Plotting Bode plot with complex roots and zeros with hand

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Introduction

Given second order transfer function can be generalized following form;

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n}$$

where ξ is **damping ratio** and ω_n is natural frequency.

When $\xi > 1$ system response said over-damped, and it can easily separate two first order system cascaded.

Then transfer function rewritten in following form;

$$H(s) = \frac{K}{(s + \alpha)(s + \beta)}$$

Then body plot of that transfer function is straight-forward.

-Regulate the transfer function

$$H(s) = \frac{\frac{K}{\alpha\beta}}{(1 + \frac{s}{\alpha})(1 + \frac{s}{\beta})}$$

-Find the dc-gain (gain at $\omega = 0$)

$$G_{dc} = \frac{K}{\alpha\beta}$$

Since there is no zero (no s in the numerator) in the following function also no pole at zero (no zero in the denominator), plot came with straight line before first pole hit.

-Plot the asymptotes and bode

When $\xi = 1$ we said system response critically-damped. Denominator can rewrite as $(s+\omega_n)^2$.That means now we have double pole in a particular location. Bode plot algorithm is still same just make sure know we have $40\frac{dB}{decade}$

Complex poles and zeros

Complex poles

All points in second order term with $\xi < 1$ is we plot asymptotes like $\xi = 1$ case. With closing ξ to one (from zero), transfer function start behave;

$$H(s) = \frac{\omega_n}{(s+\omega_n)^2}$$

The can plot above function bode plot easily. So after that we just should edit a little by looking ξ, damping ratio.

$$H(s) = \frac{1}{s^2 + 2\xi s + 1}$$

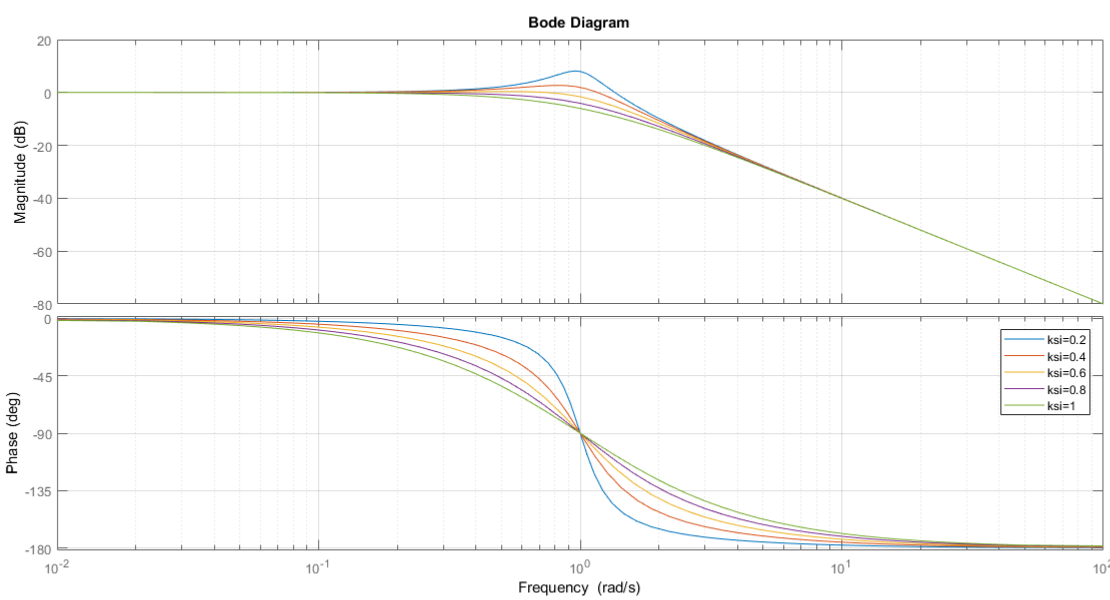

Figure 1: Bode plot of H(s) with different ξ

$\xi = 1$, we draw asymptotes and we did -3 dB approximation (for poles). However with $\xi < \frac{1}{\sqrt{2}}$ actual gain underline the other side of the asymptotes (positive over-shoot characteristic for poles). Cut-off frequency also changes slightly with changing $\xi < \frac{1}{\sqrt{2}}$

