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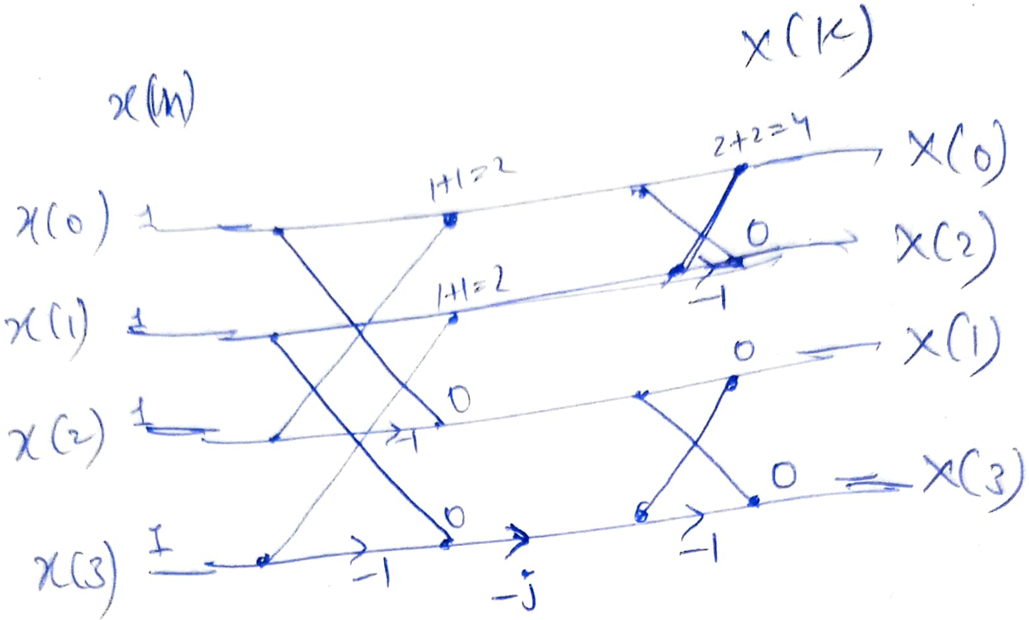
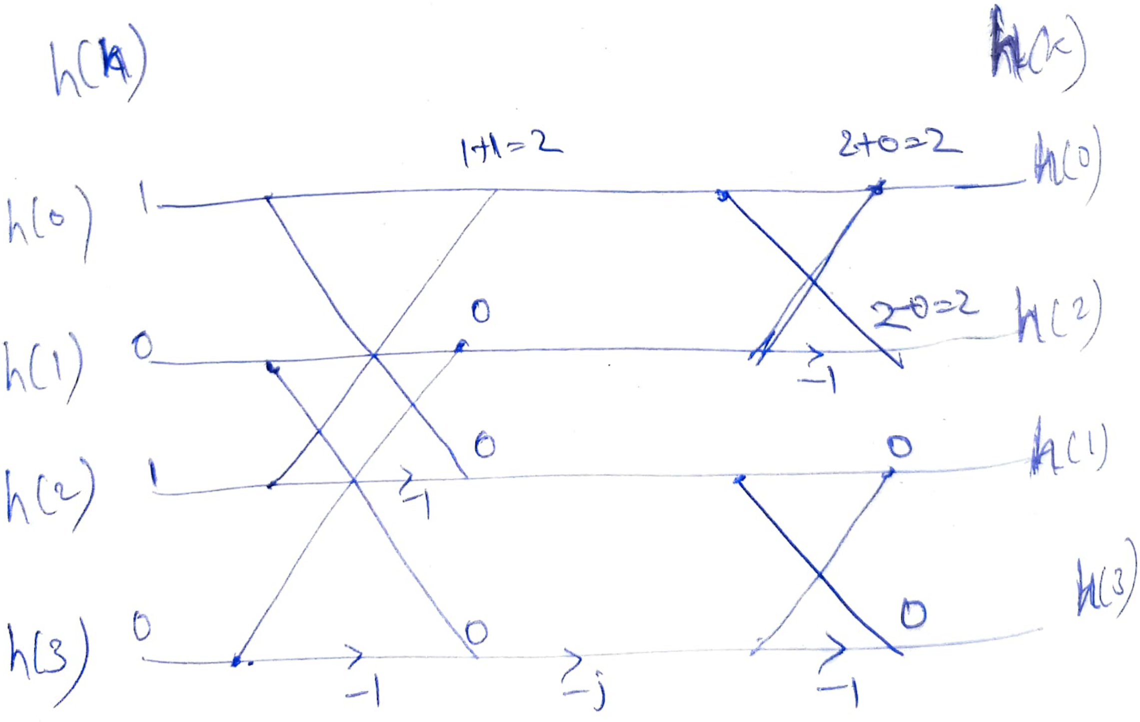
2020101074

