

EE302 Homework 4

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Question 1

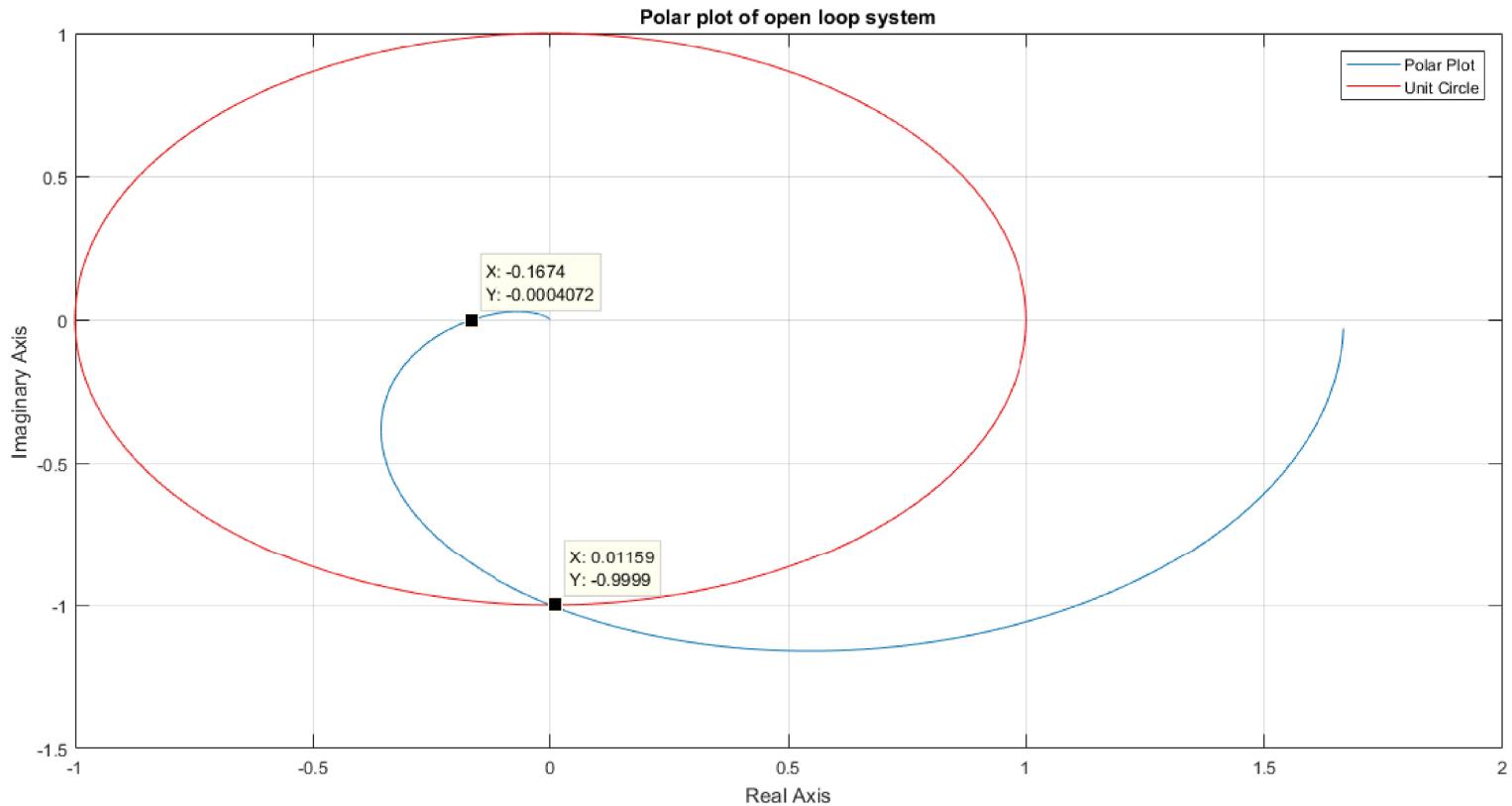


Figure 1: Polar plot of the system

The point that cross the real axis is $x_1 = -0.1674$. Then the gain margin ;

$$GM = 20\log\left(\frac{1}{|a|}\right) = 15.5249 \text{ dB}$$

The point where plot cross the unity circle is $x_2 = 0.01159$ and $y_2 = -1$. Then the phase margin is the angle between that point and the origin.

