

End Exam

▼ TIV and TV

For TIV Systems:

1. No time scaling
2. Coeff. should be constant
3. Any added/subtracted term in the system relationship (except i/p and o/p) must be constant or zero



split systems are always Tv

▼ Linearity

should follow LOA and LOH

>> System linearity is independent of time scaling

>> System linearity is independent of coefficient used in system relationship

but when there is $y = x^n(t)$ then its not linear $n > 1$

$$6. \underbrace{y(t)}_{\text{o/p}} = \underbrace{-2t}_{\text{time}} + \underbrace{x(t)}_{\text{i/p}}$$

dependent

$$x(t) \rightarrow \text{sys.} \rightarrow y(t) = \underbrace{2t}_{\text{time}} + x(t)$$

X a) LOA

$$x_1(t) \rightarrow \text{sys.} \rightarrow y_1(t) = 2t + x_1(t)$$

$$x_2(t) \rightarrow \text{sys.} \rightarrow y_2(t) = 2t + x_2(t)$$

$$y_1(t) + y_2(t) = \underbrace{4t}_{\text{NL}} + x_1(t) + x_2(t)$$

$$x_1(t) + x_2(t) \rightarrow \text{sys.} \rightarrow R(t) = \underbrace{2t}_{\text{NL}} + x_1(t) + x_2(t)$$

$$7. \underbrace{y(t)}_{\text{o/p}} = \underbrace{2}_{\text{time}} + \underbrace{x(t)}_{\text{i/p}}$$

ind

$$x(t) \rightarrow \text{sys.} \rightarrow y(t) = \underbrace{(2)}_{\text{time}} + x(t)$$

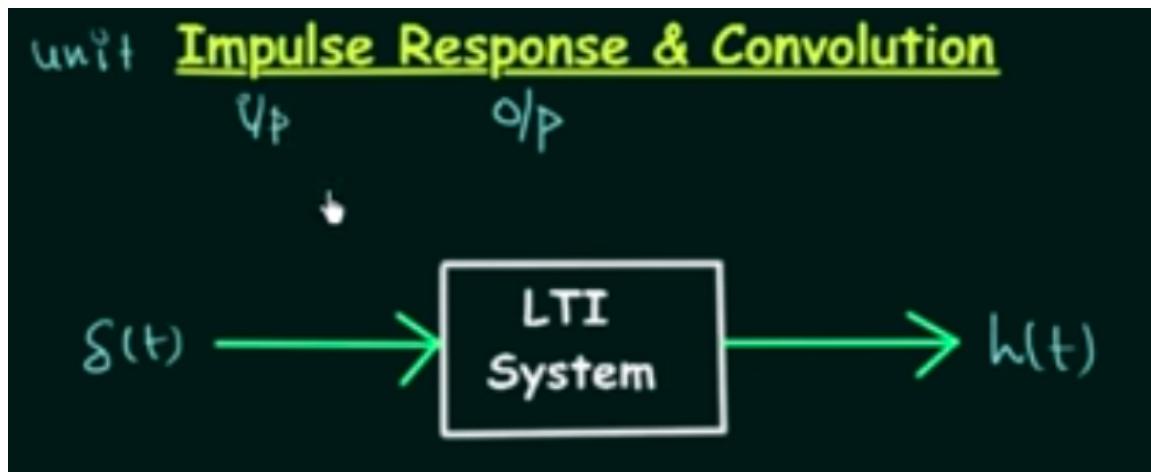
a) LOH

$$x(t) \rightarrow \text{sys.} \rightarrow y(t) \rightarrow 'k' \rightarrow ky(t)$$

$$ky(t) = \underbrace{2k}_{\text{time}} + kx(t)$$

$$kx(t) \rightarrow \text{sys.} \rightarrow y(t) = \underbrace{2}_{\text{time}} + kx(t)$$

