

EE302 Homework 1

Nail Tosun - 2094563 -Section 5
Electric and Electronic Engineering Department, METU

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2) Since system is first order there is no overshoot. By solving the differential equation analytically;

$$T(s) = \frac{\frac{4}{s+1}}{1 + \frac{4}{s+1}}$$

$$T(s) = \frac{4}{s+5}$$

$$Y(s) = \frac{4}{s(s+5)}$$

$$y(t) = \frac{4}{5} - \frac{4}{5}e^{-5t} \quad t > 0$$

$$\tau = \frac{1}{a} = 0.2sec$$

$$T_s = 4\tau = 0.8sec$$

$$e(\infty) = 1 - 0.8 = 0.2$$

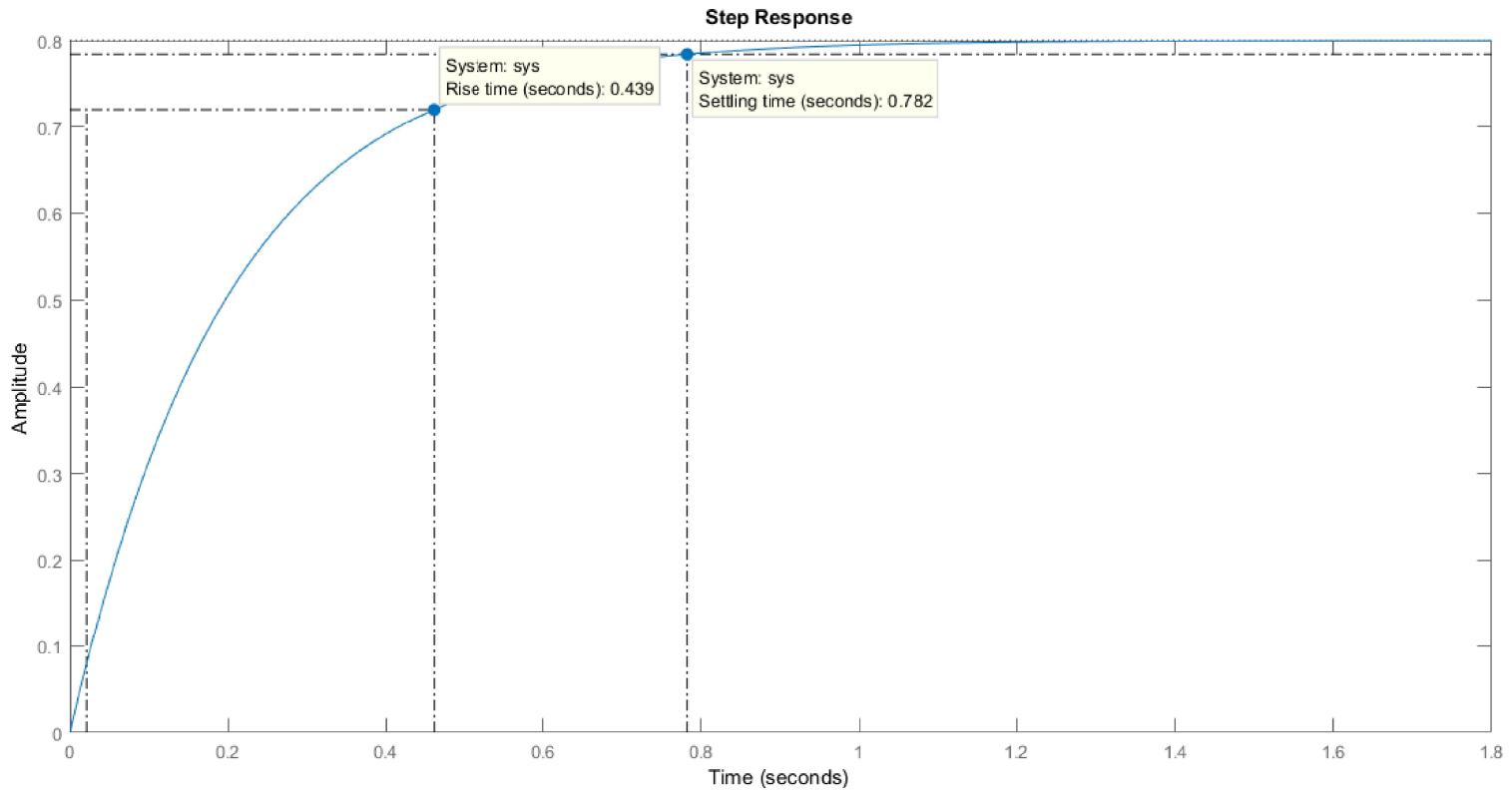


Figure 1: Step response of the system

The results are as expected.

$$T(s) = \frac{\frac{20s+4}{5s(s+1)}}{1 + \frac{20s+4}{5s(s+1)}}$$

$$T(s) = \frac{20s+4}{5s^2 + 25s + 4}$$

$$Y(s) = \frac{20s+4}{s(5s^2 + 25s + 4)}$$

System has 2 negative real pole and 1 negative real zero.

$$Pole_1 = -4.83 \quad Pole_2 = -0.165 \quad Zero = -0.2$$

Since one of the pole is so close to zero we can do pole-zero cancellation.
 Then we can solve like first order system.