$$PM = 90^\circ$$

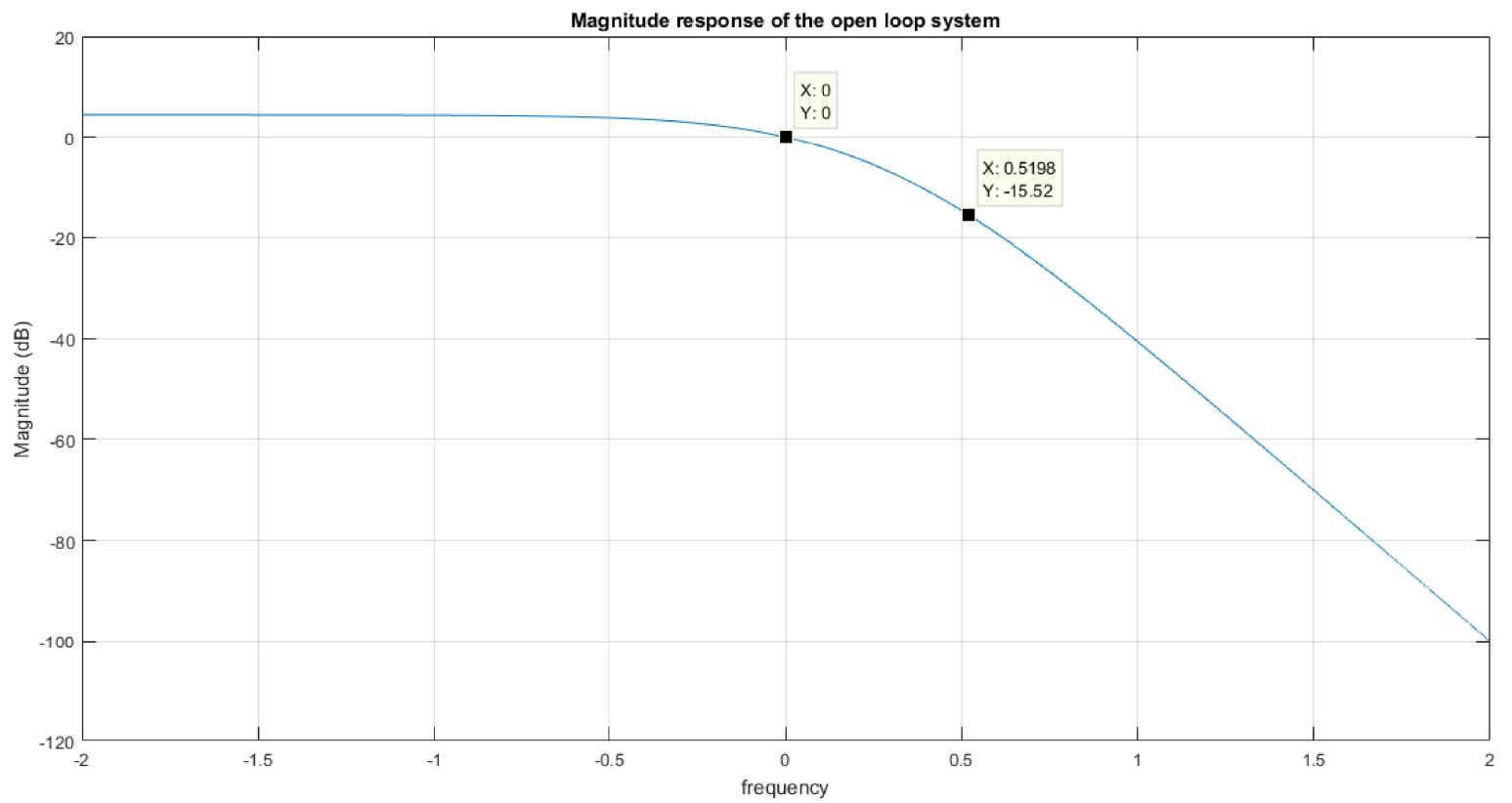


Figure 2: Magnitude response of the system

Gain cross-over frequency is the frequency that system has 0 dB gain.

$$f_{gain_{cross-over}} = 0 \frac{rad}{sec} (dc)$$

To find gain margin of the system, first we find phase cross-over frequency.