Assignment-3

Q1. $y(n) = x(n) * h(n)$



$X(K) = \{4, 0, 0, 0\}$



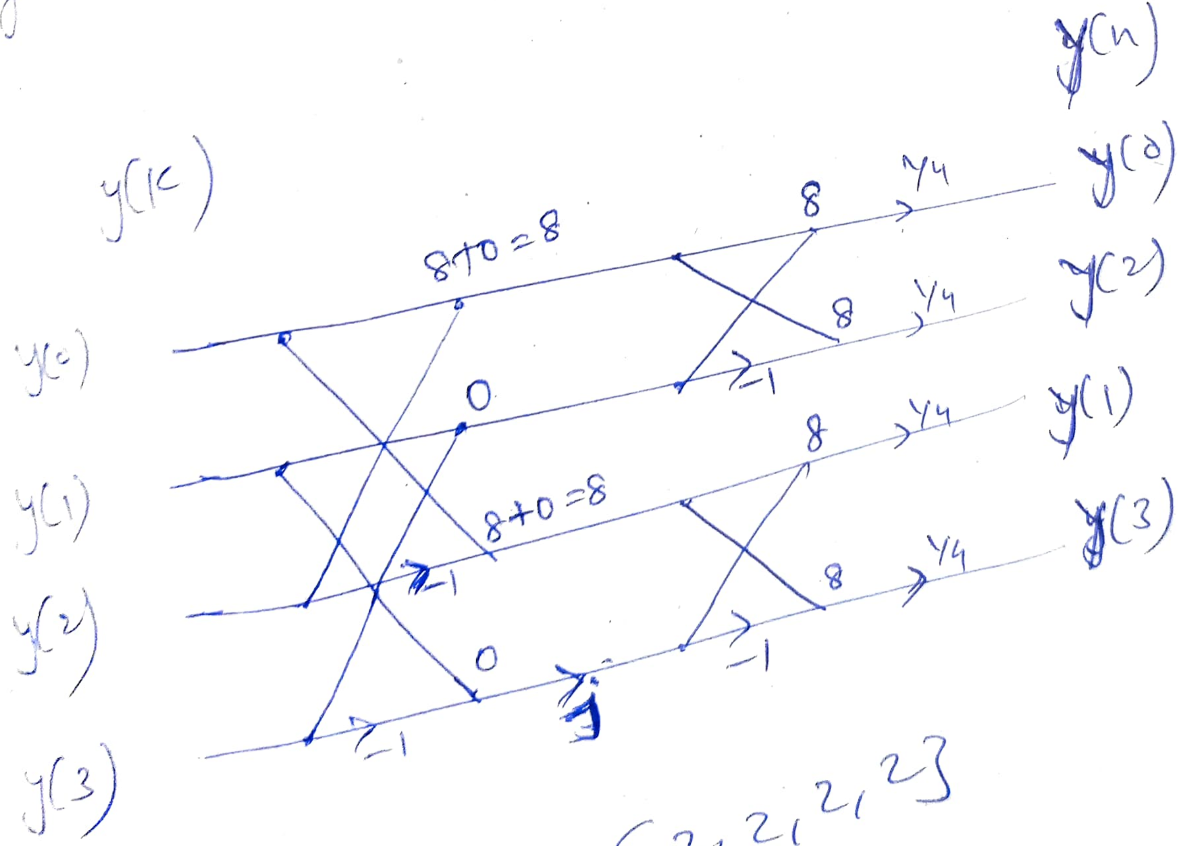
$h(K) = \{2, 0, 2, 0\}$

$$y(k) = x(k) \cdot h(k)$$

$$= \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$y(k) = \{8, 0, 0, 0\}$$