▼ Impulse response



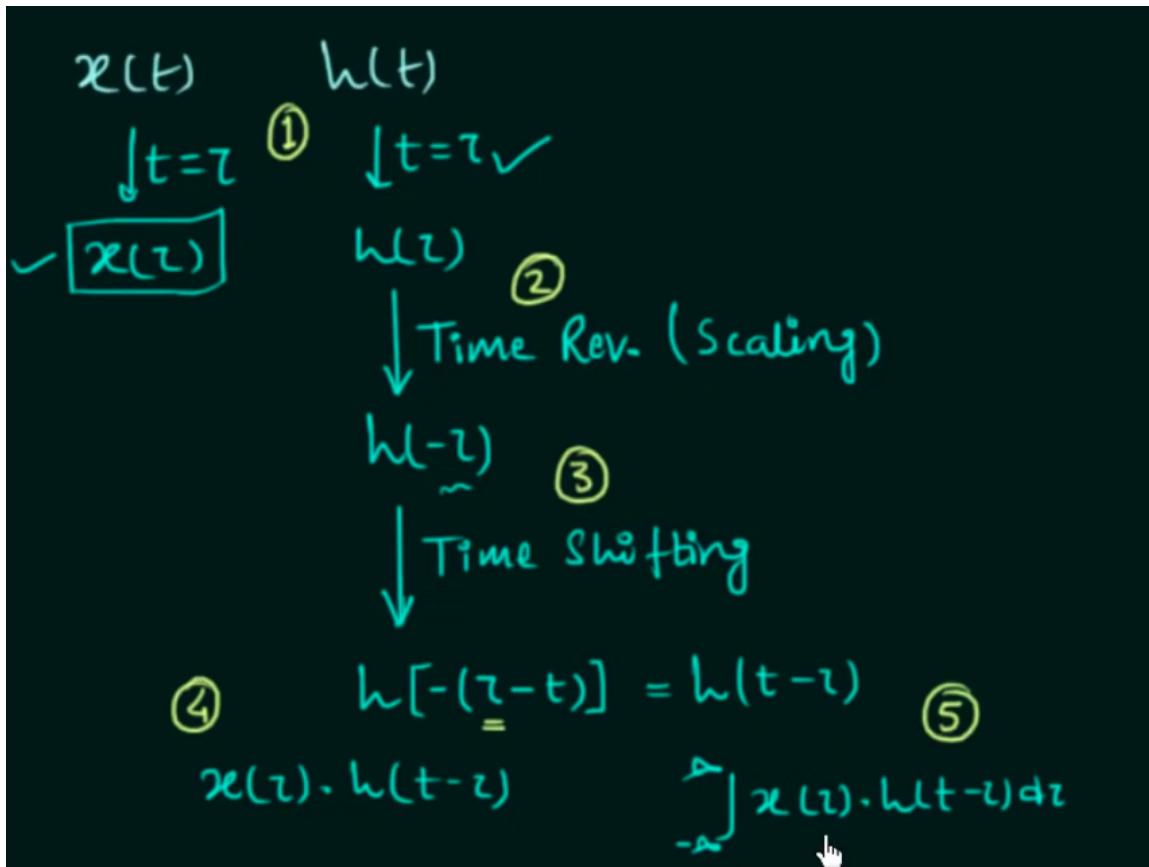
when ever a unit impulse is supplied to an LTI system the response we get is called impulse response and it is fixed for an LTI system

▼ Convolution

» A convolution is an integral that expresses the amount of overlap of one function when it is shifted over another function

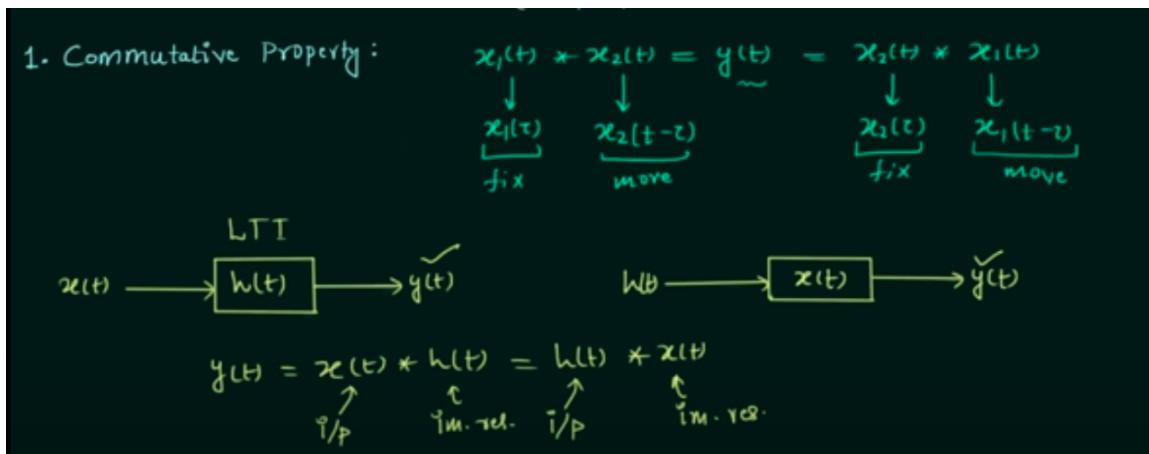
$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) d\tau$$

↑
convolution op.



