MS4303 PROJECT REPORT

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Contents

1 Project Files Instruction

In this section, I will briefly introduce all the files included in the project.

There are 11 files (include this **Report**) contained in this project. Among them, **Run.m** contains all the data (tableaux, variables, etc.) used in the project, **colsortjk.m**, **makeindex.m** and **Pivot.m** are files provided on the website, **Finda.m**, **Findq.m**, **Pivoting.m** and **Pivoting2.m** are created by myself. Besides, there are two extra diary files, **diary** and **diary2**, which contain all the commands and their outputs used in the project.

2 Project Process Analysis

In this section, I will show the specific processes and outputs of each step.

2.1 Solution to Optimality

- Problem Display

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Minimise: $z = -2x_1 - 4x_2 + 6x_4 - 5x_5 - 7x_6 - 2x_7 + 1x_8 - 2x_9 - 5x_{10}$

Constraint Inequalities:

$$
\begin{aligned}
+5x_1 - 3x_2 + 4x_3 - 5x_4 + 1x_5 + 1x_6 - 2x_7 + 1x_8 - 2x_9 + 3x_{10} &\leq -48.11166538 \\
-1x_1 + 4x_2 + 4x_3 + 4x_4 - 5x_5 + 4x_6 - 1x_7 + 1x_8 + 1x_9 + 4x_{10} &\leq 182.27087185 \\
-2x_1 + 5x_2 - 1x_3 + 2x_4 + 3x_5 + 4x_6 + 3x_7 - 1x_8 - 1x_{10} &\leq 147.45636480 \\
-2x_1 + 4x_2 - 4x_3 - 3x_4 + 4x_5 + 4x_6 - 4x_7 - 2x_8 + 3x_9 - 1x_{10} &\leq 50.04439649 \\
-3x_1 - 1x_2 + 1x_3 + 2x_4 + 2x_5 - 2x_6 - 5x_7 - 1x_8 - 2x_{10} &\leq -32.73834288 \\
-4x_1 - 2x_2 - 1x_3 + 4x_4 + 5x_5 - 5x_8 - 2x_9 - 2x_{10} &\leq -45.46840390 \\
+4x_1 - 4x_2 - 3x_3 - 3x_4 + 3x_5 + 4x_6 + 2x_7 - 4x_8 - 1x_9 + 2x_{10} &\leq -82.83038298 \\
-1x_1 - 3x_2 - 2x_3 - 3x_4 + 1x_5 - 3x_6 + 5x_7 - 1x_8 - 4x_9 + 2x_{10} &\leq -167.30089590 \\
+1x_1 + 4x_2 - 4x_3 - 4x_4 - 1x_5 + 1x_6 - 2x_7 + 2x_8 + 2x_9 - 2x_{10} &\leq 9.43730093 \\
-3x_1 + 2x_2 + 3x_4 + 4x_5 - 3x_6 - 1x_7 + 5x_8 + 2x_9 + 2x_{10} &\leq 123.10851935
\end{aligned}
$$

$x_i \geq 0, i = 1, \ldots, n.$

Figure 1: Problem

- Generate a Simplex tableau T_0 from the LP problem

```
T0 =
```

```
0   -2.0000   -4.0000         0    6.0000   -5.0000   -7.0000   -2.0000    1.0000   -2.0000   -5.0000
-48.1117    5.0000   -3.0000    4.0000   -5.0000    1.0000    1.0000   -2.0000    1.0000   -2.0000    3.0000
182.2709   -1.0000    4.0000    4.0000    4.0000   -5.0000    4.0000   -1.0000    1.0000    1.0000    4.0000
147.4564   -2.0000    5.0000   -1.0000    2.0000    3.0000    4.0000    3.0000   -1.0000         0   -1.0000
50.0444   -2.0000    4.0000   -4.0000   -3.0000    4.0000    4.0000   -4.0000   -2.0000    3.0000   -1.0000
-32.7383   -3.0000   -1.0000    1.0000    2.0000    2.0000   -2.0000   -5.0000   -1.0000         0   -2.0000
-45.4684   -4.0000   -2.0000   -1.0000    4.0000    5.0000         0         0   -5.0000   -2.0000   -2.0000
-82.8304    4.0000   -4.0000   -3.0000   -3.0000    3.0000    4.0000    2.0000   -4.0000   -1.0000    2.0000
-167.3009   -1.0000   -3.0000   -2.0000   -3.0000    1.0000   -3.0000    5.0000   -1.0000   -4.0000    2.0000
9.4373    1.0000    4.0000   -4.0000   -4.0000   -1.0000    1.0000   -2.0000    2.0000    2.0000   -2.0000
123.1085   -3.0000    2.0000         0    3.0000    4.0000   -3.0000   -1.0000    5.0000    2.0000    2.0000

m =

10

format rat

T0=[T0 [zeros(1,m);eye(m)]]

T0 =

Columns 1 through 8

0            -2            -4             0             6            -5            -7            -2
-9478/197          5            -3             4            -5             1             1            -2
8749/48           -1             4             4             4            -5             4            -1
21971/149         -2             5            -1             2             3             4             3
2252/45           -2             4            -4            -3             4             4            -4
-3503/107         -3            -1             1             2             2            -2            -5
-12231/269         -4            -2            -1             4             5             0             0
-9277/112          4            -4            -3            -3             3             4             2
-18905/113         -1            -3            -2            -3             1            -3             5
2935/311           1             4            -4            -4            -1             1            -2
15881/129          -3             2             0             3             4            -3            -1

Columns 9 through 16

1            -2            -5             0             0             0             0             0
1            -2             3             1             0             0             0             0
1             1             4             0             1             0             0             0
-1             0            -1             0             0             1             0             0
-2             3            -1             0             0             0             1             0
-1             0            -2             0             0             0             0             1
-5            -2            -2             0             0             0             0             0
-4            -1             2             0             0             0             0             0
-1            -4             2             0             0             0             0             0
2             2            -2             0             0             0             0             0
5             2             2             0             0             0             0             0

Columns 17 through 21

0             0             0             0             0
0             0             0             0             0
0             0             0             0             0
0             0             0             0             0
0             0             0             0             0
0             0             0             0             0
1             0             0             0             0
0             1             0             0             0
0             0             1             0             0
0             0             0             1             0
0             0             0             0             1
```

- Use the provided **Pivot.m** and my own **Pivoting.m** (DSM) m-files to transform T_0 to a canonical form tableau T_B
  - **Pivoting.m** file display

```
function Tout=Pivoting(T)

% DSM

[~,pos]=min(T(2:end,1));pos=pos+1;

r=pos;
negcols=find(T(r,2:end)<0);
negcols=negcols+1;
colrat=T(1,negcols)./T(r,negcols);
[~,pos]=max(colrat);
c=negcols(pos);

Tout=Pivot(T,r,c);
```

- The **succession of tableaux** from T_0 to T_B is shown in the Section 3
- Tableau T_B

```
TB =

Columns 1 through 5

246.2199    1.6383         0         0    9.9495
18.4876    0.2325         0         0    1.2699
1.3038    1.9594         0         0   -0.9940
37.2164    0.3681         0         0    2.2360
22.7731   -0.2955         0    1.0000    1.1291
61.8394    0.9192         0         0    5.6459
66.6736    2.2207         0         0    6.0190
13.9028   -1.3209    1.0000         0    0.2464
8.7308    1.1864         0         0   -0.5924
3.8777    0.2750         0         0   -0.1790
43.5202   -8.1998         0         0    3.2142

Columns 6 through 10

0         0         0         0         0
0         0    1.0000         0         0
0         0         0    1.0000         0
0         0         0         0    1.0000
0         0         0         0         0
0         0         0         0         0
0         0         0         0         0
0         0         0         0         0
0    1.0000         0         0         0
1.0000         0         0         0         0
0         0         0         0         0

Columns 11 through 15

-9.9457    0.1373   -0.6607    1.5296   -0.7891
-0.8175   -0.0971   -0.0996    0.1556   -0.2288
-1.0432    0.1534   -0.1839    0.1576   -0.2997
-1.8923   -0.1960   -0.2110    0.0980   -0.2604
0.0094    0.0101   -0.0035    0.0527   -0.0218
-4.7865   -0.3332   -0.2010    0.7735   -1.3851
-6.6298   -0.0133   -0.3426    0.4268   -1.5899
0.3134   -0.0208    0.0966    0.0187    0.1591
0.2731    0.0862    0.0285    0.0654   -0.0877
-0.7469    0.0713   -0.1618    0.1295   -0.0266
13.3631   -0.4569    1.7814   -1.1874    1.3154

Columns 16 through 20

0         0    0.5570   -1.3857    0.1585
0         0    0.0743   -0.1582    0.0167
0         0    0.0539   -0.2130    0.2958
0         0    0.1002   -0.4385   -0.0269
0         0   -0.0552   -0.0841   -0.1513
1.0000         0    0.6728   -0.7114    0.7586
0    1.0000    0.2854   -1.2089    1.2678
```

```
0         0   -0.1362    0.1560   -0.0638
0         0    0.1357   -0.0749    0.1217
0         0   -0.0286   -0.1011   -0.0243
0         0    0.3981    1.6516   -0.8185

Column 21

0
0
0
0
0
0
0
0
0
0
1.0000
```

- Use **colsortjk.m** m-file to transform T_B to a canonical form tableau T_C which has the basic columns in standard order on the right of the tableau

```
TC=colsortjk(TB)

TC =

Columns 1 through 11

246.2199    1.6383    9.9495   -9.9457    0.1373   -0.6607    1.5296   -0.7891    0.5570   -1.3857    0.1585
18.4876    0.2325    1.2699   -0.8175   -0.0971   -0.0996    0.1556   -0.2288    0.0743   -0.1582    0.0167
1.3038    1.9594   -0.9940   -1.0432    0.1534   -0.1839    0.1576   -0.2997    0.0539   -0.2130    0.2958
37.2164    0.3681    2.2360   -1.8923   -0.1960   -0.2110    0.0980   -0.2604    0.1002   -0.4385   -0.0269
22.7731   -0.2955    1.1291    0.0094    0.0101   -0.0035    0.0527   -0.0218   -0.0552   -0.0841   -0.1513
61.8394    0.9192    5.6459   -4.7865   -0.3332   -0.2010    0.7735   -1.3851    0.6728   -0.7114    0.7586
66.6736    2.2207    6.0190   -6.6298   -0.0133   -0.3426    0.4268   -1.5899    0.2854   -1.2089    1.2678
13.9028   -1.3209    0.2464    0.3134   -0.0208    0.0966    0.0187    0.1591   -0.1362    0.1560   -0.0638
8.7308    1.1864   -0.5924    0.2731    0.0862    0.0285    0.0654   -0.0877    0.1357   -0.0749    0.1217
3.8777    0.2750   -0.1790   -0.7469    0.0713   -0.1618    0.1295   -0.0266   -0.0286   -0.1011   -0.0243
43.5202   -8.1998    3.2142   13.3631   -0.4569    1.7814   -1.1874    1.3154    0.3981    1.6516   -0.8185

Columns 12 through 21

0         0         0         0         0         0         0         0         0         0
1.0000         0         0         0         0         0         0         0         0         0
0    1.0000         0         0         0         0         0         0         0         0
0         0    1.0000         0         0         0         0         0         0         0
0         0         0    1.0000         0         0         0         0         0         0
0         0         0         0    1.0000         0         0         0         0         0
0         0         0         0         0    1.0000         0         0         0         0
0         0         0         0         0         0    1.0000         0         0         0
0         0         0         0         0         0         0    1.0000         0         0
0         0         0         0         0         0         0         0    1.0000         0
0         0         0         0         0         0         0         0         0    1.0000
```

- Use the provided **Pivot.m** and my own **Pivoting2.m** (SM) m-files to transform T_C to an optimal form tableau and name it T_S
  - **Pivoting2.m** file display

```
function Tout=Pivoting2(T)

% SM

[~,pos]=min(T(1,2:end));pos=pos+1;
```

```
c=pos;
posrows=find(T(2:end,c)>0);
posrows=posrows+1;
colrat=T(posrows,1)./T(posrows,c);
[~,pos]=min(colrat);
r=posrows(pos);

Tout=Pivot(T,r,c);
```

- The **succession of tableaux** from T_C to T_S is shown in the **Section 3**
- Tableau T_S

```
TS =

Columns 1 through 11

344.9156         0    9.6574    6.5233    0.2050    1.4348    0.5968         0    1.7922         0    0.1465
35.9918          0    1.3794    1.4188   -0.1234    0.2376    0.0601         0    0.2966         0   -0.1233
14.8536    1.0000   -0.5187    1.1803    0.0728    0.1601    0.0172         0    0.2113         0    0.0701
71.2160          0    2.0747    3.5287   -0.1660    0.4844   -0.1937         0    0.5298         0   -0.0306
33.3316          0    0.8841    1.3984    0.0488    0.1672    0.0006         0    0.0861         0   -0.0988
160.1769         0    6.3787    6.4715   -0.5214    1.6053    0.3763         0    1.9330         0   -0.2099
185.9420         0    6.9493    9.4679   -0.2116    2.0424   -0.3242         0    1.7870         0    0.3216
16.1709          0   -0.3705   -0.4069    0.0698   -0.0106    0.1373         0   -0.0812         0    0.0858
49.8261          0    0.8594    2.9546   -0.2216    0.7582    0.0848    1.0000    0.6557         0   -0.9764
7.2219           0   -0.1463    0.1791    0.0718   -0.0574    0.0561         0    0.0083         0   -0.0059
60.4117          0   -1.3135   11.5980    0.2610    1.2699   -0.7012         0    0.7678    1.0000    0.6303

Columns 12 through 21

0    1.3530         0         0         0         0         0    4.9353         0    1.2372
1.0000   -1.2319         0         0         0         0         0    2.0420         0    0.0294
0   -0.3482         0         0         0         0         0    1.6128         0    0.0282
0    0.2680    1.0000         0         0         0         0    1.9238         0    0.3873
0    0.2084         0    1.0000         0         0         0    0.6447         0    0.1070
0   -9.1822         0         0    1.0000         0         0   13.3700         0   -0.1476
0   -7.6541         0         0         0    1.0000         0   13.1131         0    0.3391
0    0.1620         0         0         0         0    1.0000    0.4913         0   -0.0513
0  -10.1058         0         0         0         0         0   11.1858         0   -0.7964
0    0.4659         0         0         0         0         0   -0.2374    1.0000    0.1105
0    6.3197         0         0         0         0         0   -0.9016         0    1.3797
```

- The optimal solution vector **x** is

```
Xs =

14.8536
0
0
0
0
0
49.8261
0
60.4117
0
35.9918
0
71.2160
33.3316
160.1769
185.9420
16.1709
0
7.2219
0
```

