EE302 Homework 1

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2) Since system is first order there is no overshoot. By solving the differential equation analytically;

$$T(s) = \frac{\frac{4}{s+1}}{1 + \frac{4}{s+1}}$$

$$T(s) = \frac{4}{s+5}$$

$$Y(s) = \frac{4}{s(s+5)}$$

$$y(t) = \frac{4}{5} - \frac{4}{5}e^{-5t} \ \ t > 0$$

$$\tau = \frac{1}{a} = 0.2sec$$

$$T_s = 4\tau = 0.8sec$$

$$e(\infty) = 1 - 0.8 = 0.2$$

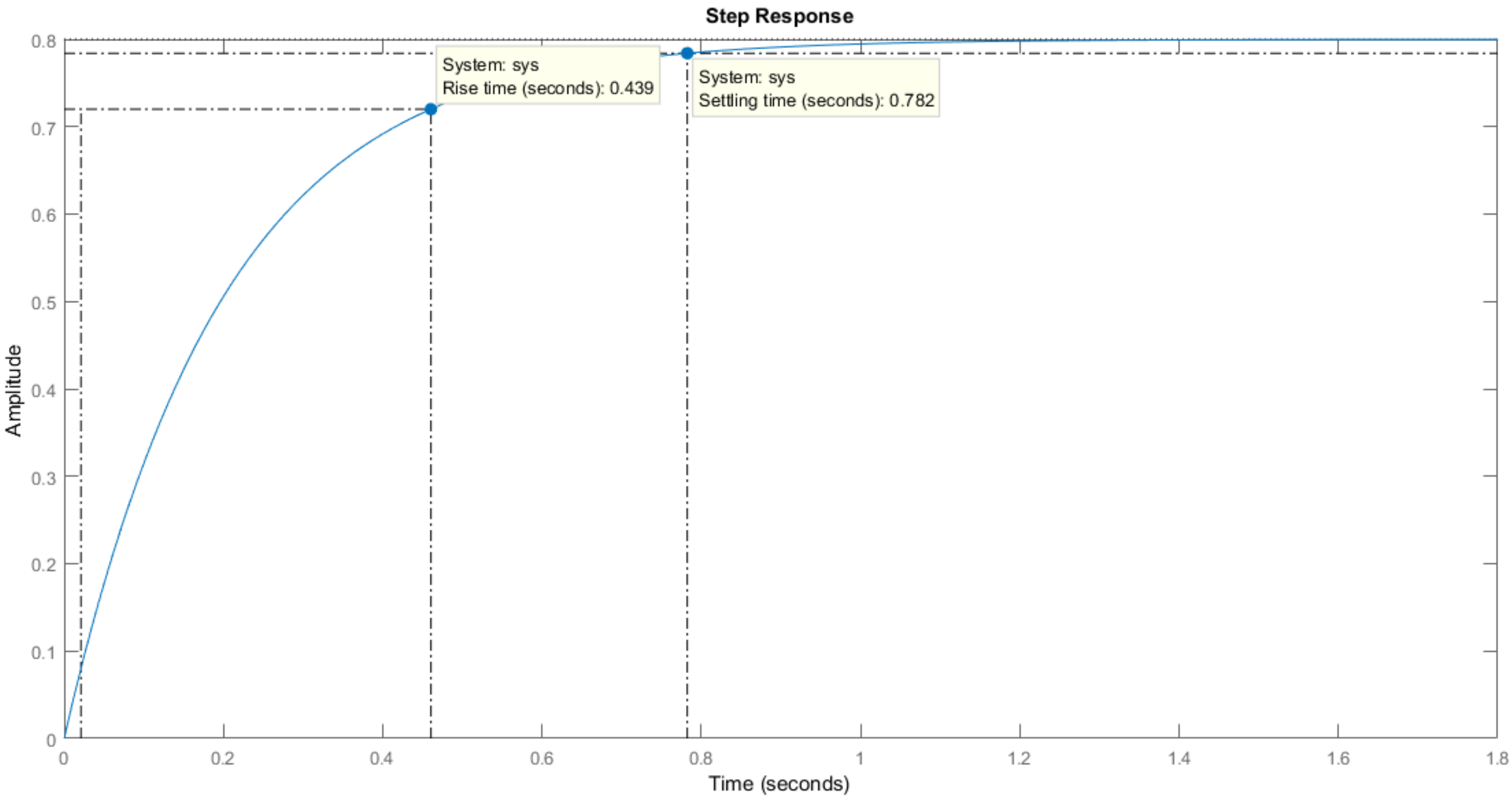

Figure 1: Step response of the system

The results are as expected.

$$T(s) = \frac{\frac{20s+4}{5s(s+1)}}{1 + \frac{20s+4}{5s(s+1)}}$$

$$T(s) = \frac{20s+4}{5s^2+25s+4}$$

$$Y(s) = \frac{20s+4}{s(5s^2+25s+4)}$$

System has 2 negative real pole and 1 negative real zero.

$$Pole_1 = -4.83 \, Pole_2 = -0.165 \, Zero = -0.2$$

Since one of the pole is so close to zero we can do pole-zero cancellation. Then we can solve like first order system.

