

STEPS & FORMULAE FOR REGRESSION ANALYSIS

Simple Linear Regression Model:

$$y = \beta_0 + \beta_1 x + \varepsilon$$

where:

β_0 is the y-intercept of the regression line

β_1 is the slope of the regression line

ε is the error term.

Estimated Simple Linear Regression Equation:

"y hat"

$$\hat{y} = b_0 + b_1 x$$

where:

\hat{y} is the predicted value of y for a given x value.

b_0 is the y intercept of the line.

b_1 is the slope of the line.

Least Squares Method:

Use Sample data to find
the line of regression

$$\hat{y} = b_0 + b_1x$$

where:

\hat{y} is the predicted grade on exam

b_0 is the y intercept of the line.

b_1 is the slope of the line.

x is number of hours studied

Least Squares Method:

$$\min \sum (y_i - \hat{y}_i)^2$$

where:

y_i = observed value of the dependent
variable for the i th observation

\hat{y}_i = predicted value of the dependent
variable for the i th observation

Example: $x=3$ hours studied

\hat{y}_i = approx. 69

Calculating the Slope:

$$b_1 = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

where:

x_i = value of independent variable for i th observation

y_i = value of dependent variable for i th observation

\bar{x} = mean value for independent variable

\bar{y} = mean value for dependent variable

Calculating the y – intercept:

$$b_0 = \bar{y} - b_1\bar{x}$$

x_i	y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x})^2$
2	69	-2.8	-8.8	24.64	7.84

$\sum x_i = 48$	$\sum y_i = 778$		320.6	67.6
$\bar{x} = 48/10$	$\bar{y} = 778/10$		$\sum(x_i - \bar{x})(y_i - \bar{y})$	$\sum(x_i - \bar{x})^2$
= 4.8	= 77.8			

$$b_1 = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$\hat{y} = b_0 + b_1x$$

Coefficient of Determination:

How well does the regression line fit the data?

$$r^2 = SSR/SST$$

where:

SSR = sum of squares due to regression = $\sum(\hat{y}_i - \bar{y})^2$

SST = total sum of squares = $\sum(y_i - \bar{y})^2$

SSE = sum of squares due to error = $\sum(y_i - \hat{y}_i)^2$

$$SST = SSR + SSE$$

x_i	y_i	Predicted Grades $\hat{y}_i = 55.048 + 4.74x_i$	Error $y_i - \hat{y}_i$	Squared Error $(y_i - \hat{y}_i)^2$	Deviation $y_i - \bar{y}$	Squared Deviation $(y_i - \bar{y})^2$
2	69	64.528	4.472	19.9988	-8.8	77.44

$$SSE = 79.1215$$

$$SST = 1599.6$$

Coefficient of Determination:

$$r^2 = SSR/SST = 1520.4785/1599.6 = .9505$$

$$SST = SSR + SSE$$

$$SSR = SST - SSE$$

$$\begin{aligned} SSR &= 1599.6 - 79.1215 \\ &= 1520.4785 \end{aligned}$$

r^2 = percent of variability in y
can be explained by x

r^2 = 95.05% of the variability in
grades can be explained by
the number of hours studied