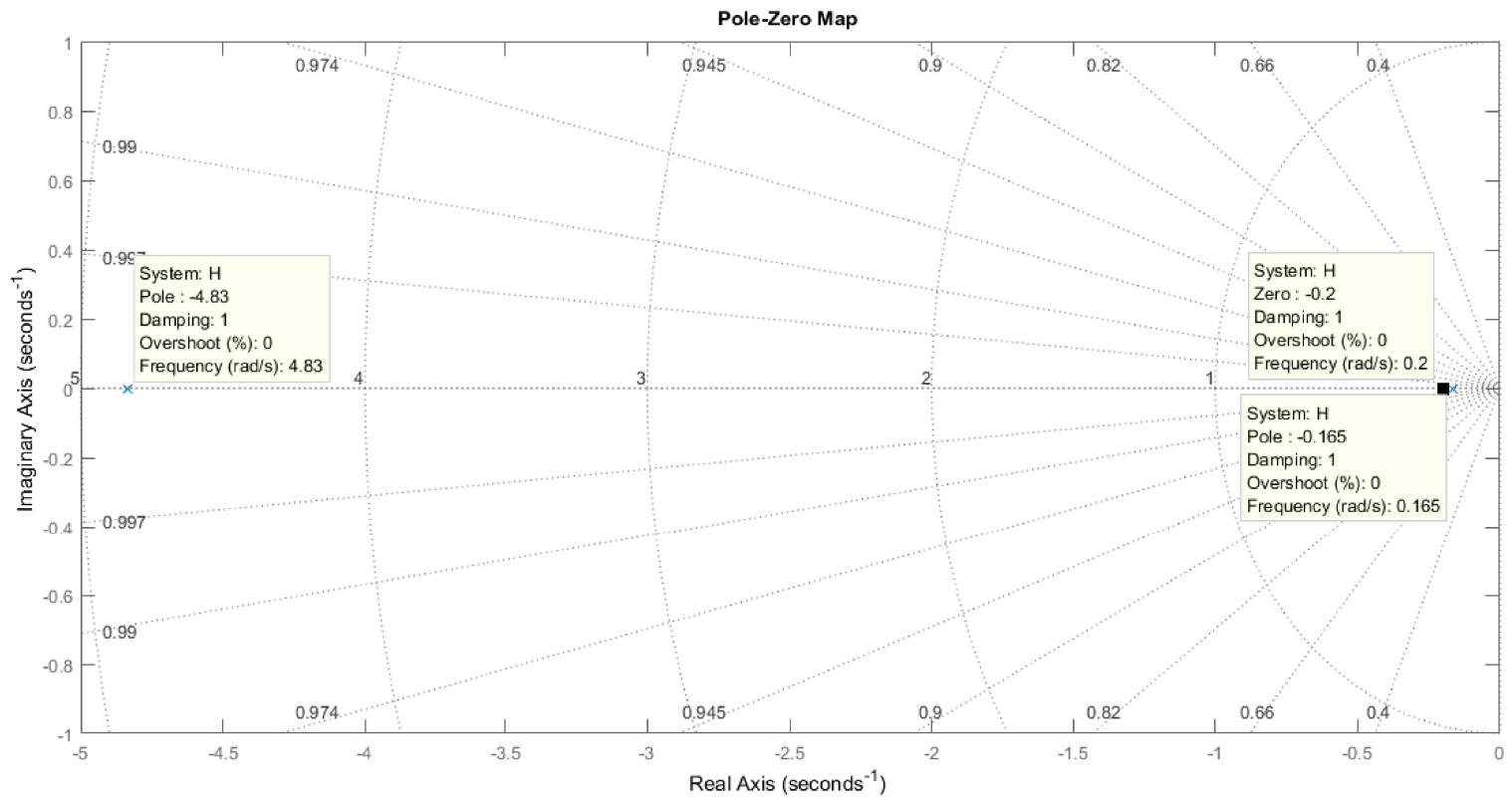


Figure 2: Pole-zero diagram of the system

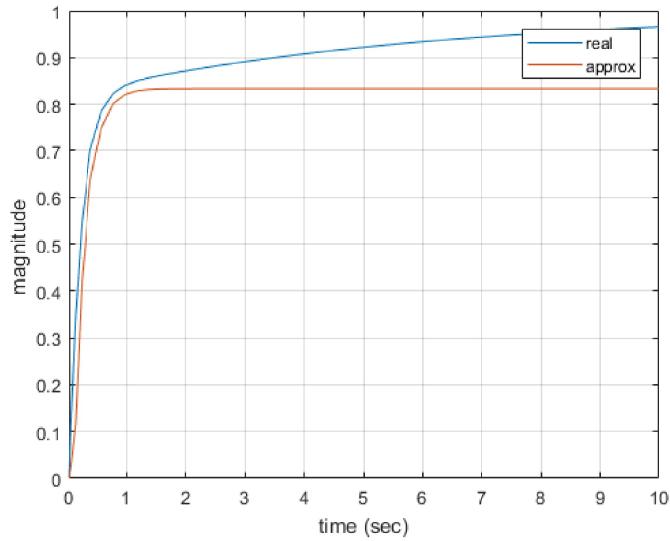


Figure 3: Step response of the system

$$T(s) = \frac{1}{0.25 + 1}$$

$$y(t) = 1 - e^{-4t}$$

$$\tau = 0.25 \text{ sec}$$

$$t_{settle} = 4\tau = 1 \text{ sec}$$

Since it is an overdamped system (before approximation) there is no overshoot.

