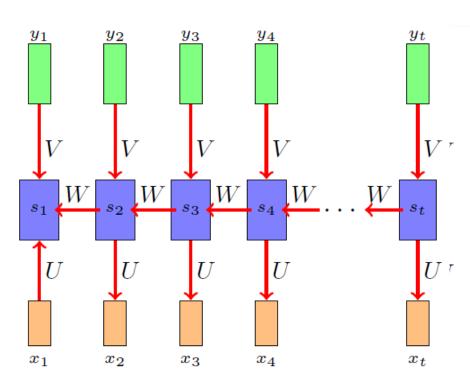
LSTM & GRU

DSE 3151 DEEP LEARNING

Dr. Rohini Rao & Dr. Abhilash K Pai

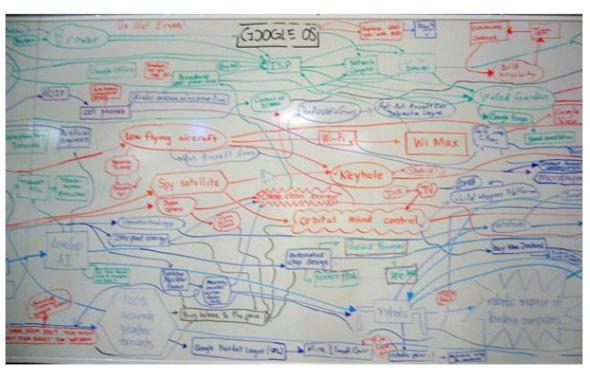
Dept. of Data Science and Computer Applications

MIT Manipal



- The state (s_i) of an RNN records information from all previous time steps.
- At each new timestep the old information gets morphed by the current input.
- After 't' steps the information stored at time step tk (for some k < t) gets completely morphed.
- It would be impossible to extract the original information stored at time step t k.
- Also, there is the Vanishing gradients problem!

The white board analogy:



Source: CS7015 Deep Learning, Dept. of CSE, IIT Madras

 Consider a scenario where we have to evaluate the expression on a whiteboard:

Evaluate "
$$ac(bd+a) + ad$$
"
given that $a=1$, $b=3$, $c=5$, $d=11$

 Normally, the evaluation in white board would look like:

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$
 $ac(bd + a) = 170$
 $ad = 11$
 $ac(bd + a) + ad = 181$

 Now, if the white board has space to accommodate only 3 steps, the above evaluation cannot fit in the required space and would lead to loss of information.

Evaluate "
$$ac(bd+a) + ad$$
"
given that $a=1$, $b=3$, $c=5$, $d=11$

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$
 $ac(bd + a) = 170$
 $ad = 11$
 $ac(bd + a) + ad = 181$

A solution is to do the following:

Evaluate "
$$ac(bd+a) + ad$$
"
given that $a=1$, $b=3$, $c=5$, $d=11$

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$
 $ac(bd + a) = 170$
 $ad = 11$
 $ac(bd + a) + ad = 181$

- A solution is to do the following:
 - Selectively write:

$$ac = 5$$
$$bd = 33$$

Evaluate "
$$ac(bd+a) + ad$$
"
given that $a=1$, $b=3$, $c=5$, $d=11$

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$
 $ac(bd + a) = 170$
 $ad = 11$
 $ac(bd + a) + ad = 181$

- A solution is to do the following:
 - Selectively write:

$$ac = 5$$
$$bd = 33$$

• Selectively read:

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$

Evaluate "
$$ac(bd+a) + ad$$
" given that $a=1$, $b=3$, $c=5$, $d=11$

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$
 $ac(bd + a) = 170$
 $ad = 11$
 $ac(bd + a) + ad = 181$

- A solution is to do the following:
 - Selectively write:

$$ac = 5$$

 $bd = 33$

Selectively read:

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$

Now the board is full

Evaluate "
$$ac(bd+a) + ad$$
" given that $a=1$, $b=3$, $c=5$, $d=11$

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$
 $ac(bd + a) = 170$
 $ad = 11$
 $ac(bd + a) + ad = 181$

- A solution is to do the following:
 - Selectively write:

$$ac = 5$$
$$bd = 33$$

• Selectively read:

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$

Now the board is full

• So, Selectively forget:

$$ac = 5$$

$$bd + a = 34$$

Evaluate "
$$ac(bd+a) + ad$$
" given that $a=1$, $b=3$, $c=5$, $d=11$

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$
 $ac(bd + a) = 170$
 $ad = 11$
 $ac(bd + a) + ad = 181$

- A solution is to do the following:
 - Selectively write:

$$ac = 5$$
 $bd = 33$

Selectively read:

