DSA Assignment - 2

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Q1 (a) $x[n] = \cos(0.03\pi n)$

if its periodic,

$x[n] = x[n+T]$

$$\cos(0.03\pi n) = \cos(0.03\pi(n+T))$$

here as $x[n]$ is discrete,

$T = kN$, $N \in \mathbb{N}$ & N = period,
$k \in \mathbb{N}$, k can be 1, 2, 3 ...

as $\cos x = \cos(x + 2\pi f)$, $f \in \mathbb{N}$

$$\therefore \; 0.03\pi kN = 2\pi f$$

$$\therefore \; N = \frac{2}{0.03} * f/k$$

here as $N \in \mathbb{N}$, and we are finding the fundamental period

hence, $N = 200m$, $m \in \mathbb{N}$

& ~~for~~ fundamental period, $\underline{\underline{N_0 = 200}}$

Hence, $\boxed{\text{periodic, period} = 200}$

1b) $x[n] = \cos\left(\frac{\pi}{4} n^2\right)$

Answer : periodic, fundamental period = 4

Solution :

$$x[n] = x[n+T]$$

T = period

$$\cos\left(\frac{\pi}{4} n^2\right) = \cos\left(\frac{\pi}{4}(n+T)^2\right)$$

$$= \cos\left(\frac{\pi}{4} n^2 + \frac{\pi}{4}\left(2Tn + T^2\right)\right)$$

as $\cos(x + 2\pi k) = \cos(x) \quad \forall x, k \in \mathbb{Z}$

$$\therefore \quad \frac{\pi}{4}\left(T^2 + 2nT\right) = 2\pi k$$

$$k, n \in \mathbb{Z}$$

$$T^2 + 2nT = 8k$$

now as $x[n]$ is discrete, T is integer.

take $T = 4a$, $a \in \mathbb{Z}$

as then $T^2 + 2nT$ is of the form $8k$

hence, fundamental period,

$$T_0 = 4 \times 1 = 4$$

(1c) $x[n] = 5$ [Ans = periodic, 1]

This is a DC signal,

hence it is periodic,

here $x[n] = x[n+t]$

for $t \in \mathbb{Z}$

$\therefore$ fundamental period = 1

(1d) $x[n] = \cos(5\pi n) + \cos \frac{4}{5}\pi n$

Ans: periodic, 10

for any signal $x[n] = x_1[n] + x_2[n]$

period = LCM(period $(x_1[n])$, period $(x_2[n])$)

hence we will find individual period.

$\rightarrow \quad x_1[n] = \cos 5\pi n$

similar to 1a,

$5\pi T = 2\pi m \qquad m \in Z, \; T \in N$

$\therefore T = \frac{2}{5} m$

but $T \in N$, $\cancel{\frac{m}{5} \in Z}$

hence fundamental period when $m = 5$,

$\boxed{T = 2}$

$\rightarrow \quad x_2[n] = \cos \frac{4}{5}\pi n$

$\frac{4}{5}\pi T = 2\pi m$

$T = \frac{5}{2} m$

$\therefore T_0 = 5$

$\therefore$ fundamental period for $x[n] = LCM(2, 5)$

$= 10$

1e) $x[n] = \sin(5\pi n + 2)$

~~similar~~ Ans: periodic, 2

this is similar to 1a,

so, $5\pi T = 2\pi m \quad \underline{m \in Z}$

$T = \frac{2}{5} m$

$\therefore$ fundamental period $\underline{T_0 = 2}$

1f) $x[n] = \cos(n + \pi)$

~~similar~~ Ans: not periodic

similar to 1a,

$T = 2\pi m \quad , \quad m \in Z$

here T can't be an integer for any m,

$\therefore$ $\underline{\text{Aperiodic}}$

Q2 ~~First~~ odd & even parts

(a) $e^{(a\pi n/b)j}$

$$x_e(n) = \frac{x[n] + x[-n]}{2}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

$$\therefore x_e[n] = \frac{e^{j\frac{a\pi}{b}n} + e^{j\frac{a\pi}{b}(-n)}}{2}$$

$$= \cos\frac{a\pi n}{b}$$

$$x_o[n] = \frac{e^{j\frac{a\pi n}{b}} - e^{-ja\pi b/n}}{2}$$

$$= j\sin\frac{a\pi n}{b}$$

(b) $a\cos(b\pi n + 1)$

$$x[n] = a\left[\underbrace{\cos b\pi n \cos 1}_{\text{even part}} - \underbrace{\sin b\pi n \sin 1}_{\text{odd part}}\right]$$

(as this part is even in $x[n]$)

(as this part is odd in $x[n]$)

$$\therefore \boxed{\begin{aligned} x_e[n] &= a \cos 1 \cos 6\pi n \\ x_o[n] &= -a \sin 1 \sin \pi 6 n \end{aligned}}$$

Q3 **Energy or power signals :**

(a) $x[n] = \left(\frac{1}{4}\right)^n u(n)$.