$$t_{rise} = \frac{2}{a} = 0.5 \text{ sec}$$

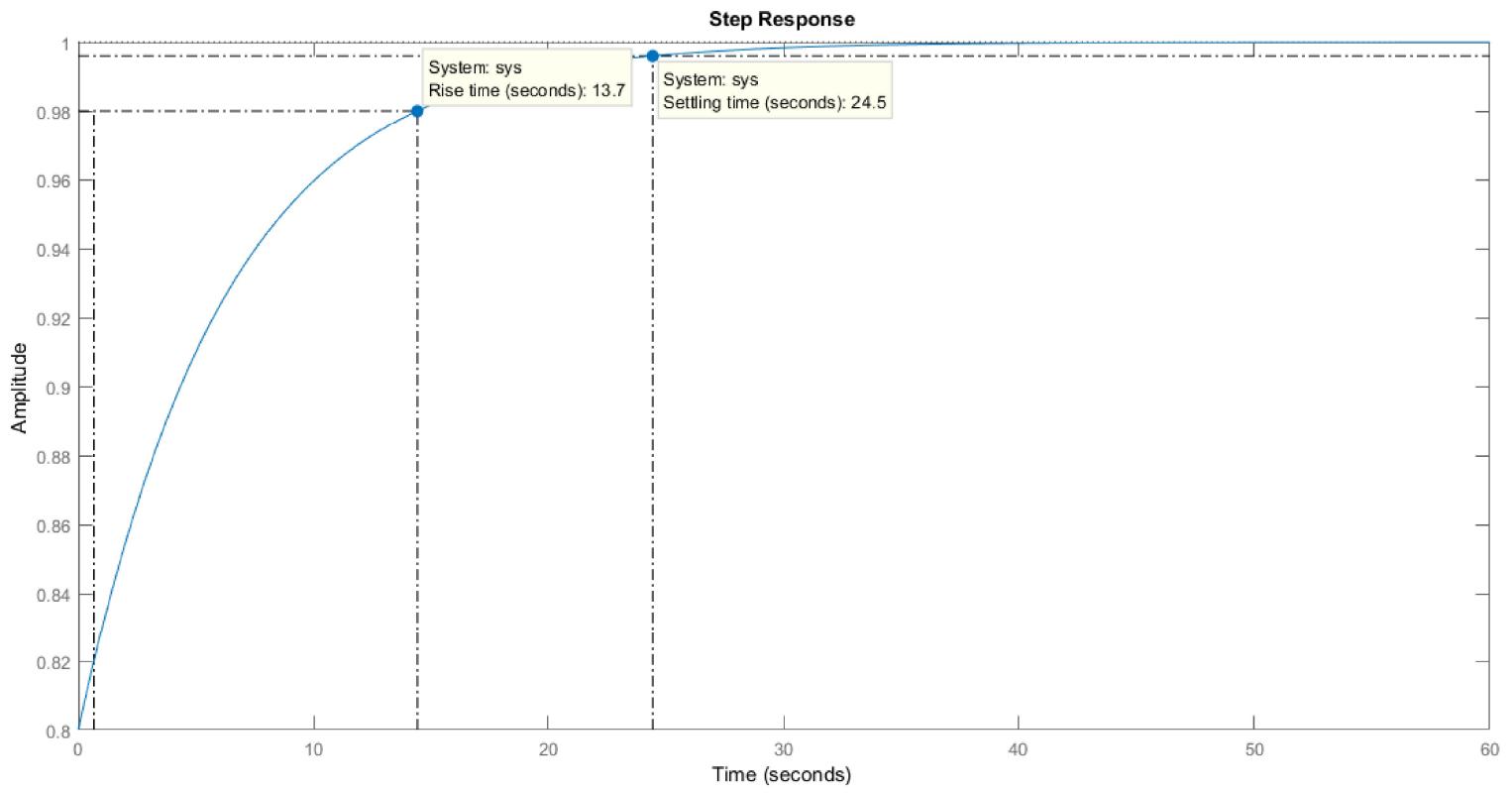


Figure 4: Step response of the system with PI controller where $P=4$ and $I=0.8$

d)

$$D(s) = \frac{4s + 10}{s}$$

$$G(s) = \frac{4s + 10}{(s + 1)}$$

$$T(s) = \frac{G(s)}{1 + G(s)}$$

$$T(s) = \frac{4s + 10}{s^2 + 5s + 10}$$

$$Y(s) = \frac{4s + 10}{s(s^2 + 5s + 10)}$$

$$Y(s) = \frac{1}{s} + \frac{Bs + C}{s^2 + 5s + 10}$$

$$Y(s) = \frac{1}{s} - \frac{s + 1}{s^2 + 5s + 10}$$

$$Y(s) = \frac{1}{s} - \frac{s + 1}{(s + 2.5)^2 + 3.75}$$

$$Y(s) = \frac{1}{s} - \frac{s + 2.5}{(s + 2.5)^2 + 3.75} - \frac{1.5}{(s + 2.5)^2 + 3.75}$$

