hw8

ME179P W21

Naimul Hoque

Perm Number:

3119773

E8.1(ii) Answer:

$$H_M^N = H_S^N H_M^S \rightarrow \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1/2 & \sqrt{3}/2 & 0 & -\sqrt{3} \\ \sqrt{3}/2 & 1/2 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 & 1 \\ 0 & 1 & 0 & -1 \\ -\frac{\sqrt{3}}{2} & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{H_M^N = \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & -1/2 & 0 \\ -1/4 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{4} & -\frac{1-3\sqrt{3}}{2} \\ \sqrt{3}/4 & 1/2 & 3/4 & \frac{\sqrt{3}-3}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

Exercise 8.2: Change of reference frame

E8.2(i) Answer:

Planar Displacement: $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$

$$\boxed{H = \begin{bmatrix} 0, & 0, & -1, & 1 \\ 1, & 0, & 0, & 3 \\ 0, & 0, & 1, & 1 \\ 0, & 0, & 0, & 1 \end{bmatrix}}$$

$$R = Rot_z(\pi/2)\, Rot_x(-\pi/2)$$

$$R = \begin{bmatrix} \cos\pi/2 & -\sin\pi/2 & 0 \\ \sin\pi/2 & \cos\pi/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\pi/2 & \sin\pi/2 \\ 0 & -\sin\pi/2 & \cos\pi/2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Do not
write here!

Name:

Naimul Hoque

Perm Number:

3119773

3

hw8

ME179P W21

E8.2(ii) Answer:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation accross y-axis a distance of 1.

$$A' = H_0^1 A^0$$

$$-(R_1^0)^T O_1^0 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} 0 & 1 & 0 & -3 \\ 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A' = H_0^1 A^0 = \begin{bmatrix} 0 & 1 & 0 & -3 \\ 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{A' = \begin{bmatrix} 0 & 0 & 0 & -2 \\ 0 & 0 & -2 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

Naimul Hoque

Perm Number:

3119773

hw8

ME179P W21

E8.3(ii) Answer:

$X=0, Y=0$

$\theta=0$

$$H=\begin{bmatrix}\cos(0) & -\sin(0) & 0\\ \sin(0) & \cos(0) & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I_3$$

E8.3(iii) Answer:

$R=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$v=\begin{bmatrix}x\\y\end{bmatrix}$

2x2 block matrix with form $\begin{bmatrix}A & B\\ 0 & D\end{bmatrix}$ has inverse $\begin{bmatrix}A^{-1} & -A^{-1}BD^{-1}\\ 0 & D^{-1}\end{bmatrix}$

$B=\begin{bmatrix}x\\y\end{bmatrix}$

$$H=\left[\begin{array}{cc|c}\cos\theta, & \sin\theta, & x\\ -\sin\theta, & \cos\theta, & y\\ \hline 0, & 0 & 1\end{array}\right]$$

$A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta, & \cos\theta\end{bmatrix}$

$-A^{-1}B$

$D^{-1}=1=D$

$$A^{-1}=\frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$$

$$A^{-1}=\frac{1}{\cos^2\theta+\sin^2\theta}\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

$$\boxed{H^{-1}=\begin{bmatrix}R^T & -R^Tv\\ 0_{1\times2} & 1\end{bmatrix}}$$

* It is a Planar displacement matrix

$$A^{-1}=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}, \quad -A^{-1}B=\begin{bmatrix}-\cos\theta & \sin\theta\\ -\sin\theta & -\cos\theta\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=-R^Tv$$

Do not write here!