Ans : **Energy signal**

Solution :

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$x(n) = \begin{cases} \left(\frac{1}{4}\right)^n \cdot 1 \ ; & n \geq 0 \\ 0 \ ; & n < 0 \end{cases}$$

$$\therefore E = \sum_{n=-\infty}^{-1} 0 + \sum_{n=0}^{\infty} \left(\left(\frac{1}{4}\right)^n\right)^2$$

$$E = \sum_{n=0}^{\infty} \left(\frac{1}{16}\right)^n = \frac{1}{1-1/16}$$

$$= 16/15$$

$\therefore$ E is finite $\Rightarrow$ **Energy signal**

(b) $x(n) = a^n u(n)$, $a \in R$

$$x(n) = \begin{cases} a^n \cdot 1 ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$$

$$E = \sum_{n=-\infty}^{\infty} |a^n|^2$$

$$= \sum_{n=-\infty}^{-1} 0 + \sum_{n=0}^{\infty} |a^n|^2$$

$$= 1 + a^2 + a^4 + \cdots$$

if $a \in (-1, 1)$

$$E = \frac{1}{1-a^2} \longrightarrow \text{finite}$$

if $a \in R - (-1, 1)$

$E \rightarrow$ not finite

$\therefore$ for $a \in (-1,1) \longrightarrow$ Energy signal

for $a \in R - (-1,1)$

~~$P = \frac{1}{2N+1} \sum_{n=0}$~~

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |a^n u(n)|^2$$

~~for n =~~

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{-1} 0 +$$

$$\lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} |a^n|^2$$

as $|a| \geq 1$

$$\therefore \text{sum of GP} = \frac{1 \cdot ((a^2)^{N+1} - 1)}{a^2 - 1}$$

$$\therefore P = \lim_{N \to \infty} \frac{a^{2(N+1)} - 1}{(a^2-1)(2N+1)}$$

$\therefore$ P is also not finite.

$\therefore$ for $|a| < 1 \longrightarrow$ Energy

$|a| > 1 \longrightarrow$ NENP

(c) $x(n) = a^n \delta(n)$, $a \in R$

$$\delta(n) = \begin{cases} 1 ; & n = 0 \\ 0 ; & \text{otherwise} \end{cases}$$

$$E = \sum_{n=-\infty}^{\infty} |(a^n \delta(n))|^2$$

$$= [a^0]^2 = 1$$

$\therefore$ Energy is finite $\Rightarrow$ Energy signal

(d) $x(n) = \sin\left(\frac{n\pi}{4}\right)$

- here this is a periodic signal with fundamental period 8,

we know that, (from the book),

all bounded periodic signals are power signals & this is a periodic & bounded signal hence Power signal.

[also told in lectures]

Q4 $x_a(t) = 5 \sin 200\pi t$

⇒ to get : min. sampling rate to avoid ~~[illegible]~~ aliasing

~~$x_a(t) = 5 \sin 200\pi t$~~

~~[illegible]~~

$f_s = 2 f_m$ (min sampling rate to avoid aliasing)

~~[illegible]~~ so, $\omega_m = 200\pi$

$\therefore f_m = 100$

$\therefore \boxed{f_s = 200}$

⇒ if $f_s = 250\,Hz$, the DTS :