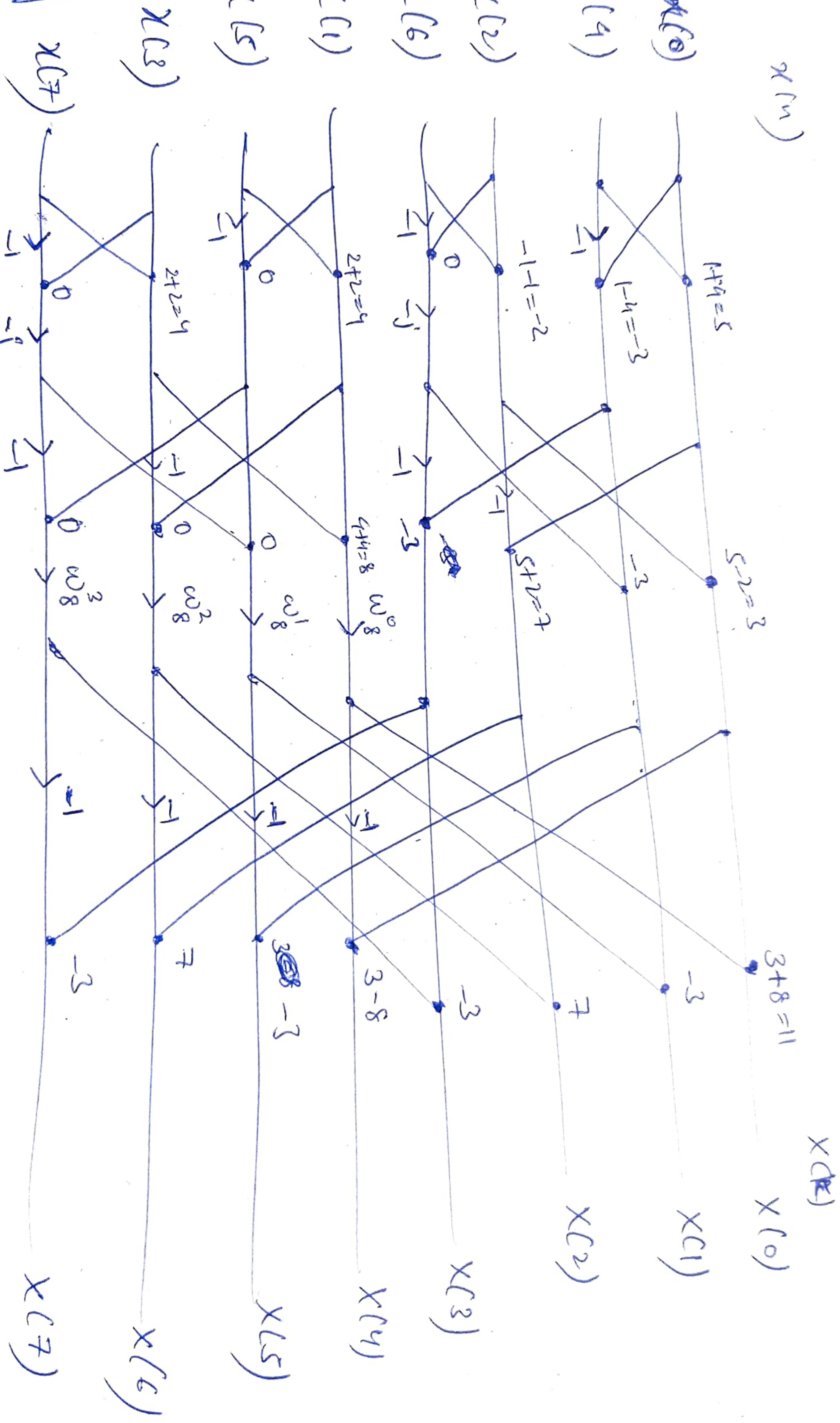
get $y(n)$ by IDFT on $y(k)$



$$y(n) = \{2, 2, 2, 2\}$$

Q2 x[k) DIT FFT 8 point

N=8



X(k) = {11, -3, 7, -3, -5, -3, 7, -3}

Q3 Z transform & ROC

(i) $x(n) = \left(\frac{1}{4}\right)^n u(n)$

$$x(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n u(n)\, z^{-n}$$

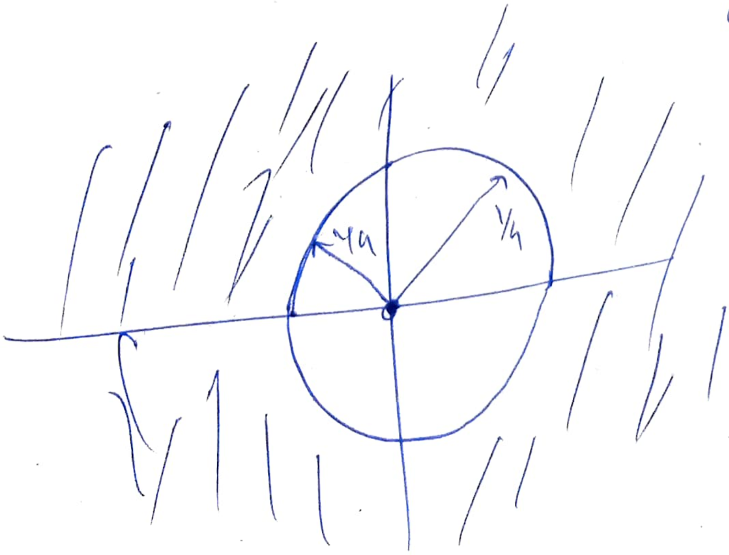
$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n}$$

$$= 1 + \left(\frac{z^{-1}}{4}\right) + \left(\frac{z^{-1}}{4}\right)^2 + \dots$$

$$\boxed{x(z) = \frac{1}{1 - \frac{z^{-1}}{4}}}$$

for ROC $\left|\frac{z^{-1}}{4}\right| < 1$

