EE302 Homework 1

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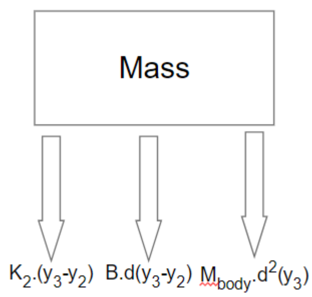

Figure 1: Free body diagram of body mass

Assuming $y_3 > y_2 > y_1$ By Newton's second law;

$$M_{body}\ddot{y_3} = -B(\dot{y_3} - \dot{y_2}) - K_2(y_3 - y_2)$$

In Laplace domain with zero initial conditions;

$$(M_{body}s^2 + Bs + K_2)Y_3(s) = (-Bs - K_2)Y_2(s)$$

$$Y_2(s) = \frac{M_{body}s^2 + Bs + K_2}{-Bs - K_2}Y_3(s)$$

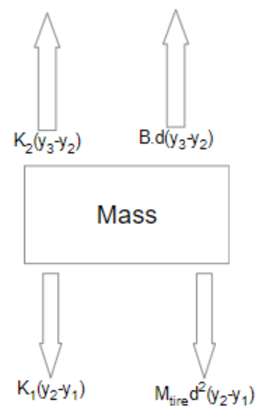

Figure 2: Free body diagram of tire mass

By Newton's second law;

$$M_{tire}\ddot{y_2} = B(\dot{y_3} - \dot{y_2}) + K_2(y_3 - y_2) - K_1(y_2 - y_1)$$

In Laplace domain with zero initial conditions;

$$(M_{tire}s^2 + Bs + K_2 + K_1)Y_2(s) = (Bs + K_2)Y_3(s) + K_1Y_1(s)$$

Putting $Y_2(s) = \frac{M_{body}s^2+Bs+K_2}{-Bs-K_2}Y_3(s)$ into the equation;

$$\frac{(M_{body}s^2 + Bs + K_2)(M_{tire}s^2 + Bs + K_2 + K_1)}{-Bs - K_2}Y_3(s) = (Bs+K_2)Y_3(s)+K_1Y_1(s)$$

$$T(s) = \frac{Y_3(s)}{Y_1(s)}$$

$$\frac{(M_{body}s^2 + Bs + K_2)(M_{tire}s^2 + Bs + K_2 + K_1) + (Bs + K_2)^2}{-Bs - K_2}Y_3(s) = K_1Y_1(s)$$

$$T(s) = \frac{-K_1(Bs + K_2)}{(M_{body}s^2 + Bs + K_2)(M_{tire}s^2 + Bs + K_2 + K_1) + (Bs + K_2)^2}$$

For the state equations; Let $z = \begin{vmatrix} y_2 \\ \dot{y_2} \\ y_3 \\ \dot{y_3} \end{vmatrix}$ then ,

$$\dot{z} = \begin{vmatrix} \dot{y_2} \\ \ddot{y_2} \\ \dot{y_3} \\ \ddot{y_3} \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ \frac{k_2 - k_1}{M_{tire}} & \frac{-b}{M_{tire}} & \frac{-k_2}{M_{tire}} & \frac{b}{M_{tire}} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{M_{body}} & \frac{b}{M_{body}} & \frac{-k_2}{M_{body}} & \frac{-b}{M_{body}} \end{vmatrix} \begin{vmatrix} y_2 \\ y_3 \\ \dot{y_2} \\ \dot{y_3} \end{vmatrix} + \begin{vmatrix} 0 \\ \frac{k_1}{M_{tire}} \\ 0 \\ 0 \end{vmatrix} y_1$$

$$y_3 = \begin{vmatrix} 0 & 0 & 1 & 0 \end{vmatrix} \begin{vmatrix} y_2 \\ \dot{y_2} \\ y_3 \\ \dot{y_3} \end{vmatrix} + [0] y_1$$

2)

$$\tau_0 = J\ddot{\theta}_o + B\dot{\theta}_o$$

$$\tau_0 = K_\tau i_m$$

$$K_\theta(\theta_r - \theta_o) = V_f$$

In Laplace domain;

$$V_f(s) = I_f(s)(R_f + sL_f)$$

$$V_g(s) - V_b(s) = K_g I_f(s) = (R_g + R_m) I_m(s) + (sL_g + sL_m) I_m(s) where V_b(s) = K_b \dot{\theta}_o$$