$x(t) = 5 \sin 200\pi t$

$x(n) = 5 \sin \left(200 \times \frac{1}{250}\right) n$

⇒ $= \boxed{5 \sin \frac{4\pi n}{5}}$

Q5 $x_a(t) = 3\cos 80\pi t + 5\sin 40\pi t$

$- 10\cos 160\pi t$

find Nyquist rate

$$\omega_m = \max(30\pi, 40\pi, 160\pi)$$

$$= 160\pi$$

$$f_m = 80 \text{ Hz}$$

$$\therefore f_s \geq 2 \cdot f_m$$

$$\min f_s = 2f_m = 160 \text{ Hz}$$

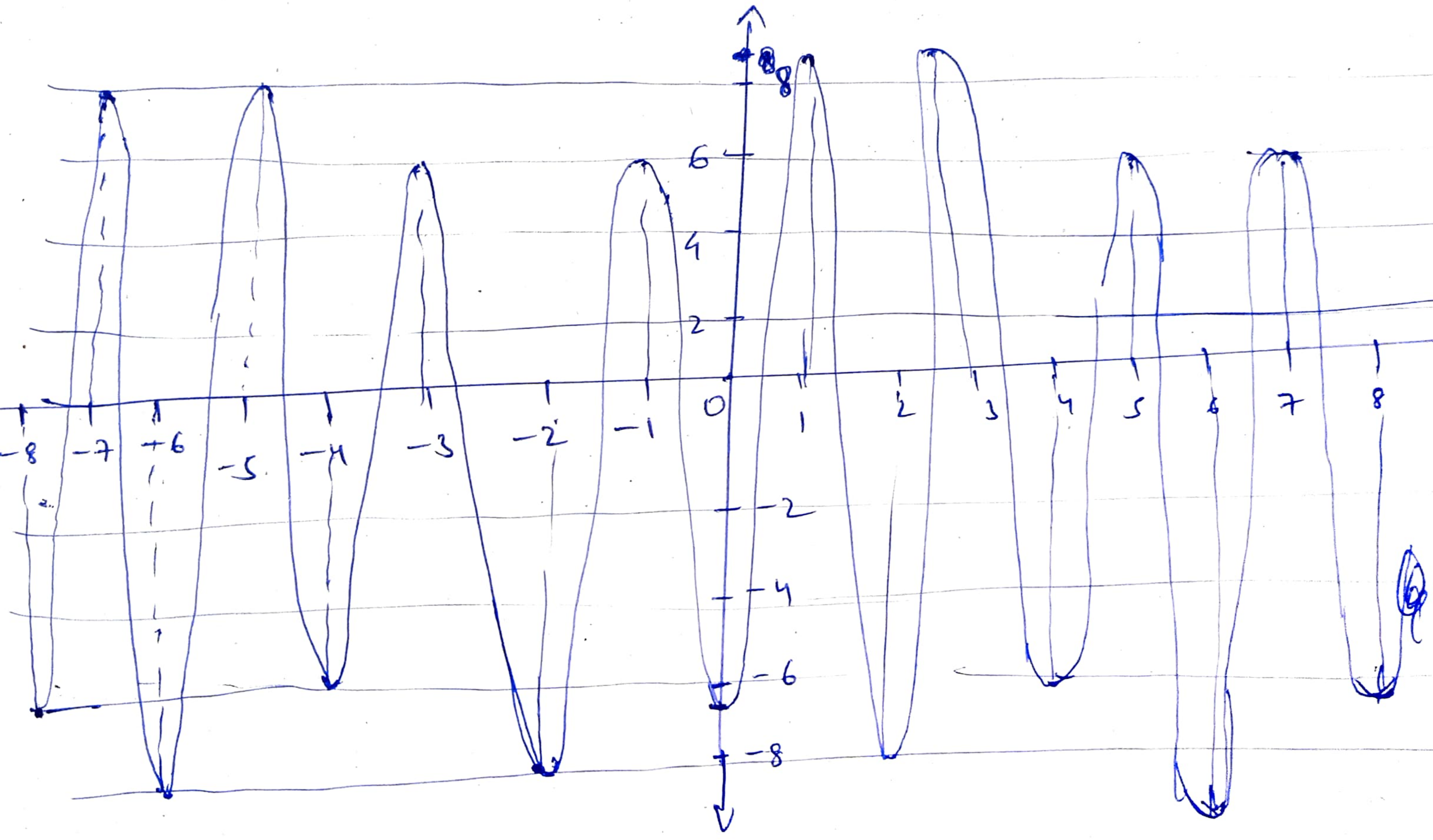
$$x_d[n] = x_a(n/f_s)$$

$$= 3\cos\frac{80\pi n}{160} + 5\sin\frac{40\pi n}{100}$$

$$- 10\cos\frac{160\pi n}{160}$$

$$\boxed{x_d[n] = 3\cos\frac{\pi n}{2} + 5\sin\frac{2\pi n}{5} - 10\cos\pi n}$$

Waveform



Q6 $x_a(t) = 5\cos 2000\pi t + 5\sin 6000\pi t - 7\cos 12000\pi t$

analog signal

$\Rightarrow$ find : ~~fs~~ min. f_s

$f_s = 2 f_m$

(f_m = max$_m$ f in signal)

$\omega_1 = 2000\pi$, $\omega_2 = 6000\pi$

$\omega_3 = 12000\pi$

$f_1 = \dfrac{2000\pi}{2\pi} = 1k\ Hz$

$f_2 = 3k\ Hz$ $\quad f_3 = 6k$

$\therefore f_s = 2 \times 6k = 12k\ Hz$

$\Rightarrow f_s = 5k$, DTS ? , Analog signal ?

