EE302 Homework 4

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Question 1

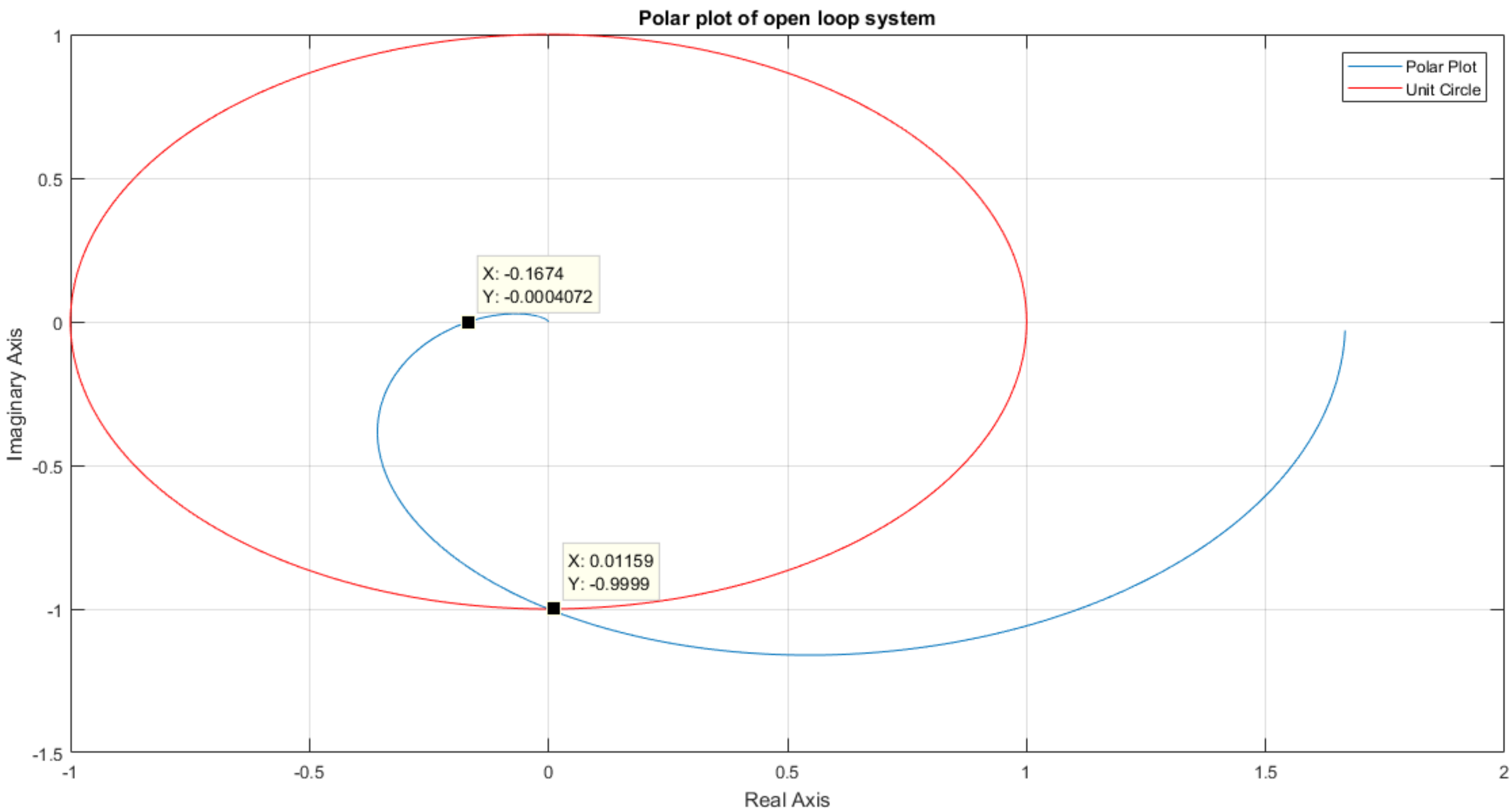

Figure 1: Polar plot of the system

The point that cross the real axis is $x_1 = -0.1674$. Then the gain margin ;

$$GM = 20log(\frac{1}{|a|}) = 15.5249\, dB$$

The point where plot cross the unity circle is $x_2 = 0.01159$ and $y_2 = -1$. Then the phase margin is the angle between that point and the origin.