$$y(t) = 1 - e^{-\frac{5t}{2}} (\cos(1.9t) - 0.77\sin(1.9t))$$

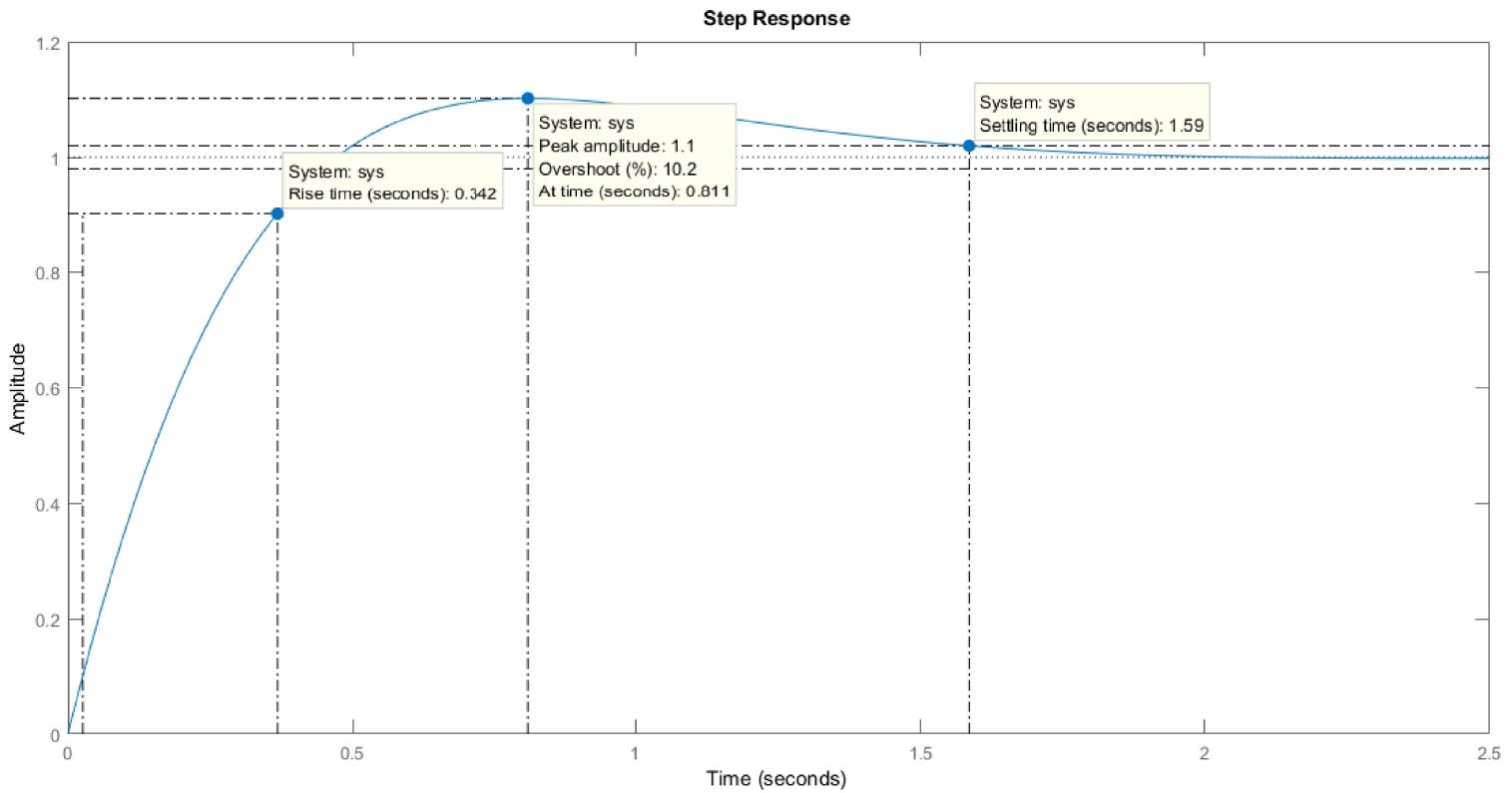


Figure 5: Step response of the system with PI controller P=4 I=10

Without controller unit i have significant steady state error. Using PI controller help about this issue. When i used I=0.8 system is to slow to response. However when i increase I to 10 system response time significantly improved. (Settling time 24.5 sec to 1.59 sec). But with increasing I there is overshooting problem but trade-off is okey.

3)

$$\frac{G(s)}{1+G(s)} = \frac{Ks+b}{s^2+as+b}$$

$$G(s)(s^2 + (a-k)s) = Ks + b$$

$$G(s) = \frac{Ks+b}{s(s+a-k)}$$

b)

$$K_v = \frac{1}{\lim_{s \rightarrow 0} sG(s)} = 0.04$$

$$\lim s \rightarrow 0 \frac{s^2r + sm}{s^2 + ns} = 25 \text{ by L'Hospital}$$

$$\frac{m}{n} = 25$$

$$T(s) = \frac{G(s)}{1+G(s)}$$

$$T(s) = \frac{rs + m}{s^2 + (n+r)s + m}$$

$$m = 25$$

$$n = 1$$

$$Y(s) = \frac{A}{s} + \frac{\delta(s+3)}{s^2 + 6s + 25} + \frac{\beta 5}{s^2 + 6s + 25}$$