$x_d(n) = x_a(nT) = x_a(n/f_s)$

$= 5\cos \dfrac{2\pi}{5} n + 5\sin \dfrac{6\pi}{5} n -$

$7\cos \dfrac{12\pi}{5} n$

$$= 5\cos\frac{2\pi n}{5} + 5\sin\left(2n\pi - \frac{4\pi n}{5}\right) - 7\cos\left(2n\pi + \frac{2\pi n}{5}\right)$$

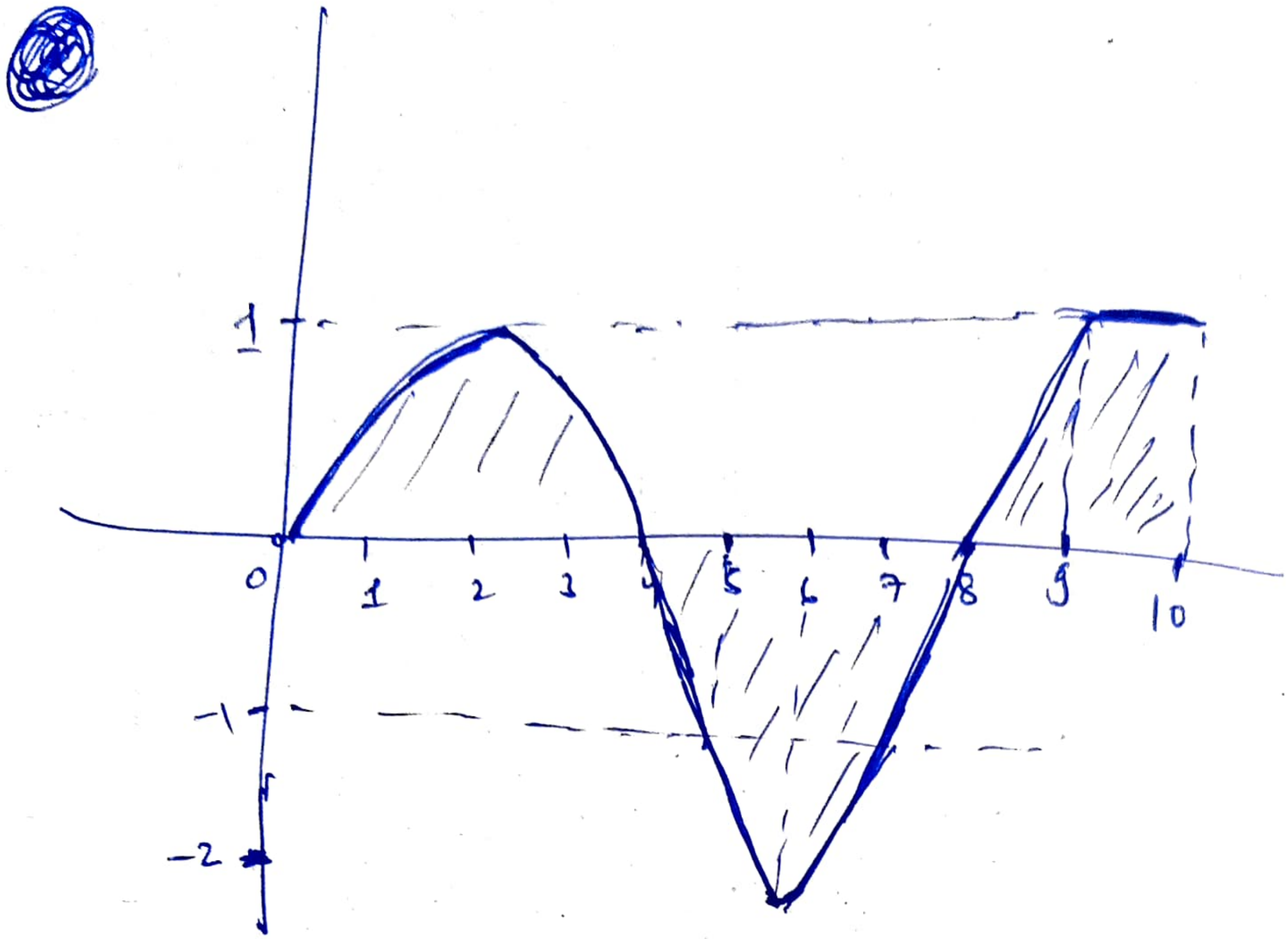
$$= \boxed{-2\cos\frac{2n\pi}{5} \mp 5\sin\frac{4n\pi}{5}}$$