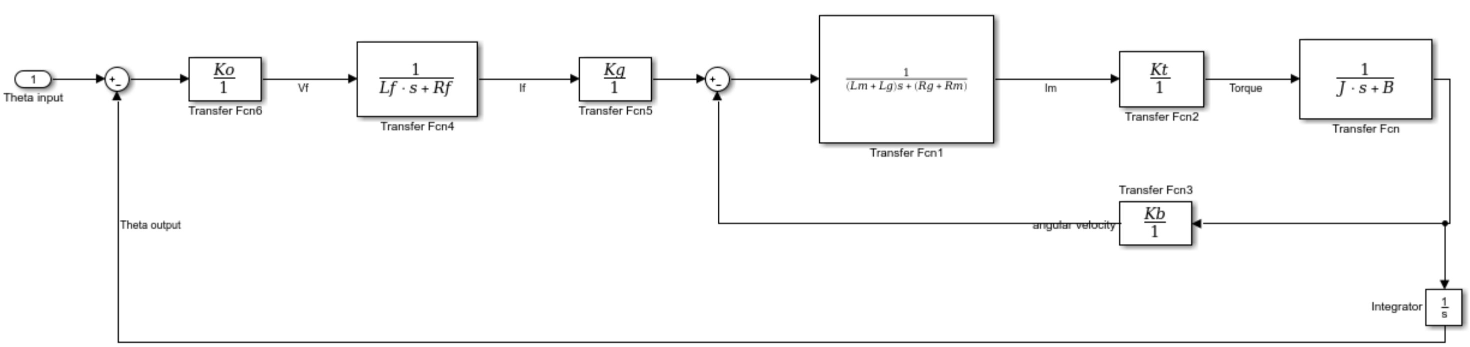

Figure 3: Body diagram of the system

$$A(s) = \frac{\frac{K_\tau}{(Js+B)(R_g+R_m+s(L_g+L_m)}}{1 + \frac{K_b K_\tau}{(Js+B)(R_g+R_m+s(L_g+L_m))}} = \frac{K_\tau}{(Js+B)(R_g + R_m + s(L_g + L_m) + K_b K_\tau}$$

$$B(s) = \frac{\frac{K_\tau K_g K_\theta}{(Js+B)(R_g+R_m+s(L_g+L_m)s}}{1+\frac{K_b K_\tau}{(Js+B)(R_g+R_m+s(L_g+L_m)s)}}$$

B(s) is the transfer function of the system. Needed state variables number is 4 since transfer function has maximum s^4. Only second one is suitable for state variable. Because first one has input element in the vector. State variable must be linearly independent since $\tau_o = K_{tau} I_m$ fourth one is also not possible. Derivative of state variable should be linear combination of that state variable. $\dddot{\theta}_o$ is not in any equation so this is also not possible. Then state equation is following;

$$\dot{x} = \begin{vmatrix} \dot{I_m} \\ \dot{I_f} \\ \dot{\theta_o} \\ \ddot{\theta_0} \end{vmatrix} = \begin{vmatrix} \frac{R_f}{L_f} & 0 & \frac{K_\theta}{L_f} & 0 \\ \frac{K_g}{L_g+L_m} & -\frac{R_g+R_m}{L_g+L_m} & 0 & \frac{-K_b}{L_g+L_m} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K_\tau}{J} & 0 & \frac{B}{J} \end{vmatrix} \begin{vmatrix} I_m \\ I_f \\ \theta_o \\ \dot{\theta_0} \end{vmatrix} + \begin{vmatrix} \frac{K_\theta}{L_f} \\ 0 \\ 0 \\ 0 \end{vmatrix} \theta_r$$

3)

For mechanical system;

$$F = M\ddot{x} + B\dot{x}$$

in Laplace domain;

$$F(s) = (Ms^2 + Bs)X(s)$$

In cylinder this F creates torque. Actually F is result of τ_2;

$$Fr = \tau_2 - K\theta_2 \; where \; \theta_2 = \frac{x}{r}$$

$$T_2(s) = (Ms^2 r + Bsr + \frac{K}{r})X(s)$$

For electrical system;

$$V(s) - E_a(s) = (sL + R_a)I(s)$$

$$I(s) = \frac{V(s) - E_a(s)}{sL + R_a}$$

$$T_m(s) = K_a I(s)$$

$$T_m(s) = \frac{K_a(V(s) - E_a(s))}{sL + R_a}$$

When we do block diagram simplification for obtaining transfer function;