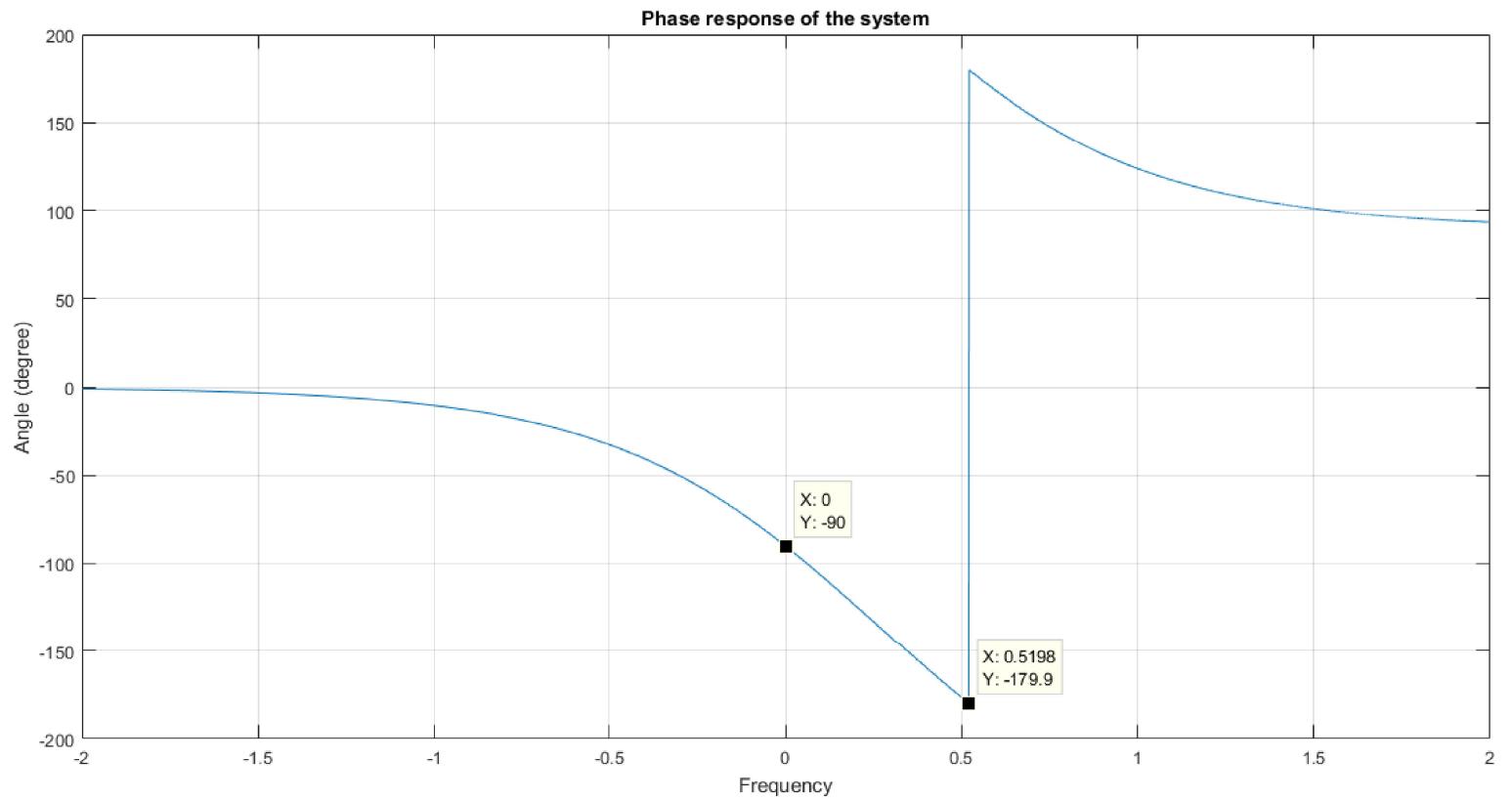


Figure 3: Phase response of the system

$$f_{phase\,cross-over} = 0.5198 \frac{rad}{sec}$$

Then the gain margin and phase margin is following

$$GM = 15.52 dB$$

$$PM = 90^\circ$$

In both method i find exactly same gain and phase margin.