$$\omega_{n'} = \omega_n \sqrt{1 - 2\xi^2}$$

Complex zeros

$$H(s) = s^2 + 2\xi s + 1$$

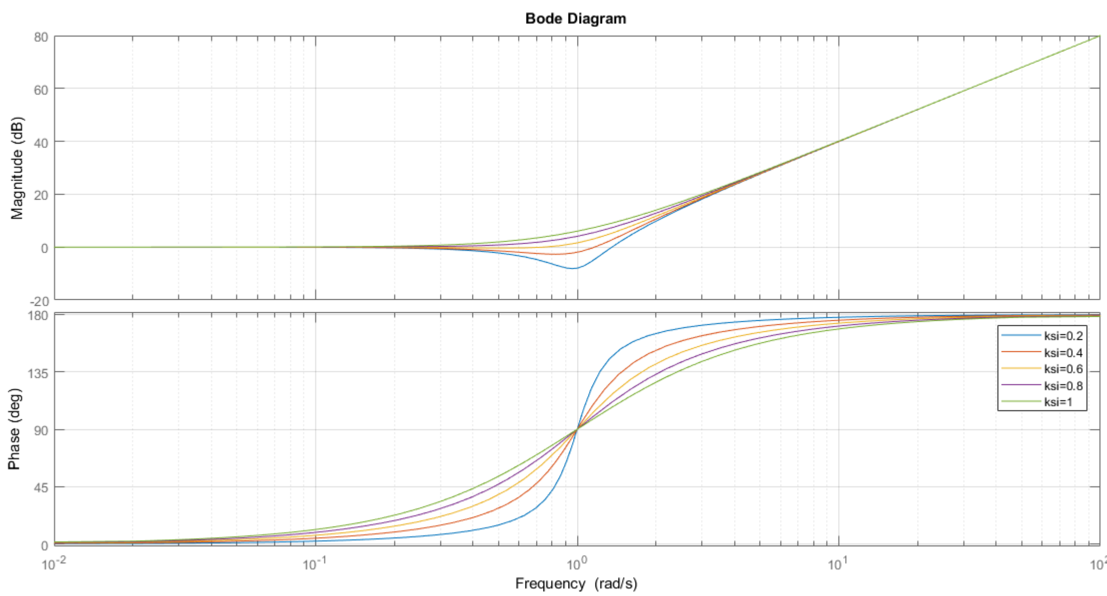

Figure 2: Bode plot of H(s) with different ξ

Similarly, $\xi = 1$, we draw asymptotes and we did +3 dB approximation (for poles). Now the actual gain again is at other side of the asymptote (negative over-shoot characteristic for zeros)

Example

$$H(s) = 128\frac{s^2+4}{(s+32)(s^2+8s+64)}$$

Behaviour at DC

System is type-0, no pole at $\omega = 0$ also no zero at $\omega = 0$ therefore straight line comes from the left side of the graph

DC gain

Normalizing the transfer function;

$$H(s) = \frac{1}{4}\frac{1+\frac{s^2}{4}}{(1+\frac{s}{32})(1+\frac{s}{8}+\frac{s^2}{64})}$$

$$G_{dc} = 20log(\frac{1}{4}) = -12\,dB$$

Second order Behaviour

Second order pole

$$s^2 + 2\xi\omega_n + \omega_n^2 = s^2 + 8s + 64$$

$$\omega_n = 8$$

$$\xi = 0.5$$

Double pole at $\omega = 8$

Second order zero

$$s^2 + 2\xi\omega_n + \omega_n^2 = s^2 + 8$$

$$\omega_n = 2$$

$$\xi = 0$$

Double zero at $\omega = 2$
Matlab plot;