$$n + r = 6$$

$$r = 5$$

c)

$$T(s) = \frac{5s + 25}{s^2 + 6s + 25}$$

$$Y(s) = \frac{5s + 25}{s(s^2 + 6s + 25)}$$

$$T(s) = \frac{5s+15}{(s+3)^2+4^2} + \frac{10}{(s+3)^2+4^2}$$

$$25A=25$$

$$A=1$$

$$\delta=5$$

$$\beta=\frac{5}{8}$$

d)

$$M_p = e^{\frac{-\pi}{tan\phi}}$$

$$\omega_n=5 \; \omega_d=4$$

$$\phi = arctan(\frac{4}{3})$$

$$M_p=0.105$$

4)

$$T(s) = \frac{\frac{G(s)}{s}}{1+\frac{G(s)}{s}}$$

$$T(s) = \frac{4}{s^2+2s+4}$$

$$\omega_n=2$$

$$\omega_d=\sqrt{3}$$

$$tan(\phi)=\sqrt{3}$$

$$M_p = e^{-\frac{\pi}{tan(\phi)}}$$

$$M_p=0.163$$

$$e_{unit} = \lim_{s \rightarrow 0} G(s)$$

$$e_{unit} = \lim_{s \rightarrow 0} \frac{4}{s(s+2)} = 0 \text{ Type1 system}$$

$$e_{ramp} = \lim_{s \rightarrow 0} sG(s) = 2$$

$$\lim_{s \rightarrow \inf} \frac{4K_i}{s+2} = 8$$

$$K_i=4$$

$$T(s) = \frac{16}{s^2 + 2s + 16}$$

$$\tan(\phi) = \frac{\sqrt{15}}{1}$$

$$M_p = 0.444$$

With increasing K_i increases overshoot percentage.