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$

Now the board is full

• So, Selectively forget:

$$ac = 5$$

 $ac(bd + a) = 170$
 $bd + a = 34$

Evaluate "
$$ac(bd+a) + ad$$
" given that $a=1$, $b=3$, $c=5$, $d=11$

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$
 $ac(bd + a) = 170$
 $ad = 11$
 $ac(bd + a) + ad = 181$

- A solution is to do the following:
 - Selectively write:

$$ac = 5$$
 $bd = 33$

Selectively read:

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$

Now the board is full

• So, Selectively forget:

$$ac = 5$$

 $ac(bd + a) = 170$
 $ad = 11$

Evaluate "
$$ac(bd+a) + ad$$
" given that $a=1$, $b=3$, $c=5$, $d=11$

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$
 $ac(bd + a) = 170$
 $ad = 11$
 $ac(bd + a) + ad = 181$

- A solution is to do the following:
 - Selectively write:

$$ac = 5$$
$$bd = 33$$

• Selectively read:

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$

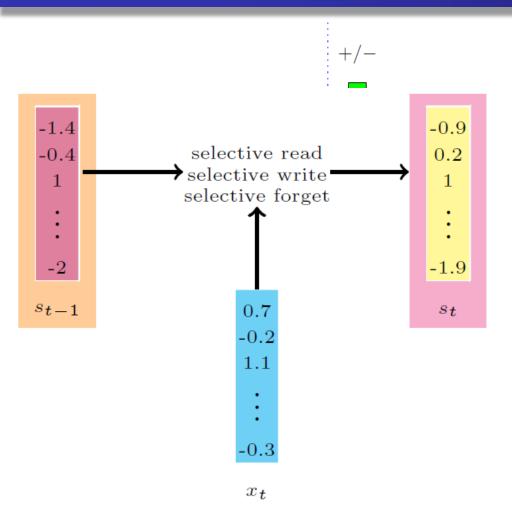
Now the board is full

• So, Selectively forget:

$$ac(bd + a) + ad = 181$$

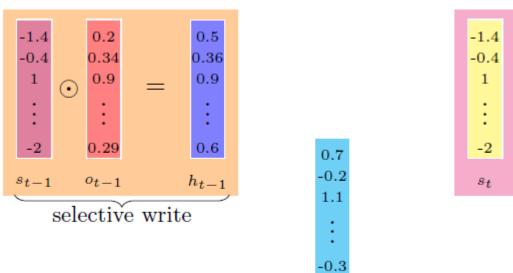
 $ac(bd + a) = 170$
 $ad = 11$

Since the RNN also has a finite state size, we need to figure out a way to allow it to selectively read, write and forget



- RNN reads the document from left to right and after every word updates the state.
- By the time we reach the end of the document the information obtained from the first few words is completely lost.
- In our improvised network, ideally, we would like to:
 - **Forget** the information added by stop words (a, the, etc.)
 - **Selectively read** the information added by previous sentiment bearing words (awesome, amazing, etc.)
 - **Selectively write** new information from the current word to the state.

Source: CS7015 Deep Learning, Dept. of CSE, IIT Madras



The RNN has to learn o_{t-1} along with other parameters (W,U,V)

 x_t

$$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$$

$$h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$$

New parameters to be learned are: W_o , U_o , b_o

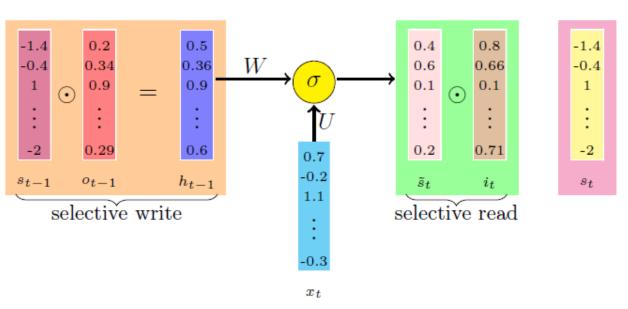
 O_t is called the output gate as it decides how much to pass (write) to the next time step.

Selectively write:

In an RNN, the state s, is defined as follows:

$$s_t = \sigma(Ws_{t-1} + Ux_t)$$
 (ignoring bias)

- Instead of passing s_{t-1} as it is we need to pass (write) only some portions of it.
- To do this, we introduce a vector o_{t-1} which decides what fraction of each element of s_{t-1} should be passed to the next state.
- Each element of o_{t-1} (restricted to be between 0 and 1) gets multiplied with s_{t-1}
- How does RNN know what fraction of the state to pass on?