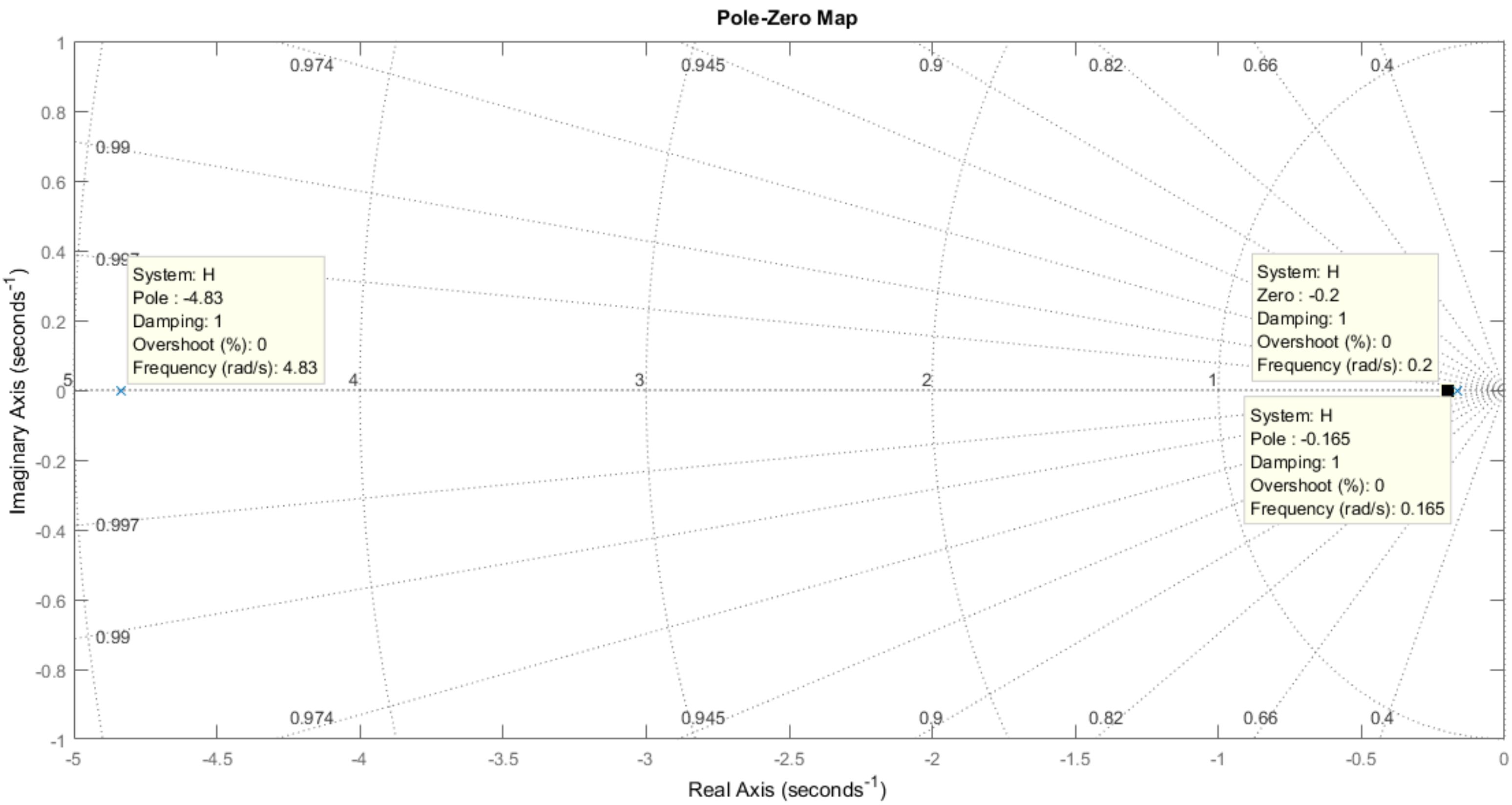

Figure 2: Pole-zero diagram of the system

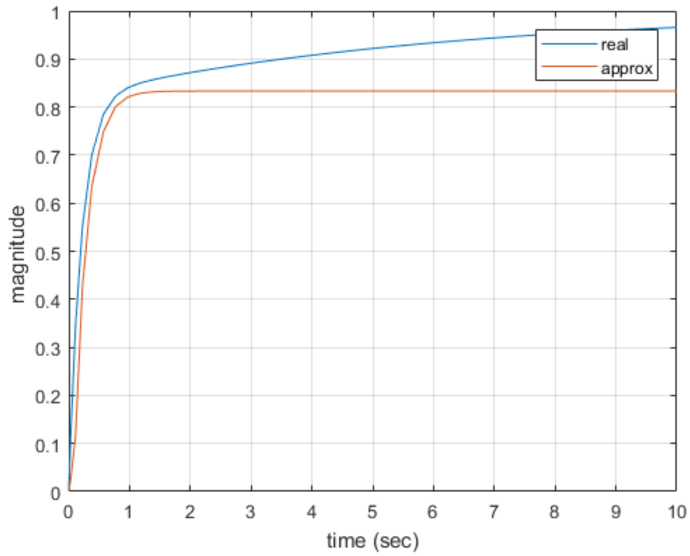

Figure 3: Step response of the system

$$T(s) = \frac{1}{0.25 + 1}$$

$$y(t) = 1 - e^{-4t}$$

$$\tau = 0.25sec$$

$$t_{settle} = 4\tau = 1sec$$

Since it is an overdamped system (before approximation) there is no overshoot.