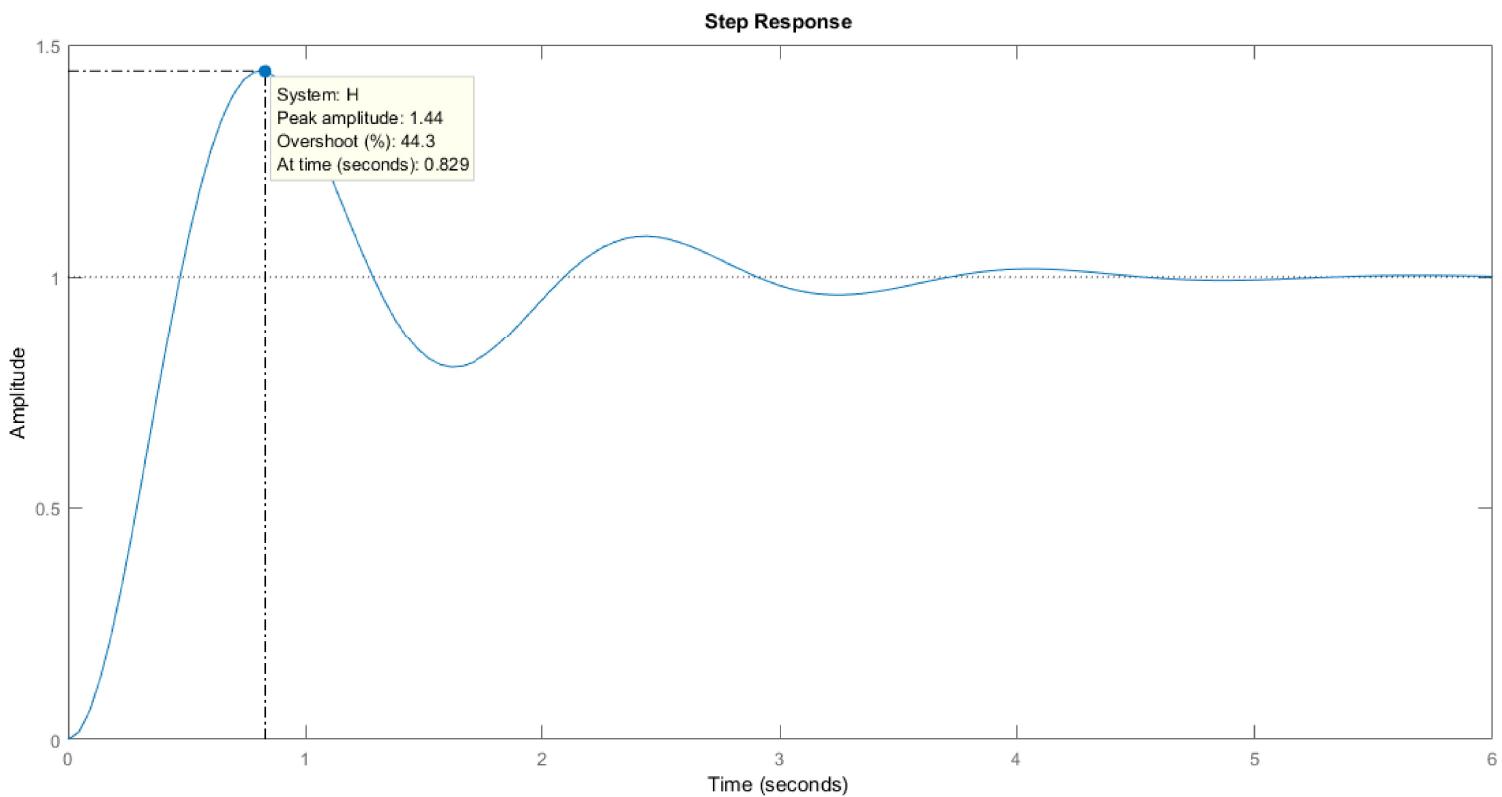


Figure 6: Step response $K_i = 4$

5)
a)

$$\begin{aligned}
A(s) &= \frac{100(s+2)}{s^2 - 1} \\
(E(s)G_c(s) + D(s))A(s) &= Y(s) \\
E(s) &= R(s) - Y(s) \\
((R(s) - Y(s))G_c(s) + D(s))A(s) &= Y(s) \\
R(s)G_c(s)A(s) + D(s)A(s) &= Y(s)(1 + G_c(s)A(s)) \\
G_r(s) &= \frac{G_c(s)A(s)}{1 + G_c(s)A(s)} \\
G_d(s) &= \frac{A(s)}{1 + G_c(s)A(s)}
\end{aligned}$$

b)

$$\begin{aligned}
C(s) &= G_c(s)A(s)E(s) + D(s)A(s) \\
C(s) &= R(s) - E(s) \\
E(s) &= \frac{R(s) - D(s)A(s)}{1 + A(s)G_c(s)} \text{ where } G_c(s) = 1 \text{ and } R(s) = \frac{1}{s} \\
E(s) &= \frac{\frac{1}{s} - \frac{100(s+2)}{s(s^2-1)}}{1 + \frac{100(s+2)}{s^2-1}} \\
e_r(\infty) &= \lim_{s \rightarrow 0} sE(s) \\
e_r(\infty) &= \lim_{s \rightarrow 0} \frac{1 - \frac{100(s+2)}{s^2-1}}{1 + \frac{100(s+2)}{s^2-1}} = -\frac{199}{201}
\end{aligned}$$