$$x'_a(t) = x_d(t f_s)$$

$$= \boxed{-2\cos 2000\pi t \mp 5\sin 4000\pi t}$$

Q7 analog waveform $x(t)$

$$x(t) = \begin{cases} \sin \frac{\pi t}{4} , & 0 \le t \le 4 \\ 4 - t , & 4 \le t \le 6 \\ t - 8 , & 6 \le t \le 9 \\ 1 , & 9 \le t \le 10 \end{cases}$$

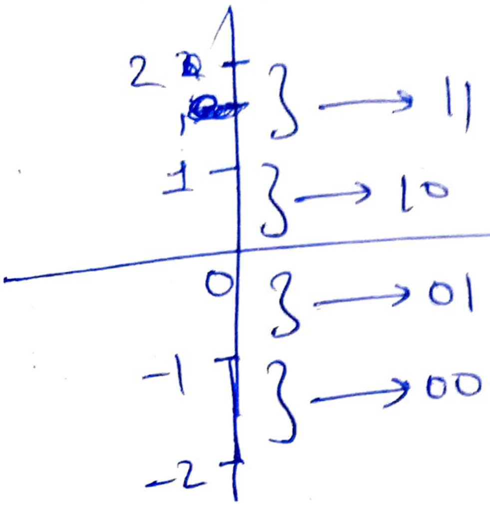


(a) sampled points $= \frac{1}{fs}, \frac{2}{fs}, \frac{3}{fs} \ldots$

$fs = 1000$

$\therefore$ sample points $= \frac{1}{1000}, \frac{2}{1000}, \frac{3}{1000}, \ldots$

(b) sketch quantisation intervals



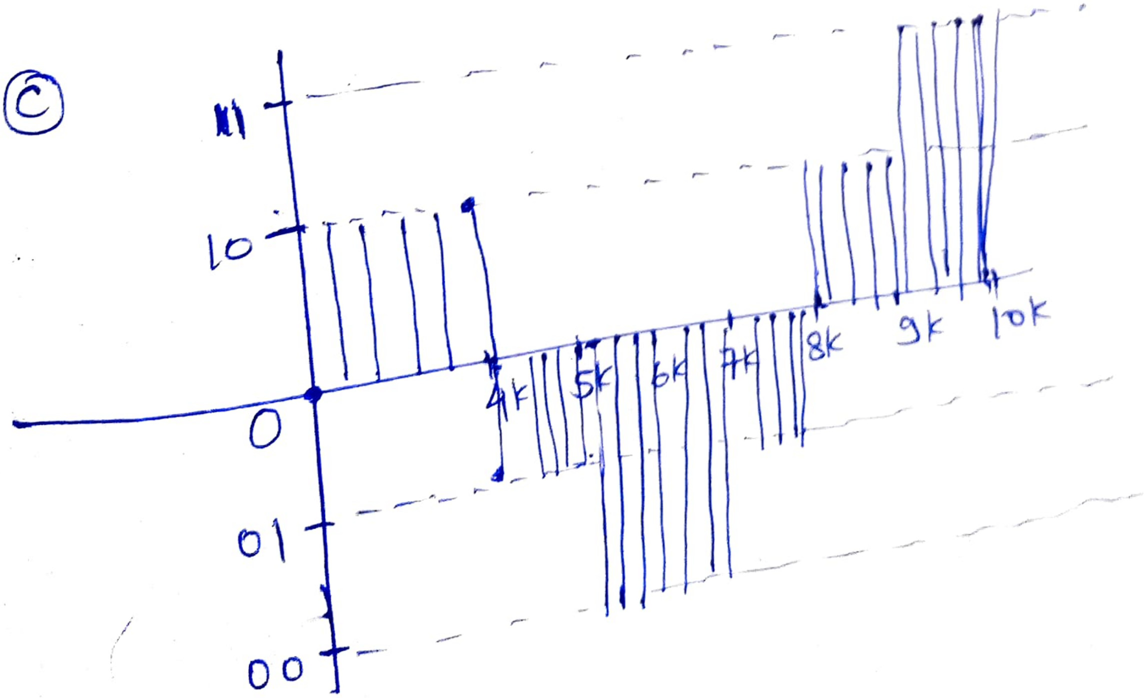
$[-2,-1) \to 00$

$[-1,0) \to 01$

$[0,1) \to 10$

$[1,2) \to 11$

(c)