- The optimal **z-value** is

```
z =

-344.9156
```

2.2 Sensitivity Analysis

2.2.1 Aa

- Pick the **last non-basic** column in T_S tableau

```
lnbcol =

21
```

- Calculate the row ratios for the $lnbcol$ column in T_S

```
rat=TS(:,1)./TS(:,21)

rat =

1.0e+03 *

0.2788
1.2222
0.5271
0.1839
0.3114
-1.0855
0.5483
-0.3152
-0.0626
0.0653
0.0438
```

- Find all the non-negative row numbers for the column $lnbcol$ in T_S

```
posrows=find(TS(2:end,lnbcol)>0)+1

posrows =

2
3
4
5
7
10
11
```

- Select the row ratios of the non-negative rows

```
posrats=rat(posrows)

posrats =

1.0e+03 *

1.2222
0.5271
0.1839
```

```
0.3114
0.5483
0.0653
0.0438
```

- Get the minimum row ratio mrr

```
mrr=min(posrats)

mrr =

43.7870
```

- Get the value **X**

```
X=mrr/2

X =

21.8935
```

- Copy T_S to a new tableau **TtAa** and subtract X times **TtAa(:,lnbcol)** from LH column of **TtAa**

```
TtAa(:,1)=TtAa(:,1)-X*TtAa(:,lnbcol)

TtAa =

Columns 1 through 4

317.8295         0    9.6574    6.5233
35.3470          0    1.3794    1.4188
14.2366    1.0000   -0.5187    1.1803
62.7373          0    2.0747    3.5287
30.9883          0    0.8841    1.3984
163.4076         0    6.3787    6.4715
178.5170         0    6.9493    9.4679
17.2940          0   -0.3705   -0.4069
67.2619          0    0.8594    2.9546
4.8024           0   -0.1463    0.1791
30.2058          0   -1.3135   11.5980

Columns 5 through 8

0.2050    1.4348    0.5968         0
-0.1234   0.2376    0.0601         0
0.0728    0.1601    0.0172         0
-0.1660   0.4844   -0.1937         0
0.0488    0.1672    0.0006         0
-0.5214   1.6053    0.3763         0
-0.2116   2.0424   -0.3242         0
0.0698   -0.0106    0.1373         0
-0.2216   0.7582    0.0848    1.0000
0.0718   -0.0574    0.0561         0
0.2610    1.2699   -0.7012         0

Columns 9 through 12

1.7922         0    0.1465         0
0.2966         0   -0.1233    1.0000
0.2113         0    0.0701         0
0.5298         0   -0.0306         0
0.0861         0   -0.0988         0
1.9330         0   -0.2099         0
1.7870         0    0.3216         0
-0.0812        0    0.0858         0
0.6557         0   -0.9764         0
0.0083         0   -0.0059         0
```

```
0.7678    1.0000    0.6303         0

Columns 13 through 16

1.3530         0         0         0
-1.2319         0         0         0
-0.3482         0         0         0
0.2680    1.0000         0         0
0.2084         0    1.0000         0
-9.1822         0         0    1.0000
-7.6541         0         0         0
0.1620         0         0         0
-10.1058         0         0         0
0.4659         0         0         0
6.3197         0         0         0

Columns 17 through 20

0         0    4.9353         0
0         0    2.0420         0
0         0    1.6128         0
0         0    1.9238         0
0         0    0.6447         0
0         0   13.3700         0
1.0000         0   13.1131         0
0    1.0000    0.4913         0
0         0   11.1858         0
0         0   -0.2374    1.0000
0         0   -0.9016         0

Column 21

1.2372
0.0294
0.0282
0.3873
0.1070
-0.1476
0.3391
-0.0513
-0.7964
0.1105
1.3797
```

- New optimal solution **x** and optimal **z-value**

```
XAa =

14.2366
0
0
0
0
0
67.2619
0
30.2058
0
35.3470
0
62.7373
30.9883
163.4076
178.5170
17.2940
0
4.8024
0


ZAa =
```

```
-317.8295
```

- Copy tableau T_S to a new tableau **Ttesta** and check result of SA by adding an extra row **newrowa** to **Ttesta**
  - **newrowa** display

```
newrowa =

Columns 1 through 6

21.8935         0         0         0         0         0

Columns 7 through 12

0         0         0         0         0         0

Columns 13 through 18

0         0         0         0         0         0

Columns 19 through 21

0         0    1.0000
```

  - Tableau **Ttesta** display

```
Ttesta=TS;
Ttesta=[Ttesta;newrowa]

Ttesta =

Columns 1 through 4

344.9156         0    9.6574    6.5233
35.9918          0    1.3794    1.4188
14.8536    1.0000   -0.5187    1.1803
71.2160          0    2.0747    3.5287
33.3316          0    0.8841    1.3984
160.1769         0    6.3787    6.4715
185.9420         0    6.9493    9.4679
16.1709          0   -0.3705   -0.4069
49.8261          0    0.8594    2.9546
7.2219           0   -0.1463    0.1791
60.4117          0   -1.3135   11.5980
21.8935          0         0         0

Columns 5 through 8

0.2050    1.4348    0.5968         0
-0.1234    0.2376    0.0601         0
0.0728    0.1601    0.0172         0
-0.1660    0.4844   -0.1937         0
0.0488    0.1672    0.0006         0
-0.5214    1.6053    0.3763         0
-0.2116    2.0424   -0.3242         0
0.0698   -0.0106    0.1373         0
-0.2216    0.7582    0.0848    1.0000
0.0718   -0.0574    0.0561         0
0.2610    1.2699   -0.7012         0
0         0         0         0

Columns 9 through 12

1.7922         0    0.1465         0
0.2966         0   -0.1233    1.0000
0.2113         0    0.0701         0
0.5298         0   -0.0306         0
```

```
 0.0861         0   -0.0988         0
 1.9330         0   -0.2099         0
 1.7870         0    0.3216         0
-0.0812         0    0.0858         0
 0.6557         0   -0.9764         0
 0.0083         0   -0.0059         0
 0.7678    1.0000    0.6303         0
 0         0         0         0

 Columns 13 through 16

 1.3530         0         0         0
-1.2319         0         0         0
-0.3482         0         0         0
 0.2680    1.0000         0         0
 0.2084         0    1.0000         0
-9.1822         0         0    1.0000
-7.6541         0         0         0
 0.1620         0         0         0
-10.1058         0         0         0
 0.4659         0         0         0
 6.3197         0         0         0
 0         0         0         0

 Columns 17 through 20

 0         0    4.9353         0
 0         0    2.0420         0
 0         0    1.6128         0
 0         0    1.9238         0
 0         0    0.6447         0
 0         0   13.3700         0
 1.0000         0   13.1131         0
 0    1.0000    0.4913         0
 0         0   11.1858         0
 0         0   -0.2374    1.0000
 0         0   -0.9016         0
 0         0         0         0

 Column 21

 1.2372
 0.0294
 0.0282
 0.3873
 0.1070
-0.1476
 0.3391
-0.0513
-0.7964
 0.1105
 1.3797
 1.0000
```

- Pivot **Ttesta** then we get a new tableau called **TtestAa**

```
TtestAa=Pivot(Ttesta,12,21)
Row 12 and Col 21 selected.

TtestAa =

Columns 1 through 4

317.8295         0    9.6574    6.5233
35.3470         0    1.3794    1.4188
14.2366    1.0000   -0.5187    1.1803
62.7373         0    2.0747    3.5287
30.9883         0    0.8841    1.3984
163.4076         0    6.3787    6.4715
178.5170         0    6.9493    9.4679
```

```
17.2940         0   -0.3705   -0.4069
67.2619         0    0.8594    2.9546
4.8024          0   -0.1463    0.1791
30.2058         0   -1.3135   11.5980
21.8935         0         0         0

Columns 5 through 8

0.2050    1.4348    0.5968         0
-0.1234    0.2376    0.0601         0
0.0728    0.1601    0.0172         0
-0.1660    0.4844   -0.1937         0
0.0488    0.1672    0.0006         0
-0.5214    1.6053    0.3763         0
-0.2116    2.0424   -0.3242         0
0.0698   -0.0106    0.1373         0
-0.2216    0.7582    0.0848    1.0000
0.0718   -0.0574    0.0561         0
0.2610    1.2699   -0.7012         0
0         0         0         0

Columns 9 through 12

1.7922         0    0.1465         0
0.2966         0   -0.1233    1.0000
0.2113         0    0.0701         0
0.5298         0   -0.0306         0
0.0861         0   -0.0988         0
1.9330         0   -0.2099         0
1.7870         0    0.3216         0
-0.0812         0    0.0858         0
0.6557         0   -0.9764         0
0.0083         0   -0.0059         0
0.7678    1.0000    0.6303         0
0         0         0         0

Columns 13 through 16

1.3530         0         0         0
-1.2319         0         0         0
-0.3482         0         0         0
0.2680    1.0000         0         0
0.2084         0    1.0000         0
-9.1822         0         0    1.0000
-7.6541         0         0         0
0.1620         0         0         0
-10.1058         0         0         0
0.4659         0         0         0
6.3197         0         0         0
0         0         0         0

Columns 17 through 20

0         0    4.9353         0
0         0    2.0420         0
0         0    1.6128         0
0         0    1.9238         0
0         0    0.6447         0
0         0   13.3700         0
1.0000         0   13.1131         0
0    1.0000    0.4913         0
0         0   11.1858         0
0         0   -0.2374    1.0000
0         0   -0.9016         0
0         0         0         0

Column 21

0
0
0
0
```

```
0
0
0
0
0
0
0
1.0000
```

- Compare the first column of **TtAa** and **TtestAa**, we can find that except the new row, all the other rows have the same elements.
- Correct result!

2.2.2 Ab

- Find the **last basic** column in T_S tableau

```
lbcol =

20
```

- Find the row (**rowAb**) that defines the variable x_{19}

```
rowAb =

10
```

- The columns in **rowAb** that contain $<$ **-sqrt(eps)** elements are:

```
candcols=find(TS(rowAb,2:end)<(-sqrt(eps)))+1

candcols =

3     6    11    19
```

- Calculate the ratios of t he costs (Row 1) in each of these columns with the number in that column in **rowAb**

```
costrats=TS(1,candcols)./TS(rowAb,candcols)
```

```
costrats =

-66.0130  -25.0000  -25.0000  -20.7879
```

- The least in magnitude of these ratios is the 4th one

```
leastratAb=max(costrats)

leastratAb =

-20.7879
```

- So we can get the best non-basic column is 19
- Now we have to increase the non-basic variable x_{18} corr to column 19
- Calculate **mrrAb** for this column

```
ratAb=TS(:,1)./TS(:,19)

ratAb =

69.8871
17.6254
9.2098
37.0191
51.7000
11.9803
14.1798
32.9150
4.4544
-30.4191
-67.0050

posrowsAb=find(TS(2:end,19)>0)+1

posrowsAb =

2
3
4
5
6
7
8
9

posratsAb=ratAb(posrowsAb)

posratsAb =

17.6254
9.2098
37.0191
51.7000
11.9803
14.1798
32.9150
```

```
4.4544

mrrAb=min(posratsAb)

mrrAb =

4.4544
```

- Copy T_S to a new tableau **TtAb** and subtract X times **TtAb(:,lbcol)** from LH column of **TtAb**

```
TtAb=TS;
TtAb(:,1)=TtAb(:,1)-mrrAb/2*TtAb(:,lbcol)

TtAb =

Columns 1 through 5

344.9156         0    9.6574    6.5233    0.2050
35.9918          0    1.3794    1.4188   -0.1234
14.8536    1.0000   -0.5187    1.1803    0.0728
71.2160          0    2.0747    3.5287   -0.1660
33.3316          0    0.8841    1.3984    0.0488
160.1769         0    6.3787    6.4715   -0.5214
185.9420         0    6.9493    9.4679   -0.2116
16.1709          0   -0.3705   -0.4069    0.0698
49.8261          0    0.8594    2.9546   -0.2216
4.9947           0   -0.1463    0.1791    0.0718
60.4117          0   -1.3135   11.5980    0.2610

Columns 6 through 10

1.4348    0.5968         0    1.7922         0
0.2376    0.0601         0    0.2966         0
0.1601    0.0172         0    0.2113         0
0.4844   -0.1937         0    0.5298         0
0.1672    0.0006         0    0.0861         0
1.6053    0.3763         0    1.9330         0
2.0424   -0.3242         0    1.7870         0
-0.0106    0.1373         0   -0.0812         0
0.7582    0.0848    1.0000    0.6557         0
-0.0574    0.0561         0    0.0083         0
1.2699   -0.7012         0    0.7678    1.0000

Columns 11 through 15

0.1465         0    1.3530         0         0
-0.1233    1.0000   -1.2319         0         0
0.0701         0   -0.3482         0         0
-0.0306         0    0.2680    1.0000         0
-0.0988         0    0.2084         0    1.0000
-0.2099         0   -9.1822         0         0
0.3216          0   -7.6541         0         0
0.0858          0    0.1620         0         0
-0.9764         0  -10.1058         0         0
-0.0059         0    0.4659         0         0
0.6303          0    6.3197         0         0

Columns 16 through 20

0         0         0    4.9353         0
0         0         0    2.0420         0
0         0         0    1.6128         0
0         0         0    1.9238         0
0         0         0    0.6447         0
1.0000         0         0   13.3700         0
0    1.0000         0   13.1131         0
0         0    1.0000    0.4913         0
0         0         0   11.1858         0
```

```
0         0         0   -0.2374    1.0000
0         0         0   -0.9016         0

Column 21

1.2372
0.0294
0.0282
0.3873
0.1070
-0.1476
0.3391
-0.0513
-0.7964
0.1105
1.3797
```

- The new optimal solution **x** and optimal **z-value** are:

```
XAb =

14.8536
0
0
0
0
0
49.8261
0
60.4117
0
35.9918
0
71.2160
33.3316
160.1769
185.9420
16.1709
0
4.9947
0

ZAb =

-344.9156
```

- Now copy T_S to a new tableau **Ttestb** and add a new row **newrowb** to this tableau
  - **newrowb** display

```
newrowb=zeros(1,21);
newrowb(1)=TtAb(rowAb,1);
newrowb(20)=1

newrowb =

Columns 1 through 4

4.9947         0         0         0

Columns 5 through 8

0         0         0         0

Columns 9 through 12

0         0         0         0

Columns 13 through 16
```

```
0         0         0         0

Columns 17 through 20

0         0         0    1.0000

Column 21

0
```