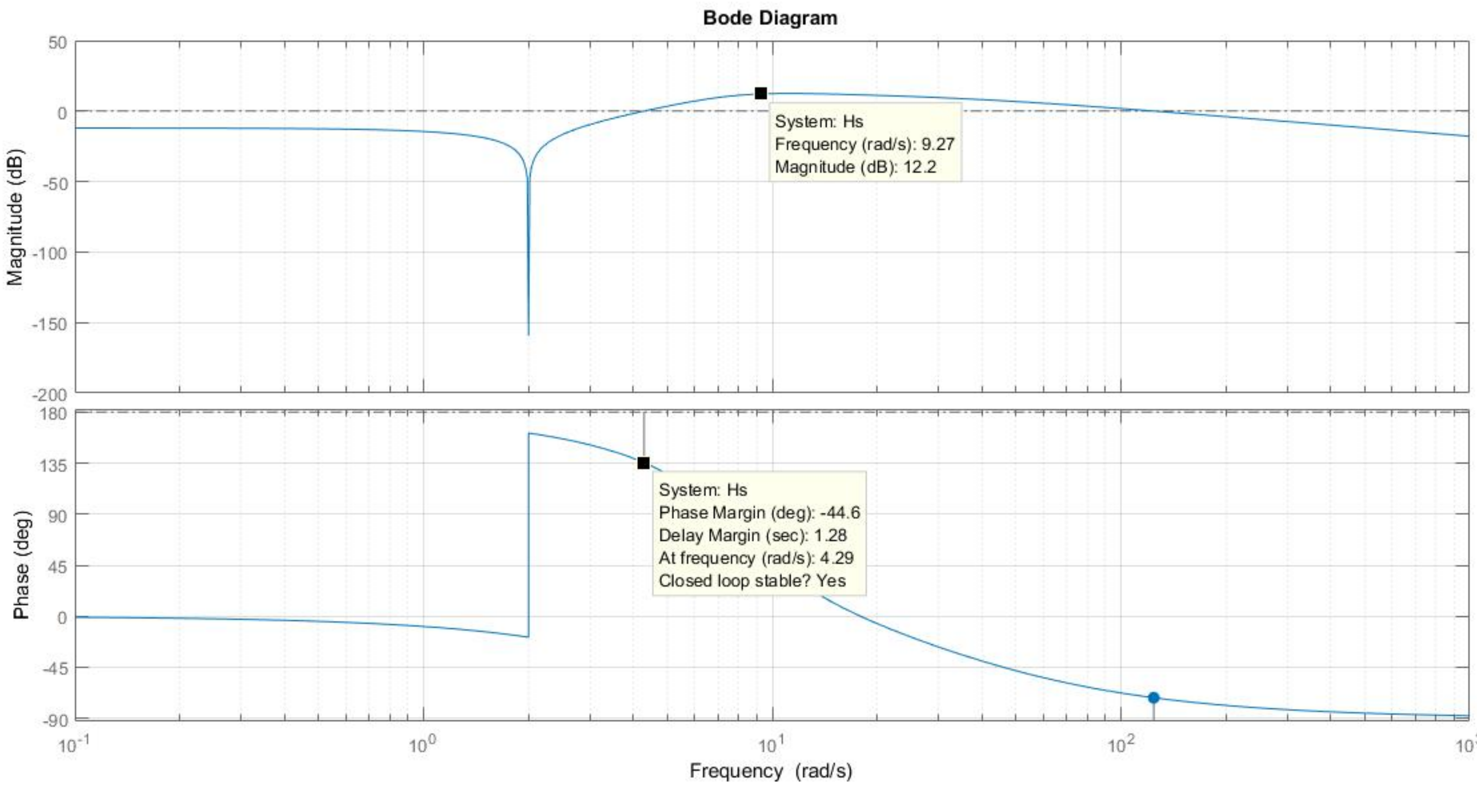

Figure 3: Bode plot of H(s) with different ξ

By hand;