

**Solutions**

**Note: Do not refer to the processor configuration in the case study notes for this tutorial. A smaller system will be used instead.**

**8.1 Cache**

1. Given a processor system with the following characteristics

- Processor has a direct-mapped cache with 32 cache blocks and a cache size of 512 bytes.
- Cache Memory Access time = 5ns.
- Cache Hit rate = 0.9
- 64Kbyte DRAM used as the main memory.
- DRAM Memory access time = 200ns

a. In doing cache mapping analysis, how many **blocks** would the main memory be partitioned to?

[Suggested Solution]

Cache block size =  $512/32 = 16$  Bytes

Number of main memory blocks =  $64\text{KByte}/16\text{Byte} = (2^{16})/(2^4) = 2^{12} = 4096$

b. What is the format of a memory address as seen by the cache (i.e. determine the sizes of the tag, block and offset fields)?

[Suggested Solution]

Cache Block Size = 16  $\Rightarrow$  Offset Field = 4 bits

Number of blocks in the cache = 32  $\Rightarrow$  Block Field = 5 bits

Main Memory Size = 64KByte  $\Rightarrow$  16 address bits

Tag Field bits = Main Memory Address bits – Offset – Block = 7

$\Rightarrow$  TAG:BLOCK:OFFSET = 7:5:4

c. CPU needs to read a byte from main memory address 0x0DB63.

- i. Which cache block would CPU look at to search for the required data?
- ii. How many main memory blocks could potentially be mapped to the same cache block as that of 0x0DB63?
- iii. How does the CPU know if the cache block identified in (i) above contains the data that it needs?
- iv. What is the purpose of the 'offset' field in the cache mapping?

**Solutions****[Suggested Solution]**

- i.  $0x0DB63 = 0000\ 1101\ 1011\ 0110\ 0011$   
 $\Rightarrow$  Block 10110b = Block 22
- ii. Number of MM blocks = 4096  
 Number of cache blocks = 32  
 $\Rightarrow$  each cache block is a potential destination to  $4096/32 = 128$  MM blocks.
- iii. By looking at the TAG value entry of the corresponding cache block  
 For 0x0DB63, the TAG value for the cache block should be equal to **00001101 101b**
- iv. Offset refers to the offset of the byte of interest from the cache block boundary. For 0x0DB63, the byte is the byte 3 of block 22. First byte of the block is byte 0.

d. What is the effective access time of the memory in this system?

**[Suggested Solution]**

Assume cache memory and main memory access do not overlap.

$$EAT = H * \text{Cache}_{\text{Access}} + (1-H) * (\text{Cache}_{\text{Access}} + \text{Mem}_{\text{Access}}) = 0.9 * 5\text{ns} + (1-0.9) * (5\text{ns} + 200\text{ns}) = 25\text{ns}.$$

## 8.2 Virtual Memory

2. In a processor system with the following characteristics,
- 1 MByte Virtual memory space
  - 64 Kbyte DRAM as main memory
  - Paging scheme used for virtual memory management, Page Table as shown in Table 8.2
  - Virtual Page size = 1 KByte
  - TLB with 4 entries

**Table 8.2 – Page Table**

| Virtual Page Number | Valid Bit | Page Frame Number |
|---------------------|-----------|-------------------|
| 0                   | 1         | 1                 |
| 1                   | 1         | 2                 |
| 2                   | 0         | -                 |
| 3                   | 1         | 16                |
| 4                   | 1         | 9                 |

- a. How many bits are required for each virtual address?

[Suggested Solution]

$$1\text{Mbyte} = 2^{20} \text{ byte} \Rightarrow 20 \text{ bits}$$

- b. How many bits are required for each physical address?

[Suggested Solution]

$$64 \text{ KByte} = 2^6 * 2^{10} = 2^{16} \Rightarrow 16 \text{ bits}$$

- c. What is the maximum number of entries in the page table in Table 7.2?

[Suggested Solution]

$$\text{Number of virtual pages} = 2^{20} / 2^{10} = 2^{10} = 1024$$

$$\Rightarrow \text{Max number of entries} = 1024$$

- d. What is the maximum number of valid entries in the page table in Table 7.2?

[Suggested Solution]

$$\text{Number of Physical Frames} = 2^{16} / 2^{10} = 64$$

$$\Rightarrow \text{Max number of valid entries} = 64$$

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e. With reference to Table 7.2, answer the following. Indicate when a page fault occurs.

- (i) The compiler mapped the UART routine to virtual address 0x005F0 – 0x006FF, where in the DRAM would you be able to find the UART routine?

[Suggested Solution]

0x005F0 → virtual page number 1 → mapped to page frame number 2

0x006FF → virtual page number 1 → mapped to page frame number 2

| Virtual Address (20 bits) | Physical Address (16 bits) |
|---------------------------|----------------------------|
| 0000 0000 0101 1111 0000  | 0000 1001 1111 0000        |
| 0000 0000 0110 1111 FFFF  | 0000 1010 1111 FFFF        |

Physical Address = 0x09F0 – 0x0AFF

- (ii) The compiler mapped the I2C routine to virtual address 0x009C0 - 0x009DF, where in the DRAM would you be able to find the I2C routine?

[Suggested solution]

0x009C0 → virtual page number 2 → page fault occurs (valid bit =0).

0x009DF → virtual page number 2 → page fault occurs (valid bit =0).

- (iii) What happens when there is a page fault?

[Suggested Solution]

Page fault => the required data/code is not in the main memory

=> OS needs to retrieve the required information from the storage memory and update the paging table accordingly.

f. What memory are the Page Table and TLB resided?

[Suggested Solution]

Main Page Table => Main Memory

TLB => Internal Fast Memory close to CPU

g. What is the function and effect of a TLB?

[Suggested Solution]

**Solutions**

- ⇒ Cache the frequently used Page table information
- ⇒ Leads to shorter access time for paging information => increase in system performance.

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(Not necessary to be covered during tutorial)

[Optional, but students are encouraged to attempt these questions]

3. Consider a system with the following characteristics.

- Direct mapped cache of 32 cache blocks and cache block size of 32 bytes
  - Cache uses Physical Address for address mapping
  - Virtual Memory page size 2048 bytes
  - Virtual Memory size is 1Mbyte. Physical Memory size is 64KByte
  - Extracts of Page Table (valid entries)
    - Virtual Page 0 → Physical Frame 9
    - Virtual Page 1 → Physical Frame 3
    - Virtual Page 2 → Physical Frame 5
    - Virtual Page 3 → Physical Frame 2
    - Virtual Page 4 → Physical Frame 7
  - The main program is 5KByte in size and starts at virtual address 0x01006
- (i) Assuming that the compiler allocates the program sequentially in the virtual memory, what is the physical address of the start and end of the main program?

[Suggested Solution]

Virtual Address (Start) = 0x01006, end = 0x02406. (5KByte size)  
 0x1006 => 0001 0000 0000 0110 => Page 2 => Physical Frame 5 (101b)  
 Physical address = 0010 1000 0000 0110 = **0x2806**  
 0x2406 => 0010 0100 0000 0110 => Page 4 => Physical Frame 7 (111b)  
 Physical Address = 0011 1100 0000 0110 => **0x3C06**

- (ii) Which cache block should the CPU check in the cache for the start of the main program? What is the corresponding TAG value used to check for cache hit/miss?

[Suggested Solution]

Physical Address used for tagging. Hence, TAG:BLK:OFFSET = 6:5:5  
 Start address = 0x2806 = 0010 1000 0000 0110  
 TAG value = 001010 = **0xA**, BLK = 000000 = **0**