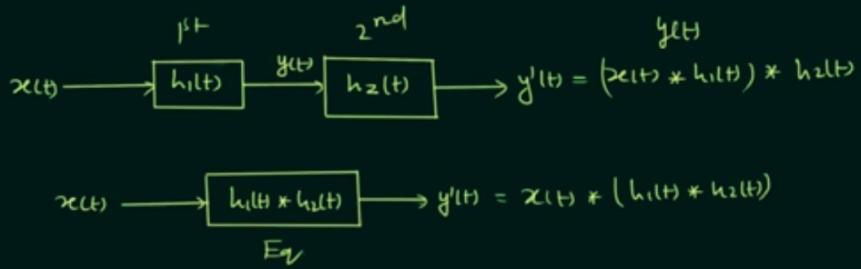
▼ Properties of Convolution [not done fully]

1. Commutative



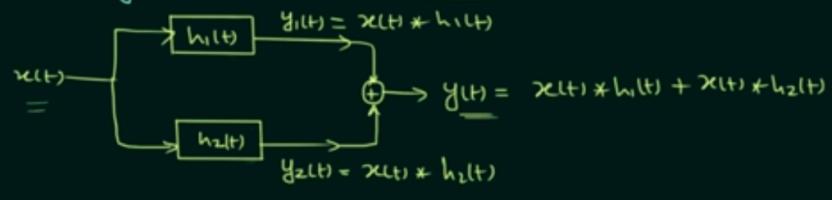
2. Associative

2. Associative property:

$$(x_1(t) * x_2(t)) * x_3(t) = y(t) = x_1(t) * (x_2(t) * x_3(t))$$


3. Distributive

3. Distributive property:

$$x_1(t) * (x_2(t) + x_3(t)) = y(t) = x_1(t) * x_2(t) + x_1(t) * x_3(t)$$


$$x(t) \xrightarrow{h_1(t) + h_2(t)} y(t) = x(t) * (h_1(t) + h_2(t))$$

4.

▼ Properties of Z transform

1. Linearity

Correction: ROC is = intersection of all other ROCs

1) Linearity:

$$x_1[n] \Leftrightarrow X_1(z), \text{ ROC} = R_1$$

$$x_2[n] \Leftrightarrow X_2(z), \text{ ROC} = R_2$$

$$\alpha x_1[n] \Leftrightarrow \alpha X_1(z), \text{ ROC} = R_1$$

$$\beta x_2[n] \Leftrightarrow \beta X_2(z), \text{ ROC} = R_2$$

$$\begin{aligned} \alpha x_1[n] + \beta x_2[n] &\Leftrightarrow \alpha X_1(z) + \beta X_2(z), \\ \text{ROC} &\geq R_1 \cap R_2 \end{aligned}$$

Proof :- H.W.

2. Time shifting

2) Time Shifting:

$$x[n] \Leftrightarrow X(z), \text{ ROC} = R$$

$$x[n - n_0] \Leftrightarrow z^{-n_0} X(z), \text{ ROC} = \text{Same}$$

3. Time scaling

3) Time Scaling:

$$x[n] \Leftrightarrow X(z), \text{ ROC} = R$$

$$x\left[\frac{n}{m}\right] \Leftrightarrow X(z^m), \text{ ROC} = R^{\frac{1}{m}}$$

4. Time Reversal

4) Time Reversal:

$$x[n] \Leftrightarrow X(z), \text{ ROC} = R$$

~~$$x[-n] \Leftrightarrow X(z^{-1}), \text{ ROC} = R^{-1}$$~~

5. scaling of Z

5) Scaling of Z:

$$x[n] \Leftrightarrow X(z), \text{ ROC} = R$$

$$\underbrace{\alpha^n x[n]}_{\text{Scaling}} \Leftrightarrow X\left(\frac{z}{\alpha}\right), \text{ ROC} = |\alpha|R$$