(d) $\underbrace{10\,10\,10\,10\ldots\ldots 10}_{4000 \text{ times}}\ \underbrace{01\,01\,01\,01\ldots 01}_{1000 \text{ times}}$

$\underbrace{00\ldots\ldots 00}_{2000 \text{ times}}\ 01 \ldots\ldots \underbrace{1010\ldots 10}_{1000 \text{ times}}$

Repeats 1000 times

$\underbrace{11\ldots\ldots 11}_{1000 \text{ times}}$

(e) resulting bit-rate

$= \text{bits} \times f_s$

$= 2 \times 1000$

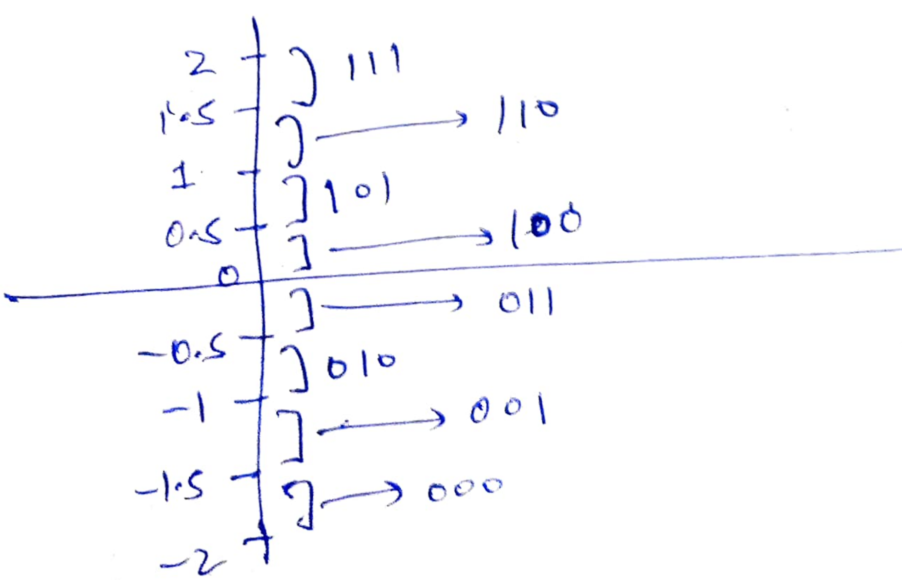
$= 2000$ bits/s

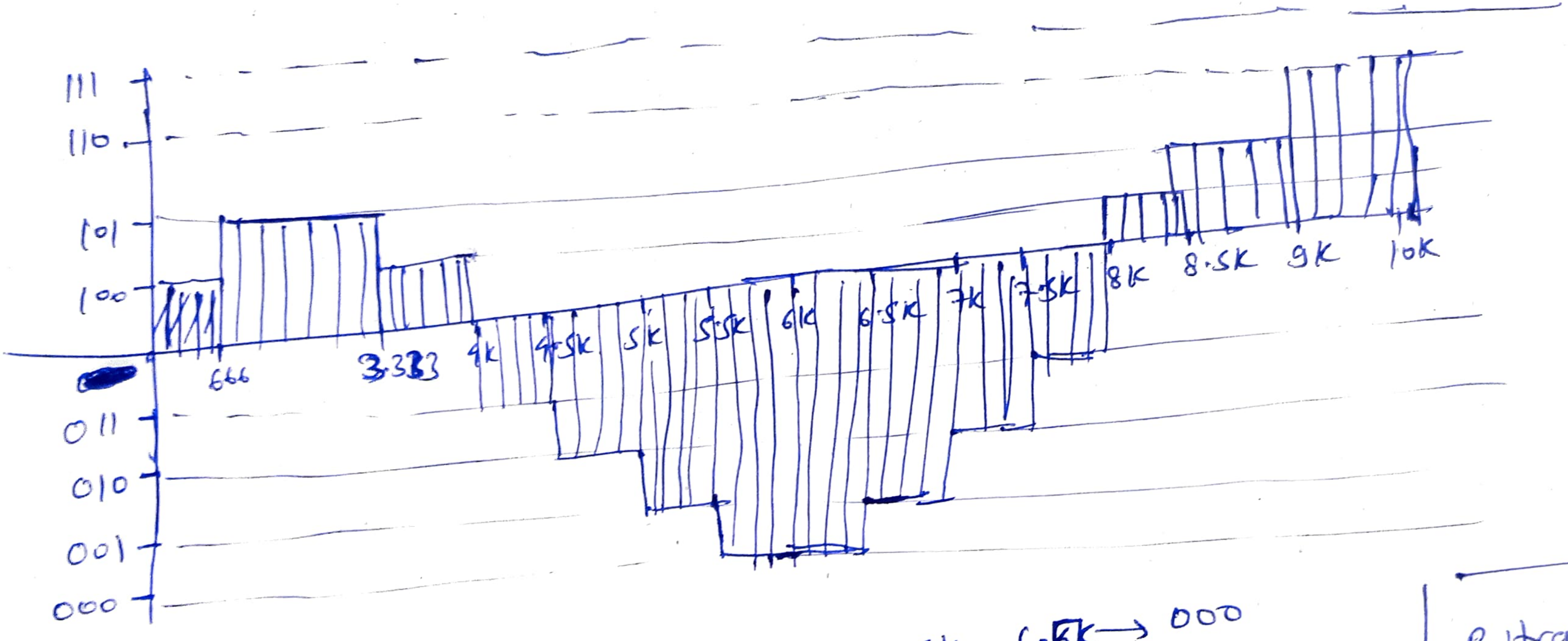
(f) Quantisation error

$$= \left[\frac{\text{Upper bound} - \text{lower bound}}{2^n}\right]\frac{1}{2}$$

$$= \frac{2-(-2)}{2^{n+1}} = \frac{4}{8} = \frac{1}{2}$$

(g)





Bits stream :

0 - 666 : 100

667 - 3333 : 101

3333 - 4k → 100

~~4k - 5k :~~

4k - 4.5k → 011

4.5k - 5k → 010

5k - 5.5k → 001

5.5k - 6.5k → 000

6.5k - 7k → 001

7k - 7.5k → 010

7.5k - 8k → 011

8k - 8.5k → 100

8.5k - 9k → 101

9k - 10k → 110

Bitrate → 3 x 1000
(bits x f_s)

= 3000

Quantisation error

$= \frac{2-(-2)}{2^4}$

= 0.25

(h) 3-bit is better

reason : lower quantisation error