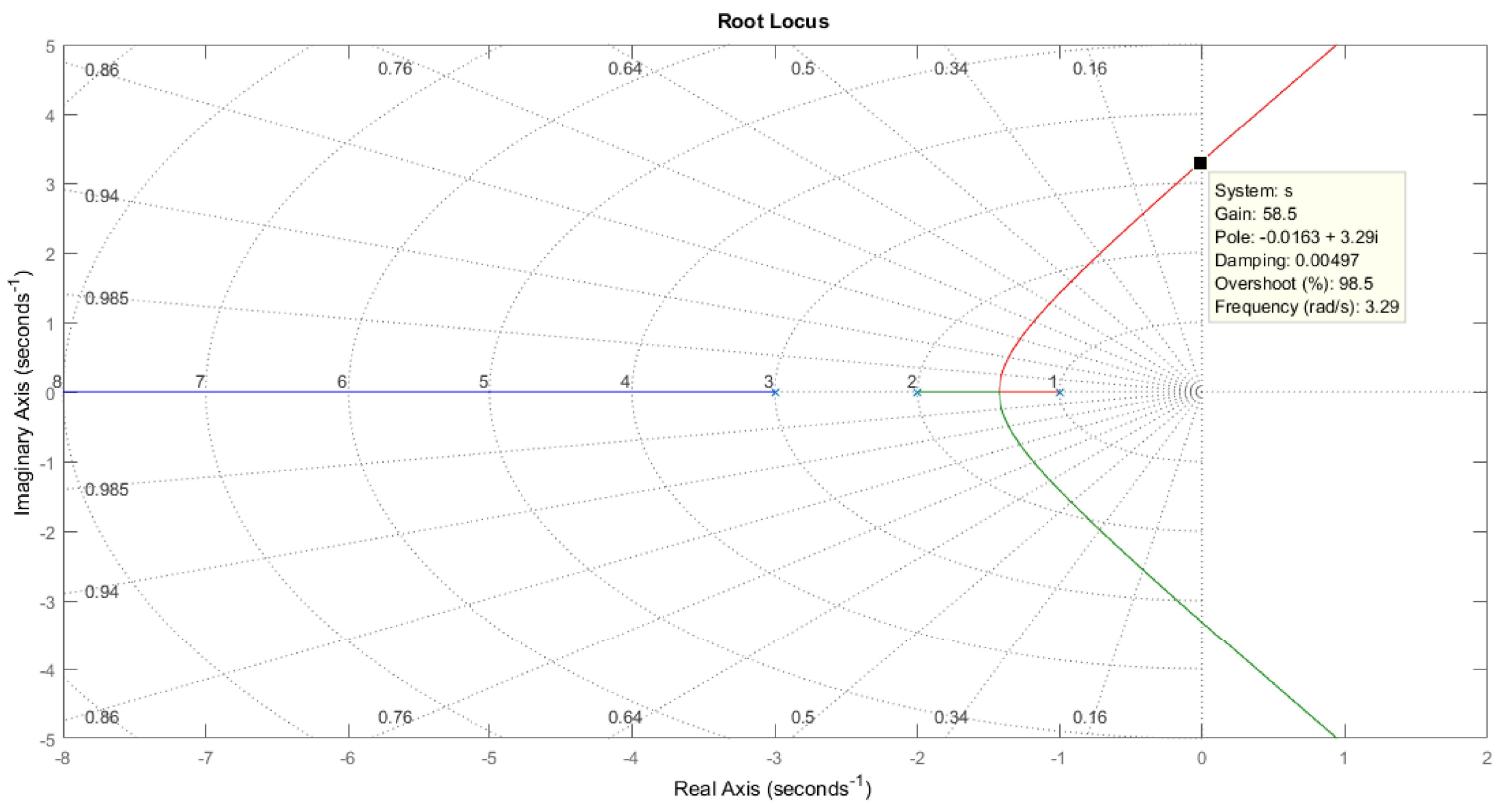


Figure 4: Root-locus

$$K_{original} = 10$$

$$K_{unstable} = 58.5$$

$$GM = 20 \log\left(\frac{K_{unstable}}{K_{original}}\right) = 15.3431 \text{ dB}$$