Source: CS7015 Deep Learning, Dept. of CSE, IIT Madras

Selectively read:

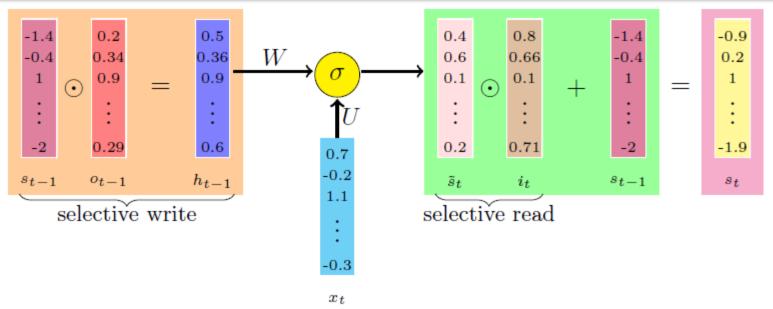
 We will now use h_{t-1} and x_t to compute the new state at the time step t :

$$\tilde{s_t} = \sigma(Wh_{t-1} + Ux_t + b)$$

- Again, to pass only useful information from $\tilde{s_t}$ to s_t , we selectively read from it before constructing the new cell state.
- To do this we introduce another gate called as the input gate:

$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$

• And use $i_t \odot \tilde{s_t}$ to selectively read the information.

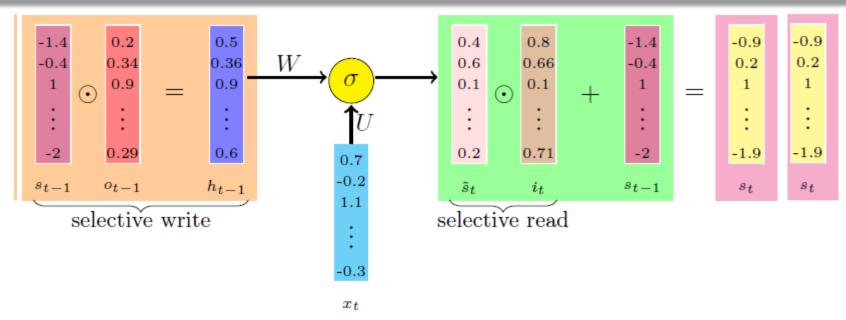


Source: CS7015 Deep Learning, Dept. of CSE, IIT Madras

Selectively forget

• How do we combine s_{t-1} and $\tilde{s_t}$ to get the new state?

$$s_t = s_{t-1} + i_t \odot \tilde{s_t}$$



Source: CS7015 Deep Learning, Dept. of CSE, IIT Madras

Selectively forget

• How do we combine s_{t-1} and $\tilde{s_t}$ to get the new state?

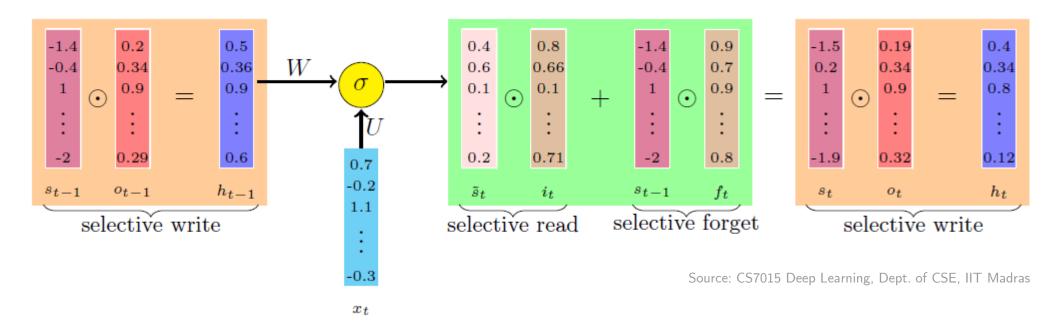
$$s_t = s_{t-1} + i_t \odot \tilde{s_t}$$

$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s_t}$$

- But we may not want to use the whole of s_{t-1} but forget some parts of it.
- To do this a forget gate is introduced:

$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

LSTM (Long Short-Term Memory)