4

hw8

ME179P W21

Name:

Naimul Hoque

Perm Number:

3119773

Exercise 8.3: The set of planar displacements

E8.3(i) Answer:

$$H = \begin{bmatrix} \cos\theta & -\sin\theta & x \\ \sin\theta & \cos\theta & y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & x \\ \sin\theta & \cos\theta & y \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta, & -2\sin\theta\cos\theta, & x\cos\theta - y\sin\theta + x \\ 2\sin\theta\cos\theta, & -\sin^2\theta + \cos^2\theta, & x\sin\theta + y\cos\theta + y \\ 0, & 0, & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta, & -\sin(2\theta), & x\cos\theta - y\sin\theta + x \\ \sin(2\theta), & -\sin^2\theta + \cos^2\theta, & x\sin\theta + y\cos\theta + y \\ 0, & 0, & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(2\theta), & -\sin(2\theta), & x\cos\theta - y\sin\theta + x \\ \sin(2\theta), & \cos(2\theta), & x\sin\theta + y\cos\theta + y \\ 0, & 0, & 1 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} \cos(2\theta), & -\sin(2\theta), & x' \\ \sin(2\theta), & \cos(2\theta), & y' \\ 0, & 0, & 1 \end{bmatrix}}$$ ✓

$-\sin^2\theta = -1 + \cos^2\theta$

$2\cos^2\theta - 1$

$= \cos(2\theta)$

$x(1+\cos\theta) - y\sin\theta$

write here!

Name: Naimul Hoque

Perm Number: 3119773

1

hw8

ME179P W21

Homework Assignment #8

- Use a **DARK** pen or pencil, and write **INSIDE** the answer boxes provided.
- Write your name and perm number **CLEARLY** at the top of **EVERY** page, inside the boxes provided.

Exercise 8.1: Frame displacements

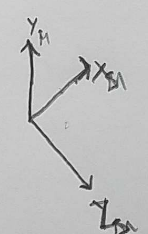
E8.1(i) Answer:

$$H_M^S = \begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 & 1 \\ 0 & 1 & 0 & -1 \\ -\frac{\sqrt{3}}{2} & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H = \begin{bmatrix} R & v \\ 0 & 1 \end{bmatrix}$$

$$H_S^N = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1/2 & -\sqrt{3}/2 & 0 & -\sqrt{3} \\ \sqrt{3}/2 & 1/2 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

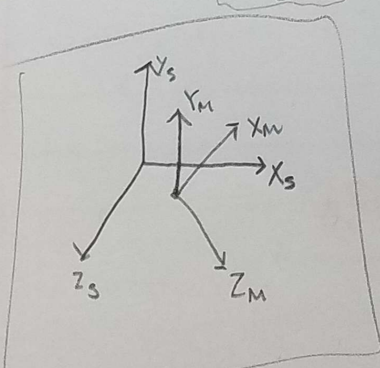
$$R = \begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & 1/2 \end{bmatrix} = Rot_y(\pi/3) \rightarrow$$

$$V = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

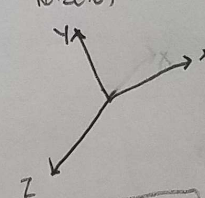


$$R = \begin{bmatrix} 0 & 0 & -1 \\ -1/2 & \sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \end{bmatrix} \quad \text{For } H_S^N \rightarrow R^T$$

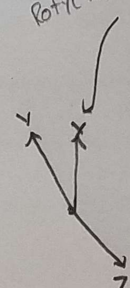
$$R^T = \begin{bmatrix} 0 & -1/2 & \sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \\ -1 & 0 & 0 \end{bmatrix}$$

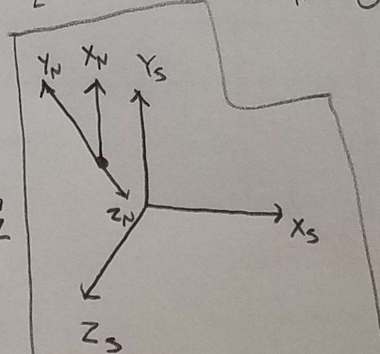


$Rot_z(\pi/6)$



$Rot_y(\pi/2)$





$Rot_z(\pi/6)\, Rot_y(\pi/2)$

$$-R^T \cdot O_N^S = -R^T \cdot \begin{bmatrix} 0 \\ -\sqrt{3} \\ 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3} \\ 1 \\ 0 \end{bmatrix} = V$$