$$t_{rise} = \frac{2}{a} = 0.5sec$$

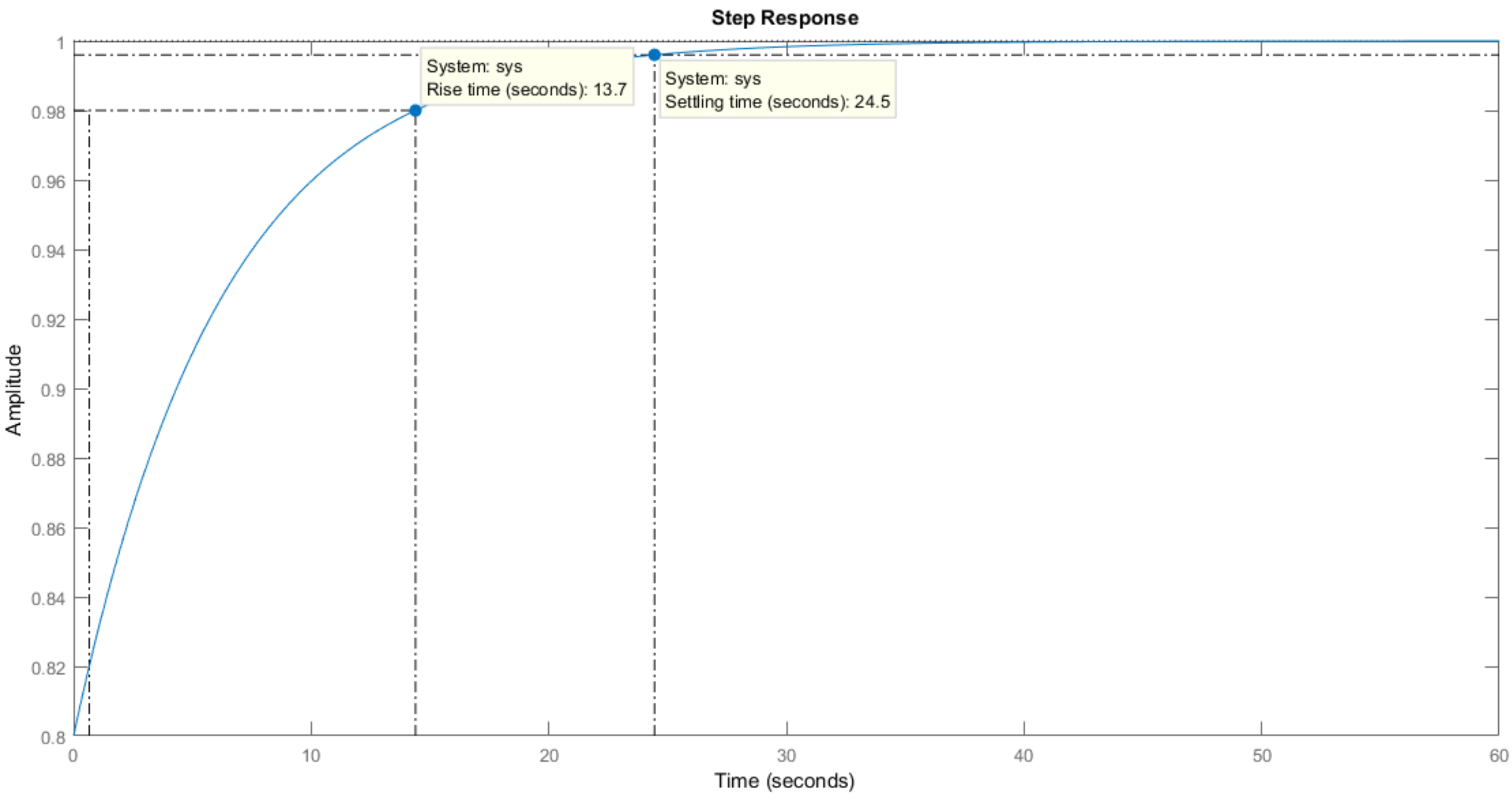

Figure 4: Step response of the system with PI controller where P=4 and I=0.8

d)

$$D(s) = \frac{4s + 10}{s}$$

$$G(s) = \frac{4s + 10}{(s + 1)}$$

$$T(s) = \frac{G(s)}{1 + G(s)}$$

$$T(s) = \frac{4s + 10}{s^2 + 5s + 10}$$

$$Y(s) = \frac{4s + 10}{s(s^2 + 5s + 10)}$$
$$Y(s) = \frac{1}{s} + \frac{Bs + C}{s^2 + 5s + 10}$$
$$Y(s) = \frac{1}{s} - \frac{s + 1}{s^2 + 5s + 10}$$
$$Y(s) = \frac{1}{s} - \frac{s + 1}{(s + 2.5)^2 + 3.75}$$
$$Y(s) = \frac{1}{s} - \frac{s + 2.5}{(s + 2.5)^2 + 3.75} - \frac{1.5}{(s + 2.5)^2 + 3.75}$$
$$y(t) = 1 - e^{-\frac{5t}{2}}(cos(1.9t) - 0.77sin(1.9t))$$