$$PM = 90^\circ$$

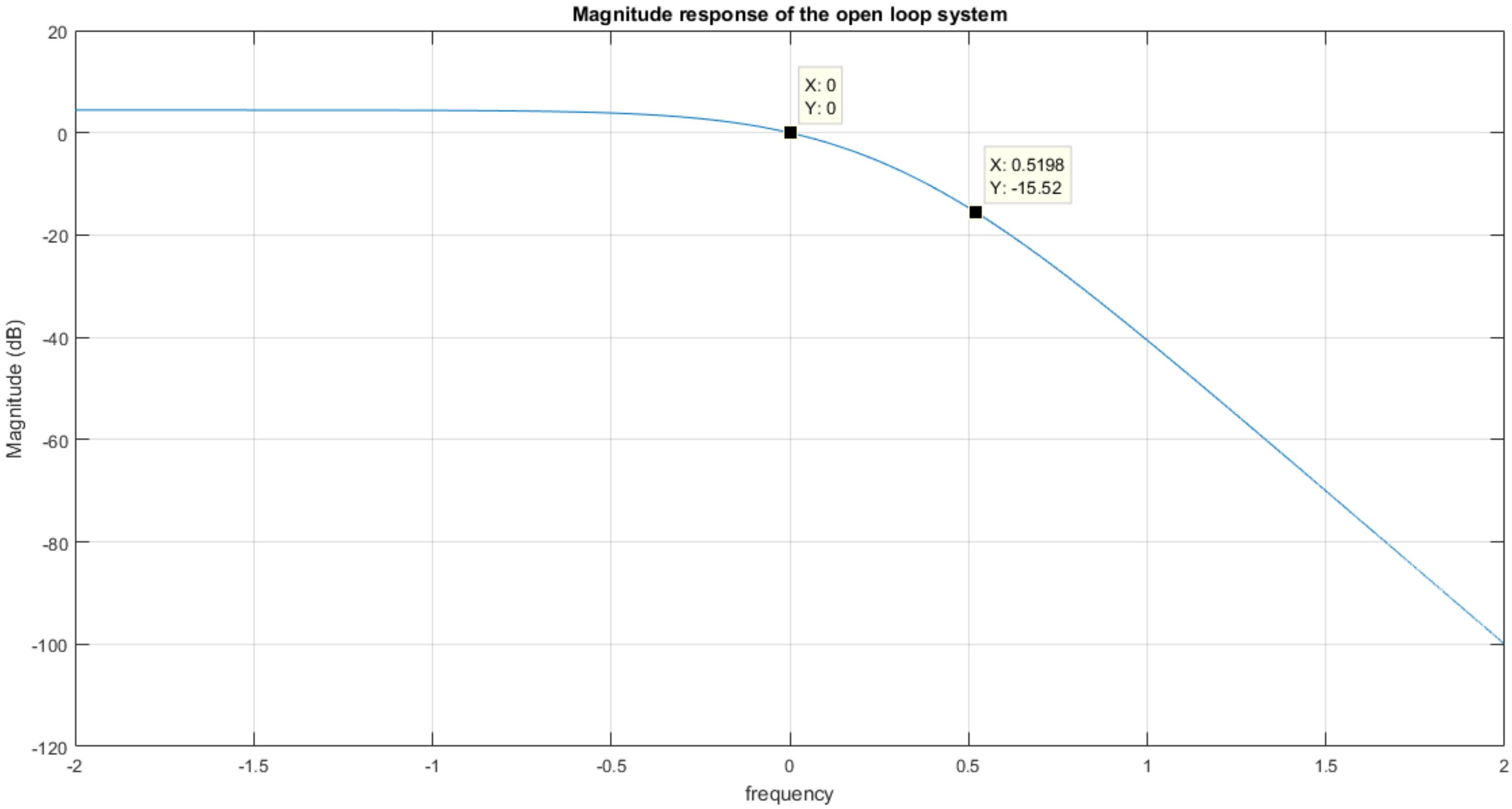

Figure 2: Magnitude response of the system

Gain cross-over frequency is the frequency that system has 0 dB gain.

$$f_{gain_{cross-over}} = 0\,\frac{rad}{sec}\,(dc)$$

To find gain margin of the system, first we find phase cross-over frequency.

