Assignment-3

If $\mu = 55$, $\sigma_{4a} = 4$, $\sigma_{4\beta} = 10$, $\sigma_4 = c$ 15, In this which is better

1. Introduction

In statistics, the **normal distribution** is one of the most important probability distributions used to describe real-world data. It represents how values are spread around a central value called the **mean (μ)**. The shape of a normal distribution is determined not only by the mean but also by another important parameter called the **standard deviation (σ)**.

Standard deviation measures how widely data values are spread around the mean. If the standard deviation is small, data values are closely clustered around the mean. If the standard deviation is large, values are more spread out. Therefore, even if multiple datasets have the same mean, they can have different shapes depending on their standard deviation.

In this assignment, we analyze three normal distributions that all have the same mean but different standard deviations:

- $\mu = 55$
- $\sigma_a = 4$
- $\sigma_\beta = 10$
- $\sigma c = 15$

The objective is to compare these distributions and determine which one is "better" depending on statistical interpretation and practical applications.

2. Understanding Mean and Standard Deviation

Mean (μ)

The mean is the average value of a dataset. It represents the center point of the distribution. In this case, all three distributions have the same mean of 55. This means all three curves will be centered at 55 on the horizontal axis.

Standard Deviation (σ)

Standard deviation shows how far data values deviate from the mean.

- Smaller σ → narrow curve → values close to mean
- Larger σ → wider curve → values more spread out

Standard deviation controls the **width and height** of the bell curve:

- Small σ → tall and narrow curve

- Large σ → short and wide curve

3. Description of the Three Distributions

Distribution A ($\sigma_a = 4$)

This distribution has the smallest standard deviation.

Characteristics:

- Data tightly clustered around mean
- Less variability
- More consistent values
- Narrow bell curve
- Higher peak

This distribution indicates that most values lie very close to 55, meaning there is very little variation in data.

Distribution B ($\sigma_\beta = 10$)

This distribution has a moderate standard deviation.

Characteristics:

- Moderate spread
- Balanced variability
- Moderate peak height
- Typical real-world distribution

This distribution represents a balanced case where values vary but not excessively.

Distribution C ($\sigma c = 15$)

This distribution has the largest standard deviation.

Characteristics:

- Data widely spread
- High variability
- Flatter curve
- Lower peak

This distribution indicates that values are scattered over a wide range and not concentrated near the mean.

4. Graphical Interpretation (Conceptual)

If all three distributions are plotted on the same graph:

- All curves would be centered at **55**
- $\sigma_a = 4 \rightarrow$ narrowest curve
- $\sigma_\beta = 10 \rightarrow$ medium width curve
- $\sigma c = 15 \rightarrow$ widest curve

Thus:

- Smaller $\sigma \rightarrow$ steeper curve
- Larger $\sigma \rightarrow$ flatter curve

5. Empirical Rule Application

The **Empirical Rule (68–95–99.7)** helps understand how data spreads around the mean.

For $\sigma_a = 4$

- 68% → 51 to 59
- 95% → 47 to 63
- 99.7% → 43 to 67

Values stay very close to mean.