Gates:

$$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$$

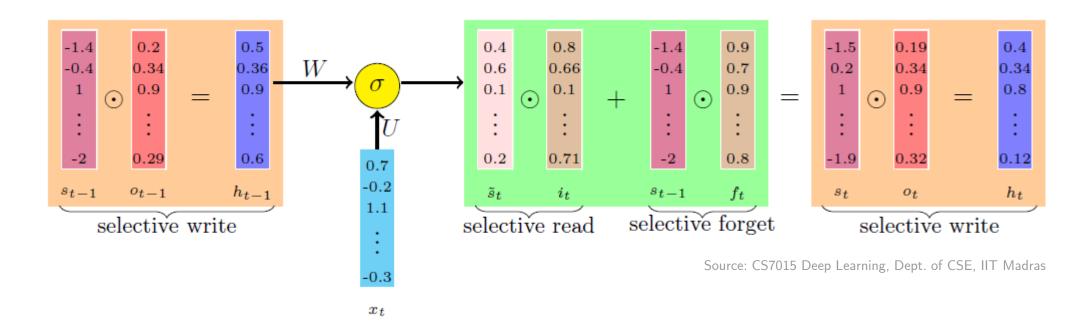
$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$

$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

States:

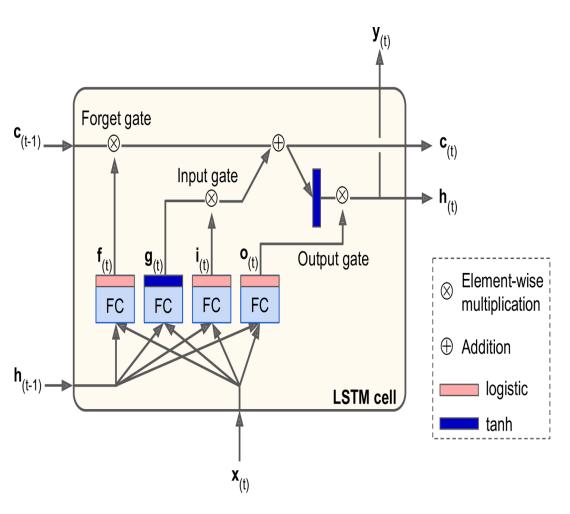
Long-term memory
$$\begin{split} \tilde{s_t} &= \sigma(W h_{t-1} + U x_t + b) \\ & \underbrace{ s_t} = f_t \odot s_{t-1} + i_t \odot \tilde{s_t} \\ & \underbrace{ h_t} = o_t \odot \sigma(s_t) \text{ and } rnn_{out} \\ & = h_t \end{split}$$
 Short-term memory

LSTM (Long Short-Term Memory)



- LSTM has many variants which include different number of gates and also different arrangement of gates.
- A popular variant of LSTM is the Gated Recurrent Unit (GRU).

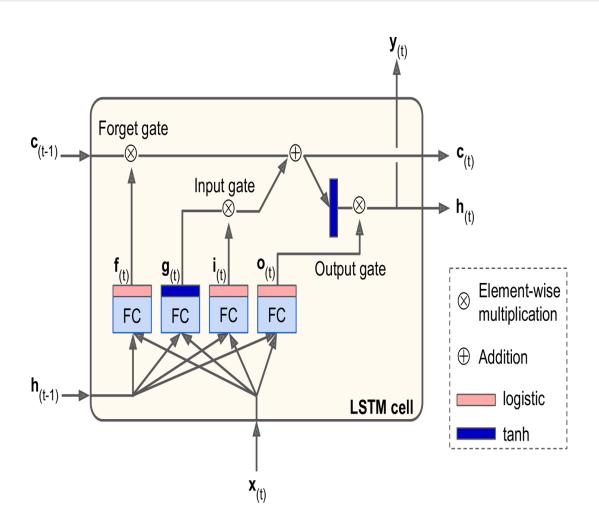
LSTM Cell



- Neuron called a Cell
- FC are fully connected layers
- Long Term state $c_{(t-1)}$ traverses through forget gate forgetting some memories and adding some new memories
- Long term state $c_{(t-1)}$ is passed through tanh and then filtered by an output gate, which produces short term state $h_{(t)}$
- Update gate- $g_{(t)}$ takes current input $x_{(t)}$ and previous short term state $h_{(t-1)}$
- Important parts of output g_(t) goes to long term state

Note: c_t is same as s_t

LSTM Cell



 Gating Mechanism - regulates information that the network stores

Other 3 layers are gate controllers

- Forget gate f_(t) controls which part of the long—term state should be erased
- Input gate $i_{(t)}$ controls which part of $g_{(t)}$ should be added to long term state
- Output gate o_(t) controls which parts of long term state should be read and output at this time state
 - both to h_(t) and to y_(t)

LSTM computations

$$\mathbf{i}_{(t)} = \sigma(\mathbf{W}_{xi}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hi}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{i})$$

$$\mathbf{f}_{(t)} = \sigma(\mathbf{W}_{xf}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hf}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{f})$$

$$\mathbf{o}_{(t)} = \sigma(\mathbf{W}_{xo}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{ho}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{o})$$

$$\mathbf{g}_{(t)} = \tanh(\mathbf{W}_{xg}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hg}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{g})$$

$$\mathbf{c}_{(t)} = \mathbf{f}_{(t)} \otimes \mathbf{c}_{(t-1)} + \mathbf{i}_{(t)} \otimes \mathbf{g}_{(t)}$$

$$\mathbf{y}_{(t)} = \mathbf{h}_{(t)} = \mathbf{o}_{(t)} \otimes \tanh(\mathbf{c}_{(t)})$$

An LSTM cell can learn to recognize an important input (role of the input gate), store it in long term state, preserve it for as long as possible it is needed (role of forget gate), and extract it whenever it is needed.