Question 2

```

%%
%Compensator design
desired_pm = 65;
current_pm = 49.9;
margin = -2;
phi_max = (desired_pm-current_pm + margin)*pi/180;
alfa = (1-sin(phi_max))/(1+sin(phi_max));
pointer = 10^log10(alfa);
%%
%Finding compensator and plotting its bode
wgc = 0.3136 %pointer frequency value
T = 1/(sqrt(alfa)*wgc)
s = tf([1 0],[1]);
Gc = (1+s*T)/(1-T*alfa*s);
%%
s = tf([1 0],[1]);
Gc = (1+s*T)/(1-T*alfa*s);

```

Figure 5: Magnitude response of the system using experimental data

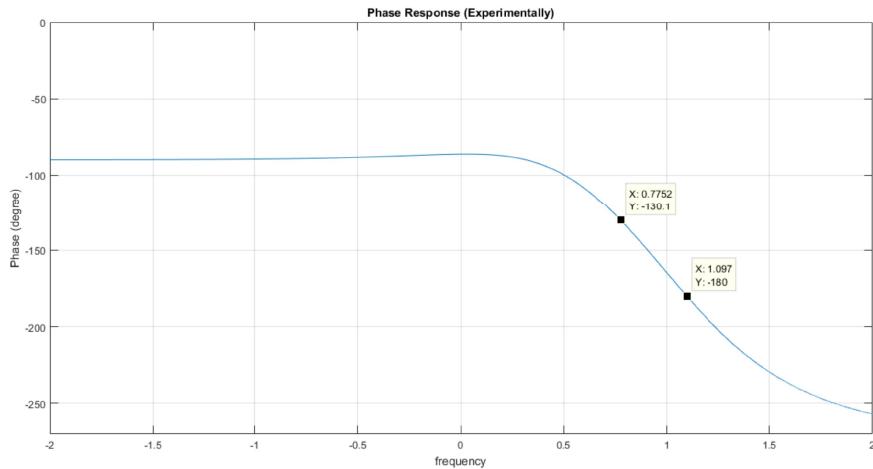


Figure 6: Phase response of the system using experimental data

$$f_{gain_{cross-over}} = 0.7752 \frac{rad}{sec}$$

$$f_{phase_{cross-over}} = 1.097 \frac{rad}{sec} (dc)$$

$$GM = 10.64 dB$$

$$PM = 49.9^\circ$$

Transfer function of the compensator is following;

$$G_c = \frac{1 + 1.297s}{1 + 1.64s}$$

Matlab script to find its compensator parameters;

```
%  
%Compensator design  
desired_pm = 55;  
current_pm = 49.9;  
margin = -2;  
phi_max = (desired_pm-current_pm + margin)*pi/180;  
alfa = (1-sin(phi_max))/(1+sin(phi_max));  
pointer = 10*log10(alfa);  
%  
%Finding compensator and plotting its bode  
wgc = 0.8136 %pointer frequency value  
T = 1/(sqrt(alfa)*wgc)  
s = tf([1 0],[1]);  
Gc = (1+T*s)/(1+T*alfa*s)  
%  
s = tf([1 0],[1]);  
Gc = (1+T*s)/(1+T*alfa*s)
```