6. Convolution in Time is multiplication in Z

8) Convolution in Time:

$$x_1[n] \Leftrightarrow X_1[z], \text{ ROC} = R_1$$

$$x_2[n] \Leftrightarrow X_2[z], \text{ ROC} = R_2$$

~~$x_1[n] * x_2[n] \Leftrightarrow X_1[z] \cdot X_2[z],$~~

$$R_{\text{OC}} \geq R_1 \cap R_2$$

7. Multiplication in time is Convolution in Z

9) Multiplication in Time:

$$x_1[n] \cdot x_2[n] \Leftrightarrow \frac{1}{2\pi j} \{ X_1[z] * X_2[z] \},$$
$$R_{\text{OC}} \geq R_1 \cap R_2$$



8. Diff in time

10) Differentiation in Time:

↓
Successive Diff. or First Diff.

$$\frac{d x[n]}{d n} = \frac{x[n] - x[n-1]}{n - (n-1)} = x[n] - x[n-1]$$

$$\rightarrow x[n] \Leftrightarrow X(z), \text{ ROC} = R$$

$$\checkmark x[n] - x[n-1] \Leftrightarrow (1 - z^{-1}) X(z)$$

$$\Leftrightarrow \left(\frac{z-1}{z}\right) X(z), \text{ ROC} = R^*$$

9. Diff in Z

11) Differentiation in z-Domain:

$$x[n] \Leftrightarrow X(z), \text{ ROC} = R$$

$$\checkmark n x[n] \Leftrightarrow -z \boxed{\frac{d X(z)}{d z}}, \text{ ROC} = R$$

▼ IZT

\underline{IZT} \underline{ROL} \underline{IDTFT} $\int x(e^{j\omega}) e^{j\omega n} d\omega$

 $\underline{X(z)}$

\rightarrow • Contour Integration
 residue method

• Table lookup or using property
 $\frac{2}{2^n} \rightarrow u(n)$

• Series expansion/ long division method

$\frac{z^2 - z^2}{z^2 + 1}$. Partial fraction Table lookup

1. Table look up method: dekh ke krlo
 2. Partial fractions
 3. long division
 4. contour integration
- ▼ ZT, IZT in LTI, filter design

Let LTI

$$y(n) = - \sum_{k=1}^N a_k y(n-k) + \sum_{k=0}^M b_k x(n-k) \quad (1)$$

Poles $\frac{1}{T_k}$ \rightarrow All pole system
 All zeros \rightarrow 1 pole + zeros

$$y(n) = 3y(n-2) + 2y(n-1) + 3x(n) + x(n-1)$$

$$\frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}}$$

values at which $H(z) = 0 \rightarrow$ zeroes, $H(z) \rightarrow \infty \rightarrow$ poles

all zero system: output depends only on present and past inputs , i.e. $a_k=0$
 such systems are always stable as there are no poles

IIR [infinite impulse response] :

if $b_k=0$, all pole system

- sometimes not stable hence
- IIR can have both poles and zeroes
- FIR will always have no poles

▼ Digital Filter

→ we need M, A, D to implement i.e. realise eqns like:

$$y(n) = \frac{3x(n-1) + 4x(n-2) + 3y(n-1)}{1}$$

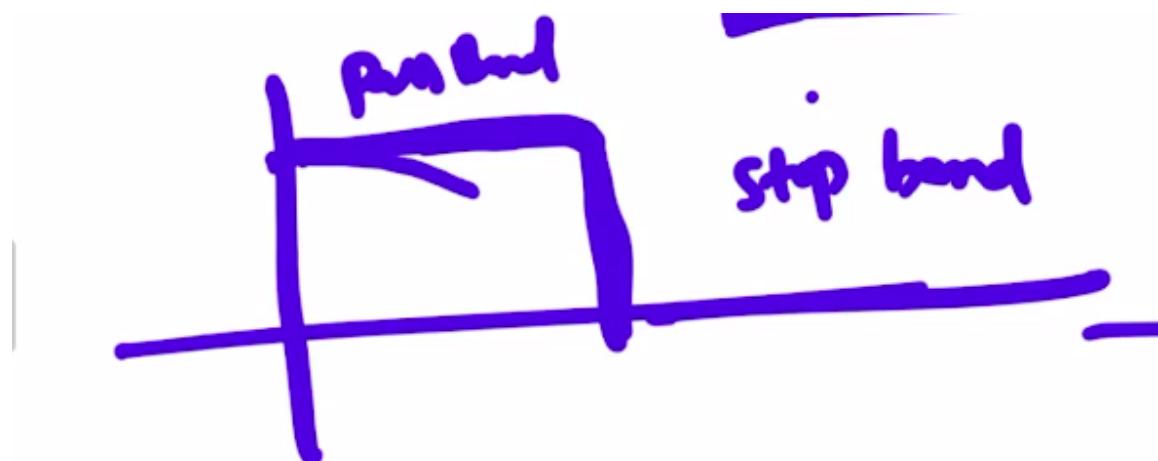
realization
M, A, D

$$\rightarrow Y(z) = \frac{3z^{-1}X(z) + 4z^{-2}X(z)}{1 + 3z^{-1}} \rightarrow ①$$

