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Math 31 Related Rates Quiz

1. The displacement of a particle, in *cm*, is described by the function $S = -\frac{2}{3}t^3 + 10t^2 - 48t - 5$.

a.) What is the average velocity from $t = 3$ s to $t = 6$ s?

$S = \frac{-2}{3}(3)^3 + 10(3)^2 - 48(3) - 5 = -77$

$S = \frac{-2}{3}(6)^3 + 10(6)^2 - 48(6) - 5 = -77$

$V_{ave} = \frac{-77 - (-77)}{6 - 3} = \frac{0}{3} = 0\text{cm/s}$

b.) What is the velocity of the particle at any time, t?

$S' = -2t^2 + 20t - 48 = V$

c.) What is the velocity at time $t = 2.5$ s?

$V = -2(2.5)^2 + 20(2.5) - 48$

$V = -10.5\text{cm/s}$

d.) When the velocity is *zero*?

$0 = -2t^2 + 20t - 48$

$0 = -2(t^2 - 10t + 24)$

$0 = -2(t-4)(t-6)$

Velocity is 0 at t = 4,6 seconds

e.) What is the acceleration at any time, t?

$V' = a = -4t + 20$

f.) What is the acceleration at time $t = 4.5$ s?

$a = -4(4.5) + 20$

$a = 2\text{cm/s}^2$

g.) When the acceleration is *zero*?

$0 = -4t + 20$

$t = 5\text{s}$

Acceleration is 0 at t = 5s

2. A spherical balloon is being inflated so that the volume is increasing at a rate of 7 m^3/min.

a.) How fast is the **radius** of the balloon increasing when the diameter is 5 m?

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$7 = 4\pi(2.5)^2 \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{7}{25\pi} \text{ m/min}}$

b.) How fast is the **surface area** increasing at this same instant?

$SA = 4\pi r^2$

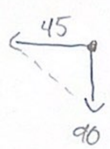
$\frac{dSA}{dt} = 8\pi r \frac{dr}{dt}$

$\frac{dSA}{dt} = 8\pi(2.5)\left(\frac{7}{25\pi}\right)$

$\frac{dSA}{dt} = \frac{140}{25}$

$\boxed{\frac{dSA}{dt} = \frac{28}{5} \text{ m}^2\text{/min}}$

3. Two cars start at the same point. One travels south at 90 km/h and the other travels west at 45 km/h. At what rate is the distance between them increasing two hours later? State your answer as a reduced radical or to the nearest tenth.



$c^2 = a^2 + b^2$

$\cancel{2}c \frac{dc}{dt} = \cancel{2}a \frac{da}{dt} + \cancel{2}b \frac{db}{dt}$

$(90\sqrt{5})\left(\frac{dc}{dt}\right) = (180)(90) + (90)(45)$

$90 \cdot 2 = 180$

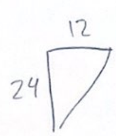
$45 \cdot 2 = 90$

$c = \sqrt{180^2 + 90^2}$

$c = \sqrt{40500} = 90\sqrt{5}$

$\boxed{\frac{dc}{dt} = \frac{225}{\sqrt{5}} \text{ km/h}}$

4. A conical glass vase has a height of 24 cm and a radius of 12 cm at the top. If the vase is being filled at a rate of 17 cm^3/s, find the rate at which the water level is rising when the ~~diameter~~ radius of the water is ~~20 cm.~~ 10cm



$\frac{r}{h} = \frac{12}{24} = \frac{1}{2}$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$= \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$17 = \frac{1}{4}\pi(20)^2 \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{17}{100\pi} \text{ cm/s}}$

5. A landscape company is pouring rock chips into a conical pile with a constant ratio of 2:5 between the radius and height. The volume of the rock chips is increasing at a rate of 1.8 m^3/min. At what rate is the height increasing when the radius is 3 m? Give your answer in exact value and rounded to the nearest hundredth.

$$\frac{r}{h} = \frac{2}{5}$$

$$r = \frac{2h}{5}$$

$$h = \frac{5r}{2}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \left(\frac{2h}{5}\right)^2 h$$

$$= \frac{1}{3}\pi \left(\frac{4h^2}{25}\right) h$$

$$= \frac{4}{75}\pi h^3$$

$$\frac{dV}{dt} = \frac{4}{25}\pi h^2 \frac{dh}{dt}$$

$$1.8 = \frac{4}{25}\pi \left(\frac{15}{2}\right)^2 \frac{dh}{dt}$$

$$1.8 = 9\pi \frac{dh}{dt}$$

$$\boxed{\frac{1}{5\pi} \text{ m/min} = \frac{dh}{dt} \quad (0.06) \text{ m/min}}$$

6. A rectangular swimming pool (with a horizontal bottom) is being drained. If its length and width are 25 m and 20 m and the water level is falling at the rate of 0.5 m/min, how fast is the water draining out of the pool?

$$V = lwh$$

$$V = 25(20)h$$

$$V = 500h$$

$$\frac{dV}{dt} = 500\frac{dh}{dt}$$

$$\frac{dV}{dt} = 500(-0.5)$$

$$\boxed{\frac{dV}{2t} = -250 \text{ m}^3/\text{min}}$$