$$|z| > 1/4$$



ROC = outside the circle of radius 1/4.

(ii) $x(n) = [5(2^n) - 4(3^n)]u(n)$

$$x(z) = \sum_{n=0}^{\infty} (5(2^n) - 4(3^n)) z^{-n}$$

$$= 5\sum_{n=0}^{\infty} (2z^{-1})^n - 4\sum_{n=0}^{\infty} (3z^{-1})^n$$

$$= \frac{5}{1-2z^{-1}} - \frac{4}{1-3z^{-1}}$$

for both to converge both need to be finite, hence

$$|2z^{-1}| < 1 \quad \& \quad |3z^{-1}| < 1$$

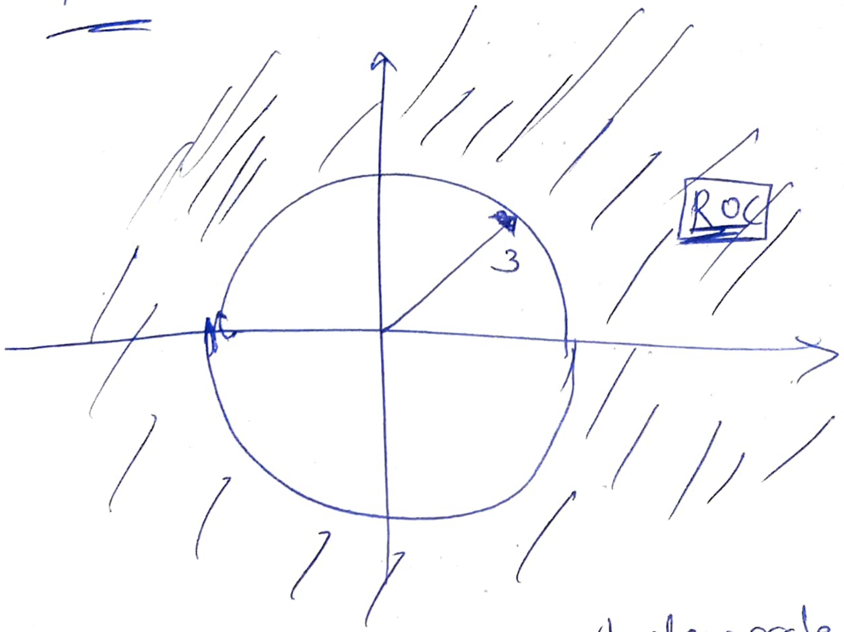
i.e ROC of $x(z) = R_1 \cap R_2$

where R_1, R_2 are ROCs of $5(2^n)$ & $4(3^n)$

$\therefore$ $|z| > 2$ & $|z| > 3$

$\therefore$ $\boxed{|z| > 3}$

ROC :