Q8) if $x[n] \longrightarrow y[n]$

if $x[n] = x[n-k]$

then $y[n] = y[n-k]$

i.e $\cancel{y[n]}$ $x[n-k] \longrightarrow y[n-k]$

a) $y(n) = a/x(n)$

for $x(n-k)$ input, $\cancel{y(n) = \frac{a}{x(n-k)}}$

output $= \dfrac{a}{x(n-k)}$

also $y(n-k) = \dfrac{a}{x(n-k)}$

$\therefore$ ~~y(n-k) = a~~ Hence time invariant

(b) $y(n) = 3x(n) + 5x(n-2)$

if $x[n] = x[n-k]$

$y[n] = 3x[n] + 5x[n-2]$

$= 3x[n-k] + 5x[n-2-k]$

$y[n-k] = 3x[n-k] + 5x[n-2-k]$

$\therefore \; g[n] = y[n-k]$

$\therefore$ time invariant

(c) $y(n) = x(-n)$

if $x[n] = x[n-k]$

$\therefore \quad x[-n] = x[-n-k]$

$y_1[n] = x_1[-n] = x[-n-k]$

$y[n-k] = x[-(n-k)]$

$y(n-k) = x[k-n]$

$\Rightarrow \quad y_1[n] \neq y(n-k]$

hence not time invariant.

(d) $y(n) = n\,x(n)$

if $x_1[n] = x[n-k]$

then, $y_1[n] = n \cdot x_1[n]$

$= n \cdot x[n-k]$

now, $y[n-k] = (n-k)\,x[n-k]$

$y_1[n] \neq y[n-k] \Rightarrow$ not time invariant e $y(n) = x(5n)$

if $x_1[n] = x[n-k]$

$\Rightarrow \; y_1[n] = x_1[5n]$

$y_1[n] = x[5n-k]$

now, $y[n-k] = x[5(n-k)]$

$\neq x[5n-k]$

$\therefore$ not time invariant.