– **Ttestb** display

```
Ttestb=TS;
Ttestb=[Ttestb;newrowb]

Ttestb =

Columns 1 through 4

344.9156          0    9.6574    6.5233
35.9918           0    1.3794    1.4188
14.8536    1.0000   -0.5187    1.1803
71.2160           0    2.0747    3.5287
33.3316           0    0.8841    1.3984
160.1769          0    6.3787    6.4715
185.9420          0    6.9493    9.4679
16.1709           0   -0.3705   -0.4069
49.8261           0    0.8594    2.9546
7.2219          0   -0.1463    0.1791
60.4117           0   -1.3135   11.5980
4.9947          0         0         0

Columns 5 through 8

0.2050    1.4348    0.5968         0
-0.1234    0.2376    0.0601         0
0.0728    0.1601    0.0172         0
-0.1660    0.4844   -0.1937         0
0.0488    0.1672    0.0006         0
-0.5214    1.6053    0.3763         0
-0.2116    2.0424   -0.3242         0
0.0698   -0.0106    0.1373         0
-0.2216    0.7582    0.0848    1.0000
0.0718   -0.0574    0.0561         0
0.2610    1.2699   -0.7012         0
0         0         0         0

Columns 9 through 12

1.7922         0    0.1465         0
0.2966         0   -0.1233    1.0000
0.2113         0    0.0701         0
0.5298         0   -0.0306         0
0.0861         0   -0.0988         0
1.9330         0   -0.2099         0
1.7870         0    0.3216         0
-0.0812         0    0.0858         0
0.6557         0   -0.9764         0
0.0083         0   -0.0059         0
0.7678    1.0000    0.6303         0
0         0         0         0

Columns 13 through 16

1.3530         0         0         0
-1.2319         0         0         0
-0.3482         0         0         0
0.2680    1.0000         0         0
0.2084         0    1.0000         0
-9.1822         0         0    1.0000
-7.6541         0         0         0
0.1620         0         0         0
```

```
-10.1058         0         0         0
 0.4659         0         0         0
 6.3197         0         0         0
      0         0         0         0

Columns 17 through 20

      0         0    4.9353         0
      0         0    2.0420         0
      0         0    1.6128         0
      0         0    1.9238         0
      0         0    0.6447         0
      0         0   13.3700         0
 1.0000         0   13.1131         0
      0    1.0000    0.4913         0
      0         0   11.1858         0
      0         0   -0.2374    1.0000
      0         0   -0.9016         0
      0         0         0    1.0000

Column 21

 1.2372
 0.0294
 0.0282
 0.3873
 0.1070
-0.1476
 0.3391
-0.0513
-0.7964
 0.1105
 1.3797
 0
```

- Now pivot this tableau **Ttestb**, then we get a new tableau **TtestAb**

```
TtestAb=Pivot(Ttestb,12,20)
Row 12 and Col 20 selected.

TtestAb =

Columns 1 through 4

344.9156         0    9.6574    6.5233
 35.9918         0    1.3794    1.4188
 14.8536    1.0000   -0.5187    1.1803
 71.2160         0    2.0747    3.5287
 33.3316         0    0.8841    1.3984
160.1769         0    6.3787    6.4715
185.9420         0    6.9493    9.4679
 16.1709         0   -0.3705   -0.4069
 49.8261         0    0.8594    2.9546
  2.2272         0   -0.1463    0.1791
 60.4117         0   -1.3135   11.5980
  4.9947         0         0         0

Columns 5 through 8

 0.2050    1.4348    0.5968         0
-0.1234    0.2376    0.0601         0
 0.0728    0.1601    0.0172         0
-0.1660    0.4844   -0.1937         0
 0.0488    0.1672    0.0006         0
-0.5214    1.6053    0.3763         0
-0.2116    2.0424   -0.3242         0
 0.0698   -0.0106    0.1373         0
-0.2216    0.7582    0.0848    1.0000
 0.0718   -0.0574    0.0561         0
 0.2610    1.2699   -0.7012         0
      0         0         0         0
```

```
Columns 9 through 12

1.7922         0    0.1465         0
0.2966         0   -0.1233    1.0000
0.2113         0    0.0701         0
0.5298         0   -0.0306         0
0.0861         0   -0.0988         0
1.9330         0   -0.2099         0
1.7870         0    0.3216         0
-0.0812         0    0.0858         0
0.6557         0   -0.9764         0
0.0083         0   -0.0059         0
0.7678    1.0000    0.6303         0
0         0         0         0

Columns 13 through 16

1.3530         0         0         0
-1.2319         0         0         0
-0.3482         0         0         0
0.2680    1.0000         0         0
0.2084         0    1.0000         0
-9.1822         0         0    1.0000
-7.6541         0         0         0
0.1620         0         0         0
-10.1058         0         0         0
0.4659         0         0         0
6.3197         0         0         0
0         0         0         0

Columns 17 through 20

0         0    4.9353         0
0         0    2.0420         0
0         0    1.6128         0
0         0    1.9238         0
0         0    0.6447         0
0         0   13.3700         0
1.0000         0   13.1131         0
0    1.0000    0.4913         0
0         0   11.1858         0
0         0   -0.2374         0
0         0   -0.9016         0
0         0         0    1.0000

Column 21

1.2372
0.0294
0.0282
0.3873
0.1070
-0.1476
0.3391
-0.0513
-0.7964
0.1105
1.3797
0
```

- Compare the first columns of **TtAb** and **TtestAb**

```
ans =

344.9156
35.9918
14.8536
71.2160
33.3316
160.1769
185.9420
```

```
16.1709
49.8261
4.9947
60.4117

ans =

344.9156
35.9918
14.8536
71.2160
33.3316
160.1769
185.9420
16.1709
49.8261
2.2272
60.4117
4.9947
```

- Correct result!

2.2.3 Ac

- Find the **first non-basic** column in T_S tableau

```
fnbcol =


3
```

- Calculate the mrr value **mrrAc** of this column

```
ratAc=TS(:,1)./TS(:,fnbcol)

ratAc =

35.7152
26.0932
-28.6372
34.3254
37.7010
25.1111
26.7568
-43.6475
57.9775
-49.3652
-45.9933

posrowsAc=find(TS(2:end,fnbcol)>0)+1

posrowsAc =

2
4
5
6
7
9

posratsAc=ratAc(posrowsAc)

posratsAc =
```

```
26.0932
34.3254
37.7010
25.1111
26.7568
57.9775

mrrAc=min(posratsAc)

mrrAc =

25.1111
```

- From **mrrAc**, we can select **row 6** and column **fnbcol** as the pivoting point. Then copy T_S to a new temp tableau **TtActemp** and pivot it to another tableau **TtAc1**

```
TtActemp=TS;
TtAc1=Pivot(TtActemp,6,3)
Row 6 and Col 3 selected.

TtAc1 =

Columns 1 through 11

102.4079         0         0   -3.2746    0.9943   -0.9956    0.0270         0   -1.1344         0    0.4643
1.3546         0         0    0.0194   -0.0107   -0.1096   -0.0213         0   -0.1214         0   -0.0779
27.8784    1.0000         0    1.7066    0.0304    0.2907    0.0478         0    0.3685         0    0.0531
19.1171         0         0    1.4237    0.0036   -0.0377   -0.3161         0   -0.0989         0    0.0377
11.1307         0         0    0.5014    0.1210   -0.0553   -0.0515         0   -0.1818         0   -0.0697
25.1111         0    1.0000    1.0145   -0.0817    0.2517    0.0590         0    0.3030         0   -0.0329
11.4363         0         0    2.4174    0.3564    0.2935   -0.7342         0   -0.3190         0    0.5503
25.4742         0         0   -0.0310    0.0395    0.0826    0.1591         0    0.0311         0    0.0736
28.2455         0         0    2.0827   -0.1513    0.5419    0.0341    1.0000    0.3953         0   -0.9481
10.8955         0         0    0.3275    0.0598   -0.0206    0.0648         0    0.0526         0   -0.0107
93.3949         0         0   12.9306    0.1536    1.6004   -0.6237         0    1.1659    1.0000    0.5871

Columns 12 through 21

0   15.2549         0         0   -1.5140         0         0  -15.3069         0    1.4606
1.0000    0.7537         0         0   -0.2162         0         0   -0.8491         0    0.0614
0   -1.0949         0         0    0.0813         0         0    2.7000         0    0.0162
0    3.2546    1.0000         0   -0.3253         0         0   -2.4249         0    0.4353
0    1.4810         0    1.0000   -0.1386         0         0   -1.2084         0    0.1275
0   -1.4395         0         0    0.1568         0         0    2.0960         0   -0.0231
0    2.3495         0         0   -1.0895    1.0000         0   -1.4529         0    0.4999
0   -0.3714         0         0    0.0581         0    1.0000    1.2678         0   -0.0599
0   -8.8687         0         0   -0.1347         0         0    9.3845         0   -0.7765
0    0.2553         0         0    0.0229         0         0    0.0692    1.0000    0.1071
0    4.4290         0         0    0.2059         0         0    1.8515         0    1.3493
```

- The row that defines the variable x_2 is 6

```
rowAc =

6
```

- The columns in row 6 that contain — significantly, i.e. $<$ -sqrt(eps) — negative elements are:

```
candcolsAc=find(TtAc1(rowAc,2:end)<(-sqrt(eps)))+1
```

```
candcolsAc =

5    11    13    21
```

- Calculate the ratios of the costs (Row 1) in each of these columns with the (negative) number in that column in row 6

```
costratsAc=TtAc1(1,candcolsAc)./TtAc1(rowAc,candcolsAc)

costratsAc =

-12.1649  -14.1071  -10.5973  -63.1373
```

- The least in magnitude of these ratios is the 3rd one

```
leastratAc=max(costratsAc)

leastratAc =

-10.5973
```

- So the best non-basic column is 13
- The task now is to increase the non-basic variable x_{12} corr to column 13
- First calculate the mrr for this column

```
ratAc2=TtAc1(:,1)./TtAc1(:,13)

ratAc2 =

6.7131
1.7972
-25.4622
5.8739
7.5155
-17.4442
4.8675
-68.5987
-3.1848
42.6795
21.0873

posrowsAc2=find(TtAc1(2:end,13)>0)+1

posrowsAc2 =

2
```

```
4
5
7
10
11

posratsAc2=ratAc2(posrowsAc2)

posratsAc2 =

1.7972
5.8739
7.5155
4.8675
42.6795
21.0873

mrrAc2=min(posratsAc2)

mrrAc2 =

1.7972
```

- Copy tableau **TtAc1** to a new tableau **TtAc2** and subtract **mrrAc2/2** times **TtAc2(:,13)** from LH column of **TtAc2**

```
TtAc2=TtAc1;
TtAc2(:,1)=TtAc2(:,1)-mrrAc2/2*TtAc2(:,13)

TtAc2 =

Columns 1 through 11

88.6995         0         0   -3.2746    0.9943   -0.9956    0.0270         0   -1.1344         0    0.4643
0.6773         0         0    0.0194   -0.0107   -0.1096   -0.0213         0   -0.1214         0   -0.0779
28.8622    1.0000         0    1.7066    0.0304    0.2907    0.0478         0    0.3685         0    0.0531
16.1925         0         0    1.4237    0.0036   -0.0377   -0.3161         0   -0.0989         0    0.0377
9.7998         0         0    0.5014    0.1210   -0.0553   -0.0515         0   -0.1818         0   -0.0697
26.4047         0    1.0000    1.0145   -0.0817    0.2517    0.0590         0    0.3030         0   -0.0329
9.3250         0         0    2.4174    0.3564    0.2935   -0.7342         0   -0.3190         0    0.5503
25.8079         0         0   -0.0310    0.0395    0.0826    0.1591         0    0.0311         0    0.0736
36.2151         0         0    2.0827   -0.1513    0.5419    0.0341    1.0000    0.3953         0   -0.9481
10.6661         0         0    0.3275    0.0598   -0.0206    0.0648         0    0.0526         0   -0.0107
89.4149         0         0   12.9306    0.1536    1.6004   -0.6237         0    1.1659    1.0000    0.5871

Columns 12 through 21

0   15.2549         0         0   -1.5140         0         0  -15.3069         0    1.4606
1.0000    0.7537         0         0   -0.2162         0         0   -0.8491         0    0.0614
0   -1.0949         0         0    0.0813         0         0    2.7000         0    0.0162
0    3.2546    1.0000         0   -0.3253         0         0   -2.4249         0    0.4353
0    1.4810         0    1.0000   -0.1386         0         0   -1.2084         0    0.1275
0   -1.4395         0         0    0.1568         0         0    2.0960         0   -0.0231
0    2.3495         0         0   -1.0895    1.0000         0   -1.4529         0    0.4999
0   -0.3714         0         0    0.0581         0    1.0000    1.2678         0   -0.0599
0   -8.8687         0         0   -0.1347         0         0    9.3845         0   -0.7765
0    0.2553         0         0    0.0229         0         0    0.0692    1.0000    0.1071
0    4.4290         0         0    0.2059         0         0    1.8515         0    1.3493
```

- New **x** and **z-value** are:

```
XAc =

28.8622
```

```
26.4047
0
0
0
0
36.2151
0
89.4149
0
0.6773
0
16.1925
9.7998
0
9.3250
25.8079
0
10.6661
0

ZAc =

-88.6995
```

- Now we need to check the result. First add a new row **newrowc**

```
newrowc=zeros(1,21);
newrowc(1)=TtAc2(rowAc,1);
newrowc(fnbcol)=1

newrowc =

Columns 1 through 11

26.4047         0    1.0000         0         0         0         0         0         0         0         0

Columns 12 through 21

0         0         0         0         0         0         0         0         0         0
```