shaded region : outside circle of radius 3

(iii) $x(n) = n a^n u(n)$

$$x(z) = \sum_{n=0}^{\infty} n a^n z^{-n}$$

$$x(z) = 0 + 1 \cdot a z^{-1} + 2 \cdot (a z^{-1})^2 + 3 \cdot (a z^{-1})^n + \dots$$

$$x(z)\, a z^{-1} = 1 \cdot (a z^{-1})^2 + 2 \cdot (a z^{-1})^3 + \dots$$

$$x(z)(1 - a z^{-1}) = 1 \cdot a z^{-1} + 1 \cdot (a z^{-1})^2 + 1 \cdot (a z^{-1})^3 + \dots$$

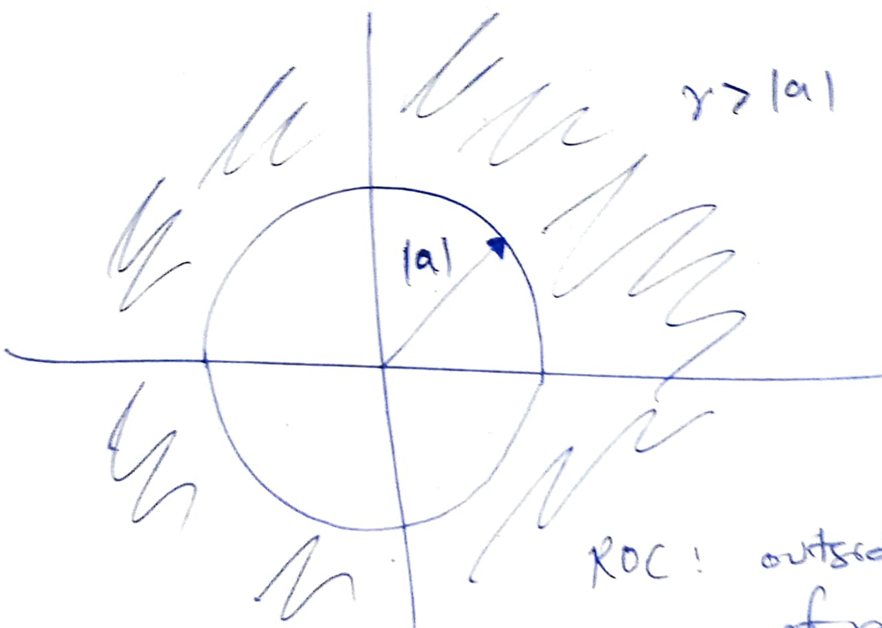
$$x(z) = \frac{a z^{-1}}{1 - a z^{-1}} \times \frac{1}{1 - a z^{-1}}$$

$$x(z) = \frac{az^{-1}}{(1-az^{-1})^2}$$

but the sum of infinite GP is finite only when,

$$|az^{-1}| < 1$$

$$\therefore \ |z| > |a|$$



ROC : outside circle of radius $|a|$.

(iv) $x(n) = \{3, 4, \underset{\uparrow}{8}, 7, 0, 4\}$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= 0 + x(-2) z^{2} + x(-1) z^{1} +$$

$$x(0) + x(1) z^{-1} + x(2) z^{-2}$$

$$+ x(3) z^{-3} + 0$$

$$x(z) = 3z^{2} + 4z + 8 + 7z^{-1} + 0 +$$

$$4z^{-3}$$

ROC = $C - \{0\}$

(entire plane except origin)

(1) $x(n) = a^n u(n) + b^n u(-n-2)$

$$x(z) = \sum_{n=-\infty}^{\infty} a^n u(n) z^{-n} + \sum_{n=-\infty}^{\infty} b^n u(-n-2) z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n z^{-n} + \sum_{n=-\infty}^{-2} b^n z^{-n}$$

$$x(z) = \underbrace{\frac{1}{1-az^{-1}}}_{p(z)} + \underbrace{\frac{z^2/b^2}{1-z/b}}_{q(z)}$$