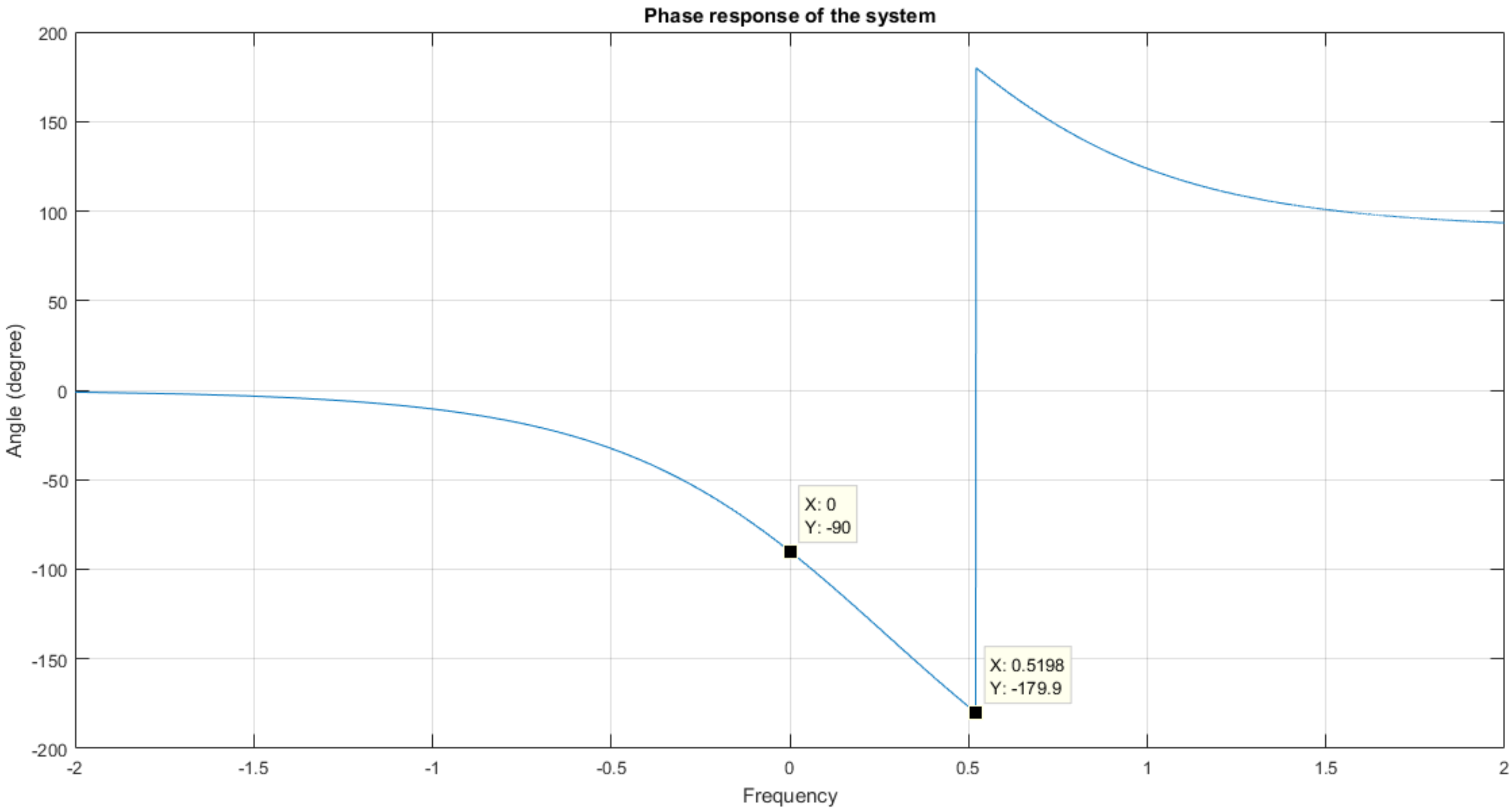

Figure 3: Phase response of the system

$$f_{phase_{cross-over}} = 0.5198\,\frac{rad}{sec}$$

Then the gain margin and phase margin is following

$$GM = 15.52\,dB$$

$$PM = 90^{\circ}$$

In both method i find exactly same gain and phase margin.