- Copy tableau T_S to a new tableau **Ttestc** and add this new row to **Ttestc**

```
Ttestc=TS;
Ttestc=[Ttestc;newrowc]
```

```
Ttestc =

Columns 1 through 11

344.9156         0    9.6574    6.5233    0.2050    1.4348    0.5968         0    1.7922         0    0.1465
35.9918          0    1.3794    1.4188   -0.1234    0.2376    0.0601         0    0.2966         0   -0.1233
14.8536    1.0000   -0.5187    1.1803    0.0728    0.1601    0.0172         0    0.2113         0    0.0701
71.2160          0    2.0747    3.5287   -0.1660    0.4844   -0.1937         0    0.5298         0   -0.0306
33.3316          0    0.8841    1.3984    0.0488    0.1672    0.0006         0    0.0861         0   -0.0988
160.1769         0    6.3787    6.4715   -0.5214    1.6053    0.3763         0    1.9330         0   -0.2099
185.9420         0    6.9493    9.4679   -0.2116    2.0424   -0.3242         0    1.7870         0    0.3216
16.1709          0   -0.3705   -0.4069    0.0698   -0.0106    0.1373         0   -0.0812         0    0.0858
49.8261          0    0.8594    2.9546   -0.2216    0.7582    0.0848    1.0000    0.6557         0   -0.9764
7.2219           0   -0.1463    0.1791    0.0718   -0.0574    0.0561         0    0.0083         0   -0.0059
60.4117          0   -1.3135   11.5980    0.2610    1.2699   -0.7012         0    0.7678    1.0000    0.6303
26.4047          0    1.0000         0         0         0         0         0         0         0         0

Columns 12 through 21

0    1.3530         0         0         0         0         0    4.9353         0    1.2372
1.0000   -1.2319         0         0         0         0         0    2.0420         0    0.0294
0   -0.3482         0         0         0         0         0    1.6128         0    0.0282
0    0.2680    1.0000         0         0         0         0    1.9238         0    0.3873
0    0.2084         0    1.0000         0         0         0    0.6447         0    0.1070
0   -9.1822         0         0    1.0000         0         0   13.3700         0   -0.1476
0   -7.6541         0         0         0    1.0000         0   13.1131         0    0.3391
0    0.1620         0         0         0         0    1.0000    0.4913         0   -0.0513
0  -10.1058         0         0         0         0         0   11.1858         0   -0.7964
0    0.4659         0         0         0         0         0   -0.2374    1.0000    0.1105
0    6.3197         0         0         0         0         0   -0.9016         0    1.3797
0         0         0         0         0         0         0         0         0         0
```

- Pivot this tableau to **TtestAc**

```
TtestAc=Pivot(Ttestc,12,3)
Row 12 and Col 3 selected.

TtestAc =

Columns 1 through 11

89.9154          0         0    6.5233    0.2050    1.4348    0.5968         0    1.7922         0    0.1465
-0.4297          0         0    1.4188   -0.1234    0.2376    0.0601         0    0.2966         0   -0.1233
28.5493    1.0000         0    1.1803    0.0728    0.1601    0.0172         0    0.2113         0    0.0701
16.4333          0         0    3.5287   -0.1660    0.4844   -0.1937         0    0.5298         0   -0.0306
9.9871           0         0    1.3984    0.0488    0.1672    0.0006         0    0.0861         0   -0.0988
-8.2513          0         0    6.4715   -0.5214    1.6053    0.3763         0    1.9330         0   -0.2099
2.4468           0         0    9.4679   -0.2116    2.0424   -0.3242         0    1.7870         0    0.3216
25.9535          0         0   -0.4069    0.0698   -0.0106    0.1373         0   -0.0812         0    0.0858
27.1338          0         0    2.9546   -0.2216    0.7582    0.0848    1.0000    0.6557         0   -0.9764
11.0848          0         0    0.1791    0.0718   -0.0574    0.0561         0    0.0083         0   -0.0059
95.0940          0         0   11.5980    0.2610    1.2699   -0.7012         0    0.7678    1.0000    0.6303
26.4047          0    1.0000         0         0         0         0         0         0         0         0

Columns 12 through 21

0    1.3530         0         0         0         0         0    4.9353         0    1.2372
1.0000   -1.2319         0         0         0         0         0    2.0420         0    0.0294
0   -0.3482         0         0         0         0         0    1.6128         0    0.0282
0    0.2680    1.0000         0         0         0         0    1.9238         0    0.3873
0    0.2084         0    1.0000         0         0         0    0.6447         0    0.1070
0   -9.1822         0         0    1.0000         0         0   13.3700         0   -0.1476
0   -7.6541         0         0         0    1.0000         0   13.1131         0    0.3391
0    0.1620         0         0         0         0    1.0000    0.4913         0   -0.0513
0  -10.1058         0         0         0         0         0   11.1858         0   -0.7964
0    0.4659         0         0         0         0         0   -0.2374    1.0000    0.1105
0    6.3197         0         0         0         0         0   -0.9016         0    1.3797
0         0         0         0         0         0         0         0         0         0
```

- We can see that this new tableau need one DSM pivot

```
TtestAc1=Pivoting(TtestAc)
Row 6 and Col 13 selected.

TtestAc1 =

Columns 1 through 11

88.6995         0         0    7.4768    0.1281    1.6714    0.6523         0    2.0770         0    0.1155
0.6773         0         0    0.5506   -0.0535    0.0222    0.0096         0    0.0373         0   -0.0952
28.8622    1.0000         0    0.9349    0.0925    0.0993    0.0029         0    0.1380         0    0.0781
16.1925         0         0    3.7175   -0.1812    0.5313   -0.1827         0    0.5863         0   -0.0367
9.7998         0         0    1.5452    0.0369    0.2036    0.0092         0    0.1300         0   -0.1036
0.8986         0         0   -0.7048    0.0568   -0.1748   -0.0410         0   -0.2105         0    0.0229
9.3250         0         0    4.0733    0.2230    0.7042   -0.6379         0    0.1756         0    0.4966
25.8079         0         0   -0.2927    0.0606    0.0177    0.1439         0   -0.0471         0    0.0821
36.2151         0         0   -4.1679    0.3522   -1.0086   -0.3294    1.0000   -1.4717         0   -0.7453
10.6661         0         0    0.5074    0.0453    0.0241    0.0752         0    0.1064         0   -0.0165
89.4149         0         0   16.0521   -0.0978    2.3747   -0.4421         0    2.0982    1.0000    0.4858
26.4047         0    1.0000         0         0         0         0         0         0         0         0

Columns 12 through 21

0         0         0         0    0.1473         0         0    6.9054         0    1.2154
1.0000         0         0         0   -0.1342         0         0    0.2483         0    0.0492
0         0         0         0   -0.0379         0         0    1.1057         0    0.0338
0         0    1.0000         0    0.0292         0         0    2.3139         0    0.3830
0         0         0    1.0000    0.0227         0         0    0.9481         0    0.1037
0    1.0000         0         0   -0.1089         0         0   -1.4561         0    0.0161
0         0         0         0   -0.8336    1.0000         0    1.9681         0    0.4621
0         0         0         0    0.0176         0    1.0000    0.7271         0   -0.0539
0         0         0         0   -1.1006         0         0   -3.5291         0   -0.6340
0         0         0         0    0.0507         0         0    0.4409    1.0000    0.1030
0         0         0         0    0.6883         0         0    8.3004         0    1.2781
0         0         0         0         0         0         0         0         0         0
```

- Compare the first columns of tableau **TtAc2** and **TtestAc1**

```
ans =

88.6995
0.6773
28.8622
16.1925
9.7998
26.4047
9.3250
25.8079
36.2151
10.6661
89.4149

ans =

88.6995
0.6773
28.8622
16.1925
9.7998
0.8986
9.3250
25.8079
36.2151
10.6661
89.4149
26.4047
```

- Correct result!

2.2.4 B

- Find the **last non-basic slack** column in tableau T_C

```
lnbslackcol =

21
```

- Use my own **Finda.m** m-file to find the range of **a**
  - **Finda.m** display

```
b=TS(2:end,1);
s=TS(2:end,lnbslackcol);
pos=find(s>0);
neg=find(s<0);
posratsB=-b(pos)./s(pos);
negratsB=-b(neg)./s(neg);
minposval=max(posratsB);
maxnegval=min(negratsB);
```

  - Then we can get two numbers, which are **minposval** and **maxnegval** respectively

- The range of **a**:

$$-43.7870 \leq a \leq 62.5650 \tag{1}$$

- Assign the value of **a** to half of **maxnegval**

```
a=maxnegval/2

a =

31.2825
```

- Add **a** times col. 21 of T_S to LH col, then we get a new tableau **TtB**

```
TtB=TS;
TtB(:,1)=TtB(:,1)+a*TtB(:,lnbslackcol)

TtB =

Columns 1 through 12

383.6175         0    9.6574    6.5233    0.2050    1.4348    0.5968         0    1.7922         0    0.1465         0
36.9130          0    1.3794    1.4188   -0.1234    0.2376    0.0601         0    0.2966         0   -0.1233    1.0000
15.7353    1.0000   -0.5187    1.1803    0.0728    0.1601    0.0172         0    0.2113         0    0.0701         0
83.3308          0    2.0747    3.5287   -0.1660    0.4844   -0.1937         0    0.5298         0   -0.0306         0
36.6797          0    0.8841    1.3984    0.0488    0.1672    0.0006         0    0.0861         0   -0.0988         0
155.5608         0    6.3787    6.4715   -0.5214    1.6053    0.3763         0    1.9330         0   -0.2099         0
196.5511         0    6.9493    9.4679   -0.2116    2.0424   -0.3242         0    1.7870         0    0.3216         0
14.5661          0   -0.3705   -0.4069    0.0698   -0.0106    0.1373         0   -0.0812         0    0.0858         0
```

```
24.9131         0    0.8594    2.9546   -0.2216    0.7582    0.0848    1.0000    0.6557         0   -0.9764         0
10.6790         0   -0.1463    0.1791    0.0718   -0.0574    0.0561         0    0.0083         0   -0.0059         0
103.5712         0   -1.3135   11.5980    0.2610    1.2699   -0.7012         0    0.7678    1.0000    0.6303         0

Columns 13 through 21

1.3530         0         0         0         0         0    4.9353         0    1.2372
-1.2319         0         0         0         0         0    2.0420         0    0.0294
-0.3482         0         0         0         0         0    1.6128         0    0.0282
0.2680    1.0000         0         0         0         0    1.9238         0    0.3873
0.2084         0    1.0000         0         0         0    0.6447         0    0.1070
-9.1822         0         0    1.0000         0         0   13.3700         0   -0.1476
-7.6541         0         0         0    1.0000         0   13.1131         0    0.3391
0.1620         0         0         0         0    1.0000    0.4913         0   -0.0513
-10.1058         0         0         0         0         0   11.1858         0   -0.7964
0.4659         0         0         0         0         0   -0.2374    1.0000    0.1105
6.3197         0         0         0         0         0   -0.9016         0    1.3797
```

- New **x** and **z-value** are:

```
XB =

15.7353
0
0
0
0
0
24.9131
0
103.5712
0
36.9130
0
83.3308
36.6797
155.5608
196.5511
14.5661
0
10.6790
0

ZB =

-383.6175
```

- Now check by adding **a** units of supply to value in row 11 of column 1 in T_C

```
TtestB=TC;
TtestB(11,1)=TtestB(11,1)+a

TtestB =

Columns 1 through 11

246.2199    1.6383    9.9495   -9.9457    0.1373   -0.6607    1.5296   -0.7891    0.5570   -1.3857    0.1585
18.4876    0.2325    1.2699   -0.8175   -0.0971   -0.0996    0.1556   -0.2288    0.0743   -0.1582    0.0167
1.3038    1.9594   -0.9940   -1.0432    0.1534   -0.1839    0.1576   -0.2997    0.0539   -0.2130    0.2958
37.2164    0.3681    2.2360   -1.8923   -0.1960   -0.2110    0.0980   -0.2604    0.1002   -0.4385   -0.0269
22.7731   -0.2955    1.1291    0.0094    0.0101   -0.0035    0.0527   -0.0218   -0.0552   -0.0841   -0.1513
61.8394    0.9192    5.6459   -4.7865   -0.3332   -0.2010    0.7735   -1.3851    0.6728   -0.7114    0.7586
66.6736    2.2207    6.0190   -6.6298   -0.0133   -0.3426    0.4268   -1.5899    0.2854   -1.2089    1.2678
13.9028   -1.3209    0.2464    0.3134   -0.0208    0.0966    0.0187    0.1591   -0.1362    0.1560   -0.0638
8.7308    1.1864   -0.5924    0.2731    0.0862    0.0285    0.0654   -0.0877    0.1357   -0.0749    0.1217
3.8777    0.2750   -0.1790   -0.7469    0.0713   -0.1618    0.1295   -0.0266   -0.0286   -0.1011   -0.0243
74.8027   -8.1998    3.2142   13.3631   -0.4569    1.7814   -1.1874    1.3154    0.3981    1.6516   -0.8185

Columns 12 through 21
```

```
0         0         0         0         0         0         0         0         0         0
1.0000         0         0         0         0         0         0         0         0         0         0
0    1.0000         0         0         0         0         0         0         0         0
0         0    1.0000         0         0         0         0         0         0         0
0         0         0    1.0000         0         0         0         0         0         0
0         0         0         0    1.0000         0         0         0         0         0
0         0         0         0         0    1.0000         0         0         0         0
0         0         0         0         0         0    1.0000         0         0         0
0         0         0         0         0         0         0    1.0000         0         0
0         0         0         0         0         0         0         0    1.0000         0
0         0         0         0         0         0         0         0         0    1.0000
```

- Pivot **TtestB** to optimality with **Pivoting2.m** (SM), see **Section 3**
- Now we get a new tableau called **TtestBopt**. Compare first columns of tableau **TtB** and **TtestBopt**

```
ans =

383.6175
36.9130
15.7353
83.3308
36.6797
155.5608
196.5511
14.5661
24.9131
10.6790
103.5712

ans =

383.6175
36.9130
103.5712
83.3308
36.6797
155.5608
196.5511
14.5661
15.7353
10.6790
24.9131
```

- Correct result!