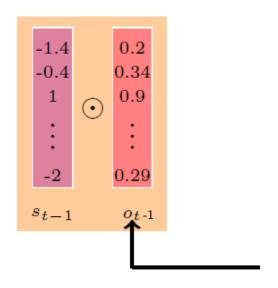
 \mathbf{W}_{xi} , \mathbf{W}_{xf} , \mathbf{W}_{xo} , \mathbf{W}_{xg} are the weight matrices of each of the four layers for their connection to the input vector $\mathbf{x}_{(t)}$.

 \mathbf{W}_{hi} , \mathbf{W}_{hf} , \mathbf{W}_{ho} , and \mathbf{W}_{hg} are the weight matrices of each of the four layers for their connection to the previous short-term state $\mathbf{h}_{(t-1)}$.

 \mathbf{b}_{i} , \mathbf{b}_{o} , and \mathbf{b}_{g} are the bias terms for each of the four layers.

-1.4 -0.4 1 \vdots -2 s_{t-1}

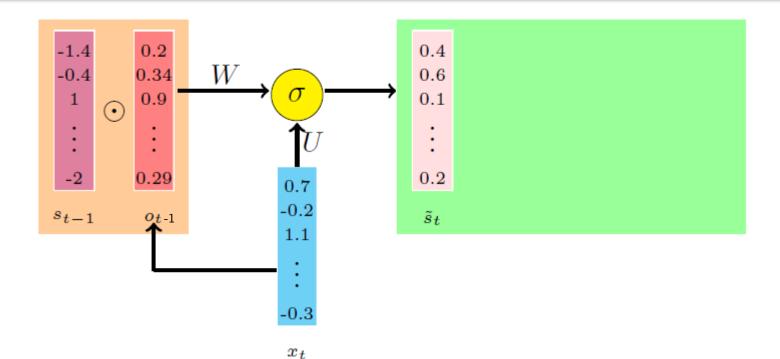
Gates: States:



Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

States:

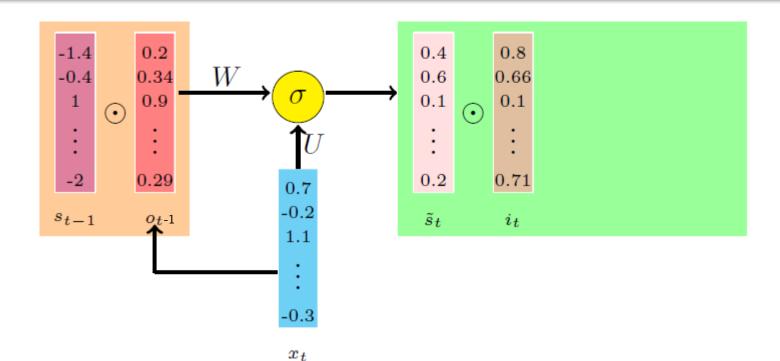


Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

States:

$$\tilde{s_t} = \sigma(W(o_t \odot s_{t-1}) + Ux_t + b)$$

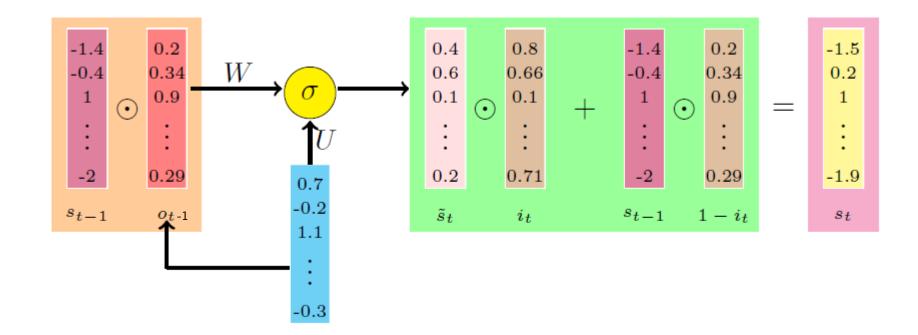


Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

States:

