Lab 2A: Discrete-Time Signals in the Frequency-Domain *

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September 14, 2015

1 Lab Objective

The objective of this lab is to study the frequency-representation of discrete-time signals (sequences) and to compute their magnitude and phase spectra using MATLAB.

2 Introduction

2.1 Introduction to Discrete-time Fourier Transform

The discrete-time Fourier transform (DTFT) $X(e^{j\omega})$ of a sequence $x[n]$ is defined by

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \tag{1}$$

Note that $X(e^{j\omega})$ is continuous function of ω and must be evaluated for all ω. This is computationally intractable and thus must be evaluated at prescribed set of frequencies $\omega = \omega_k$. This is called frequency griding.

If $x[n]$ has finite length say, $N_1 \leq n \leq N_2$, the DTFT can be computed at a prescribed set of frequencies $\omega = \omega_k$ by adjusting (1) as follows:

$$X(e^{j\omega_k}) = \sum_{n=N_1}^{N_2} x[n]e^{-j\omega_k n} \tag{2}$$

The Matlab function

```
[H,W] = freqz(x,1,L)
```

provides a way to compute (2) with $N_1 = 0$, x is a vector of the samples of $x[n]$ and L is prescribed set of equally spaced frequency points. H is the frequency response corresponding to the frequency points W in rad/s. For

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faster computation, L should be chosen as a power of 2, such as 256 or 512. Matlab default is 512.

For any arbitrary N_1 and N_2, Equation (2) can be computed after a change of variable $m = n - N_1$ to get

$$X(e^{j\omega_k}) = \sum_{n=N_1}^{N_2} x[n]e^{-j\omega_k n} = \sum_{m=0}^{N_2-N_1} x[m+N_1]e^{-j\omega_k(m+N_1)} \tag{3}$$

$$= e^{-j\omega_k N_1} \sum_{m=0}^{N_2-N_1} x[m+N_1]e^{-j\omega_k m} \tag{4}$$

This can be achieved by using the following Matlab lines of code:

```
X = freqz(x,1,L);
X=exp(-j*L*N1).*X;
```

To explore any of matlab command you will come across in this and subsequent labs, type "help" followed by the "command name" at the command prompt.

To introduce the concepts, the programs below illustrate how Matlab can be employed to compute the DTFT of a discrete-time sequence.

Example 1. *Evaluate the DTFT of the exponential sequence*

$$x[n] = \begin{cases} \alpha^n; & -5 \le n \le 10 \\ 0; & otherwise \end{cases} \tag{5}$$

for $\alpha = 0.5$

```
% Program for evaluating the DTFT of an exponential sequence
%
% Generate the exponential sequence
n=-5:10;
a=0.5;
u=[ones(1,16)];
x=a.^n.*u;
% Ploting the exponential sequence
figure(1);
stem(n,x);
ylabel('Amplitude');
xlabel('Time index n');
title('Exponential Sequence');
%
%Computing the DTFT of the exponential sequence using "freqz"
w=linspace(-pi,pi,512);
h = freqz([x], 1, w);
%Plotting the frequency response
figure(2);
subplot(2,2,1)
plot(w/pi,real(h));grid
title('Real part')
```

```
xlabel('\omega/\pi'); ylabel('Amplitude')
subplot(2,2,2)
plot(w/pi,imag(h));grid
title('Imaginary part')
xlabel('\omega/\pi'); ylabel('Amplitude')
subplot(2,2,3)
plot(w/pi,abs(h));grid
title('Magnitude Spectrum')
xlabel('\omega/\pi'); ylabel('Magnitude')
subplot(2,2,4)
plot(w/pi,angle(h));grid
title('Phase Spectrum')
xlabel('\omega/\pi'); ylabel('Phase, radians')
%
%Computing the DTFT of the exponential sequence accounting for noncausality
g = exp(-j*w*n(1)).*h;
%Plotting the frequency response
figure(3);
subplot(2,2,1)
plot(w/pi,real(g));grid
title('Real part')
xlabel('\omega/\pi'); ylabel('Amplitude')
subplot(2,2,2)
plot(w/pi,imag(g));grid
title('Imaginary part')
xlabel('\omega/\pi'); ylabel('Amplitude')
subplot(2,2,3)
plot(w/pi,abs(g));grid
title('Magnitude Spectrum')
xlabel('\omega/\pi'); ylabel('Magnitude')
subplot(2,2,4)
plot(w/pi,angle(g));grid
title('Phase Spectrum')
xlabel('\omega/\pi'); ylabel('Phase, radians')
```