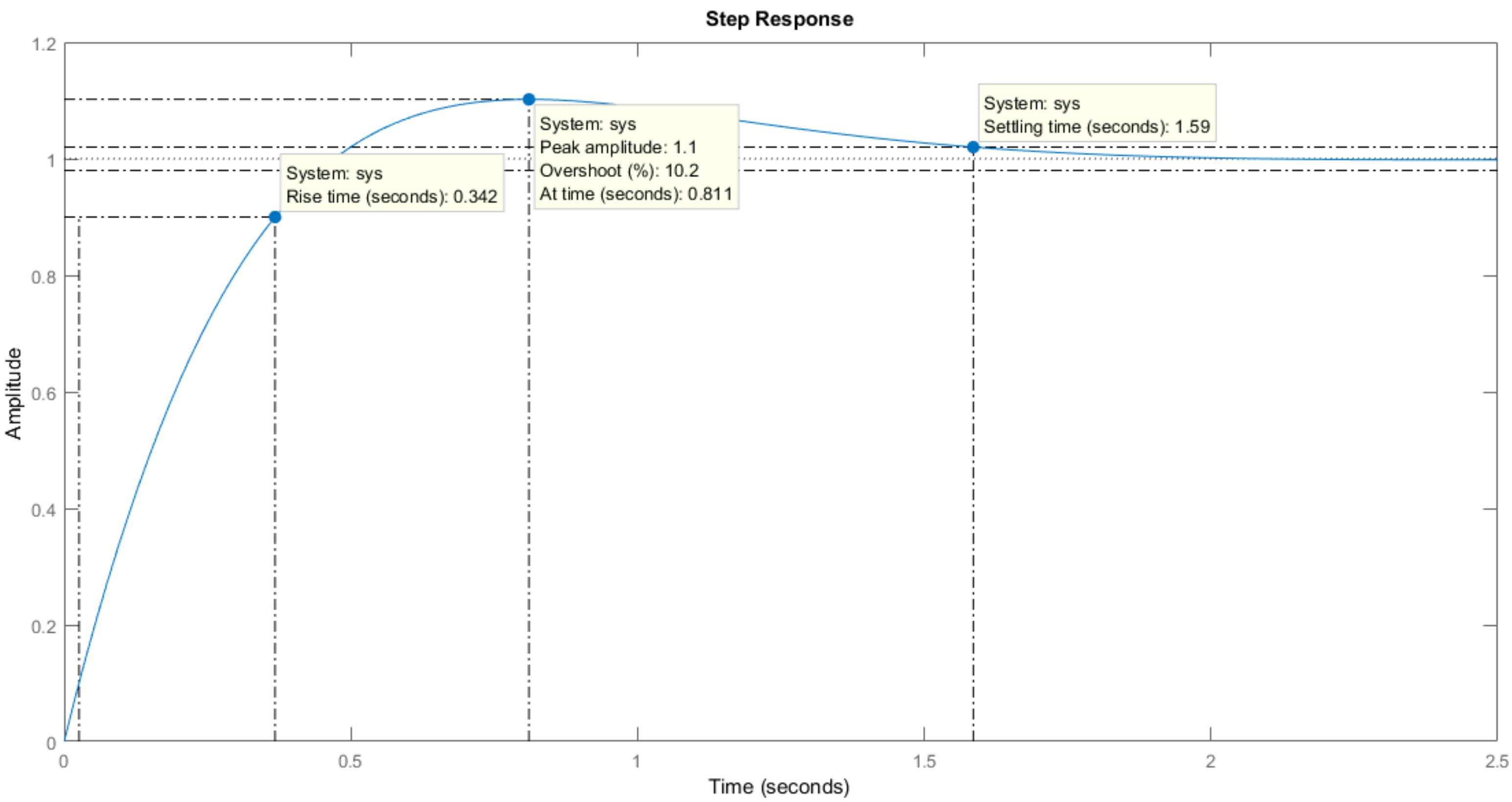

Figure 5: Step response of the system with PI controller P=4 I=10

Without controller unit i have significant steady state error. Using PI controller help about this issue. When i used I=0.8 system is to slow to response. However when i increase I to 10 system response time significantly improved. (Settling time 24.5 sec to 1.59 sec). But with increasing I there is overshooting problem but trade-off is okey.

3)

$$\frac{G(s)}{1+G(s)} = \frac{Ks+b}{s^2+as+b}$$
$$G(s)(s^2+(a-k)s) = Ks+b$$
$$G(s) = \frac{Ks+b}{s(s+a-k)}$$

b)

$$K_v = \frac{1}{\lim_{s\to 0} sG(s)} = 0.04$$
$$\lim s \to 0 \frac{s^2r+sm}{s^2+ns} = 25\; byL'hospital$$
$$\frac{m}{n} = 25$$
$$T(s) = \frac{G(s)}{1+G(s)}$$
$$T(s) = \frac{rs+m}{s^2+(n+r)s+m}$$
$$m = 25$$
$$n = 1$$
$$Y(s) = \frac{A}{s} + \frac{\delta(s+3)}{s^2+6s+25} + \frac{\beta 5}{s^2+6s+25}$$
$$n+r = 6$$