Figure 7: Matlab script to find compensator parameters

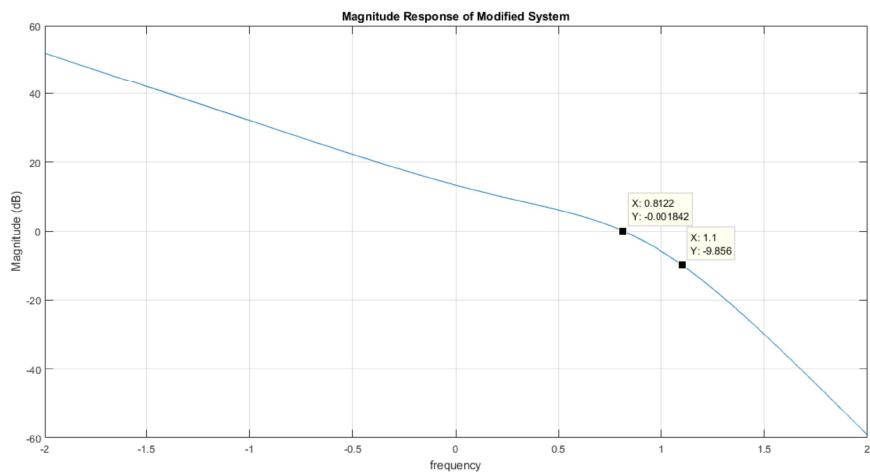


Figure 8: Magnitude response of the system with lead compensator

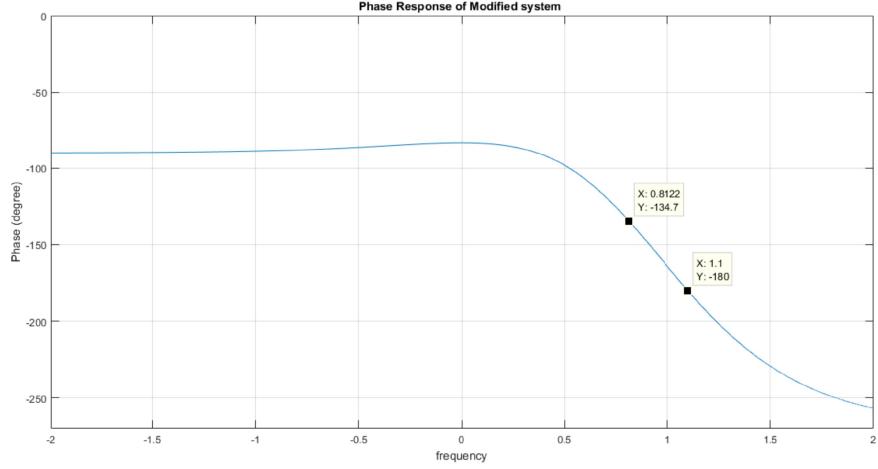


Figure 9: Phase response of the system with lead compensator

$$PM = 55.3^\circ$$

$$GM = 9.856 \text{ dB}$$

Designing phase lag compensator is following;

