3(e)

Proof of Rolle's Theorem

If ff is a function continuous on [a,b]$[a,b]$ and differentiable on (a,b)(a,b), with f(a)=f(b)=0$f(a)=f(b)=0$, then there exists some cc in (a,b)(a,b) where f'(c)=0$f'(c)=0$.

Proof:

Consider the two cases that could occur:

Case 1:

f(x)=0$f(x)=0$ for all xx in [a,b]$[a,b]$.

In this case, any value between aa and bb can serve as the cc guaranteed by the theorem, as the function is constant on [a,b]$[a,b]$ and the derivatives of constant functions are zero.

Case 2:

f(x)≠0$f(x)\neq 0$ for some xx in (a,b)(a,b).

We know by the Extreme Value Theorem, that ff attains both its absolute maximum and absolute minimum values somewhere on [a,b]$[a,b]$.

Recall by our hypothesis, f(a)=f(b)=0$f(a)=f(b)=0$, and that in this case, f(x)$f(x)$ is not zero for some xx in (a,b)(a,b). Thus, ff will have either a positive absolute maximum value at some cmaxc_{max} in (a,b)(a,b) or a negative absolute minimum value at some cminc_{min} in (a,b)(a,b) or both.

Take cc to be either cminc_{min} or cmaxc_{max}, depending on which you have.

Note then, the open interval (a,b)(a,b) contains cc, and either:

i. f(c)≥f(x)$f(c)\geq f(x)$ for all xx in (a,b)(a,b); or
ii. f(c)≤f(x)$f(c)\leq f(x)$ for all xx in (a,b)(a,b).

Either way, this means ff has a local extremum at cc.

As ff is also differentiable at cc, Fermat's Theorem applies, and concludes that f'(c)=0$f'(c)=0$.

3 c)

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$$

Here, the denominator in the left side of the Cauchy formula is not zero: g(b)-g(a) ≠ 0. If g(b) = g(a), then by Rolle's theorem, there is a point d ? (a,b), in which g'(d) = 0. Therefore, contradicts the hypothesis that g'(x) ≠ 0 for all x ? (a,b).

Now, we apply the auxiliary function.

F (x) = f (x) + λg(x)

And select λ in such a way to satisfy the given condition

F (a) = f (b). we get,

f (a) + λg(a) = f (b) + λg(b)

= f(b)-f (a) = λ[g(a)- g(b)]

$$\lambda = \frac{f(b)-f(a)}{g(b)-g(a)}$$

And the function F (x) exists in the form

$$F(x) = f(x) - \frac{f(b)-f(a)}{g(b)-g(a)}\, g(x).$$

The function F (x) is continuous in the closed interval (a≤x ≤b), differentiable in the open interval (a < x< b) and takes equal vales at the endpoints of the interval. So, it satisfies all the conditions of Rolle's theorem. Then, there is a point c exist in the interval (a,b) given as

F' (c) = 0.

It follows that

$$f'(c) - \frac{f(b)-f(a)}{b-a}\, g'(c) = 0$$

or

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$$

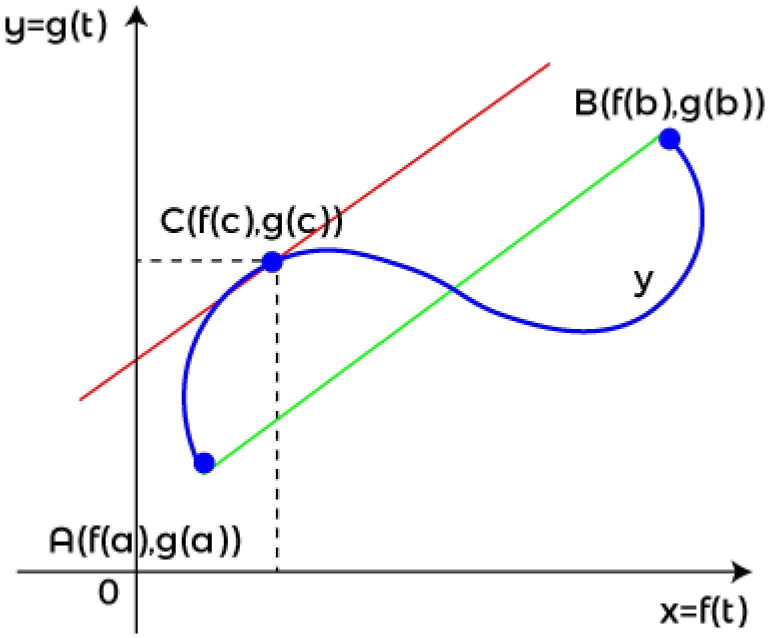
By putting g (x) = x in the given formula, we get the Lagrange formula:

$$f'(c) - \frac{f(b) - f(a)}{b - a}$$

Cauchy's mean value theorem has the given geometric meaning. Consider the parametric equations give a curve ? X = f (t) and Y = g (t), where the parameter t lies in the interval [a,b].