$$r = 5$$

c)

$$T(s) = \frac{5s+25}{s^2+6s+25}$$
$$Y(s) = \frac{5s+25}{s(s^2+6s+25)}$$

$$T(s) = \frac{5s+15}{(s+3)^2+4^2} + \frac{10}{(s+3)^2+4^2}$$
$$25A = 25$$
$$A = 1$$
$$\delta = 5$$
$$\beta = \frac{5}{8}$$

d)

$$M_p = e^{\frac{-\pi}{tan\phi}}$$
$$\omega_n = 5 \, \omega_d = 4$$
$$\phi = arctan(\frac{4}{3})$$
$$M_p = 0.105$$

4)

$$T(s) = \frac{\frac{G(s)}{s}}{1+\frac{G(s)}{s}}$$
$$T(s) = \frac{4}{s^2+2s+4}$$
$$\omega_n = 2$$
$$\omega_d = \sqrt{3}$$
$$tan(\phi) = \sqrt{3}$$
$$M_p = e^{-\frac{\pi}{tan(\phi)}}$$
$$M_p = 0.163$$
$$e_{unit} = \lim_{s\to 0} G(s)$$
$$e_{unit} = \lim_{s\to 0} \frac{4}{s(s+2)} = 0 \;\; Type1 \, system$$
$$e_{ramp} = \lim_{s\to 0} sG(s) = 2$$
$$\lim_{s\to \inf} \frac{4K_i}{s+2} = 8$$
$$K_i = 4$$

$$T(s) = \frac{16}{s^2 + 2s + 16}$$

$$tan(\phi) = \frac{\sqrt{15}}{1}$$

$$M_p = 0.444$$

With increasing K_i increases overshoot percentage.