$$\phi = -180 + \phi_{desired} + 5$$

From phase response desired magnitude is following

$$\log(\omega_{gc}) = 0.5623$$

$$\omega_{gc} = 3.61 \frac{\text{rad}}{\text{sec}}$$

$$|G(j\omega_{gc})| = 4.219 \text{ dB} \quad 20\log(\alpha) = 4.219 \text{ dB}$$

$$\alpha = 1.62$$

$$T = \frac{10}{\omega_{gc}} = 2.77$$

$$T(s) = \frac{1 + Ts}{1 + T\alpha s} = \frac{1 + 2.77s}{1 + 4.48s}$$

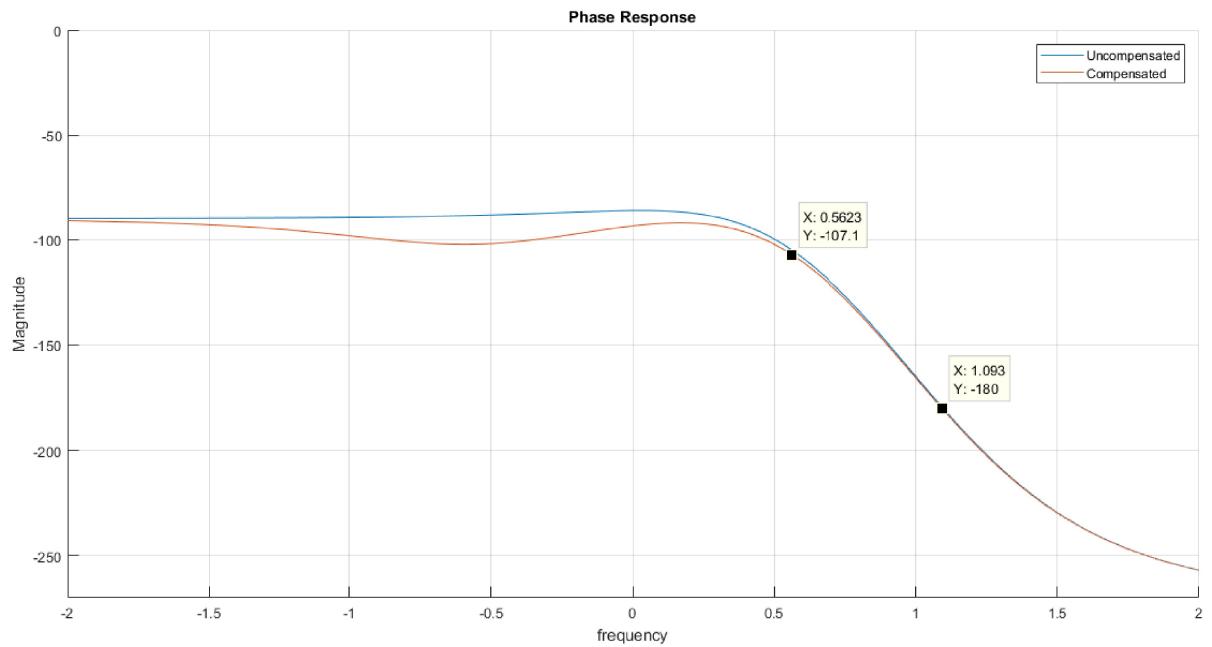


Figure 10: Compensated phase response

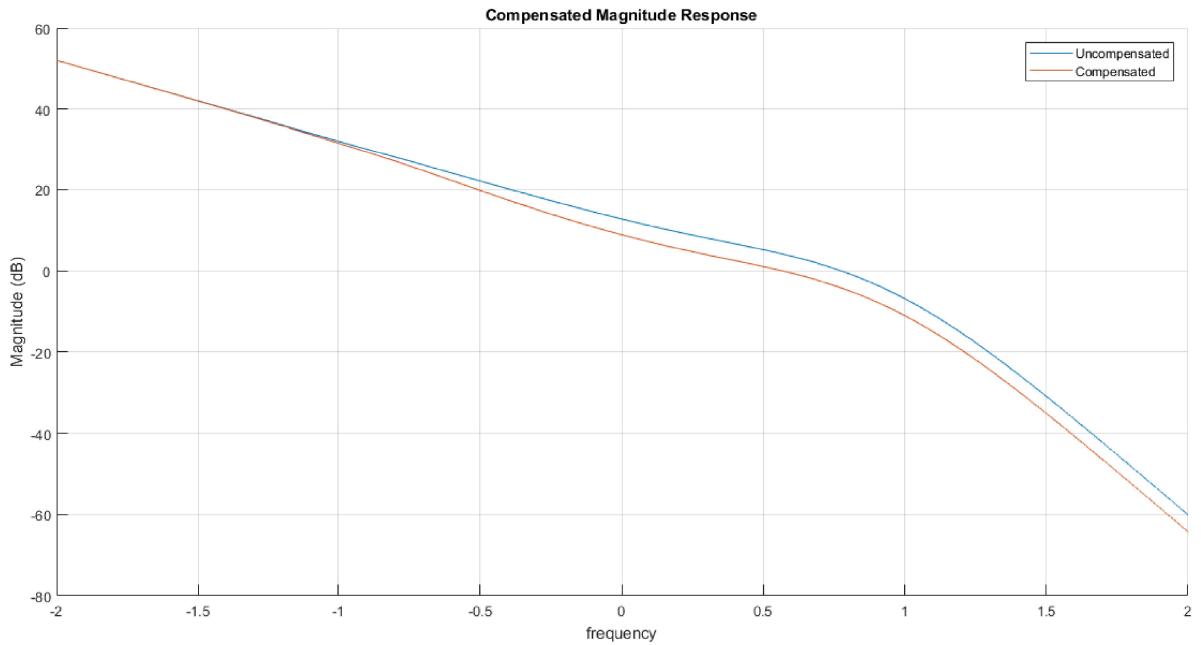


Figure 11: Compensated magnitude response

$$PM = 72.9^\circ$$

$$GM = 14.69 \text{ dB}$$

Question 3

$$e_{ss} < 0.25$$

$$G(s) = \frac{0.2}{s^2(s + 100)}$$

$$K_a = \lim_{s \rightarrow 0} s^2 K G(s)$$

$$K \geq 2000$$

In order to find gain cross-over frequency following equation must solve for ω ;