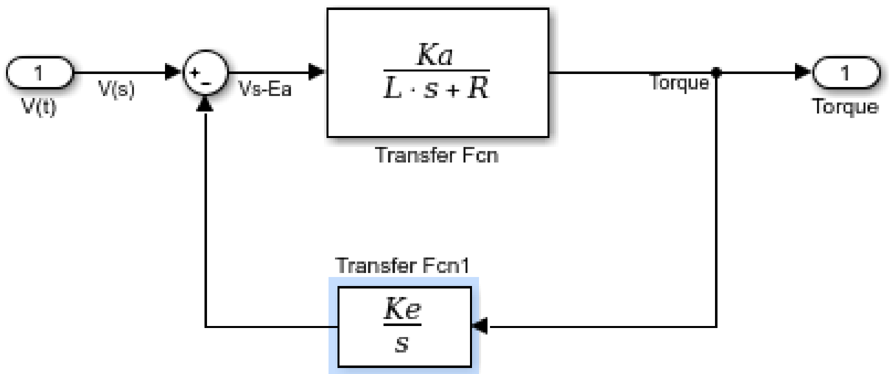

Figure 4: Electrical system block diagram

$$T_m(s) = \frac{G(s)}{1 + G(s)H(s)}$$

Where $G(s) = \frac{K_a}{sL+R}$ and $H(s) = \frac{K_e}{s}$

$$T_m(s) = \frac{K_a s}{s^2 L + sR + K_e K_a}$$

I refer to J_m to the other case (like impedance referring in transformer case) $J_m n^2$

$$T_2(s) = \frac{N_2 T_m(s)}{N_1}$$

$$T_{net}(s) = T_2(s) - K\theta_2(s) - (J + n^2 J_m)s^2\theta_2(s) \; where \; \theta_2 = \frac{x}{r}$$

$$T_{net} = Fr$$

$$F(s) = (Ms^2 + Bs)X(s)$$

$$T_{net} = (Ms^2 + Bs)X(s)r$$

$$(Ms^2 + Bs)X(s)r = \frac{N_2}{N_1}\frac{K_a s}{s^2 L + sR + K_e K_a} - \frac{KX(s)}{r} - \frac{(J + n^2 J_m)s^2 X(s)}{r^2}$$

$$T(s) = \frac{X(s)}{V(s)}$$

$$T(s) = \frac{\frac{K_m J s^2 n}{(R+sL)r(Ms^2+Bs)(Jms^2+n^2(Js^2+K))}}{1 + \frac{K_b sr(Ms^2+Bs)K_m Js^2 n}{Js^2 n(R+sL)r(Ms^2+Bs)(Jms^2+n^2(Js^2+K))}}$$

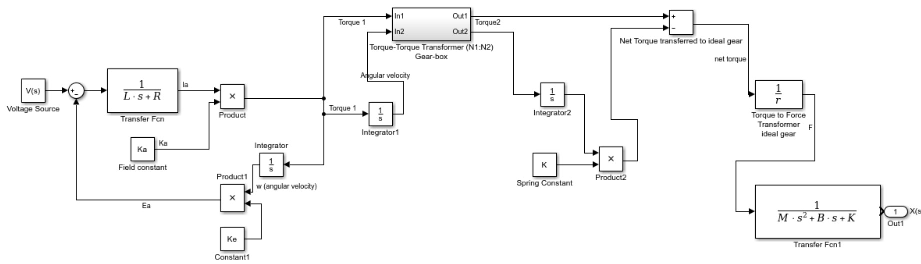

Figure 5: Overall block diagram

4)

$$G_5(s)\frac{G_2(s)+G_3(s)}{1+G_2(s)G_3(s)}+G_4(s)$$

$$G_6(s)=\frac{G_6(s)}{1+G_6(s)}$$

$$\frac{G_6(s)G_5(s)(G_2(s)+G_3(s))}{(1+G_6(s))(1+G_2(s)G_3(s))}+\frac{G_4(s)G_6(s)}{1+G_6(s)}$$

$$G_{overall}=\frac{\frac{G_6(s)G_5(s)(G_2(s)+G_3(s))}{(1+G_6(s))(1+G_2(s)G_3(s))}+\frac{G_4(s)G_6(s)}{1+G_6(s)}}{1+G_7(s)\frac{G_6(s)G_5(s)(G_2(s)+G_3(s))}{(1+G_6(s))(1+G_2(s)G_3(s))}+\frac{G_4(s)G_6(s)}{1+G_6(s)}}$$