Q9 Test linearity

~~(a) $y(n) = 5x(n) + 7x(n-1)$~~

linearity

if $x_1(n) \longrightarrow y_1(n)$ &

$x_2(n) \longrightarrow y_2(n)$

then, $a x_1(n) + b x_2(n) \longrightarrow a y_1(n) + b y_2(n)$

(a) $y(n) = 5x(n) + 7x(n-1)$

$y_1(n) = 5x_1(n) + 7x_1(n-1)$ $[x_1(n) \longrightarrow y_1(n)]$

$y_2(n) = 5x_2(n) + 7x_2(n-1)$ $[x_2(n) \longrightarrow y_2(n)]$

$a x_1(n) + b x_2(n) \longrightarrow y'(n)$

$y'(n) = a\left[5x_1(n) + 7x_1(n-1)\right] + b\left(5x_2(n) + 7x_2(n-1)\right)$

$= a\, y_1(n) + b\, y_2(n)$

Hence **linear**

[here obviously, $k\,y(n) = k\,y'(n)$

where $y(n) \longleftarrow k\,x(n)$

(b) $y(n) = 4x(n) - \frac{9}{x(n-1)}$

$$y_1(n) = 4x_1(n) - \frac{9}{x_1(n-1)}$$

$$y_2(n) = 4x_2(n) - \frac{9}{x_2(n-1)}$$

$$y_1(n) + y_2(n) = 4x_1(n) - \frac{9}{x_1(n-1)} + 4x_2(n) - \frac{9}{x_2(n-1)}$$

$$= 4\left(x_1(n) + x_2(n)\right) - 9\left(\frac{1}{x_1(n-1)} + \frac{1}{x_2(n-1)}\right)$$

& $x_1(n) + x_2(n) \longrightarrow y'(n)$

$$= 4\left(x_1(n) + x_2(n)\right) - \frac{9}{x_1(n-1) + x_2(n-1)}$$

$\therefore\ y_1(n) + y_2(n) \neq y'(n)$

$\therefore$ Non-linear

(c) $$y(n) = \sum_{m=0}^{N} b_m x(n-m) - \sum_{m=1}^{N} d_m \cancel{k} x(n-m)$$

$$= b_0 x(n-0) + \sum_{m=1}^{N} b_m (x \cancel{k}(n-m)) - \sum_{m=1}^{N} d_m x(n-m)$$

$$= b_0 x(n-0) + \sum_{m=1}^{N} (b_m - d_m) x(n-m)$$

$$=$$

$$y_1(n) = b_0 x_1(n) + \sum_{m=1}^{N} (b_m - d_m) x_1(n-m)$$

$$y_2(n) = b_0 x_2(n) + \sum_{m=1}^{N} (b_m - d_m) x_2(n-m)$$

$$y_1(n) + y_2(n) = b_0 x_1(n) + b_0 x_2(n) + \sum_{m=1}^{N} (b_m - d_m) \left(x_1(n-m) + x_2(n-m)\right)$$

$x_1(n) + x_2(n) \longrightarrow y'(n)$

$$= b_0 (x_1(n) + x_2(n)) + \sum_{m=1}^{N} (b_m - d_m)\left(x_1(n-m) + x_2(n-m)\right)$$

now check ~~for~~ homogeneity,

$$k y(n) = k\left[b_0 x(n) + \sum_{m=1}^{N} (b_m - d_m)\, x(n-m)\right]$$

$$k x(n) \longrightarrow y'(n) = \cancel{b_0 k\, b_0 x}$$

$$= k\, b_0 x(n) + k \sum_{n=1}^{N} (b_m - d_m)\, x(n-m)$$

$\therefore\; k y(n) = y'(n)$

$\therefore$ the system is linear

Q10 test causality

(a) $y(n) = ax(n) + bx(n-1)$

Impulse response,

$$h[n] = a\delta[n] + b\delta[n-1]$$

if $n<0$, $\delta[n]=0$ $\quad \delta[n-1]=0$

$\therefore h[n] = 0 \Rightarrow$ $\boxed{\text{causal}}$

$h[n] = a\delta[$

(b) $y(n) = ax(n-1) + bx(n+1)$

$x(n+1) \longrightarrow$ future input

& output depends on $x(n+1)$

$\Rightarrow$ Non-causal system

ⓒ $y(n) = \sum_{k=0}^{\infty} x(n-k)$

$y(n) = x(n-0) + x(n-1) + \ldots$

$\therefore$ $y(n)$ depends only on present & past inputs.

$\boxed{\text{Causal system}}$

ⓓ $y(n) = \sum_{k=0}^{\infty} x(n+k)$

$y(n) = x(n+0) + x(n+1) + \ldots$

$y(n)$ depends only on future & present inputs

Hence ~~Non-ca~~ $\boxed{\text{not causal}}$