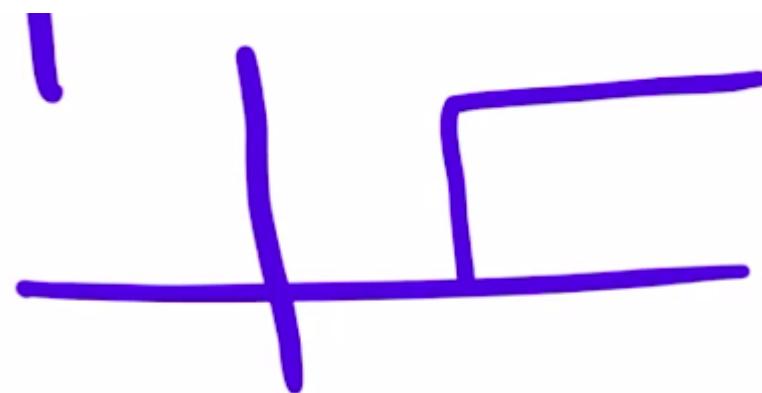
So we have been given some input signal and we want to suppress some unwanted frequencies then we apply filter



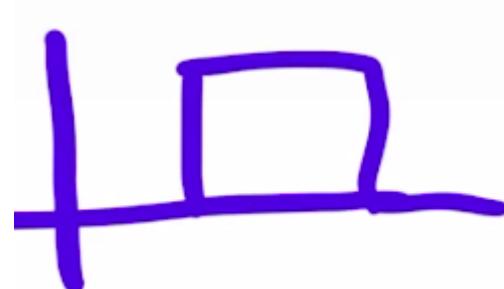
ex: low pass filter



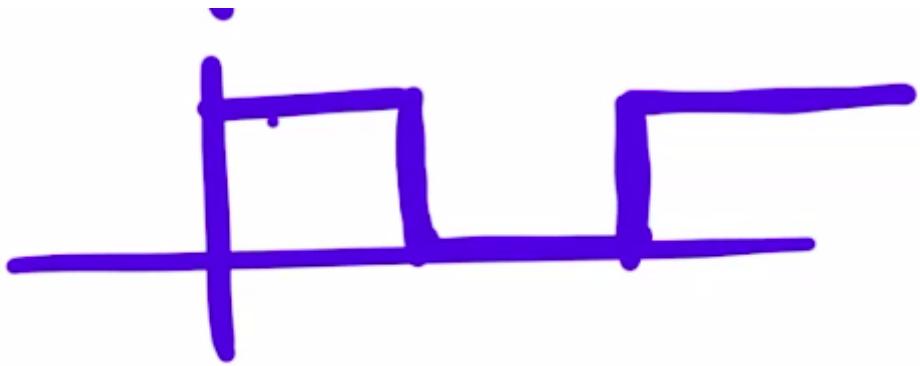
high pass filter



band pass



band rejection



so overall, all you need is come up with some a_k and b_k values and we get a filter

$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}}$$

LTI

Delay mul add

advantages:

no problem with temp or aging like with resistors

reconfiguration is easier

compact

realisation and flexibility

PROBLEM:

quantisation error can make some issues

FIR

used for stable filters but need much more values for the coeff in numeratot and we can design that easily with less values of ak and bk also

with more bks u need more delay elements, hence number of delay elements more here

IIR

may or may not be stable but a=merits above

So the filter we are using should be LTI, stable, causal.

Causal $h(n) = 0 \quad n < 0$

stability $\sum |h(n)| < \infty$

$n = -\infty$

~~not causal~~

[right side signal]

$H(z) \rightarrow$ stability

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$n = -\infty$

$$h(n)$$

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

$$\sum_{n=-\infty}^{\infty} h(n) < \infty \quad \boxed{z=1} \quad \downarrow \text{ROC}$$

Stability check unit circle.

$H(z)$ is stable if ROC also has unit circle in it.

i.e poles shud be inside unit circle for a stable IIR system