$$\tilde{s_t} = \sigma(W(o_t \odot s_{t-1}) + Ux_t + b)$$



Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

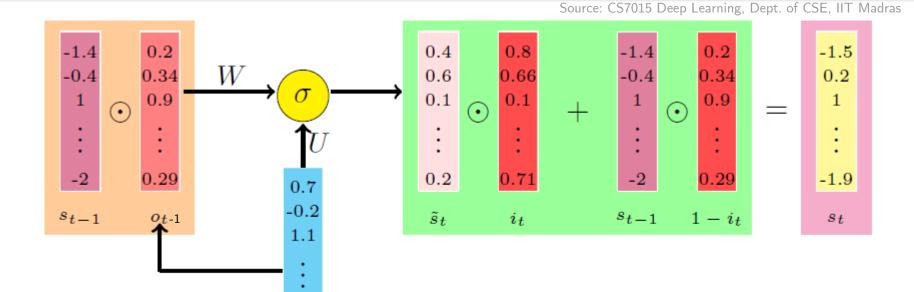
States:

$$\tilde{s_t} = \sigma(W(o_t \odot s_{t-1}) + Ux_t + b)$$

$$s_t = (1 - i_t) \odot s_{t-1} + i_t \odot \tilde{s_t}$$

Source: CS7015 Deep Learning, Dept. of CSE, IIT Madras

 x_t



Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

States:

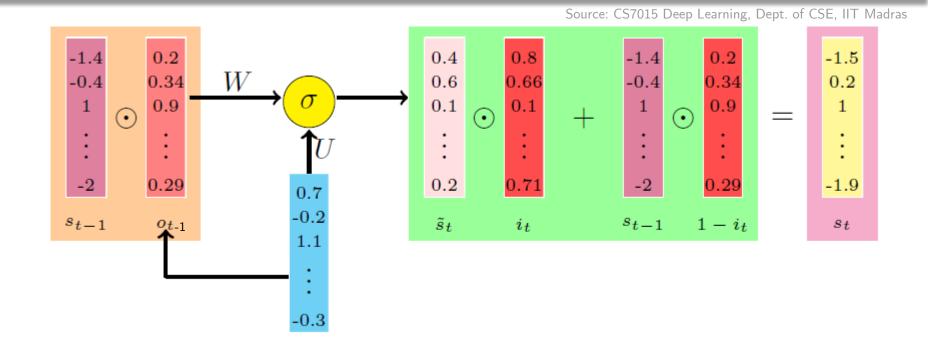
$$\tilde{s_t} = \sigma(W(o_t \odot s_{t-1}) + Ux_t + b)$$

$$s_t = (1 - i_t) \odot s_{t-1} + i_t \odot \tilde{s_t}$$

No explicit forget gate (the forget gate and input gates are tied)

-0.3

 x_t



Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

States:

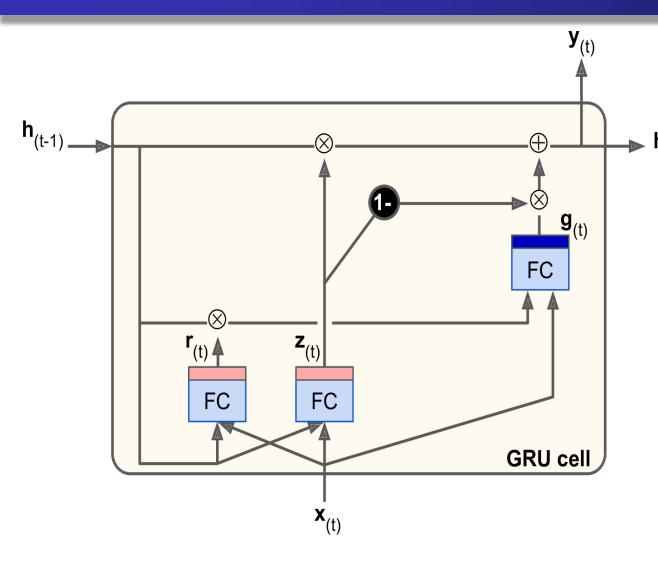
$$\tilde{s_t} = \sigma(W(o_t \odot s_{t-1}) + Ux_t + b)$$

$$s_t = (1 - i_t) \odot s_{t-1} + i_t \odot \tilde{s_t}$$

The gates depend directly on s_{t-1} and not the intermediate h_{t-1} as in the case of LSTMs

 x_t

Gated Recurrent Unit CELL (Kyunghyun Cho et al, 2014)



The main simplifications of LSTM are:

- Both state vectors (short and long term) are merged into a single vector $\mathbf{h}_{(t)}$.
- Gate controller $z_{(t)}$ controller controls both the forget gate and the input gate.
- If the gate controller outputs
 - 1, the forget gate is open and the input gate is closed.
 - 0, the opposite happens
 - whenever a memory must be written, the location where it will be stored is erased first.
- No output gate, the full state vector is output at every time step.
- Reset gate controller $r_{(t)}$ that controls which part of the previous state will be shown to the main layer $g_{(t)}$.