$\text{ROC } x(z) = \text{ROC } p(z) \cap \text{ROC } q(z)$

$\text{ROC } p(z) \rightarrow \left|\frac{a}{z}\right| < 1$

$\therefore |z| > |a|$

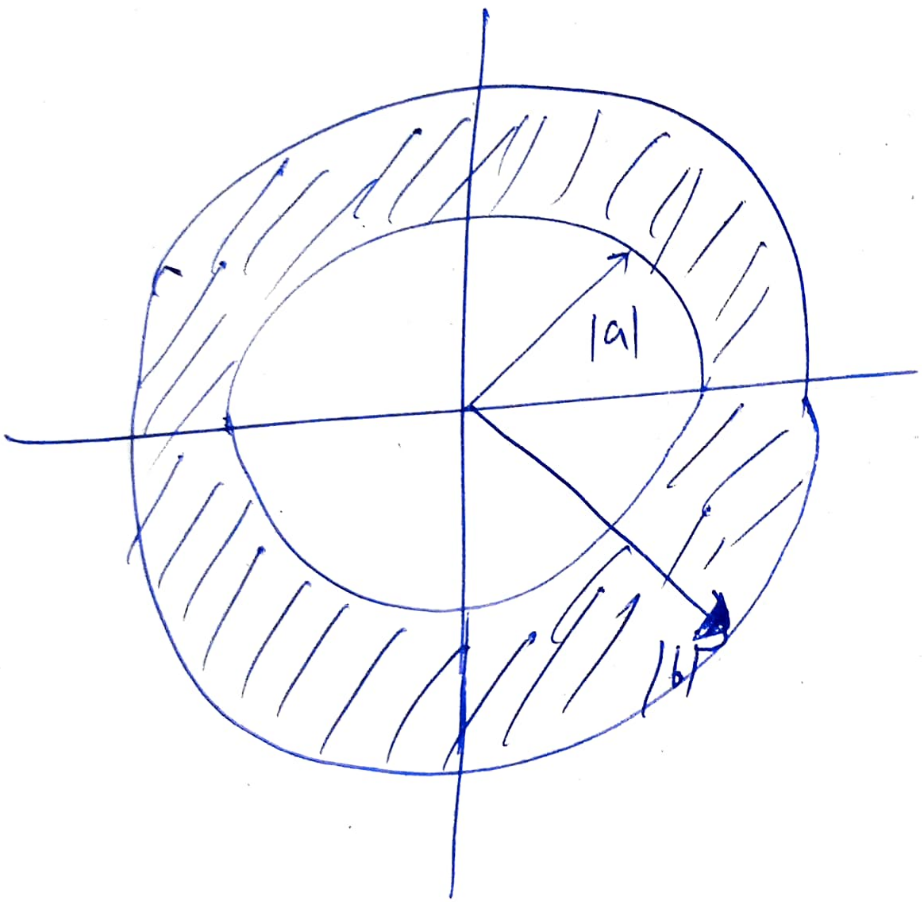
$\text{ROC } q(z) \rightarrow \left|\frac{z}{b}\right| < 1 \Rightarrow |z| < |b|$

$\therefore$ (1) $|a| > |b| \Rightarrow \text{ROC} = \phi$

(2) $|a| = |b| \Rightarrow \text{ROC} = \phi$

③ $|a| < |b|$

ROC : region b/w circle of radius $|a|$ & $|b|$.



Q4 $x_1(n) = 2\delta(n) - \delta(n-1)$

$x_2(n) = 4\delta(n) + 3\delta(n-1)$

(i) ZT of $z(x_1(n) * x_2(n))$

convolution in time domain becomes multiplication in z

$X(z) = X_1(z) \cdot X_2(z)$

$x_1(n) \longrightarrow X_1(z)$

$x_2(n) \longrightarrow X_2(z)$

$x_1(n) * x_2(n) \longrightarrow X_1(z) \cdot X_2(z)$

$$X_1(z) = \sum_{n=-\infty}^{\infty} \left(2\delta(n) - \delta(n-1)\right) z^{-n}$$