2.2.5 C

- Find the **first basic** column in tableau T_S

```
fbascol =

2
```

- The basic variable x_1 is defined in row 3

```
rowC =

3
```

- Get all the positive columns and negative columns of **rowC**

```
c=TS(1,2:end);
s=TS(rowC,2:end);
posC=find(s>0);
negC=find(s<0);
```

- We can see that the positive columns are:

```
posC =

1     3     4     5     6     8    10    18    20
```

- Now we must remove 1 from **posC**

```
posC =

3     4     5     6     8    10    18    20
```

- Use my own **Findq.m** m-file to get the range of value **q**

$$-2.0880 \le q \le 3.8852 \tag{2}$$

- Get the value q

```
q=minnegval/2

q =

-1.0440
```

- Copy tableau T_S to a new tableau **TtC** and add **q** times row 3 to top row in T_S

```
TtC=TS;
TtC(1,:)=TtC(1,:)+q*TtC(rowC,:)

TtC =

Columns 1 through 12

329.4081   -1.0440   10.1989    5.2910    0.1290    1.2676    0.5789         0    1.5716         0    0.0732         0
35.9918         0    1.3794    1.4188   -0.1234    0.2376    0.0601         0    0.2966         0   -0.1233    1.0000
14.8536    1.0000   -0.5187    1.1803    0.0728    0.1601    0.0172         0    0.2113         0    0.0701         0
71.2160         0    2.0747    3.5287   -0.1660    0.4844   -0.1937         0    0.5298         0   -0.0306         0
33.3316         0    0.8841    1.3984    0.0488    0.1672    0.0006         0    0.0861         0   -0.0988         0
160.1769        0    6.3787    6.4715   -0.5214    1.6053    0.3763         0    1.9330         0   -0.2099         0
185.9420        0    6.9493    9.4679   -0.2116    2.0424   -0.3242         0    1.7870         0    0.3216         0
16.1709         0   -0.3705   -0.4069    0.0698   -0.0106    0.1373         0   -0.0812         0    0.0858         0
49.8261         0    0.8594    2.9546   -0.2216    0.7582    0.0848    1.0000    0.6557         0   -0.9764         0
7.2219          0   -0.1463    0.1791    0.0718   -0.0574    0.0561         0    0.0083         0   -0.0059         0
60.4117         0   -1.3135   11.5980    0.2610    1.2699   -0.7012         0    0.7678    1.0000    0.6303         0

Columns 13 through 21

1.7166         0         0         0         0         0    3.2515         0    1.2078
-1.2319        0         0         0         0         0    2.0420         0    0.0294
-0.3482        0         0         0         0         0    1.6128         0    0.0282
0.2680    1.0000         0         0         0         0    1.9238         0    0.3873
0.2084         0    1.0000         0         0         0    0.6447         0    0.1070
-9.1822        0         0    1.0000         0         0   13.3700         0   -0.1476
-7.6541        0         0         0    1.0000         0   13.1131         0    0.3391
0.1620         0         0         0         0    1.0000    0.4913         0   -0.0513
-10.1058       0         0         0         0         0   11.1858         0   -0.7964
0.4659         0         0         0         0         0   -0.2374    1.0000    0.1105
6.3197         0         0         0         0         0   -0.9016         0    1.3797
```

- Manually set **TtC(1,2)** to **0**, now we can see top row of **TtC** is:

```
ans =

Columns 1 through 12

329.4081         0   10.1989    5.2910    0.1290    1.2676    0.5789         0    1.5716         0    0.0732         0

Columns 13 through 21

1.7166         0         0         0         0         0    3.2515         0    1.2078
```

- New **x** and **z-value** are:

```
XC =

14.8536
0
0
0
0
0
49.8261
0
60.4117
0
35.9918
0
71.2160
33.3316
160.1769
185.9420
16.1709
0
7.2219
0
```

```
ZC =

-329.4081
```

- Copy tableau T_C to **TtestC** and subtract **q** from **TtestC(1,2)**

```
TtestC=TC;
TtestC(1,2)=TtestC(1,2)-q

TtestC =

Columns 1 through 13

246.2199    2.6824    9.9495   -9.9457    0.1373   -0.6607    1.5296   -0.7891    0.5570   -1.3857    0.1585         0         0
18.4876    0.2325    1.2699   -0.8175   -0.0971   -0.0996    0.1556   -0.2288    0.0743   -0.1582    0.0167    1.0000         0
1.3038    1.9594   -0.9940   -1.0432    0.1534   -0.1839    0.1576   -0.2997    0.0539   -0.2130    0.2958         0    1.0000
37.2164    0.3681    2.2360   -1.8923   -0.1960   -0.2110    0.0980   -0.2604    0.1002   -0.4385   -0.0269         0         0
22.7731   -0.2955    1.1291    0.0094    0.0101   -0.0035    0.0527   -0.0218   -0.0552   -0.0841   -0.1513         0         0
61.8394    0.9192    5.6459   -4.7865   -0.3332   -0.2010    0.7735   -1.3851    0.6728   -0.7114    0.7586         0         0
66.6736    2.2207    6.0190   -6.6298   -0.0133   -0.3426    0.4268   -1.5899    0.2854   -1.2089    1.2678         0         0
13.9028   -1.3209    0.2464    0.3134   -0.0208    0.0966    0.0187    0.1591   -0.1362    0.1560   -0.0638         0         0
8.7308    1.1864   -0.5924    0.2731    0.0862    0.0285    0.0654   -0.0877    0.1357   -0.0749    0.1217         0         0
3.8777    0.2750   -0.1790   -0.7469    0.0713   -0.1618    0.1295   -0.0266   -0.0286   -0.1011   -0.0243         0         0
43.5202   -8.1998    3.2142   13.3631   -0.4569    1.7814   -1.1874    1.3154    0.3981    1.6516   -0.8185         0         0

Columns 14 through 21

0         0         0         0         0         0         0         0
0         0         0         0         0         0         0         0
0         0         0         0         0         0         0         0
1.0000         0         0         0         0         0         0         0
0    1.0000         0         0         0         0         0         0
0         0    1.0000         0         0         0         0         0
0         0         0    1.0000         0         0         0         0
0         0         0         0    1.0000         0         0         0
0         0         0         0         0    1.0000         0         0
0         0         0         0         0         0    1.0000         0
0         0         0         0         0         0         0    1.0000
```

- Now we have to pivot this tableau to its optimality by using SM, see details on **Section 3**
- After SM, we get a tableau **TtestCopt**. Now compare the first rows of tableau **TtC** and **TtestCopt**, we can see

```
ans =

Columns 1 through 13

329.4081         0   10.1989    5.2910    0.1290    1.2676    0.5789         0    1.5716         0    0.0732         0    1.7166

Columns 14 through 21

0         0         0         0         0    3.2515         0    1.2078


ans =

Columns 1 through 13

329.4081         0   10.1989    5.2910    0.1290    1.2676    0.5789         0    1.5716         0    0.0732         0    1.7166

Columns 14 through 21

0         0         0         0         0    3.2515         0    1.2078
```

- Correct result!

