E_1| - 1 = d_2(t)$$

Thus, in Ford-Fulkerson algorithm the distance $d(v)$ can decrease. This is because the algorithm does not follow any especific search principle and the first found path can be the maximal in terms of distance.

Exercise 3

Edge in all minimum cuts

Statement:

If an edge belongs to all minumum cuts, removing it will increase the cut value.

Algorithm:

1. Find the maximum flow, for example with Edmonds-Karp.
2. Remove the edge of the network and recompute maximum flow.
3. Compare flows, if the new maximum flow is less than the original maximum flow, then e belongs to all minimum cuts. Otherwise, it is not.

Correctness:

- If removing e increases the minimum cut value, it means that edge e was part of all minimum cuts.
- If removing edge e does not change the minimum cut value, it means that there was another path that could carry the same flow, so e is not in all minimum cuts.

Time complexity:

The time complexity for this algorithm is the same as the time complexity used for finding the maximum flow.

Edge in some minimum cut

Statement:

If an edge is in some minimum cut, reducing its capacity will increase the minimum cut value.

Algorithm:

1. Find the maximum flow.
2. Reduce capacity of edge in ϵ.
3. Recompute maximum flow.
4. Compare, if new maximum flow is less than the original flow, then e is in some minimum cut. Otherwise it is not.

Correctness:

- If reducing the capacity of e increases the minimum cut value, then e was part of some minimum cut.
- If reducing the capacity does not change the value of the minumum cut, then there are other paths that can carry the increased flow, so e is not in any minimum cut.

Time complexity:

The time complexity for this algorithm is the same as the time complexity used for finding the maximum flow.

In [ ]:
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!jupyter nbconvert --to html HW2.ipynb
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