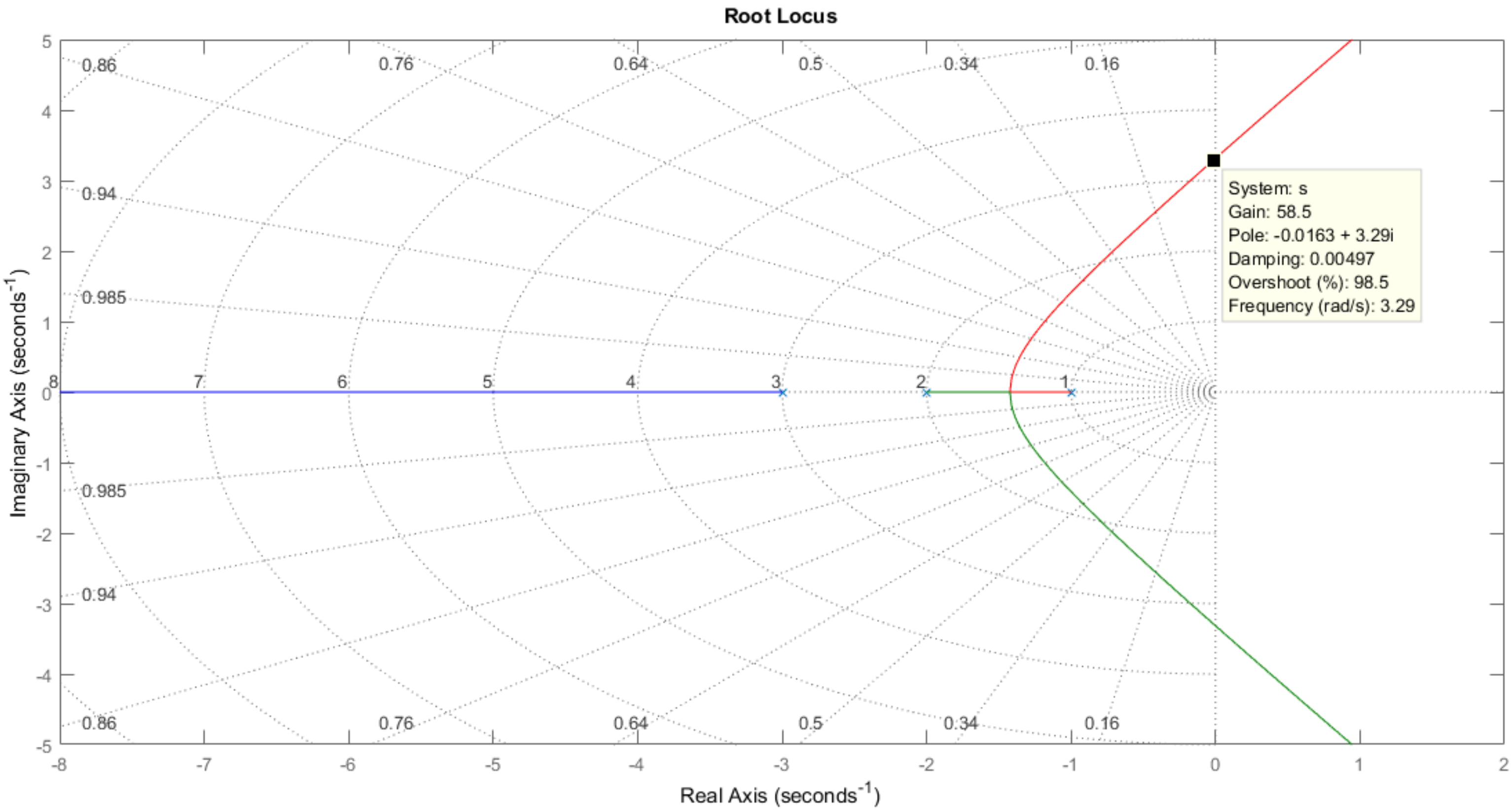

Figure 4: Root-locus

$$K_{original} = 10$$

$$K_{unstable} = 58.5$$

$$GM = 20log(\frac{K_{unstable}}{K_{original}}) = 15.3431\ dB$$

Question 2

```
%%
%Compensator design
desired_pm = 55;
current_pm = 49.9;
margin = -2;
phi_max = (desired_pm-current_pm + margin)*pi/180;
alfa = (1-sin(phi_max))/(1+sin(phi_max));
pointer = 10*log10(alfa);
%%
%Finding compansator and plotting its bode
wgc = 0.8136 %pointer frequency value
T = 1/(sqrt(alfa)*wgc)
s = tf([1 0],[1]);
Gc = (1+T*s)/(1+T*alfa*s)
%%
s = tf([1 0],[1]);
Gc = (1+T*s)/(1+T*alfa*s)
```

Figure 5: Magnitude response of the system using experimental data

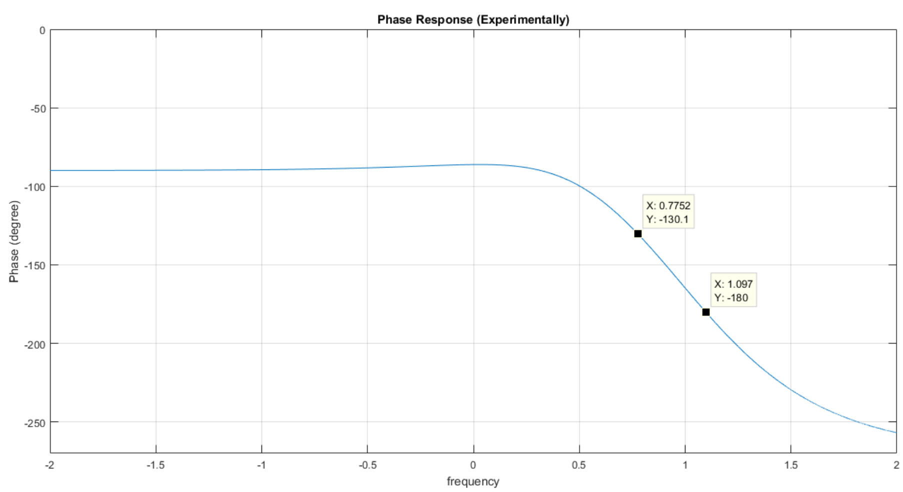

Figure 6: Phase response of the system using experimental data

$$f_{gain_{cross-over}} = 0.7752\,\frac{rad}{sec}$$

$$f_{phase_{cross-over}} = 1.097\,\frac{rad}{sec}\,(dc)$$

$$GM = 10.64\,dB$$

$$PM = 49.9°$$

Transfer function of the compensator is following;

$$G_c = \frac{1 + 1.297s}{1 + 1.64s}$$

Matlab script to find its compensator parameters;

```
%%
%Compensator design
desired_pm = 55;
current_pm = 49.9;
margin = -2;
phi_max = (desired_pm-current_pm + margin)*pi/180;
alfa = (1-sin(phi_max))/(1+sin(phi_max));
pointer = 10*log10(alfa);
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%Finding compansator and plotting its bode
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T = 1/(sqrt(alfa)*wgc)
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Gc = (1+T*s)/(1+T*alfa*s)
%%
s = tf([1 0],[1]);
Gc = (1+T*s)/(1+T*alfa*s)
```