LSTM vs GRU computation

$$\mathbf{i}_{(t)} = \sigma(\mathbf{W}_{xi}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hi}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{i})$$

$$\mathbf{f}_{(t)} = \sigma(\mathbf{W}_{xf}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hf}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{f})$$

$$\mathbf{o}_{(t)} = \sigma(\mathbf{W}_{xo}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{ho}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{o})$$

$$\mathbf{g}_{(t)} = \tanh(\mathbf{W}_{xg}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hg}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{g})$$

$$\mathbf{c}_{(t)} = \mathbf{f}_{(t)} \otimes \mathbf{c}_{(t-1)} + \mathbf{i}_{(t)} \otimes \mathbf{g}_{(t)}$$

$$\mathbf{y}_{(t)} = \mathbf{h}_{(t)} = \mathbf{o}_{(t)} \otimes \tanh(\mathbf{c}_{(t)})$$

$$\mathbf{z}_{(t)} = \sigma(\mathbf{W}_{xz}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hz}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{z})$$

$$\mathbf{r}_{(t)} = \sigma(\mathbf{W}_{xr}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hr}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{r})$$

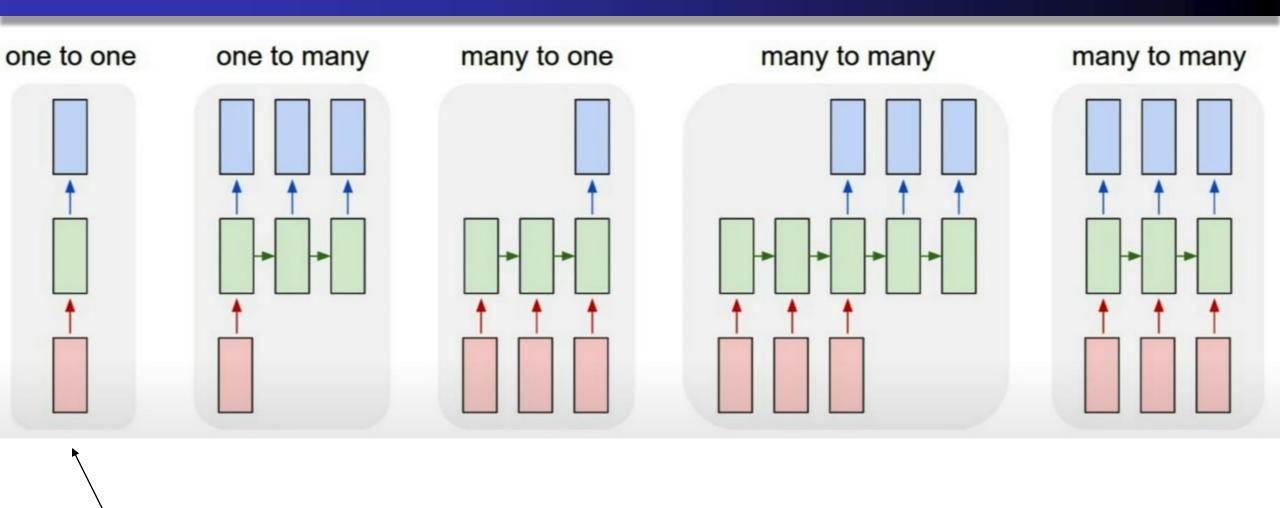
$$\mathbf{g}_{(t)} = \tanh(\mathbf{W}_{xg}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hg}^{T} \cdot (\mathbf{r}_{(t)} \otimes \mathbf{h}_{(t-1)}) + \mathbf{b}_{g})$$

$$\mathbf{h}_{(t)} = \mathbf{z}_{(t)} \otimes \mathbf{h}_{(t-1)} + (1 - \mathbf{z}_{(t)}) \otimes \mathbf{g}_{(t)}$$