c)

$$\begin{aligned}
E(s) &= \frac{\frac{1}{s}}{1 + \frac{(s+\alpha)100(s+2)}{s(s^2-1)}} \\
e_r(\infty) &= \lim_{s \rightarrow 0} sE(s) \\
e_r(\infty) &= \lim_{s \rightarrow 0} \frac{1}{1 + \frac{(s+\alpha)100(s+2)}{s(s^2-1)}} = 0
\end{aligned}$$

d)

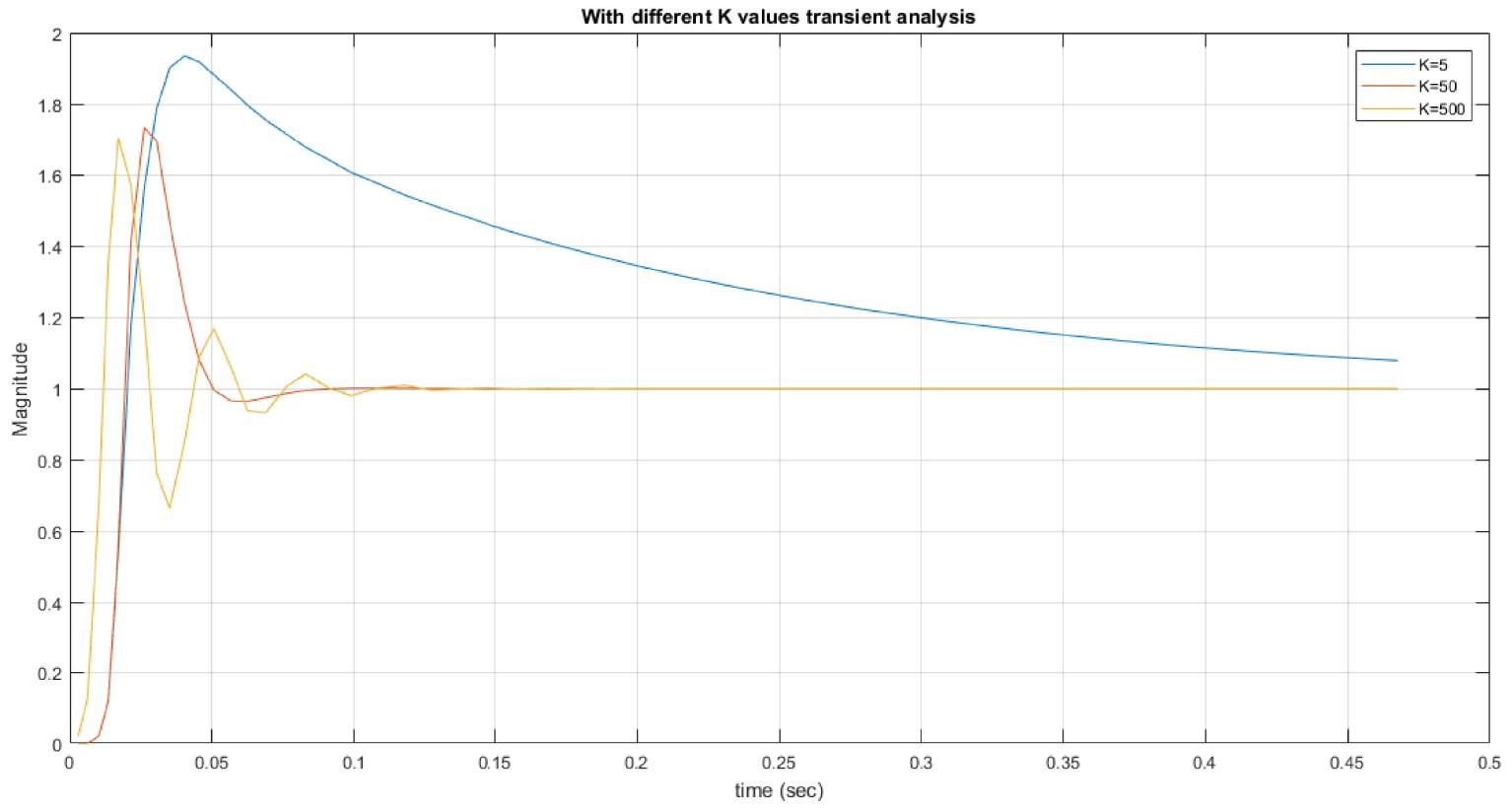


Figure 7: Transient Analysis

Increasing k means that we increased the integrator controller. With increasing integrator we have faster response means that shorter t_{rise} and $t_{settling}$. However the trade-off is oscillation frequency increased.