Figure 7: Matlab script to find compensator parameters

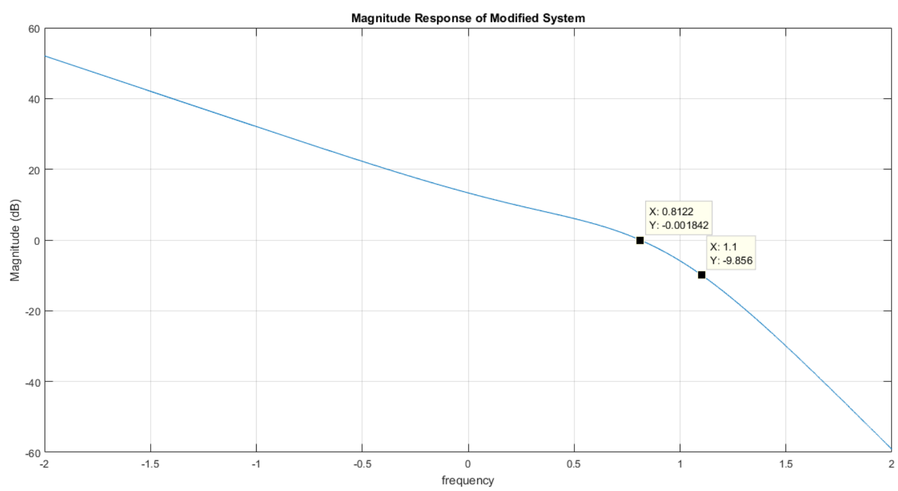

Figure 8: Magnitude response of the system with lead compensator

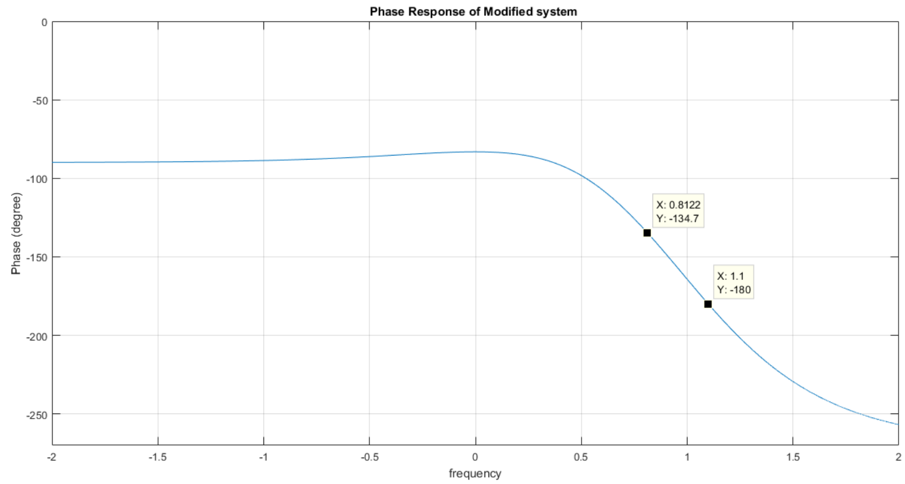

Figure 9: Phase response of the system with lead compensator

$$PM = 55.3^\circ$$

$$GM = 9.856\,dB$$

Designing phase lag compensator is following;