When we change the parameter t, the point of the curve in the given figure runs from A (f (a). g(a) to B (f(b), g (b)).

According to Cauchy's mean value theorem, there is a point (f(c), g(c)) on the curve ? where the tangent is parallel to the chord linking the two ends A and B of the curve.



3c)

Given function f(x) is continuous in [0,4] and differentiable in]0,4[

Again, f(a) = f(0) = -6

f(b) = f(4) = 6

Now, f(x) = (x - 1)(x - 2)(x - 3)

x3 - 6x2 + 11x - 6

∴ f'(x) = 3x2 - 12x + 11

∴ Mean value theorem, f'(c) = (f(b) - f(a))/(b - a)

3c2 - 12c + 11 = (6 + 6)/(4 - 0) = 12/4 =

3 3c2 - 12c + 8 = 0

c = -3.15 or 0.08453 ∈]0,4[.

Misc 32

Evaluate the definite integral

$\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$

Let $\mathrm{I} = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$ ...(1)

Using The Property, P_4

$P_4 : \int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$\therefore\ \mathrm{I} = \int_0^{\pi} \frac{(\pi - x) \tan\ (\pi - x)}{\sec(\pi - x) + \tan(\pi - x)}\, dx$$

$$\mathrm{I} = \int_0^{\pi} \frac{(\pi - x)(-\tan\ x)}{(-\sec\ x) + (-\tan x)}\, dx$$

$$\mathrm{I} = \int_0^{\pi} \frac{-(\pi - x) \tan x}{-(\sec x + \tan x)}\, dx$$

teachoo.com

$$\mathrm{I} = \int_0^{\pi} \frac{(\pi - x)\tan x}{(\sec x + \tan x)}\, dx \qquad \text{...(2)}$$

Adding (1) and (2) i.e. (1) + (2)

$$\mathrm{I} + \mathrm{I} = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx + \int_0^{\pi} \frac{\pi \tan x - x \tan x}{\sec x + \tan x}\, dx$$

$$2\mathrm{I} = \int_0^{\pi} \frac{x \tan x + \pi \tan x - x \tan x}{\sec x + \tan x}\, dx$$

$$2\mathrm{I} = \int_0^{\pi} \frac{\pi \tan x}{\sec x + \tan x}\, dx$$

$$2\mathrm{I} = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x}\, dx$$

$$\mathrm{I} = \frac{\pi}{2} \int_0^{\pi} \frac{\tan x}{\sec x + \tan x}\, dx$$

\

teachoo.com

$$= \frac{\pi}{2} \int_0^{\pi} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}\ dx$$

$$= \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1 + \sin x}\ dx$$

$$= \frac{\pi}{2} \int_0^{\pi} \frac{\sin x + 1 - 1}{1 + \sin x}\ dx$$

(Adding and subtracting 1 in numerator)

$$= \frac{\pi}{2} \int_0^{\pi} \left[\frac{1 + \sin x}{1 + \sin x} - \frac{1}{1 + \sin x}\right] dx$$

$$= \frac{\pi}{2} \int_0^{\pi} \left[1 - \frac{1}{1 + \sin x}\right] dx$$

$$= \frac{\pi}{2} \left[\int_0^{\pi} 1\ dx - \int_0^{\pi} \frac{1}{1 + \sin x}\ dx\right]$$

$$= \frac{\pi}{2} \left[[x]_0^{\pi} - \int_0^{\pi} \frac{1}{1 + \sin x} \left(\frac{1 - \sin x}{1 - \sin x}\right) dx\right]$$

(Mult. & dividing by $1 - \sin x$)

$$= \frac{\pi}{2}\left[[\pi - 0] - \int_0^{\pi} \frac{1 - \sin x}{1 - \sin^2 x}\ dx\right]$$

$$= \frac{\pi}{2}\left[\pi - \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x}\ dx\right]$$

$$= \frac{\pi}{2}\left[\pi - \int_0^{\pi}\left[\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\right] dx\right]$$

$$= \frac{\pi}{2}\left\{\pi - \int_0^{\pi}[\sec^2 x - \tan x \sec x]\, dx\right\}$$

$$= \frac{\pi}{2}\left\{\pi - \int_0^{\pi} \sec^2 x\, dx + \int_0^{\pi} \tan x \sec x\, dx\right\}$$

$$= \frac{\pi}{2}\ [\pi - [\tan x]_0^{\pi} + [\sec x]_0^{\pi}]$$

$$= \frac{\pi}{2}\{\pi - [\tan(\pi) - \tan(0)] + [\sec(\pi) - \sec(0)]\}$$

teachoo.com

$$= \frac{\pi}{2}\{\pi - [0 - 0] + [-1 - 1]\}$$

$$= \frac{\pi}{2}\{\pi - 0 + [-2]\}$$

$$= \boldsymbol{\frac{\pi}{2}(\pi - 2)}$$

1a)

2. The function f(x)=x+1 from the set of integers to itself is onto. Is it True or False?

a) True

b) False

View Answer

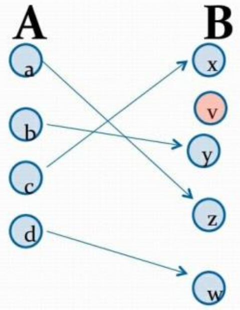
Answer: a

Explanation: For every integer “y” there is an integer “x ” such that f(x) = y.