3 Succession of Tableaux (Pivoting)

3.1 From T_0 to T_B

```
T1=Pivoting(T0)
Row 9 and Col 7 selected.

T1 =

Columns 1 through 11

390.3688    0.3333    3.0000    4.6667   13.0000   -7.3333         0  -13.6667    3.3333    7.3333   -9.6667
-103.8786    4.6667   -4.0000    3.3333   -6.0000    1.3333         0   -0.3333    0.6667   -3.3333    3.6667
-40.7970   -2.3333         0    1.3333         0   -3.6667         0    5.6667   -0.3333   -4.3333    6.6667
-75.6115   -3.3333    1.0000   -3.6667   -2.0000    4.3333         0    9.6667   -2.3333   -5.3333    1.6667
-173.0235   -3.3333         0   -6.6667   -7.0000    5.3333         0    2.6667   -3.3333   -2.3333    1.6667
78.7956   -2.3333    1.0000    2.3333    4.0000    1.3333         0   -8.3333   -0.3333    2.6667   -3.3333
-45.4684   -4.0000   -2.0000   -1.0000    4.0000    5.0000         0         0   -5.0000   -2.0000   -2.0000
-305.8982    2.6667   -8.0000   -5.6667   -7.0000    4.3333         0    8.6667   -5.3333   -6.3333    4.6667
55.7670    0.3333    1.0000    0.6667    1.0000   -0.3333    1.0000   -1.6667    0.3333    1.3333   -0.6667
-46.3297    0.6667    3.0000   -4.6667   -5.0000   -0.6667         0   -0.3333    1.6667    0.6667   -1.3333
290.4094   -2.0000    5.0000    2.0000    6.0000    3.0000         0   -6.0000    6.0000    6.0000         0

Columns 12 through 21

0         0         0         0         0         0         0   -2.3333         0         0
1.0000         0         0         0         0         0         0    0.3333         0         0
0    1.0000         0         0         0         0         0    1.3333         0         0
0         0    1.0000         0         0         0         0    1.3333         0         0
0         0         0    1.0000         0         0         0    1.3333         0         0
0         0         0         0    1.0000         0         0   -0.6667         0         0
0         0         0         0         0    1.0000         0         0         0         0
0         0         0         0         0         0    1.0000    1.3333         0         0
0         0         0         0         0         0         0   -0.3333         0         0
0         0         0         0         0         0         0    0.3333    1.0000         0
0         0         0         0         0         0         0   -1.0000         0    1.0000


T2=Pivoting(T1)
Row 8 and Col 3 selected.

T2 =

Columns 1 through 11

275.6569    1.3333         0    2.5417   10.3750   -5.7083         0  -10.4167    1.3333    4.9583   -7.9167
49.0705    3.3333         0    6.1667   -2.5000   -0.8333         0   -4.6667    3.3333   -0.1667    1.3333
-40.7970   -2.3333         0    1.3333         0   -3.6667         0    5.6667   -0.3333   -4.3333    6.6667
-113.8488   -3.0000         0   -4.3750   -2.8750    4.8750         0   10.7500   -3.0000   -6.1250    2.2500
-173.0235   -3.3333         0   -6.6667   -7.0000    5.3333         0    2.6667   -3.3333   -2.3333    1.6667
40.5583   -2.0000         0    1.6250    3.1250    1.8750         0   -7.2500   -1.0000    1.8750   -2.7500
31.0062   -4.6667         0    0.4167    5.7500    3.9167         0   -2.1667   -3.6667   -0.4167   -3.1667
38.2373   -0.3333    1.0000    0.7083    0.8750   -0.5417         0   -1.0833    0.6667    0.7917   -0.5833
17.5297    0.6667         0   -0.0417    0.1250    0.2083    1.0000   -0.5833   -0.3333    0.5417   -0.0833
-161.0415    1.6667         0   -6.7917   -7.6250    0.9583         0    2.9167   -0.3333   -1.7083    0.4167
99.2230   -0.3333         0   -1.5417    1.6250    5.7083         0   -0.5833    2.6667    2.0417    2.9167

Columns 12 through 21

0         0         0         0         0         0    0.3750   -1.8333         0         0
1.0000         0         0         0         0         0   -0.5000   -0.3333         0         0
0    1.0000         0         0         0         0         0    1.3333         0         0
0         0    1.0000         0         0         0    0.1250    1.5000         0         0
0         0         0    1.0000         0         0         0    1.3333         0         0
```

```
0         0         0         0    1.0000         0    0.1250   -0.5000         0         0
0         0         0         0         0    1.0000   -0.2500   -0.3333         0         0
0         0         0         0         0         0   -0.1250   -0.1667         0         0
0         0         0         0         0         0    0.1250   -0.1667         0         0
0         0         0         0         0         0    0.3750    0.8333    1.0000         0
0         0         0         0         0         0    0.6250   -0.1667         0    1.0000

T3=Pivoting(T2)
Row 5 and Col 4 selected.

T3 =

Columns 1 through 11

209.6917    0.0625         0         0    7.7063   -3.6750         0   -9.4000    0.0625    4.0687   -7.2813
-110.9762    0.2500         0         0   -8.9750    4.1000         0   -2.2000    0.2500   -2.3250    2.8750
-75.4017   -3.0000         0         0   -1.4000   -2.6000         0    6.2000   -1.0000   -4.8000    7.0000
-0.3021   -0.8125         0         0    1.7188    1.3750         0    9.0000   -0.8125   -4.5938    1.1563
25.9535    0.5000         0    1.0000    1.0500   -0.8000         0   -0.4000    0.5000    0.3500   -0.2500
-1.6162   -2.8125         0         0    1.4188    3.1750         0   -6.6000   -1.8125    1.3063   -2.3438
20.1922   -4.8750         0         0    5.3125    4.2500         0   -2.0000   -3.8750   -0.5625   -3.0625
19.8535   -0.6875    1.0000         0    0.1313    0.0250         0   -0.8000    0.3125    0.5437   -0.4062
18.6111    0.6875         0         0    0.1687    0.1750    1.0000   -0.6000   -0.3125    0.5562   -0.0938
15.2261    5.0625         0         0   -0.4937   -4.4750         0    0.2000    3.0625    0.6687   -1.2813
139.2347    0.4375         0         0    3.2437    4.4750         0   -1.2000    3.4375    2.5812    2.5312

Columns 12 through 21

0         0         0    0.3812         0         0    0.3750   -1.3250         0         0
1.0000         0         0    0.9250         0         0   -0.5000    0.9000         0         0
0    1.0000         0    0.2000         0         0         0    1.6000         0         0
0         0    1.0000   -0.6563         0         0    0.1250    0.6250         0         0
0         0         0   -0.1500         0         0         0   -0.2000         0         0
0         0         0    0.2437    1.0000         0    0.1250   -0.1750         0         0
0         0         0    0.0625         0    1.0000   -0.2500   -0.2500         0         0
0         0         0    0.1063         0         0   -0.1250   -0.0250         0         0
0         0         0   -0.0062         0         0    0.1250   -0.1750         0         0
0         0         0   -1.0188         0         0    0.3750   -0.5250    1.0000         0
0         0         0   -0.2312         0         0    0.6250   -0.4750         0    1.0000

T4=Pivoting(T3)
Row 2 and Col 8 selected.

T4 =

Columns 1 through 11

683.8628   -1.0057         0         0   46.0540  -21.1932         0         0   -1.0057   14.0028  -19.5653
50.4437   -0.1136         0         0    4.0795   -1.8636         0    1.0000   -0.1136    1.0568   -1.3068
-388.1528   -2.2955         0         0  -26.6932    8.9545         0         0   -0.2955  -11.3523   15.1023
-454.2957    0.2102         0         0  -34.9972   18.1477         0         0    0.2102  -14.1051   12.9176
46.1310    0.4545         0    1.0000    2.6818   -1.5455         0         0    0.4545    0.7727   -0.7727
331.3125   -3.5625         0         0   28.3437   -9.1250         0         0   -2.5625    8.2812  -10.9687
121.0797   -5.1023         0         0   13.4716    0.5227         0         0   -4.1023    1.5511   -5.6761
60.2085   -0.7784    1.0000         0    3.3949   -1.4659         0         0    0.2216    1.3892   -1.4517
48.8773    0.6193         0         0    2.6165   -0.9432    1.0000         0   -0.3807    1.1903   -0.8778
5.1374    5.0852         0         0   -1.3097   -4.1023         0         0    3.0852    0.4574   -1.0199
199.7672    0.3011         0         0    8.1392    2.2386         0         0    3.3011    3.8494    0.9631

Columns 12 through 21

-4.2727         0         0   -3.5710         0         0    2.5114   -5.1705         0         0
-0.4545         0         0   -0.4205         0         0    0.2273   -0.4091         0         0
2.8182    1.0000         0    2.8068         0         0   -1.4091    4.1364         0         0
4.0909         0    1.0000    3.1278         0         0   -1.9205    4.3068         0         0
-0.1818         0         0   -0.3182         0         0    0.0909   -0.3636         0         0
-3.0000         0         0   -2.5312    1.0000         0    1.6250   -2.8750         0         0
-0.9091         0         0   -0.7784         0    1.0000    0.2045   -1.0682         0         0
-0.3636         0         0   -0.2301         0         0    0.0568   -0.3523         0         0
-0.2727         0         0   -0.2585         0         0    0.2614   -0.4205         0         0
0.0909         0         0   -0.9347         0         0    0.3295   -0.4432    1.0000         0
-0.5455         0         0   -0.7358         0         0    0.8977   -0.9659         0    1.0000
```

```
T5=Pivoting(T4)
Row 4 and Col 10 selected.

T5 =

Columns 1 through 11

232.8611   -0.7970         0         0   11.3106   -3.1770         0         0   -0.7970         0   -6.7414
16.4059   -0.0979         0         0    1.4574   -0.5039         0    1.0000   -0.0979         0   -0.3390
-22.5203   -2.4647         0         0    1.4737   -5.6514         0         0   -0.4647         0    4.7057
32.2079   -0.0149         0         0    2.4812   -1.2866         0         0   -0.0149    1.0000   -0.9158
21.2431    0.4661         0    1.0000    0.7646   -0.5513         0         0    0.4661         0   -0.0651
64.5910   -3.4391         0         0    7.7966    1.5297         0         0   -2.4391         0   -3.3847
71.1209   -5.0792         0         0    9.6230    2.5184         0         0   -4.0792         0   -4.2556
15.4652   -0.7577    1.0000         0   -0.0520    0.3215         0         0    0.2423         0   -0.1795
10.5390    0.6371         0         0   -0.3370    0.5883    1.0000         0   -0.3629         0    0.2123
-9.5940    5.0920         0         0   -2.4445   -3.5138         0         0    3.0920         0   -0.6010
75.7852    0.3585         0         0   -1.4119    7.1913         0         0    3.3585         0    4.4884

Columns 12 through 21

-0.2115         0    0.9927   -0.4659         0         0    0.6048   -0.8949         0         0
-0.1480         0    0.0749   -0.1861         0         0    0.0834   -0.0864         0         0
-0.4743    1.0000   -0.8048    0.2894         0         0    0.1366    0.6701         0         0
-0.2900         0   -0.0709   -0.2218         0         0    0.1362   -0.3053         0         0
0.0423         0    0.0548   -0.1468         0         0   -0.0143   -0.1277         0         0
-0.5982         0    0.5871   -0.6949    1.0000         0    0.4975   -0.3464         0         0
-0.4592         0    0.1100   -0.4344         0    1.0000   -0.0066   -0.5946         0         0
0.0393         0    0.0985    0.0779         0         0   -0.1323    0.0719         0         0
0.0725         0    0.0844    0.0054         0         0    0.0993   -0.0570         0         0
0.2236         0    0.0324   -0.8332         0         0    0.2673   -0.3035    1.0000         0
0.5710         0    0.2729    0.1178         0         0    0.3736    0.2095         0    1.0000

T6=Pivoting(T5)
Row 3 and Col 9 selected.

T6 =

Columns 1 through 11

271.4881    3.4304         0         0    8.7828    6.5163         0         0         0         0  -14.8127
21.1501    0.4213         0         0    1.1469    0.6866         0    1.0000         0         0   -1.3303
48.4669    5.3043         0         0   -3.1717   12.1625         0         0    1.0000         0  -10.1274
32.9302    0.0642         0         0    2.4339   -1.1053         0         0         0    1.0000   -1.0668
-1.3455   -2.0061         0    1.0000    2.2427   -6.2198         0         0         0         0    4.6550
182.8053    9.4985         0         0    0.0607   31.1951         0         0         0         0  -28.0863
268.8247   16.5579         0         0   -3.3147   52.1313         0         0         0         0  -45.5670
3.7219   -2.0429    1.0000         0    0.7165   -2.6255         0         0         0         0    2.2744
28.1296    2.5622         0         0   -1.4881    5.0026    1.0000         0         0         0   -3.4634
-159.4557  -11.3091         0         0    7.3624  -41.1209         0         0         0         0   30.7135
-86.9913  -17.4560         0         0    9.2401  -33.6567         0         0         0         0   38.5015

Columns 12 through 21

0.6021   -1.7152    2.3732   -0.9623         0         0    0.3706   -2.0442         0         0
-0.0481   -0.2107    0.2445   -0.2471         0         0    0.0546   -0.2276         0         0
1.0208   -2.1521    1.7321   -0.6229         0         0   -0.2939   -1.4421         0         0
-0.2748   -0.0321   -0.0451   -0.2310         0         0    0.1318   -0.3268         0         0
-0.4335    1.0030   -0.7525    0.1435         0         0    0.1227    0.5444         0         0
1.8916   -5.2492    4.8119   -2.2141    1.0000         0   -0.2193   -3.8639         0         0
3.7048   -8.7789    7.1756   -2.9753         0    1.0000   -1.2055   -6.4772         0         0
-0.2081    0.5215   -0.3212    0.2289         0         0   -0.0611    0.4213         0         0
0.4430   -0.7811    0.7130   -0.2206         0         0   -0.0074   -0.5804         0         0
-2.9328    6.6545   -5.3234    1.0928         0         0    1.1760    4.1556    1.0000         0
-2.8574    7.2280   -5.5444    2.2098         0         0    1.3606    5.0529         0    1.0000

T7=Pivoting(T6)
Row 10 and Col 6 selected.

T7 =
```

```
Columns 1 through 11

246.2199    1.6383         0         0    9.9495         0         0         0         0         0   -9.9457
18.4876    0.2325         0         0    1.2699         0         0    1.0000         0         0   -0.8175
1.3038    1.9594         0         0   -0.9940         0         0         0    1.0000         0   -1.0432
37.2164    0.3681         0         0    2.2360         0         0         0         0    1.0000   -1.8923
22.7731   -0.2955         0    1.0000    1.1291         0         0         0         0         0    0.0094
61.8394    0.9192         0         0    5.6459         0         0         0         0         0   -4.7865
66.6736    2.2207         0         0    6.0190         0         0         0         0         0   -6.6298
13.9028   -1.3209    1.0000         0    0.2464         0         0         0         0         0    0.3134
8.7308    1.1864         0         0   -0.5924         0    1.0000         0         0         0    0.2731
3.8777    0.2750         0         0   -0.1790    1.0000         0         0         0         0   -0.7469
43.5202   -8.1998         0         0    3.2142         0         0         0         0         0   13.3631

Columns 12 through 21

0.1373   -0.6607    1.5296   -0.7891         0         0    0.5570   -1.3857    0.1585         0
-0.0971   -0.0996    0.1556   -0.2288         0         0    0.0743   -0.1582    0.0167         0
0.1534   -0.1839    0.1576   -0.2997         0         0    0.0539   -0.2130    0.2958         0
-0.1960   -0.2110    0.0980   -0.2604         0         0    0.1002   -0.4385   -0.0269         0
0.0101   -0.0035    0.0527   -0.0218         0         0   -0.0552   -0.0841   -0.1513         0
-0.3332   -0.2010    0.7735   -1.3851    1.0000         0    0.6728   -0.7114    0.7586         0
-0.0133   -0.3426    0.4268   -1.5899         0    1.0000    0.2854   -1.2089    1.2678         0
-0.0208    0.0966    0.0187    0.1591         0         0   -0.1362    0.1560   -0.0638         0
0.0862    0.0285    0.0654   -0.0877         0         0    0.1357   -0.0749    0.1217         0
0.0713   -0.1618    0.1295   -0.0266         0         0   -0.0286   -0.1011   -0.0243         0
-0.4569    1.7814   -1.1874    1.3154         0         0    0.3981    1.6516   -0.8185    1.0000

TB=T7

TB =

Columns 1 through 5

246.2199    1.6383         0         0    9.9495
18.4876    0.2325         0         0    1.2699
1.3038    1.9594         0         0   -0.9940
37.2164    0.3681         0         0    2.2360
22.7731   -0.2955         0    1.0000    1.1291
61.8394    0.9192         0         0    5.6459
66.6736    2.2207         0         0    6.0190
13.9028   -1.3209    1.0000         0    0.2464
8.7308    1.1864         0         0   -0.5924
3.8777    0.2750         0         0   -0.1790
43.5202   -8.1998         0         0    3.2142

Columns 6 through 10

0         0         0         0         0
0         0    1.0000         0         0
0         0         0    1.0000         0
0         0         0         0    1.0000
0         0         0         0         0
0         0         0         0         0
0         0         0         0         0
0         0         0         0         0
0    1.0000         0         0         0
1.0000         0         0         0         0
0         0         0         0         0

Columns 11 through 15

-9.9457    0.1373   -0.6607    1.5296   -0.7891
-0.8175   -0.0971   -0.0996    0.1556   -0.2288
-1.0432    0.1534   -0.1839    0.1576   -0.2997
-1.8923   -0.1960   -0.2110    0.0980   -0.2604
0.0094    0.0101   -0.0035    0.0527   -0.0218
-4.7865   -0.3332   -0.2010    0.7735   -1.3851
-6.6298   -0.0133   -0.3426    0.4268   -1.5899
0.3134   -0.0208    0.0966    0.0187    0.1591
0.2731    0.0862    0.0285    0.0654   -0.0877
-0.7469    0.0713   -0.1618    0.1295   -0.0266
```

```
13.3631   -0.4569    1.7814   -1.1874    1.3154

Columns 16 through 20

0         0    0.5570   -1.3857    0.1585
0         0    0.0743   -0.1582    0.0167
0         0    0.0539   -0.2130    0.2958
0         0    0.1002   -0.4385   -0.0269
0         0   -0.0552   -0.0841   -0.1513
1.0000    0    0.6728   -0.7114    0.7586
0    1.0000    0.2854   -1.2089    1.2678
0         0   -0.1362    0.1560   -0.0638
0         0    0.1357   -0.0749    0.1217
0         0   -0.0286   -0.1011   -0.0243
0         0    0.3981    1.6516   -0.8185

Column 21

0
0
0
0
0
0
0
0
0
0
1.0000
```