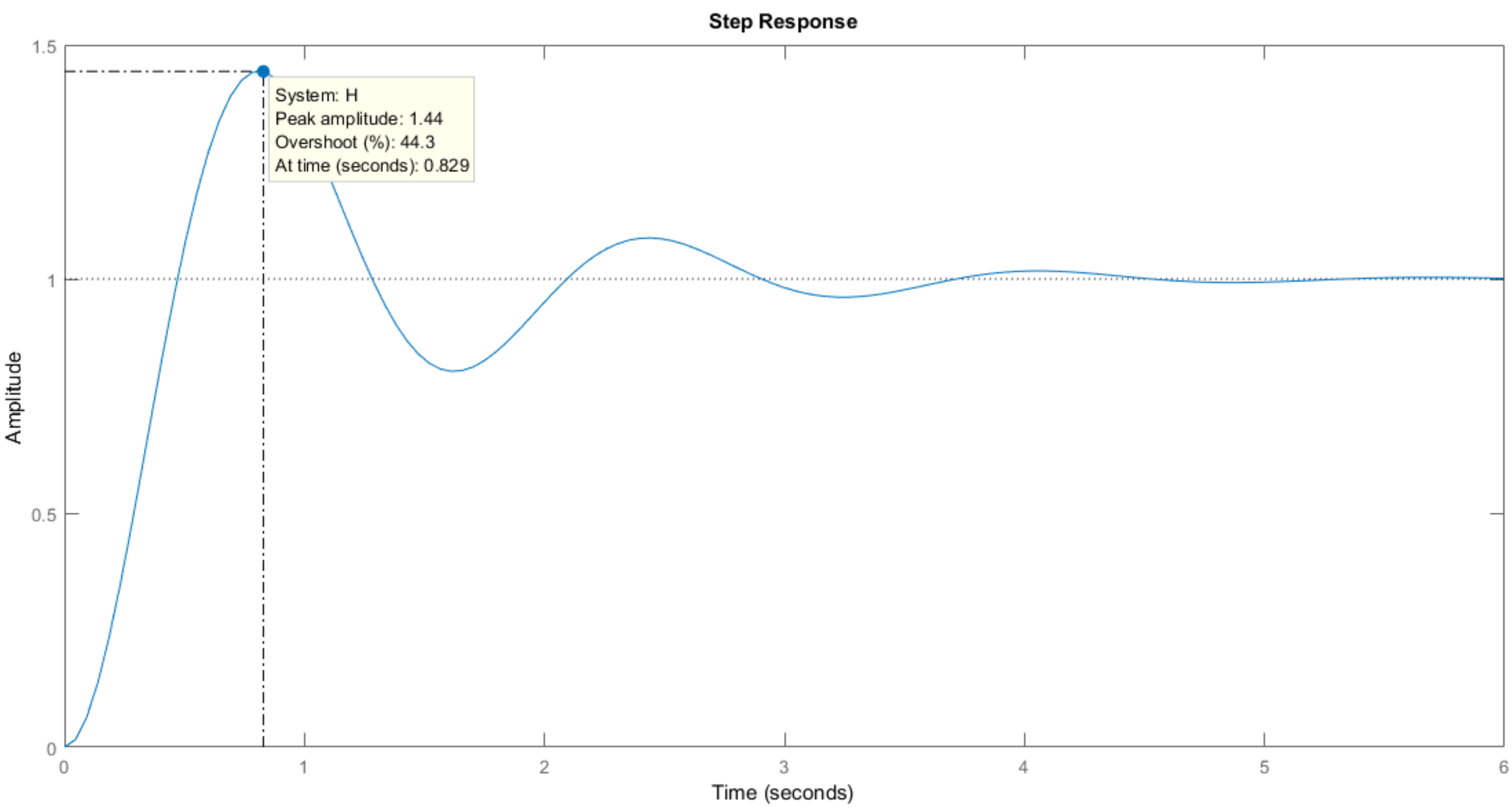

Figure 6: Step response $K_i = 4$

5)
a)

$$A(s) = \frac{100(s+2)}{s^2-1}$$
$$(E(s)G_c(s) + D(s))A(s) = Y(s)$$
$$E(s) = R(s) - Y(s)$$
$$((R(s) - Y(s))G_c(s) + D(s))A(s) = Y(s)$$
$$R(s)G_c(s)A(s) + D(s)A(s) = Y(s)(1 + G_c(s)A(s))$$
$$G_r(s) = \frac{G_c(s)A(s)}{1 + G_c(s)A(s)}$$
$$G_d(s) = \frac{A(s)}{1 + G_c(s)A(s)}$$

b)

$$C(s) = G_c(s)A(s)E(s) + D(s)A(s)$$
$$C(s) = R(s) - E(s)$$
$$E(s) = \frac{R(s) - D(s)A(s)}{1 + A(s)G_c(s)} \; where \; G_c(s) = 1 \; and \; R(s) = \frac{1}{s}$$
$$E(s) = \frac{\frac{1}{s} - \frac{100(s+2)}{s(s^2-1)}}{1 + \frac{100(s+2)}{s^2-1}}$$
$$e_r(\infty) = \lim_{s \to 0} sE(s)$$
$$e_r(\infty) = \lim_{s \to 0} \frac{1 - \frac{100(s+2)}{s^2-1}}{1 + \frac{100(s+2)}{s^2-1}} = -\frac{199}{201}$$

