Assignment-2

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Q1.1.1

$$\frac{\partial P(x,t)}{\partial t} = \frac{D\, \partial^2 P(x,t)}{\partial x^2}$$

take $D = 0.2$

$$P(x_i, t_{i+1}) = P(x_i, t_i) + D\left[P(x_{i+1}, t_i) + P(x_{i-1}, t_i) - 2P(x_i, t_i) \right]$$

I.C. $P(0,0) = 1$

$P(x,0) = 0, \quad x \neq 0$

$t = 0$,

$P(0,0) = 1, \quad P(x,0) = 0, \quad x \neq 0$

$t = 1$,

$P(0,1) = 1 + 0.2(-2)$

$= 0.6$

$P(1,1) = 0 + 0.2(1) = 0.2$

$P(-1,1) = 0 + 0.2(1) = 0.2$

All other probabilities are 0.

at $t=2$,

$$P(0,2) = 0.6 + 0.2\,(0.2+0.2-2\times 0.6)$$

$$= 0.6 - 0.2\,(0.8)$$

$$= 0.44$$

$$P(1,2) = P(-1,2) \quad \cancel{0.2+0.2}$$

$$= 0.2 + 0.2\,(0.6+0-0.4)$$

$$= 0.2 + 0.04 = 0.24$$

$$P(2,2) = P(-2,2) = 0+0.2\,(0.2+0+0)$$

$$= \underline{0.04}$$

and so on, ...

Q 1.1.2

$$p(x_i, t_{i+1}) = P(x_i, t_i)(1-2D) + D[P(x_{i+1}, t_i) + P(x_{i-1}, t_i)]$$

as $D \uparrow$ $\Rightarrow$ dependence on $P(x_i, t_i)$ decreases, & dependence sum of $p(x_{i+1}, t_i)$ & $P(x_{i-1}, t_i)$ increases

so as $D \uparrow$ $\Rightarrow$ probabilities at ~~_~~ a point is more dependent on its adjacent position,

hence, we can say it affects diffusion

ⓐ 1.2.1

2D diffusion eqn :

$$\frac{\partial u}{\partial t} = D\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

I.C. $u(x,y,0) = f(x,y)$

$u(0,y,t) = u(a,y,t) = 0 \quad 0 \le y \le b, \; t \ge 0$

$u(x,0,t) = u(x,b,t) = 0 \quad 0 \le x \le a, \; t \ge 0$

D = diffusivity of metal

Q1.2.2

Assuming we need Forward-Euler form (taking $\theta = 0$),

applying finite difference approximation on the partial derivative ~~at the points~~ at point (x,y) at t,

$$\frac{u_{x,y,t+1} - u_{x,y,t}}{\Delta t} = D\left(\frac{u_{x-1,y,t} - 2\cdot u_{x,y,t} + u_{x+1,y,t}}{(\Delta x)^2} + \frac{u_{x,y-1,t} - 2\cdot u_{x,y,t} + u_{x,y+1,t}}{(\Delta y)^2}\right)$$

~~take~~

$$u_{x,y,t+1} = \frac{D\Delta t}{(\Delta x)^2}\left(u_{x-1,y,t} - 2\cdot u_{x,y,t} + u_{x+1,y,t}\right) + \frac{D\Delta t}{(\Delta y)^2}\left(u_{x,y-1,t} - 2\cdot u_{x,y,t} + u_{x,y+1,t}\right)$$

Q1-2-3

rearranging eqn from 1-2-2,

$$u_{x,y,t+1} = k\left(u_{x-1,y,t} + u_{x+1,y,t} + u_{x,y-1,t} + u_{x,y+1,t} - 4 \cdot u_{x,y,t}\right) + u_{x,y,t}$$

$$k = \frac{D \Delta t}{(\Delta x)^2}$$

on solving, coefficient of $u_{x,y,t}$ is

$(1-4k)$.

Hence, for the process to become unstable,

time step $\Delta t \leq \frac{(\Delta x)^2}{4D}$

Explanation:

$$1-4k \geq 0 \Rightarrow 1 - 4\frac{D\Delta t}{(\Delta x)^2} \geq 0$$

$$\therefore \frac{4D\Delta t}{(\Delta x)^2} \leq 1 \Rightarrow \boxed{\Delta t \leq \frac{(\Delta x)^2}{4D}}$$