3.2 From T_C to T_S

```
TC2=Pivoting2(TC1)
Row 3 and Col 2 selected.

TC2 =

Columns 1 through 11

294.5193         0    9.8269         0    0.1955    0.5134    0.8656   -0.4768    1.1410   -0.4410    0.3340
22.1088         0    1.3149         0   -0.1010    0.0003    0.0962   -0.1885    0.1159   -0.0743    0.0139
3.5634    1.0000   -0.5633         0    0.0892   -0.0340    0.0492   -0.1493    0.0644   -0.0637    0.1758
46.2051         0    2.2444         0   -0.1899    0.0144   -0.0311   -0.1926    0.2076   -0.2552   -0.0034
23.7750         0    0.9637         0    0.0363   -0.0146    0.0678   -0.0660   -0.0368   -0.1038   -0.0998
84.6180         0    5.6606         0   -0.3169    0.3685    0.4475   -1.2153    0.9454   -0.2484    0.8201
94.8479         0    6.5730         0   -0.0752    0.4784   -0.0714   -1.2132    0.6020   -0.5073    1.1864
16.9037         0   -0.4646         0    0.0906    0.0164    0.1021   -0.0403   -0.0728    0.0453    0.1537
3.0166         0    0.1046         0   -0.0252    0.0381    0.0231    0.0876    0.0403   -0.0223   -0.0996
6.9633         0   -0.1026         0    0.0621   -0.0685    0.0721    0.0196    0.0055   -0.0204   -0.0378
5.4433         0   -0.1051    1.0000    0.0205    0.1125   -0.0587    0.0068    0.0693    0.0845    0.0466

Columns 12 through 21

0    3.3841         0         0         0         0         0         0         0    1.0084
1.0000    0.2040         0         0         0         0         0         0         0    0.0771
0    0.7580         0         0         0         0         0         0         0    0.0592
0    0.6011    1.0000         0         0         0         0         0         0    0.1885
0    0.2196         0    1.0000         0         0         0         0         0    0.0164
0    1.5295         0         0    1.0000         0         0         0         0    0.4776
0    1.4003         0         0         0    1.0000         0         0         0    0.6054
0    0.8554         0         0         0         0    1.0000         0         0    0.0433
0   -1.0263         0         0         0         0         0    1.0000         0   -0.1006
0    0.1389         0         0         0         0         0         0    1.0000    0.0667
0    0.4651         0         0         0         0         0         0         0    0.1111

TC3=Pivoting2(TC2)
Row 9 and Col 8 selected.

TC3 =
```

```
Columns 1 through 11

310.9372         0   10.3961         0    0.0582    0.7206    0.9912         0    1.3603   -0.5624   -0.2081
28.6016         0    1.5400         0   -0.1553    0.0822    0.1459         0    0.2027   -0.1223   -0.2004
8.7055    1.0000   -0.3850         0    0.0462    0.0309    0.0885         0    0.1331   -0.1018    0.0060
52.8359         0    2.4744         0   -0.2454    0.0981    0.0196         0    0.2962   -0.3042   -0.2223
26.0478         0    1.0425         0    0.0173    0.0141    0.0852         0   -0.0064   -0.1206   -0.1748
126.4682         0    7.1116         0   -0.6670    0.8967    0.7676         0    1.5046   -0.5580   -0.5616
136.6258         0    8.0216         0   -0.4247    1.0057    0.2482         0    1.1602   -0.8163   -0.1930
18.2903         0   -0.4166         0    0.0790    0.0339    0.1127         0   -0.0543    0.0351    0.1079
34.4364         0    1.1940         0   -0.2881    0.4347    0.2634    1.0000    0.4601   -0.2547   -1.1370
6.2891         0   -0.1260         0    0.0678   -0.0770    0.0670         0   -0.0035   -0.0154   -0.0156
5.2088         0   -0.1133    1.0000    0.0225    0.1095   -0.0605         0    0.0662    0.0862    0.0543

Columns 12 through 21

0   -2.2015         0         0         0         0         0    5.4424         0    0.4612
1.0000   -2.0050         0         0         0         0         0    2.1523         0   -0.1393
0   -0.9914         0         0         0         0         0    1.7046         0   -0.1122
0   -1.6548    1.0000         0         0         0         0    2.1981         0   -0.0325
0   -0.5536         0    1.0000         0         0         0    0.7534         0   -0.0593
0  -12.7086         0         0    1.0000         0         0   13.8731         0   -0.9174
0  -12.8132         0         0         0    1.0000         0   13.8491         0   -0.7871
0    0.3837         0         0         0         0    1.0000    0.4597         0   -0.0029
0  -11.7158         0         0         0         0         0   11.4155         0   -1.1479
0    0.3683         0         0         0         0         0   -0.2235    1.0000    0.0892
0    0.5449         0         0         0         0         0   -0.0777         0    0.1190

TC4=Pivoting2(TC3)
Row 11 and Col 13 selected.

TC4 =

Columns 1 through 11

331.9821         0    9.9386    4.0403    0.1491    1.1630    0.7469         0    1.6278   -0.2141    0.0115
47.7675         0    1.1233    3.6795   -0.0725    0.4851   -0.0766         0    0.4463    0.1949   -0.0005
18.1826    1.0000   -0.5911    1.8194    0.0871    0.2301   -0.0215         0    0.2536    0.0551    0.1049
68.6544         0    2.1304    3.0369   -0.1771    0.4306   -0.1640         0    0.4973   -0.0424   -0.0573
31.3398         0    0.9274    1.0160    0.0402    0.1253    0.0238         0    0.0608   -0.0330   -0.1196
247.9517         0    4.4703   23.3228   -0.1422    3.4503   -0.6424         0    3.0486    1.4529    0.7059
259.1093         0    5.3585   23.5147    0.1045    3.5804   -1.1734         0    2.7169    1.2111    1.0850
14.6226         0   -0.3368   -0.7042    0.0631   -0.0432    0.1552         0   -0.1009   -0.0256    0.0697
146.4299         0   -1.2410   21.5008    0.1958    2.7888   -1.0364    1.0000    1.8835    1.5991    0.0315
2.7684         0   -0.0495   -0.6759    0.0526   -0.1510    0.1078         0   -0.0483   -0.0737   -0.0523
9.5592         0   -0.2078    1.8352    0.0413    0.2009   -0.1109         0    0.1215    0.1582    0.0997

Columns 12 through 21

0         0         0         0         0         0         0    5.1283         0    0.9418
1.0000         0         0         0         0         0         0    1.8663         0    0.2984
0         0         0         0         0         0         0    1.5631         0    0.1042
0         0    1.0000         0         0         0         0    1.9620         0    0.3288
0         0         0    1.0000         0         0         0    0.6744         0    0.0615
0         0         0         0    1.0000         0         0   12.0600         0    1.8570
0         0         0         0         0    1.0000         0   12.0212         0    2.0101
0         0         0         0         0         0    1.0000    0.5144         0   -0.0867
0         0         0         0         0         0         0    9.7441         0    1.4098
0         0         0         0         0         0         0   -0.1709    1.0000    0.0088
0    1.0000         0         0         0         0         0   -0.1427         0    0.2183

TC5=Pivoting2(TC4)
Row 11 and Col 10 selected.

TC5 =

Columns 1 through 11

344.9156         0    9.6574    6.5233    0.2050    1.4348    0.5968         0    1.7922         0    0.1465
35.9918         0    1.3794    1.4188   -0.1234    0.2376    0.0601         0    0.2966         0   -0.1233
14.8536    1.0000   -0.5187    1.1803    0.0728    0.1601    0.0172         0    0.2113         0    0.0701
```

```
71.2160         0    2.0747    3.5287   -0.1660    0.4844   -0.1937         0    0.5298         0   -0.0306
33.3316         0    0.8841    1.3984    0.0488    0.1672    0.0006         0    0.0861         0   -0.0988
160.1769        0    6.3787    6.4715   -0.5214    1.6053    0.3763         0    1.9330         0   -0.2099
185.9420        0    6.9493    9.4679   -0.2116    2.0424   -0.3242         0    1.7870         0    0.3216
16.1709         0   -0.3705   -0.4069    0.0698   -0.0106    0.1373         0   -0.0812         0    0.0858
49.8261         0    0.8594    2.9546   -0.2216    0.7582    0.0848    1.0000    0.6557         0   -0.9764
7.2219          0   -0.1463    0.1791    0.0718   -0.0574    0.0561         0    0.0083         0   -0.0059
60.4117         0   -1.3135   11.5980    0.2610    1.2699   -0.7012         0    0.7678    1.0000    0.6303

Columns 12 through 21

0    1.3530         0         0         0         0         0    4.9353         0    1.2372
1.0000   -1.2319         0         0         0         0         0    2.0420         0    0.0294
0   -0.3482         0         0         0         0         0    1.6128         0    0.0282
0    0.2680    1.0000         0         0         0         0    1.9238         0    0.3873
0    0.2084         0    1.0000         0         0         0    0.6447         0    0.1070
0   -9.1822         0         0    1.0000         0         0   13.3700         0   -0.1476
0   -7.6541         0         0         0    1.0000         0   13.1131         0    0.3391
0    0.1620         0         0         0         0    1.0000    0.4913         0   -0.0513
0  -10.1058         0         0         0         0         0   11.1858         0   -0.7964
0    0.4659         0         0         0         0         0   -0.2374    1.0000    0.1105
0    6.3197         0         0         0         0         0   -0.9016         0    1.3797

TS=TC5

TS =

Columns 1 through 11

344.9156         0    9.6574    6.5233    0.2050    1.4348    0.5968         0    1.7922         0    0.1465
35.9918          0    1.3794    1.4188   -0.1234    0.2376    0.0601         0    0.2966         0   -0.1233
14.8536    1.0000   -0.5187    1.1803    0.0728    0.1601    0.0172         0    0.2113         0    0.0701
71.2160          0    2.0747    3.5287   -0.1660    0.4844   -0.1937         0    0.5298         0   -0.0306
33.3316          0    0.8841    1.3984    0.0488    0.1672    0.0006         0    0.0861         0   -0.0988
160.1769         0    6.3787    6.4715   -0.5214    1.6053    0.3763         0    1.9330         0   -0.2099
185.9420         0    6.9493    9.4679   -0.2116    2.0424   -0.3242         0    1.7870         0    0.3216
16.1709          0   -0.3705   -0.4069    0.0698   -0.0106    0.1373         0   -0.0812         0    0.0858
49.8261          0    0.8594    2.9546   -0.2216    0.7582    0.0848    1.0000    0.6557         0   -0.9764
7.2219           0   -0.1463    0.1791    0.0718   -0.0574    0.0561         0    0.0083         0   -0.0059
60.4117          0   -1.3135   11.5980    0.2610    1.2699   -0.7012         0    0.7678    1.0000    0.6303

Columns 12 through 21

0    1.3530         0         0         0         0         0    4.9353         0    1.2372
1.0000   -1.2319         0         0         0         0         0    2.0420         0    0.0294
0   -0.3482         0         0         0         0         0    1.6128         0    0.0282
0    0.2680    1.0000         0         0         0         0    1.9238         0    0.3873
0    0.2084         0    1.0000         0         0         0    0.6447         0    0.1070
0   -9.1822         0         0    1.0000         0         0   13.3700         0   -0.1476
0   -7.6541         0         0         0    1.0000         0   13.1131         0    0.3391
0    0.1620         0         0         0         0    1.0000    0.4913         0   -0.0513
0  -10.1058         0         0         0         0         0   11.1858         0   -0.7964
0    0.4659         0         0         0         0         0   -0.2374    1.0000    0.1105
0    6.3197         0         0         0         0         0   -0.9016         0    1.3797
```

3.3 From TtestB to TtestBopt

```
TtestB1=Pivoting2(TtestB)
Row 11 and Col 4 selected.

TtestB1 =

Columns 1 through 11

301.8929   -4.4645   12.3417         0   -0.2028    0.6651    0.6459    0.1899    0.8533   -0.1565   -0.4507
23.0635    -0.2691    1.4665         0   -0.1250    0.0094    0.0830   -0.1484    0.0986   -0.0571   -0.0334
7.1431     1.3193   -0.7431         0    0.1177   -0.0448    0.0649   -0.1970    0.0850   -0.0841    0.2319
47.8091    -0.7930    2.6912         0   -0.2607    0.0413   -0.0701   -0.0741    0.1565   -0.2047   -0.1428
```

```
22.7206   -0.2898    1.1269         0    0.0105   -0.0048    0.0535   -0.0227   -0.0555   -0.0853   -0.1507
88.6326   -2.0178    6.7972         0   -0.4969    0.4371    0.3482   -0.9140    0.8154   -0.1198    0.4654
103.7849   -1.8474    7.6136         0   -0.2400    0.5412   -0.1623   -0.9373    0.4829   -0.3895    0.8617
12.1485   -1.1286    0.1711         0   -0.0101    0.0548    0.0465    0.1282   -0.1455    0.1173   -0.0447
7.2021    1.3540   -0.6581         0    0.0955   -0.0079    0.0897   -0.1146    0.1276   -0.1086    0.1384
8.0587   -0.1833    0.0006         0    0.0458   -0.0623    0.0631    0.0469   -0.0063   -0.0087   -0.0701
5.5977   -0.6136    0.2405    1.0000   -0.0342    0.1333   -0.0889    0.0984    0.0298    0.1236   -0.0612

Columns 12 through 21

0         0         0         0         0         0         0         0         0    0.7443
1.0000         0         0         0         0         0         0         0         0    0.0612
0    1.0000         0         0         0         0         0         0         0    0.0781
0         0    1.0000         0         0         0         0         0         0    0.1416
0         0         0    1.0000         0         0         0         0         0   -0.0007
0         0         0         0    1.0000         0         0         0         0    0.3582
0         0         0         0         0    1.0000         0         0         0    0.4961
0         0         0         0         0         0    1.0000         0         0   -0.0235
0         0         0         0         0         0         0    1.0000         0   -0.0204
0         0         0         0         0         0         0         0    1.0000    0.0559
0         0         0         0         0         0         0         0         0    0.0748

TtestB2=Pivoting2(TtestB1)
Row 9 and Col 2 selected.

TtestB2 =

Columns 1 through 11

325.6406         0   10.1718         0    0.1123    0.6389    0.9417   -0.1879    1.2739   -0.5146    0.0056
24.4950         0    1.3357         0   -0.1060    0.0078    0.1008   -0.1711    0.1240   -0.0787   -0.0059
0.1256         0   -0.1019         0    0.0246   -0.0371   -0.0225   -0.0854   -0.0393    0.0217    0.0970
52.0274         0    2.3057         0   -0.2047    0.0367   -0.0176   -0.1412    0.2313   -0.2683   -0.0617
24.2618         0    0.9860         0    0.0309   -0.0065    0.0727   -0.0473   -0.0282   -0.1085   -0.1211
99.3658         0    5.8164         0   -0.3545    0.4252    0.4819   -1.0847    1.0055   -0.2816    0.6717
113.6115         0    6.7157         0   -0.1096    0.5303   -0.0399   -1.0937    0.6570   -0.5377    1.0505
18.1516         0   -0.3775         0    0.0695    0.0482    0.1213    0.0327   -0.0392    0.0267    0.0707
5.3193    1.0000   -0.4860         0    0.0706   -0.0059    0.0662   -0.0846    0.0942   -0.0802    0.1022
9.0336         0   -0.0885         0    0.0587   -0.0633    0.0752    0.0314    0.0109   -0.0234   -0.0513
8.8617         0   -0.0577    1.0000    0.0091    0.1297   -0.0482    0.0465    0.0876    0.0744    0.0015

Columns 12 through 21

0         0         0         0         0         0         0    3.2973         0    0.6769
1.0000         0         0         0         0         0         0    0.1988         0    0.0571
0    1.0000         0         0         0         0         0   -0.9744         0    0.0980
0         0    1.0000         0         0         0         0    0.5857         0    0.1296
0         0         0    1.0000         0         0         0    0.2140         0   -0.0051
0         0         0         0    1.0000         0         0    1.4903         0    0.3277
0         0         0         0         0    1.0000         0    1.3644         0    0.4682
0         0         0         0         0         0    1.0000    0.8335         0   -0.0405
0         0         0         0         0         0         0    0.7386         0   -0.0151
0         0         0         0         0         0         0    0.1354    1.0000    0.0531
0         0         0         0         0         0         0    0.4532         0    0.0656

TtestB3=Pivoting2(TtestB2)
Row 3 and Col 10 selected.

TtestB3 =

Columns 1 through 11

328.6128         0    7.7599         0    0.6942   -0.2391    0.4096   -2.2079    0.3445         0    2.3022
24.9498         0    0.9667         0   -0.0170   -0.1265    0.0194   -0.4802   -0.0182         0    0.3455
5.7760         0   -4.6870         0    1.1308   -1.7062   -1.0340   -3.9255   -1.8062    1.0000    4.4632
53.5769         0    1.0483         0    0.0986   -0.4211   -0.2950   -1.1943   -0.2533         0    1.1356
24.8887         0    0.4774         0    0.1536   -0.1916   -0.0395   -0.4733   -0.2242         0    0.3633
100.9926         0    4.4963         0   -0.0360   -0.0553    0.1906   -2.1903    0.4968         0    1.9287
116.7174         0    4.1954         0    0.4984   -0.3871   -0.5959   -3.2045   -0.3143         0    3.4505
17.9972         0   -0.2521         0    0.0393    0.0938    0.1489    0.1377    0.0091         0   -0.0487
5.7826    1.0000   -0.8620         0    0.1613   -0.1427   -0.0167   -0.3995   -0.0507         0    0.4602
9.1691         0   -0.1984         0    0.0852   -0.1033    0.0510   -0.0606   -0.0314         0    0.0533
```

```
8.4321         0    0.2909    1.0000   -0.0750    0.2566    0.0287    0.3385    0.2219         0   -0.3305

Columns 12 through 21

0   23.6653         0         0         0         0         0  -19.7614         0    2.9955
1.0000    3.6209         0         0         0         0         0   -3.3294         0    0.4119
0   45.9898         0         0         0         0         0  -44.8110         0    4.5059
0   12.3375    1.0000         0         0         0         0  -11.4355         0    1.3384
0    4.9913         0    1.0000         0         0         0   -4.6494         0    0.4840
0   12.9530         0         0    1.0000         0         0  -11.1307         0    1.5968
0   24.7299         0         0         0    1.0000         0  -22.7317         0    2.8912
0   -1.2298         0         0         0         0    1.0000    2.0318         0   -0.1610
0    3.6890         0         0         0         0         0   -2.8559         0    0.3463
0    1.0784         0         0         0         0         0   -0.9154    1.0000    0.1588
0   -3.4204         0         0         0         0         0    3.7859         0   -0.2695

TtestB4=Pivoting2(TtestB3)
Row 11 and Col 19 selected.

TtestB4 =

Columns 1 through 11

372.6256         0    9.2782    5.2197    0.3027    1.1003    0.5594   -0.4412    1.5029         0    0.5773
32.3650         0    1.2225    0.8794   -0.0830    0.0992    0.0446   -0.1826    0.1769         0    0.0549
105.5793         0   -1.2442   11.8361    0.2431    1.3310   -0.6943    0.0806    0.8207    1.0000    0.5516
79.0461         0    1.9269    3.0205   -0.1279    0.3540   -0.2083   -0.1720    0.4171         0    0.1374
35.2438         0    0.8346    1.2281    0.0615    0.1235   -0.0043   -0.0576    0.0483         0   -0.0425
125.7830         0    5.3515    2.9400   -0.2565    0.6991    0.2750   -1.1953    1.1493         0    0.9571
167.3455         0    5.9419    6.0042    0.0482    1.1536   -0.4236   -1.1723    1.0183         0    1.4662
13.4719         0   -0.4082   -0.5367    0.0796   -0.0439    0.1335   -0.0439   -0.1100         0    0.1287
12.1432    1.0000   -0.6426    0.7543    0.1047    0.0508    0.0050   -0.1442    0.1167         0    0.2109
11.2078         0   -0.1281    0.2418    0.0671   -0.0413    0.0579    0.0212    0.0222         0   -0.0266
2.2272         0    0.0768    0.2641   -0.0198    0.0678    0.0076    0.0894    0.0586         0   -0.0873

Columns 12 through 21

0    5.8118         0         0         0         0         0         0         0    1.5886
1.0000    0.6130         0         0         0         0         0         0         0    0.1748
0    5.5052         0         0         0         0         0         0         0    1.3155
0    2.0060    1.0000         0         0         0         0         0         0    0.5242
0    0.7908         0    1.0000         0         0         0         0         0    0.1529
0    2.8969         0         0    1.0000         0         0         0         0    0.8043
0    4.1930         0         0         0    1.0000         0         0         0    1.2727
0    0.6058         0         0         0         0    1.0000         0         0   -0.0163
0    1.1089         0         0         0         0         0         0         0    0.1430
0    0.2514         0         0         0         0         0         0    1.0000    0.0936
0   -0.9035         0         0         0         0         0    1.0000         0   -0.0712

TtestB5=Pivoting2(TtestB4)
Row 11 and Col 8 selected.

TtestB5 =

Columns 1 through 11

383.6175         0    9.6574    6.5233    0.2050    1.4348    0.5968         0    1.7922         0    0.1465
36.9130         0    1.3794    1.4188   -0.1234    0.2376    0.0601         0    0.2966         0   -0.1233
103.5712         0   -1.3135   11.5980    0.2610    1.2699   -0.7012         0    0.7678    1.0000    0.6303
83.3308         0    2.0747    3.5287   -0.1660    0.4844   -0.1937         0    0.5298         0   -0.0306
36.6797         0    0.8841    1.3984    0.0488    0.1672    0.0006         0    0.0861         0   -0.0988
155.5608         0    6.3787    6.4715   -0.5214    1.6053    0.3763         0    1.9330         0   -0.2099
196.5511         0    6.9493    9.4679   -0.2116    2.0424   -0.3242         0    1.7870         0    0.3216
14.5661         0   -0.3705   -0.4069    0.0698   -0.0106    0.1373         0   -0.0812         0    0.0858
15.7353    1.0000   -0.5187    1.1803    0.0728    0.1601    0.0172         0    0.2113         0    0.0701
10.6790         0   -0.1463    0.1791    0.0718   -0.0574    0.0561         0    0.0083         0   -0.0059
24.9131         0    0.8594    2.9546   -0.2216    0.7582    0.0848    1.0000    0.6557         0   -0.9764

Columns 12 through 21

0    1.3530         0         0         0         0         0    4.9353         0    1.2372
1.0000   -1.2319         0         0         0         0         0    2.0420         0    0.0294
```

```
0    6.3197         0         0         0         0         0   -0.9016         0    1.3797
0    0.2680    1.0000         0         0         0         0    1.9238         0    0.3873
0    0.2084         0    1.0000         0         0         0    0.6447         0    0.1070
0   -9.1822         0         0    1.0000         0         0   13.3700         0   -0.1476
0   -7.6541         0         0         0    1.0000         0   13.1131         0    0.3391
0    0.1620         0         0         0         0    1.0000    0.4913         0   -0.0513
0   -0.3482         0         0         0         0         0    1.6128         0    0.0282
0    0.4659         0         0         0         0         0   -0.2374    1.0000    0.1105
0  -10.1058         0         0         0         0         0   11.1858         0   -0.7964

TtestBopt=TtestB5;
```