c)

$$E(s) = \frac{\frac{1}{s}}{1 + \frac{(s+\alpha)100(s+2)}{s(s^2-1)}}$$
$$e_r(\infty) = \lim_{s \to 0} sE(s)$$
$$e_r(\infty) = \lim_{s \to 0} \frac{1}{1 + \frac{(s+\alpha)100(s+2)}{s(s^2-1)}} = 0$$

d)

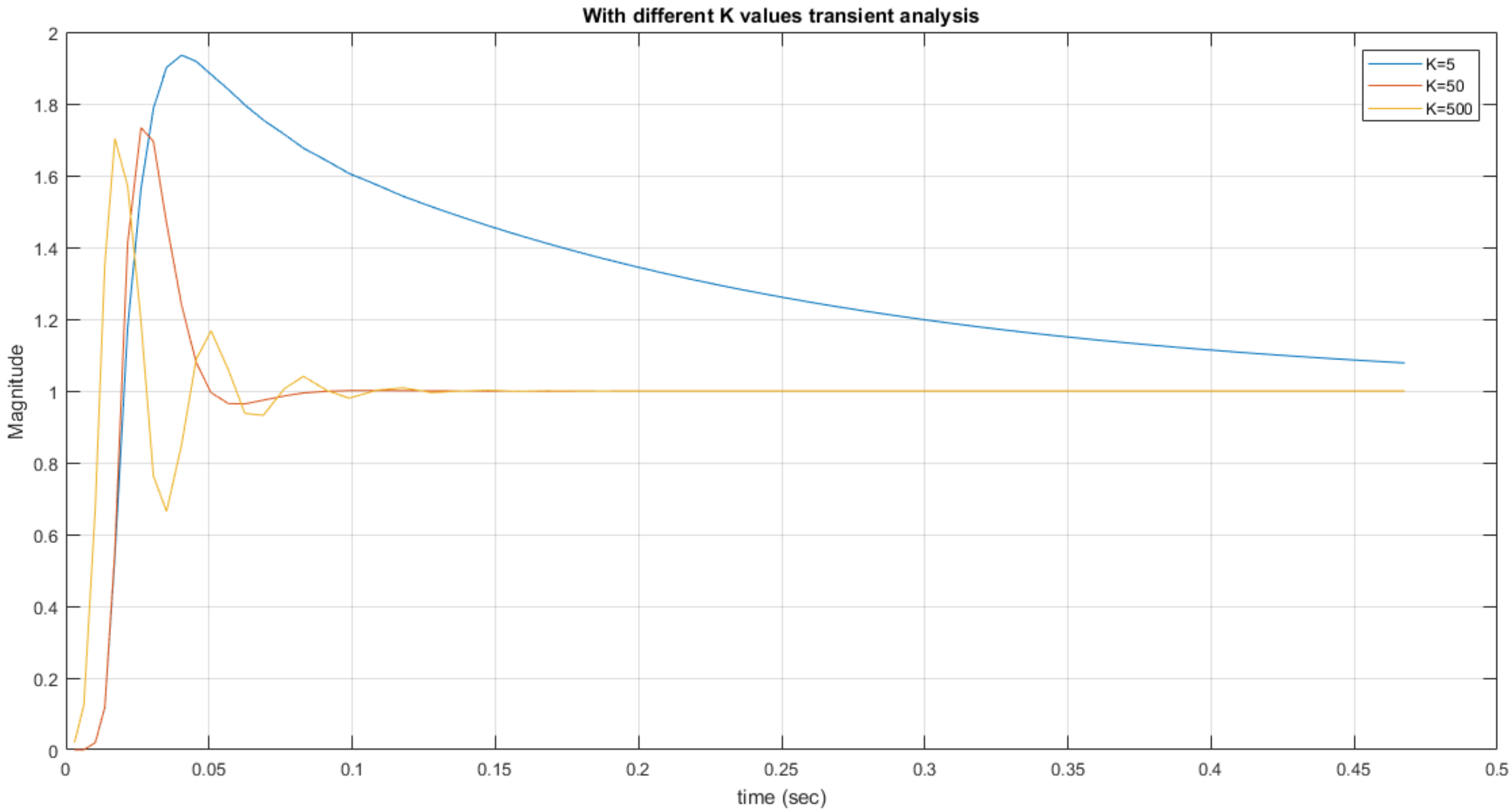

Figure 7: Transient Analysis

Increasing k means that we increased the integrator controller. With increasing integrator we have faster response means that shorter t_{rise} and $t_{settling}$. However the trade-off is oscillation frequency increased.