$$\phi = -180 + \phi_{desired} + 5$$

From phase response desired magnitude is following

$$log(\omega_{gc}) = 0.5623$$

$$\omega_{gc} = 3.61\,\frac{rad}{sec}$$

$$|G(j\omega_{gc})| = 4.219\,dB \quad 20log(\alpha) = 4.219\,dB$$

$$\alpha = 1.62$$

$$T = \frac{10}{\omega_{gc}} = 2.77$$

$$T(s) = \frac{1 + Ts}{1 + T\alpha s} = \frac{1 + 2.77s}{1 + 4.48s}$$

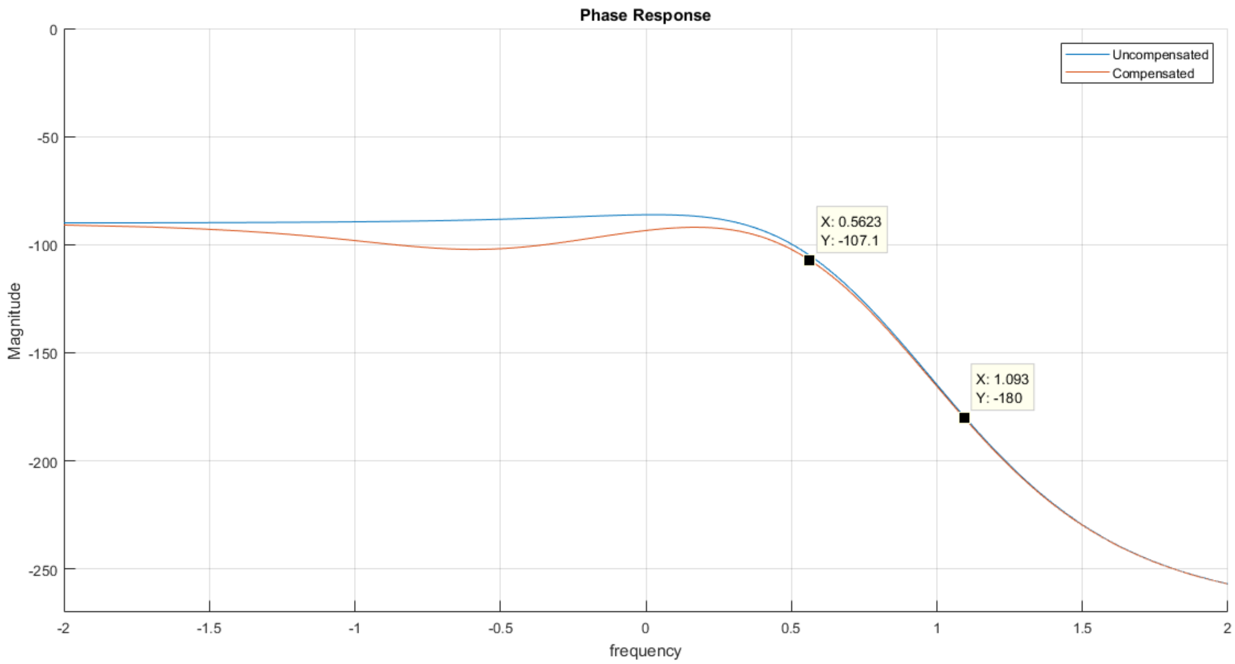

Figure 10: Compensated phase response

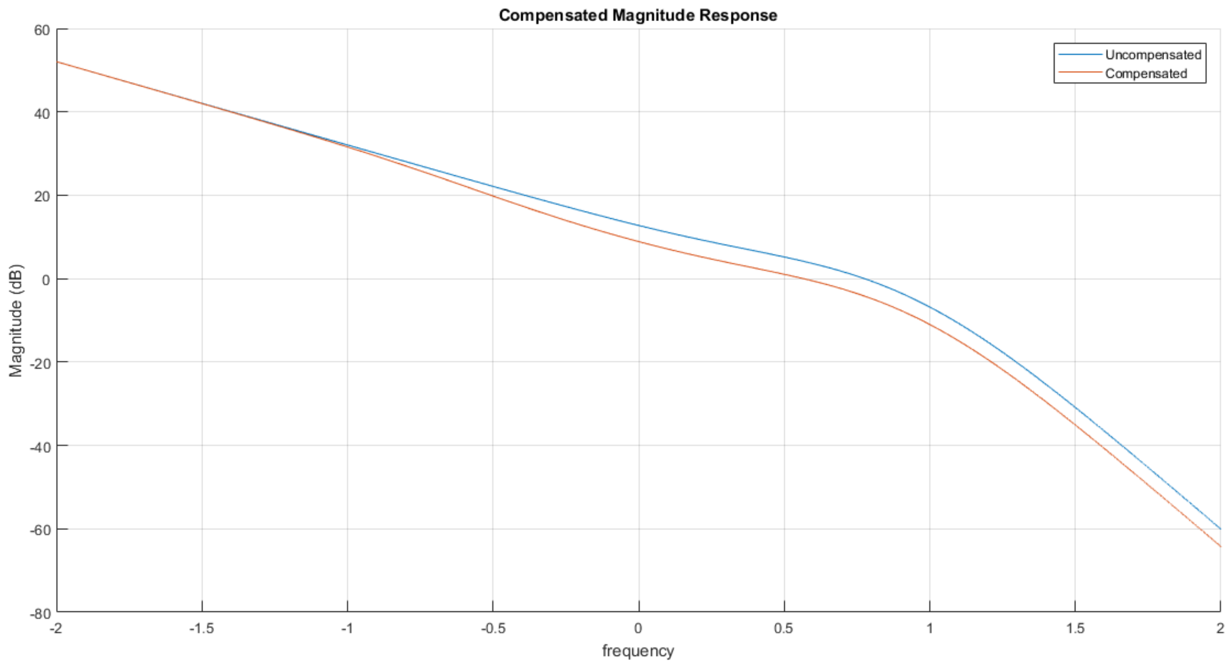

Figure 11: Compensated magnitude response

$$PM = 72.9^\circ$$
$$GM = 14.69\, dB$$

Question 3

$$e_{ss} < 0.25$$
$$G(s) = \frac{0.2}{s^2(s+100)}$$
$$K_a = \lim_{s\to 0} s^2 KG(s)$$
$$K \geq 2000$$