3 Lab

In the Lab, you will use MATLAB to compute the frequency response of sequences by modifying the example programs above. You will also display the result using MATLAB plotting capabilities.

1. Run and carefully study the MATLAB programs 1 to familiarize yourself with DTFT computation of discrete-time signals.

2. Modify program 1 to such that the prescribed frequency steps is from -3π through 3π using step size of 256. Compute the real, the imaginary, the magnitude and phase spectra of the DTFT. Is the DTFT a periodic function of ω? If it is, what is the period. Explain the type of symmetries exhibited by the four plots.

3. Assuming the sequence is delayed such that $y[n] = x[n - D]$ where D is the delay. Modify program 1 to generate magnitude and phase spectra of $y[n]$ for $D = 5$.

4. Develop a MATLAB function

```
[rxy,l]=ccrs(x,y, nx, ny)
```

that computes correlation r_{xy} between two finite length signals x and y defined over nx and ny intervals, respectively. Verify your function using the signals $x[n] = [1, 2, \underset{\uparrow}{3}, 2, 1]$ and $y[n] = [2, 1, \underset{\uparrow}{0}, -1, -2]$. Comment on the degree of similarity between the two signals. Recall that the autocorrelation sequence of a sequence $x[n]$ is defined as

$$r_{xx}[l] = \sum_{n=-\infty}^{\infty} x[n]x[n-l]; -\infty < l < \infty \tag{6}$$

The autocorrellation $r_{xx}[l]$ can be computed by exploring the relationship between correlation and convolution as

$$x[l] * x[-l] = \sum_{n=-\infty}^{\infty} x[n]x[n-l] = r_{xx}[l] \tag{7}$$

5. Modify your program in item 4 to compute the auto spectral density function $R(\omega)$ of the sequence $x[n]$. Discuss the periodicity and symmetry of $R(\omega)$. Test your code using the signal in example 1. Recall that taking the DTFT of (7) yields the following relation

$$r_{xx}[l] = x[l] * x[-l] \Longleftrightarrow R_x(\omega) = X(e^{j\omega})X(e^{j\omega}) = |X(e^{j\omega})|^2. \tag{8}$$

6. Modify your program in item 5 to compute the energy spectral density spectrum of the sequence $x[n]$. Discuss its periodicity and symmetry. Recall that for a finite energy sequence $x[n]$, the following relation known as the Parseval's relation holds

$$\mathcal{E}_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega. \tag{9}$$

Considering a small band $\Delta\omega$ centered at ω_o, the energy of the signal within the frequency band $\Delta\omega$ is given by

$$\frac{|X(e^{j\omega})|^2}{2\pi}\Delta\omega. \tag{10}$$

The function $\frac{|X(e^{j\omega})|^2}{2\pi}$ is known as the energy density spectrum of $x[n]$.

3.1 Post-Lab

Discuss your lab results and submit a detailed report of your findings.
Instructions:
Your report must include all MATLAB codes and figures. Use subplot to give 3 or 4 plots per page; label the axes of your plots accordingly e.g Time index, n on the x-axis and x[n] on the y-axis; the title should be the problem number, for example (2a). Your report and MathScripts must be uploaded to canvas on or before the due date.
Submission Due Date: October 8, 2015.