3.4 From TtestC to TtestCopt

```
TtestC1=Pivoting2(TtestC)
Row 11 and Col 4 selected.

TtestC1 =

Columns 1 through 13

278.6105   -3.4204   12.3417         0   -0.2028    0.6651    0.6459    0.1899    0.8533   -0.1565   -0.4507         0         0
21.1499   -0.2691    1.4665         0   -0.1250    0.0094    0.0830   -0.1484    0.0986   -0.0571   -0.0334    1.0000         0
4.7011    1.3193   -0.7431         0    0.1177   -0.0448    0.0649   -0.1970    0.0850   -0.0841    0.2319         0    1.0000
43.3792   -0.7930    2.6912         0   -0.2607    0.0413   -0.0701   -0.0741    0.1565   -0.2047   -0.1428         0         0
22.7425   -0.2898    1.1269         0    0.0105   -0.0048    0.0535   -0.0227   -0.0555   -0.0853   -0.1507         0         0
77.4277   -2.0178    6.7972         0   -0.4969    0.4371    0.3482   -0.9140    0.8154   -0.1198    0.4654         0         0
88.2650   -1.8474    7.6136         0   -0.2400    0.5412   -0.1623   -0.9373    0.4829   -0.3895    0.8617         0         0
12.8822   -1.1286    0.1711         0   -0.0101    0.0548    0.0465    0.1282   -0.1455    0.1173   -0.0447         0         0
7.8414    1.3540   -0.6581         0    0.0955   -0.0079    0.0897   -0.1146    0.1276   -0.1086    0.1384         0         0
6.3102   -0.1833    0.0006         0    0.0458   -0.0623    0.0631    0.0469   -0.0063   -0.0087   -0.0701         0         0
3.2567   -0.6136    0.2405    1.0000   -0.0342    0.1333   -0.0889    0.0984    0.0298    0.1236   -0.0612         0         0

Columns 14 through 21

0         0         0         0         0         0         0    0.7443
0         0         0         0         0         0         0    0.0612
0         0         0         0         0         0         0    0.0781
1.0000         0         0         0         0         0         0    0.1416
0    1.0000         0         0         0         0         0   -0.0007
0         0    1.0000         0         0         0         0    0.3582
0         0         0    1.0000         0         0         0    0.4961
0         0         0         0    1.0000         0         0   -0.0235
0         0         0         0         0    1.0000         0   -0.0204
0         0         0         0         0         0    1.0000    0.0559
0         0         0         0         0         0         0    0.0748

TtestC2=Pivoting2(TtestC1)
Row 3 and Col 2 selected.

TtestC2 =

Columns 1 through 13

290.7990         0   10.4150         0    0.1024    0.5489    0.8142   -0.3209    1.0737   -0.3745    0.1505         0    2.5927
22.1088         0    1.3149         0   -0.1010    0.0003    0.0962   -0.1885    0.1159   -0.0743    0.0139    1.0000    0.2040
3.5634    1.0000   -0.5633         0    0.0892   -0.0340    0.0492   -0.1493    0.0644   -0.0637    0.1758         0    0.7580
46.2051         0    2.2444         0   -0.1899    0.0144   -0.0311   -0.1926    0.2076   -0.2552   -0.0034         0    0.6011
23.7750         0    0.9637         0    0.0363   -0.0146    0.0678   -0.0660   -0.0368   -0.1038   -0.0998         0    0.2196
84.6180         0    5.6606         0   -0.3169    0.3685    0.4475   -1.2153    0.9454   -0.2484    0.8201         0    1.5295
94.8479         0    6.5730         0   -0.0752    0.4784   -0.0714   -1.2132    0.6020   -0.5073    1.1864         0    1.4003
16.9037         0   -0.4646         0    0.0906    0.0164    0.1021   -0.0403   -0.0728    0.0453    0.1537         0    0.8554
3.0166         0    0.1046         0   -0.0252    0.0381    0.0231    0.0876    0.0403   -0.0223   -0.0996         0   -1.0263
6.9633         0   -0.1026         0    0.0621   -0.0685    0.0721    0.0196    0.0055   -0.0204   -0.0378         0    0.1389
5.4433         0   -0.1051    1.0000    0.0205    0.1125   -0.0587    0.0068    0.0693    0.0845    0.0466         0    0.4651

Columns 14 through 21

0         0         0         0         0         0         0    0.9467
```

```
0         0         0         0         0         0         0    0.0771
0         0         0         0         0         0         0    0.0592
1.0000         0         0         0         0         0         0    0.1885
0    1.0000         0         0         0         0         0    0.0164
0         0    1.0000         0         0         0         0    0.4776
0         0         0    1.0000         0         0         0    0.6054
0         0         0         0    1.0000         0         0    0.0433
0         0         0         0         0    1.0000         0   -0.1006
0         0         0         0         0         0    1.0000    0.0667
0         0         0         0         0         0         0    0.1111

TtestC3=Pivoting2(TtestC2)
Row 11 and Col 10 selected.

TtestC3 =

Columns 1 through 13

314.9245         0    9.9491    4.4321    0.1934    1.0473    0.5542   -0.2907    1.3810         0    0.3571         0    4.6542
26.8957         0    1.2225    0.8794   -0.0830    0.0992    0.0446   -0.1826    0.1769         0    0.0549    1.0000    0.6130
7.6695    1.0000   -0.6426    0.7543    0.1047    0.0508    0.0050   -0.1442    0.1167         0    0.2109         0    1.1089
62.6467         0    1.9269    3.0205   -0.1279    0.3540   -0.2083   -0.1720    0.4171         0    0.1374         0    2.0060
30.4598         0    0.8346    1.2281    0.0615    0.1235   -0.0043   -0.0576    0.0483         0   -0.0425         0    0.7908
100.6214         0    5.3515    2.9400   -0.2565    0.6991    0.2750   -1.1953    1.1493         0    0.9571         0    2.8969
127.5307         0    5.9419    6.0042    0.0482    1.1536   -0.4236   -1.1723    1.0183         0    1.4662         0    4.1930
13.9824         0   -0.4082   -0.5367    0.0796   -0.0439    0.1335   -0.0439   -0.1100         0    0.1287         0    0.6058
4.4544         0    0.0768    0.2641   -0.0198    0.0678    0.0076    0.0894    0.0586         0   -0.0873         0   -0.9035
8.2794         0   -0.1281    0.2418    0.0671   -0.0413    0.0579    0.0212    0.0222         0   -0.0266         0    0.2514
64.4278         0   -1.2442   11.8361    0.2431    1.3310   -0.6943    0.0806    0.8207    1.0000    0.5516         0    5.5052

Columns 14 through 21

0         0         0         0         0         0         0    1.4392
0         0         0         0         0         0         0    0.1748
0         0         0         0         0         0         0    0.1430
1.0000         0         0         0         0         0         0    0.5242
0    1.0000         0         0         0         0         0    0.1529
0         0    1.0000         0         0         0         0    0.8043
0         0         0    1.0000         0         0         0    1.2727
0         0         0         0    1.0000         0         0   -0.0163
0         0         0         0         0    1.0000         0   -0.0712
0         0         0         0         0         0    1.0000    0.0936
0         0         0         0         0         0         0    1.3155

TtestC4=Pivoting2(TtestC3)
Row 9 and Col 8 selected.

TtestC4 =

Columns 1 through 13

329.4081         0   10.1989    5.2910    0.1290    1.2676    0.5789         0    1.5716         0    0.0732         0    1.7166
35.9918         0    1.3794    1.4188   -0.1234    0.2376    0.0601         0    0.2966         0   -0.1233    1.0000   -1.2319
14.8536    1.0000   -0.5187    1.1803    0.0728    0.1601    0.0172         0    0.2113         0    0.0701         0   -0.3482
71.2160         0    2.0747    3.5287   -0.1660    0.4844   -0.1937         0    0.5298         0   -0.0306         0    0.2680
33.3316         0    0.8841    1.3984    0.0488    0.1672    0.0006         0    0.0861         0   -0.0988         0    0.2084
160.1769         0    6.3787    6.4715   -0.5214    1.6053    0.3763         0    1.9330         0   -0.2099         0   -9.1822
185.9420         0    6.9493    9.4679   -0.2116    2.0424   -0.3242         0    1.7870         0    0.3216         0   -7.6541
16.1709         0   -0.3705   -0.4069    0.0698   -0.0106    0.1373         0   -0.0812         0    0.0858         0    0.1620
49.8261         0    0.8594    2.9546   -0.2216    0.7582    0.0848    1.0000    0.6557         0   -0.9764         0  -10.1058
7.2219         0   -0.1463    0.1791    0.0718   -0.0574    0.0561         0    0.0083         0   -0.0059         0    0.4659
60.4117         0   -1.3135   11.5980    0.2610    1.2699   -0.7012         0    0.7678    1.0000    0.6303         0    6.3197

Columns 14 through 21

0         0         0         0         0    3.2515         0    1.2078
0         0         0         0         0    2.0420         0    0.0294
0         0         0         0         0    1.6128         0    0.0282
1.0000         0         0         0         0    1.9238         0    0.3873
0    1.0000         0         0         0    0.6447         0    0.1070
0         0    1.0000         0         0   13.3700         0   -0.1476
0         0         0    1.0000         0   13.1131         0    0.3391
```

```
0         0         0         0    1.0000    0.4913         0   -0.0513
0         0         0         0         0   11.1858         0   -0.7964
0         0         0         0         0   -0.2374    1.0000    0.1105
0         0         0         0         0   -0.9016         0    1.3797

TtestCopt=TtestC4;
```

4 Some Matlab Files

In this section, I will demonstrate all the m-files written by myself

4.1 Pivoting.m

```
function Tout=Pivoting(T)

% Dual Simplex Method

[~,pos]=min(T(2:end,1));pos=pos+1;

r=pos;
negcols=find(T(r,2:end)<0);
negcols=negcols+1;
colrat=T(1,negcols)./T(r,negcols);
[~,pos]=max(colrat);
c=negcols(pos);

Tout=Pivot(T,r,c);
```

4.2 Pivoting2.m

```
function Tout=Pivoting2(T)

% Simplex Method

[~,pos]=min(T(1,2:end));pos=pos+1;

c=pos;
posrows=find(T(2:end,c)>0);
```

```
posrows=posrows+1;
colrat=T(posrows,1)./T(posrows,c);
[~,pos]=min(colrat);
r=posrows(pos);

Tout=Pivot(T,r,c);
```

4.3 Finda.m

```
b=TS(2:end,1);
s=TS(2:end,lnbslackcol);
pos=find(s>0);
neg=find(s<0);
posratsB=-b(pos)./s(pos);
negratsB=-b(neg)./s(neg);
minposval=max(posratsB);
maxnegval=min(negratsB);
```

4.4 Findq.m

```
%c=TS(1,2:end);
%s=TS(rowC,2:end);
%posC=find(s>0);
%negC=find(s<0);

% Delete one element from posC manually

posratsC=-c(posC)./s(posC);
negratsC=-c(negC)./s(negC);
minnegval=max(posratsC);
maxposval=min(negratsC);
```