For $\sigma_\beta = 10$

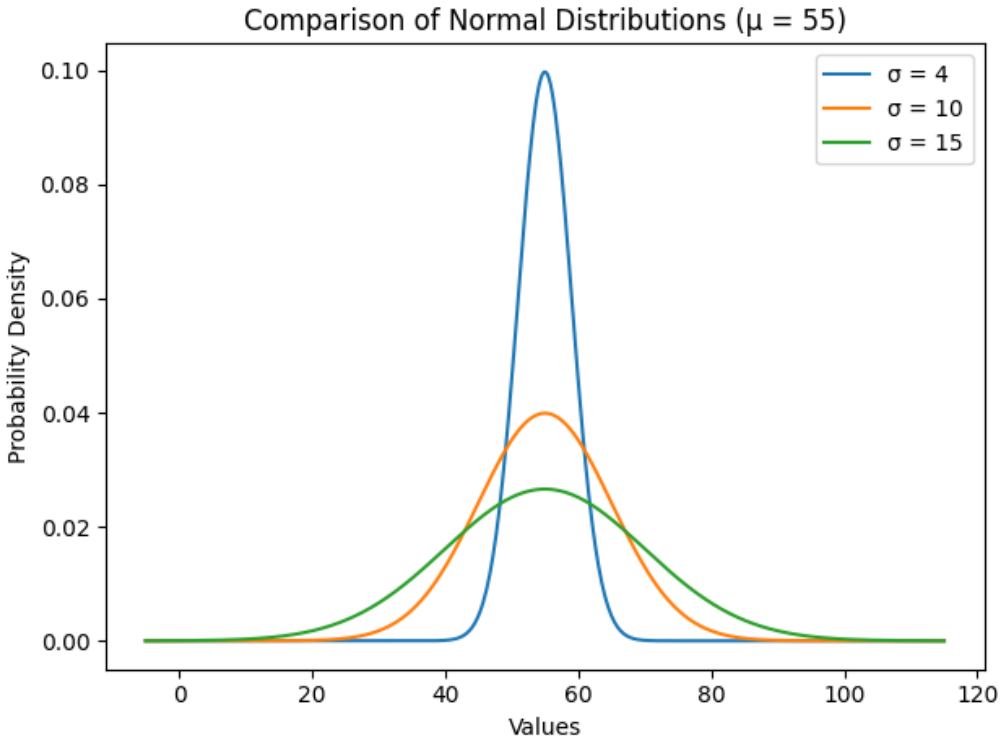
- 68% → 45 to 65
- 95% → 35 to 75
- 99.7% → 25 to 85

Moderate spread.

For $\sigma = 15$

- 68% → 40 to 70
- 95% → 25 to 85
- 99.7% → 10 to 100

Very wide spread.



6. Comparison Table

Distribution	Standard Deviation	Spread	Consistency	Shape
A	4	Very small	Very high	Narrow
B	10	Moderate	Balanced	Medium
C	15	Very large	Low	Wide

7. Which Distribution is Better?

The answer depends on what we mean by **"better."** In statistics, "better" does not always mean smaller or larger. It depends on the context.

Case 1 — If consistency is preferred

Distribution **A (σ = 4)** is best because values are close to the mean and variation is minimal.

Example: manufacturing precision parts.

Case 2 — If natural variation is expected

Distribution **B (σ = 10)** is best because it reflects realistic variability.

Example: exam marks or human heights.

Case 3 — If wide diversity is acceptable

Distribution **C (σ = 15)** is best because it allows large variation.

Example: income distribution or rainfall data.

8. Practical Interpretation

Each standard deviation serves a different purpose:

- Small σ → precise systems
- Medium σ → balanced systems
- Large σ → flexible systems

Therefore, no single distribution is universally best. The best choice depends on the type of data and purpose of analysis.

9. Real-Life Examples

Scenario	Best σ
Machine manufacturing	4
Exam scores	10
Population income	15

10. Importance of Standard Deviation in Data Analysis

Standard deviation is useful because it helps:

- Measure variability
- Compare datasets
- Detect outliers
- Evaluate reliability
- Assess consistency
- Analyze risk

Without standard deviation, we cannot understand how data is distributed.

11. Key Insight

Mean tells **where data is centered**
Standard deviation tells **how data is spread**

Both are equally important in statistics.

12. Conclusion

In this analysis, three normal distributions with the same mean ($\mu = 55$) but different standard deviations (4, 10, and 15) were compared. Although all distributions share the same center, their shapes differ significantly because standard deviation controls how widely data is spread.

Distribution A with $\sigma = 4$ is narrow and indicates high consistency, making it suitable for systems that require precision. Distribution B with $\sigma = 10$ represents moderate variation and is considered the most balanced and realistic for many natural datasets. Distribution C with $\sigma = 15$ shows wide variability and is appropriate for data that naturally varies over a large range.

Therefore, the question of which distribution is "better" does not have a single universal answer. The best distribution depends on the context and purpose of analysis. However, in most real-world scenarios, a **moderate standard deviation** is preferred because it reflects realistic variability without being too rigid or too scattered. Understanding how standard deviation affects distribution shape is essential for interpreting data accurately and making informed decisions in statistics, science, engineering, and real-world applications.