In order to find gain cross-over frequency following equation must solve for ω;

$$G(j\omega) = \frac{(0.2)2000}{j\omega^2(j\omega+100)} = 1$$

$$\omega \simeq 2\frac{rad}{sec}$$

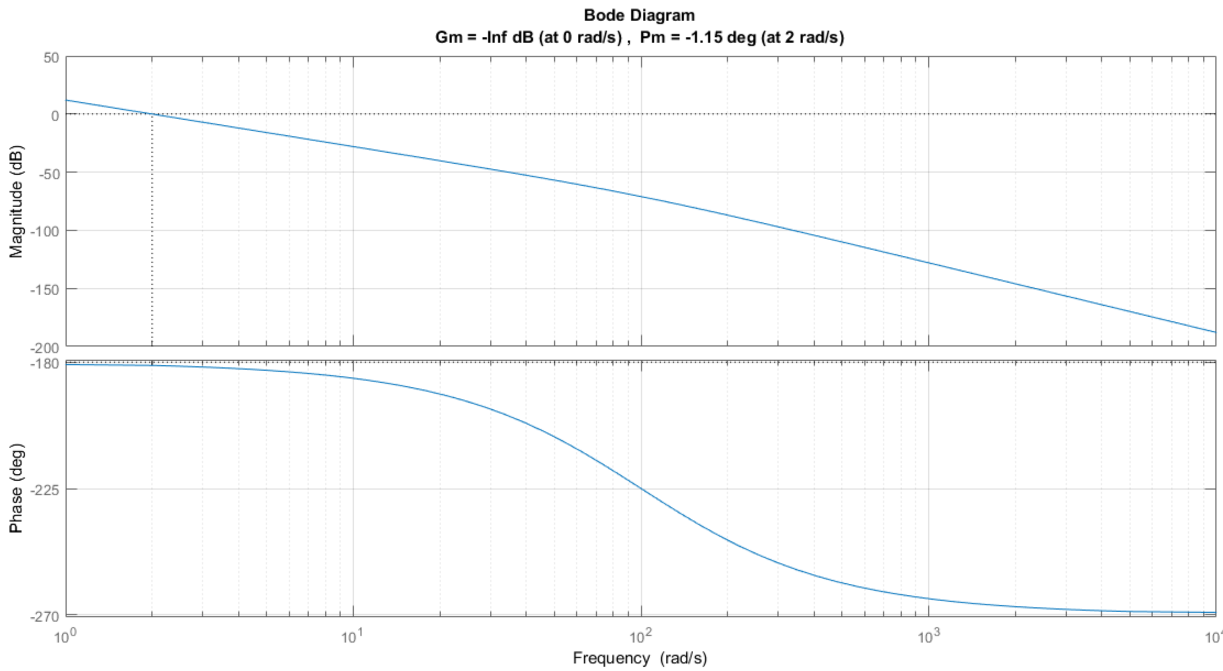

Figure 12: Analysing circuit for compensator selection

Since phase margin is too low i chose lead compensator for improving phase margin.

$$\phi_{max} = \phi_{desired} + PM + 5$$
$$\alpha = \frac{1 - sin(\phi_{max})}{1 + sin(\phi_{max})} = 0.16$$
$$|G(j\omega)| = \sqrt{\alpha} = 0.4$$
$$\frac{400}{\omega^2(\sqrt{10^4 + \omega^2})} = 0.4$$
$$\omega \simeq 3.14$$
$$T = \frac{1}{\omega\sqrt{\alpha}}$$

$$
\begin{aligned}
T &= 0.80 \\
G_c &= \frac{1 + Ts}{1 + \alpha Ts} = \frac{1 + 0.8s}{1 + 2.51s}
\end{aligned}
$$