$$G(j\omega) = \frac{(0.2)2000}{j\omega^2(j\omega + 100)} = 1$$

$$\omega \simeq 2 \frac{\text{rad}}{\text{sec}}$$

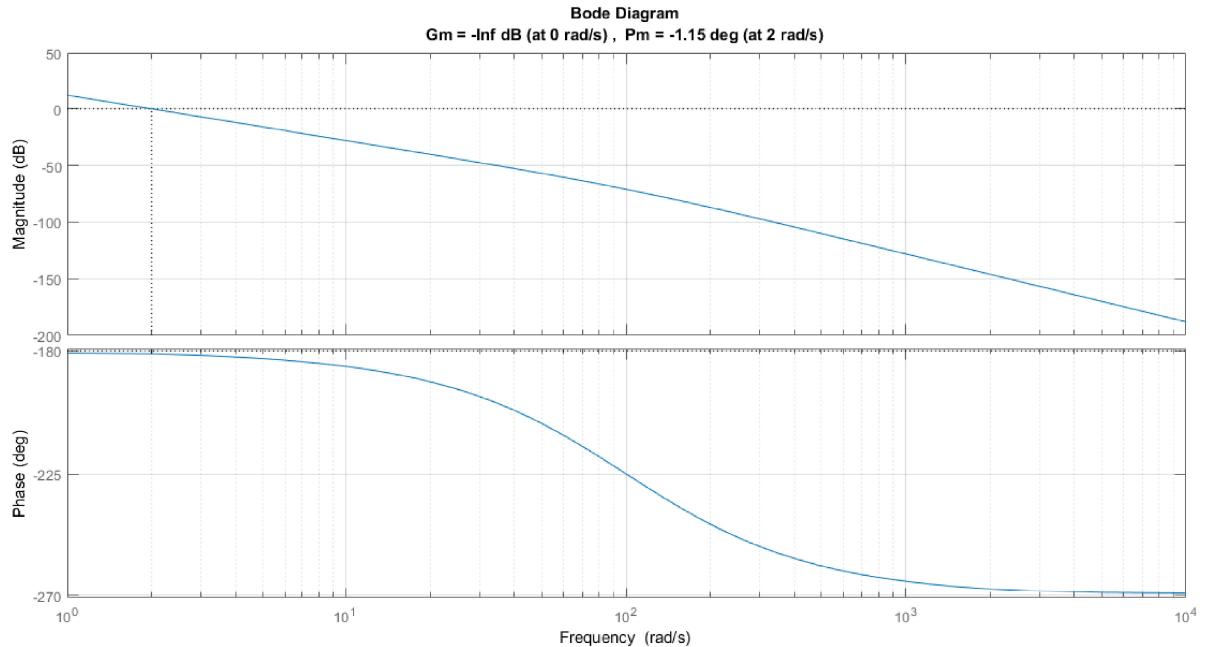


Figure 12: Analysing circuit for compensator selection

Since phase margin is too low i chose lead compensator for improving phase margin.

$$\phi_{max} = \phi_{desired} + PM + 5$$

$$\alpha = \frac{1 - \sin(\phi_{max})}{1 + \sin(\phi_{max})} = 0.16$$

$$|G(j\omega)| = \sqrt{\alpha} = 0.4$$

$$\frac{400}{\omega^2(\sqrt{10^4 + \omega^2})} = 0.4$$

$$\omega \simeq 3.14$$

$$T = \frac{1}{\omega\sqrt{\alpha}}$$

$$T=0.80$$

$$G_c=\frac{1+Ts}{1+\alpha Ts}=\frac{1+0.8s}{1+2.51s}$$