Connor Chan

$42\frac{1}{2}$ / 43

Math 31 Limits Quiz

1. Use the graph below to determine the following limits.

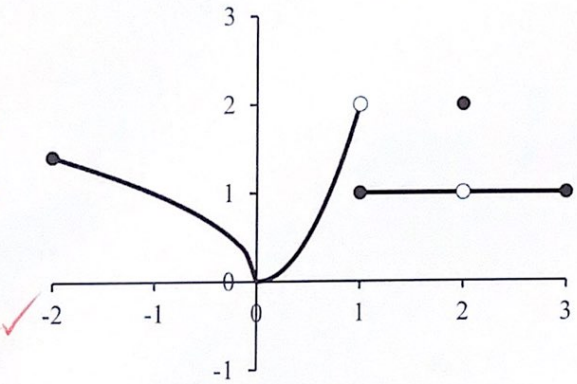
a. $\lim_{x\to -1} f(x) =$ 1
e. $\lim_{x\to 2} f(x) =$ 1

b. $\lim_{x\to 1^-} f(x) =$ 2
f. $\lim_{x\to 1^+} f(x) =$ 1

c. $\lim_{x\to 1} f(x) =$ DNE
g. $\lim_{x\to 0^+} f(x) =$ 0

d. $\lim_{x\to 0^-} f(x) =$ 0
h. $\lim_{x\to 0} f(x) =$ 0

i. $f(0) =$ 0
j. $f(2) =$ 2



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2. Find the following limits.

a. $\lim_{x\to 2} 2x$

$= 2(2)$

$\boxed{=4}$

b. $\lim_{x\to \frac{1}{3}} (3x-1)$

$= 3\left(\frac{1}{3}\right) - 1$

$\boxed{=0}$

c. $\lim_{x\to 0} (x^2 - 3x - 18)$

$= 0^2 - 3(0) - 18$

$\boxed{= -18}$

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d. $\lim_{y\to -2} \frac{y^2+5y+6}{y+2}$

$\frac{2^2 + 5(2) + 6}{2+2}$

$\boxed{=5}$

e. $\lim_{x\to -2} \frac{x^2+x-2}{x^2-4}$

$= \lim_{x\to -2} \frac{(x+2)(x-1)}{(x+2)(x-2)}$

$= \lim_{x\to -2} \frac{(x-1)}{x-2}$

$= \frac{(-2-1)}{(-2-2)} = \boxed{\frac{3}{4}}$

f. $\lim_{x\to 1} \frac{2x^2+x-3}{x-1}$

$2x^2 - 2x + 3x - 3$
$2x(x-1) + 3(x-1)$

$= \lim_{x\to 1} \frac{(2x+3)\cancel{(x-1)}}{\cancel{x-1}}$

$= \lim_{x\to 1} 2x+3$

$= 2(1)+3$

$\boxed{=5}$

✓

g. $\lim_{x\to 14} \frac{4-\sqrt{x+2}}{x-14} \frac{(4+\sqrt{x+2})}{(4+\sqrt{x+2})}$

$= \frac{16-(x+2)}{x-14(4+\sqrt{x+2})}$

$= \frac{-(x+14)}{x-14(4+\sqrt{x+2})}$

$= \frac{-1}{4+\sqrt{x+2}}$

$= \frac{-1}{4+\sqrt{14+2}} = \boxed{\frac{-1}{8}}$

h. $\lim_{x\to 8} \frac{x-8}{|x-8|}$

$\lim_{x\to 8^+} \frac{x-8}{x-8} = 1$

$\lim_{x\to 8^-} \frac{x-8}{-(x-8)} = -1$

$\therefore$

$\boxed{\lim_{x\to 8} = \text{DNE}}$

i. $\lim_{x\to \infty} \frac{3x^3 - x + 1}{x^4 - 5x^3 + 8}$

$\boxed{=0}$

ok

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/25

j. $\lim_{x\to\infty}\left(5-\frac{2}{x^2}\right)$ $\quad 5-\frac{2}{\infty^2}$ $\quad 5-0$

$\boxed{=5}$

k. $\lim_{x\to 0}\frac{\left(\frac{1}{(2+x)}-\frac{1}{2}\right)}{x}$

$=\lim_{x\to 0}\frac{\frac{2-(2+x)}{2(2+x)}}{\frac{x}{1}}$

$=\lim_{x\to 0}\frac{2-2-\cancel{x}}{2(2+x)}\cdot\frac{1}{\cancel{x}}$

$=\lim_{x\to 0}\frac{-1}{2(2+x)}=\boxed{\frac{-1}{4}}$

l. $\lim_{x\to\infty}\frac{3x^3-5x}{x^3-2x+1}$

$\boxed{=3}$

✓

3. Let

$$f(x)=\begin{cases}-\frac{1}{2}x & \text{if } x\le -2\\ -1 & \text{if } -2<x<2\\ 3-x^2 & \text{if } x\ge 2\end{cases}$$

a. Find the following limits.

i. $\lim_{x\to -2^-} f(x)$

$=\frac{-1}{2}(-2)$

$\boxed{=1}$

ii. $\lim_{x\to -2^+} f(x)$

$=\boxed{-1}$

iii. $\lim_{x\to 2^-} f(x)$

$=\boxed{-1}$

iv. $\lim_{x\to 2^+} f(x)$

$=3-2^2$

$=\boxed{-1}$

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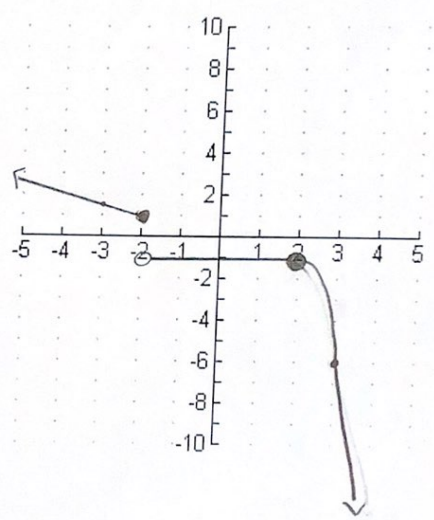
b. Sketch the graph of $f(x)$.



c. Where is $f(x)$ discontinuous?

$f(x)$ is discontinuous at $x=-2$

11

4. If $f(x) = \begin{cases} x^2 - k & x < 3 \\ 2kx & x \geq 3 \end{cases}$, find a value for k so that the function is continuous everywhere.

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$$

$$3^2 - k = 2k(3)$$

$$9 - k = 6k$$

$$9 = 7k$$

$$\boxed{k = \frac{9}{7}}$$

✓

5. Assume $\lim_{x \to b} f(x) = 7$ and $\lim_{x \to b} g(x) = -2$, find

a. $\lim_{x \to b} \left[g(x) + 2f(x) \right]$

$$= \lim_{x \to b} \left[-2 + 2(7) \right]$$

$$\boxed{= 12}$$

✓

b. $\lim_{x \to b} \frac{\sqrt{-5g(x) + \left[f(x) \right]^2}}{x}$

$$= \lim_{x \to b} \frac{\sqrt{-5(-2) + 7^2}}{x}$$

$$\boxed{= \frac{\sqrt{59}}{x}}$$

$-1/2$

$6 1/2$