1b)

Injections

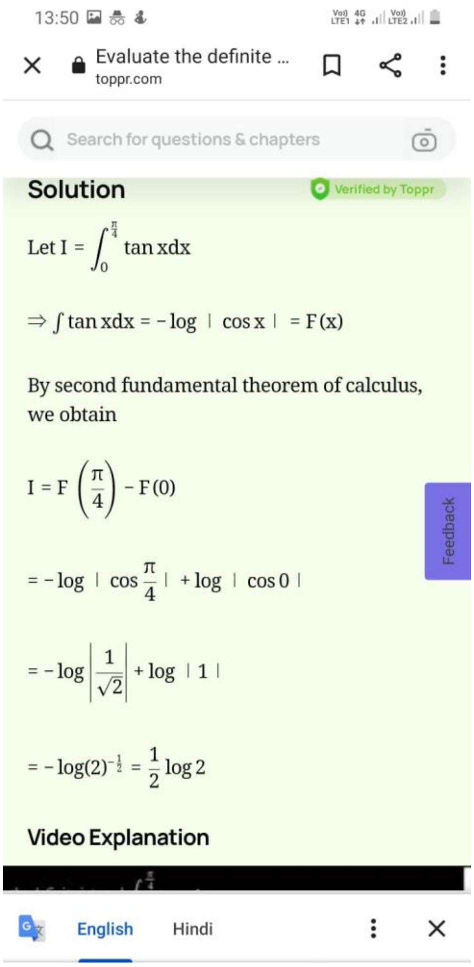
Definition: A function f is said to be *one-to-one* , or *injective*, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f. A function is said to be an *injection* if it is one-to-one.



1d)

True or False. **If a function f is not defined at x = a then it is not continuous at x = a**.
Answer : True.

1e)



13:50

Evaluate the definite ...
toppr.com

Search for questions & chapters

Solution Verified by Toppr

Let $I = \int_0^{\frac{\pi}{4}} \tan x dx$

$\Rightarrow \int \tan x dx = -\log|\cos x| = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F\left(\frac{\pi}{4}\right) - F(0)$

$= -\log\left|\cos\frac{\pi}{4}\right| + \log|\cos 0|$

$= -\log\left|\frac{1}{\sqrt{2}}\right| + \log|1|$

$= -\log(2)^{-\frac{1}{2}} = \frac{1}{2}\log 2$

Video Explanation

Feedback

English Hindi

1 8)

We know that $|x| = x$ for all $x \geq 0$ and $|x| = -x$ for all $x < 0$. Therefore,
At $x=2$,
$|x-1| = x-1$ and $|x-3| = -(x-3) = -x+3$
$\Rightarrow f(x) = (x-1) + (-x+3) = 2$
which is a constant function and the derivative of a constant function is always zero. So at $x=2$ derivative of $f(x)$ is zero.

1(9)

f0 = 3

=> a=3

=> f2 = 2+3= 5

2a)

Bounded and Unbounded Function

Let a function be defined as f(x): A →B and we can find two real numbers m and M such that m ≤ f(x) ≤ M ∀ x ε A then f(x) is called the bounded function. m and M are called the lower-bound and the upper-bound of f(x) respectively. The range of f(x) is [m, M] (see figure given below), If however, m and M or either of them is not defined (i.e. infinite) then f(x) is said to be unbounded function.

2c)

Definite Integral

$$\int_a^b f(x)\,dx$$

The definite integral of f(x) is a NUMBER and represents the area under the curve f(x) from x=a to x=b.

Indefinite Integral

$$\int f(x)\,dx$$

The indefinite integral of f(x) is a FUNCTION and answers the question, "What function when differentiated gives f(x)?"

Fundamental Theorem of Calculus

The FTC relates these two integrals in the following manner:

To compute a definite integral, find the antiderivative (indefinite integral) of the function and evaluate at the endpoints x=a and x=b.

2 ka part

Left Hand Limit

If xx approaches aa from the left side, i.e. from the values less than aa, the function is said to have a left hand limit. If pp is the left hand limit of ff as xx approaches aa, we write it as

limx→a−f(x)=p $\lim_{x\to a^-} f(x)=p$

Right Hand Limit

If xx approaches aa from the right side, i.e. from the values greater than aa, the function is said to have a right hand limit. If qq is the right hand limit of ff as xx approaches aa, we write it as

limx→a+f(x)=q $\lim_{x\to a^+} f(x)=q$

For the existence of the limit of a real valued function at a certain point, it is essential that both its left hand and right hand limits exist and have the same value.

In other words, if the left and right hand limits exist and

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

,

then f is said to have a limit at $x=a$.

On the other hand if both the left and right hand limits exist but

$$\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$$