e)

$$E(s) = -\frac{\frac{100(s+2)}{s(s^2-1)}}{1 + \frac{100(s+2)}{(s^2-1)}}$$

$$e_r(\infty) = \lim_{s \to 0} sE(s)$$

$$e_r(\infty) = \lim_{s \to 0} -\frac{\frac{100(s+2)}{s^2-1}}{1 + \frac{100(s+2)}{s^2-1}} = \frac{200}{199}$$

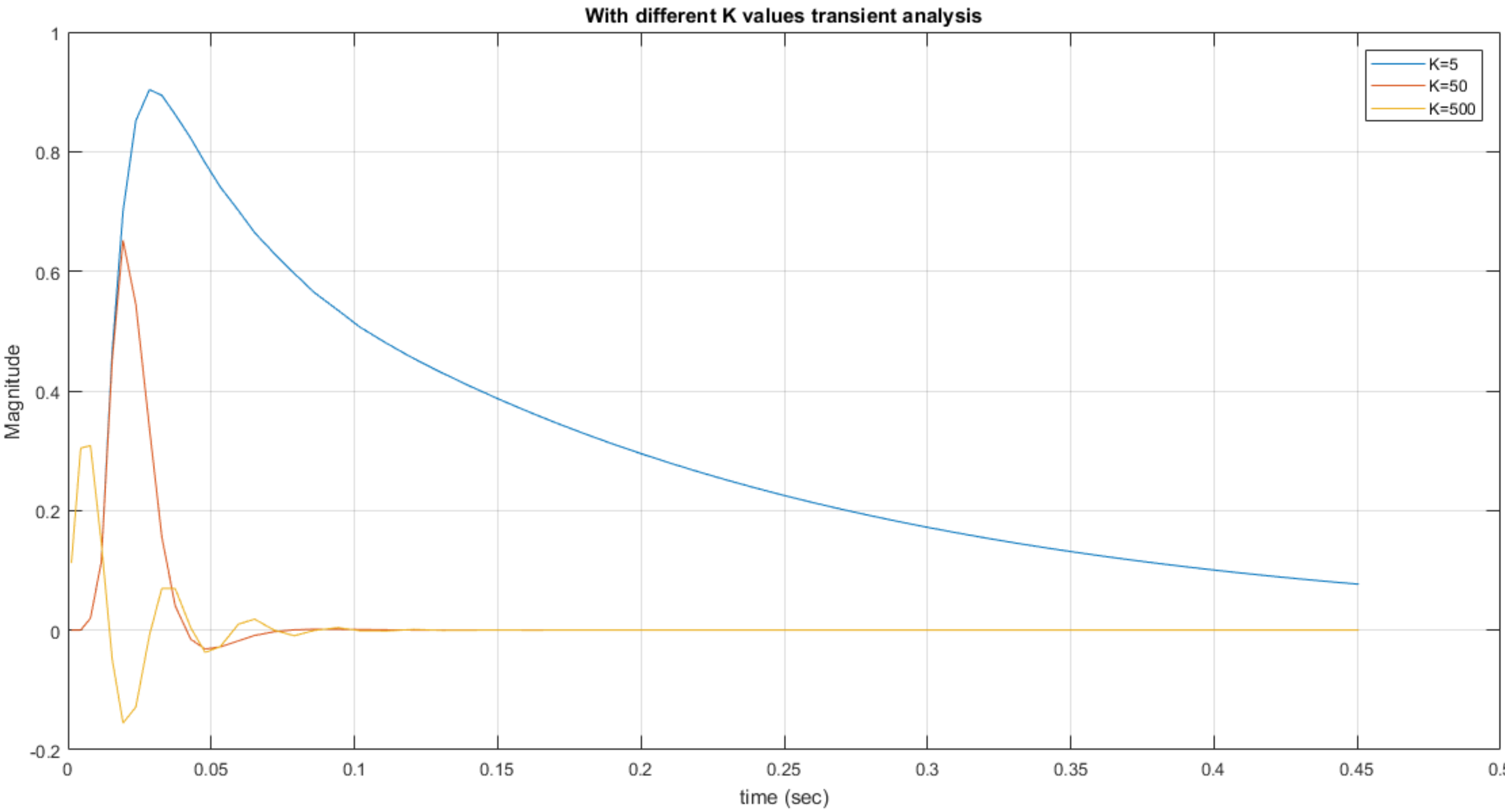

Figure 8: Transient Analysis