e)

$$E(s) = -\frac{\frac{100(s+2)}{s(s^2-1)}}{1 + \frac{100(s+2)}{(s^2-1)}}$$

$$e_r(\infty) = \lim_{s \rightarrow 0} sE(s)$$

$$e_r(\infty) = \lim_{s \rightarrow 0} -\frac{\frac{100(s+2)}{s^2-1}}{1 + \frac{100(s+2)}{s^2-1}} = \frac{200}{199}$$

With different K values transient analysis

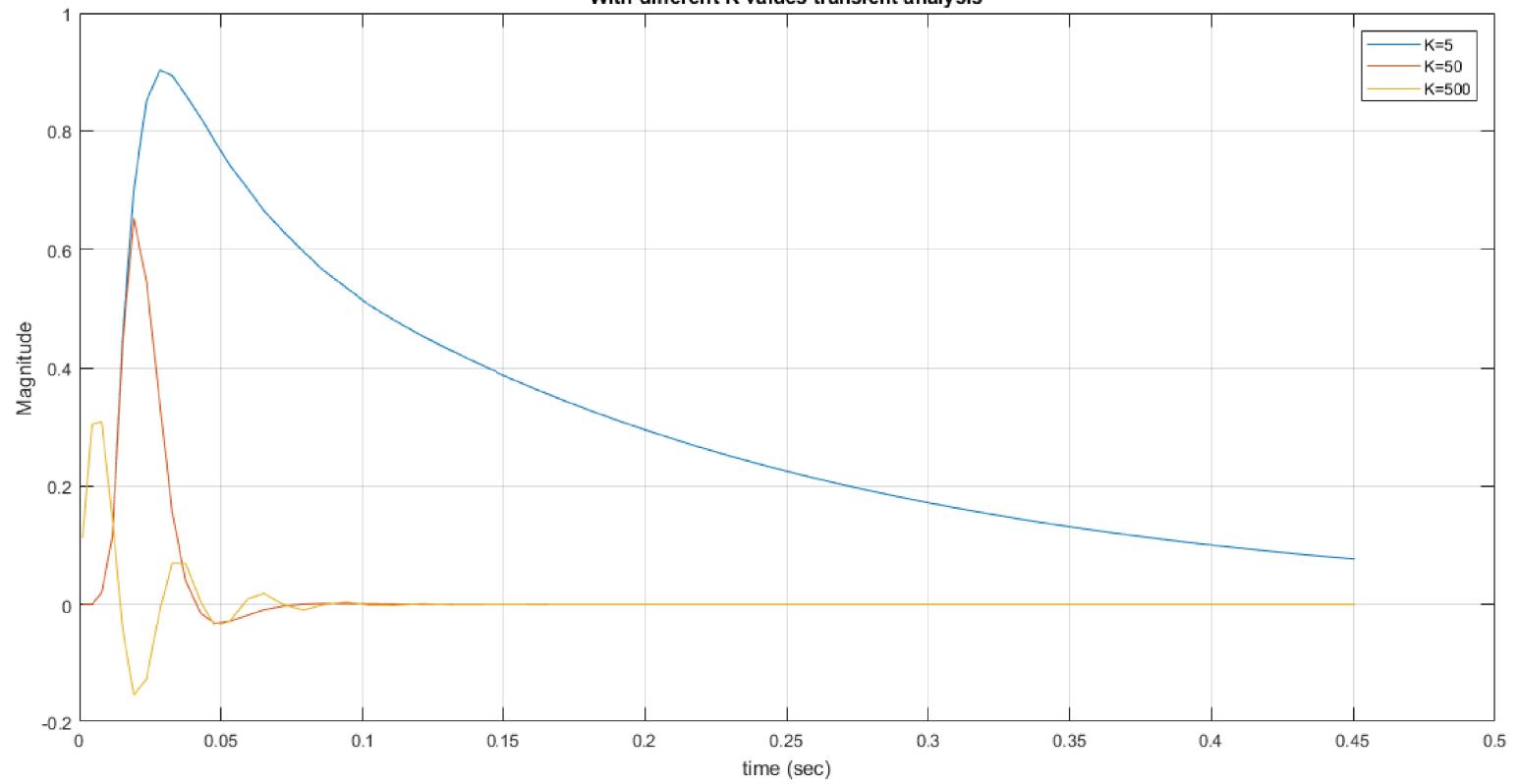


Figure 8: Transient Analysis