Assignment 2

Indranil Chakrabarty

January 2022

1 SECTION A :

1. Let us consider a situation where you want to teleport the state $\psi = \alpha|0\rangle + \beta|0\rangle$ (where $|\alpha|^2 + |\beta|^2 = 1$) with the help of a resource entangled state $|X\rangle = \frac{1}{\sqrt{1+|n|^2}}(|00\rangle + n|11\rangle)$ (Here α, β and n are complex numbers and $0 < |n| < 1$). Find out the fidelity of teleportation of an unknown quantum state from Alice's side to Bob's side. (Consider Fidelity to be the mod-square of the inner product between the state you desired and state you got)

2. Carry out the entanglement swapping at the qubits 2 and 3 of the entangled states, $\rho_{12} = |\psi\rangle\langle\psi|$, where $|\psi\rangle = \frac{|00\rangle + n|11}{\sqrt{1+|n|^2}}$ and $\rho_{34} = |\psi\rangle\langle\psi|$, where $|\psi\rangle = \frac{|00\rangle + m|11}{\sqrt{1+|m|^2}}$, $(0 < |n| < 1, 0 < |m| < 1)$.

3. What is the probability of success of Super dense coding process if we replace maximally entangled state by a non maximally entangled state $|\psi\rangle = \frac{|00\rangle + n|11}{\sqrt{1+|n|^2}}$ as a resource

2 SECTION B:

1. Consider the four bipartite quantum states given below. Some of them are density matrices, while some of them are state vectors.

   (a) $\psi_1 = |00\rangle$

   (b) $\rho_2 = \frac{1}{4}|00\rangle\langle 00| + \frac{1}{4}|01\rangle\langle 01| + \frac{1}{4}|10\rangle\langle 10| + \frac{1}{4}|11\rangle\langle 11|$

   (c) $\psi_3 = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

   (d) $\rho_4 = 0.7 \times \psi_3^\dagger \psi_3 + 0.3 \times \rho_2$

   For each of the state vectors/ density matrices given above,

   (a) If the given expression is a state vector, first find its density matrix. We will refer to this density matrix as ρ

   (b) For each density matrix ρ, find $S(\rho)$, $S(\rho^A)$ and $S(\rho^B)$ where $S(.)$ is the von Neumann entropy of a state, and ρ^A and ρ^B refer to the two subsystems.

   (c) For each of the density matrices given above, find $S_{A|B}(\rho)$ where $S_{A|B}(.)$ refers to the conditional von Neumann entropy of the state.

2. Find all the two qubit and single qubit reduced density matrices of the state $|\psi\rangle = (1/\sqrt{2})(|000\rangle + |111\rangle)$.

3. Suppose we start with a mixed state a) $\rho = \frac{1}{4}(I \otimes I)$, b) $\rho = \frac{1}{2}(|0\rangle\langle 0| + |+\rangle\langle +|) \otimes I$, c) $\rho = (p/4)I_{4\times 4} + (1-p)|\psi\rangle\langle\psi|$, where $|\psi\rangle = \frac{|00\rangle + n|11}{\sqrt{1+|n|^2}}$, $(0 < |n| < 1)$ shared between Alice and Bob. Now if Alice carries out the measurement in the $(|+\rangle, |-\rangle)$ basis, what will be the output state on Bob's side.