$$= 2 - z^{-1}$$

ROC $= \mathbb{C} - \{0\}$

$$X_2(z) = \sum_{n=-\infty}^{\infty} [4\delta(n) + 3\delta(n-1)] z^{-n}$$

$$= 4 + 3z^{-1}$$

$$ROC = \mathbb{C} - \{0\}$$

$$X(z) = X_1(z) \cdot X_2(z)$$

$$= \left(2 - \frac{1}{z}\right)\left(4 + \frac{3}{z}\right)$$

$$X(z) = 8 + \frac{2}{z} - \frac{3}{z^2}$$

$$ROC = \mathbb{C} - \{0\}$$

(ii) $x(n)$

$$X(z) = 8 + \frac{2}{z} - \frac{3}{z^2}$$

$$x(n) = 8\delta(n) + 2\delta(n-1) - 3\delta(n-2)$$

(directly converting as we saw it in part (i) of Q4 in calculation)

Q5 (i)

$h(n) = IZT\ H(z)$

$$H(z) = \frac{z+1}{z-0.5} = \frac{1+z^{-1}}{1-0.5z^{-1}}$$

$$= \frac{1}{1-0.5z^{-1}} + \frac{z^{-1}}{1-0.5z^{-1}}$$

as, $a^n u(n) \rightleftharpoons \frac{1}{1-az^{-1}}$

$\therefore h(n) = (0.5)^n u(n) + (0.5)^{n-1} u(n-1)$

as $\frac{z^{-1}}{1-a^n z^{-1}} \rightleftharpoons a^{n-1} u(n-1)$

(ii) $u(n) \rightleftharpoons \dfrac{1}{1-z^{-1}}$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z) = H(z)\left(\frac{1}{1-z^{-1}}\right)$$

$$= \frac{1+z^{-1}}{(1-0.5z^{-1})(1-z^{-1})}$$

$$= \frac{4}{1-z^{-1}} - \frac{3}{1-0.5z^{-1}}$$

$$= Y(z)$$

$$IZT(Y(z)) = y(n)$$

$$\cancel{= 4u(n) - 3(0.5)}$$

$$= \boxed{4 \cdot u(n) - 3\left(\tfrac{1}{2}\right)^n u(n)}$$

Sol System response $y(n)$

(iii) $x(n) = (0.2)^n u(n)$

$$x(n) \rightleftharpoons X(z)$$

$$X(z) = \frac{1}{1-0.2z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} \Rightarrow Y(z) = H(z) \cdot X(z)$$

$$Y(z) = \frac{1+z^{-1}}{1-0.5z^{-1}} \cdot \frac{1}{1-0.2z^{-1}}$$

$$= \frac{5}{1-0.5z^{-1}} - \frac{4}{1-0.2z^{-1}}$$

$$IZT(Y(z)) = y(n)$$

$$\boxed{y(n) = 5\left(\tfrac{1}{2}\right)^n u(n) - 4\left(\tfrac{1}{5}\right)^n u(n)}$$

Q6 $y(n) = 0.2x(n) + x(n-1) + 0.3x(n-3) + 0.5x(n-4)$

as $x(n-n_0) \underset{\text{Inverse ZT}}{\overset{ZT}{\rightleftharpoons}} z^{-n_0}X(z)$

$$Y(z) = 0.2X(z) + z^{-1}X(z) + 0.3z^{-3}X(z) + 0.5z^{-4}X(z)$$

$$Y(z) = (0.2 + z^{-1} + 0.3z^{-3} + 0.5z^{-4})X(z)$$

take $h(n)$ as the impulse response

take $H(z)$ as ZT.

$$H(z) = 0.2 + z^{-1} + 0.3z^{-3} + 0.5z^{-4}$$

taking inverse,

$$h(n) = z^{-1}(0.2 + z^{-1} + 0.3z^{-3} + 0.5z^{-4})$$

$$\therefore \; h(n) = 0.2\,\delta(n) + \delta(n-1) + 0.3\,\delta(n-3) + 0.5\,\delta(n-4)$$

$$\left[\; \delta(n) \underset{IZT}{\overset{ZT}{\rightleftharpoons}} 1 \;\right]$$

Sketch