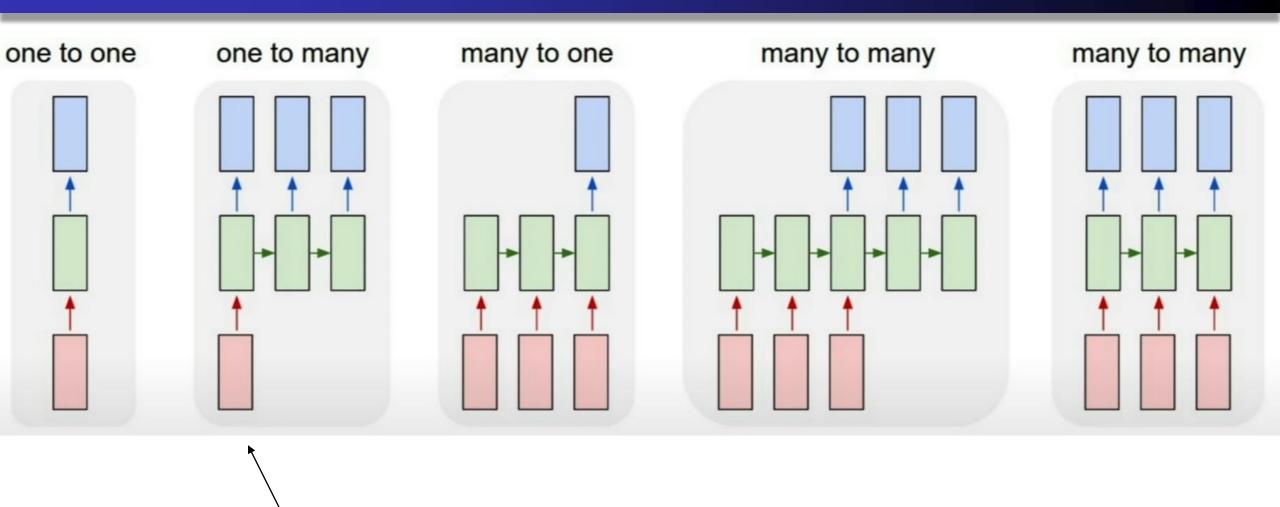
- GRU Performance is good but may have a slight dip in the accuracy
- But lesser number of trainable parameters which makes it advantageous to use

Avoiding vanishing gradients with LSTMs: Intuition

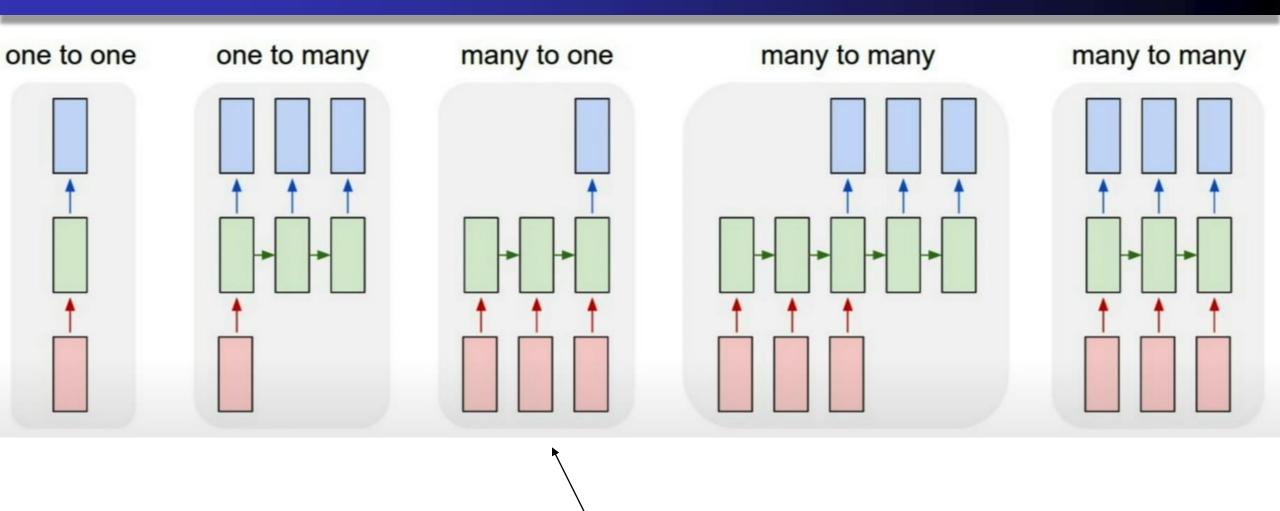
- During forward propagation the gates control the flow of information.
- They prevent any irrelevant information from being written to the state.
- Similarly during backward propagation they control the flow of gradients.
- It is easy to see that during backward pass the gradients will get multiplied by the gate.
- If the state at time t-1 did not contribute much to the state at time t then during backpropagation the gradients flowing into s_{t-1} will vanish $||f_t|| \to 0 \text{ and } ||o_{t-1}|| \to 0$
- The key difference from vanilla RNNs is that the flow of information and gradients is controlled by the gates which ensure that the gradients vanish only when they should.



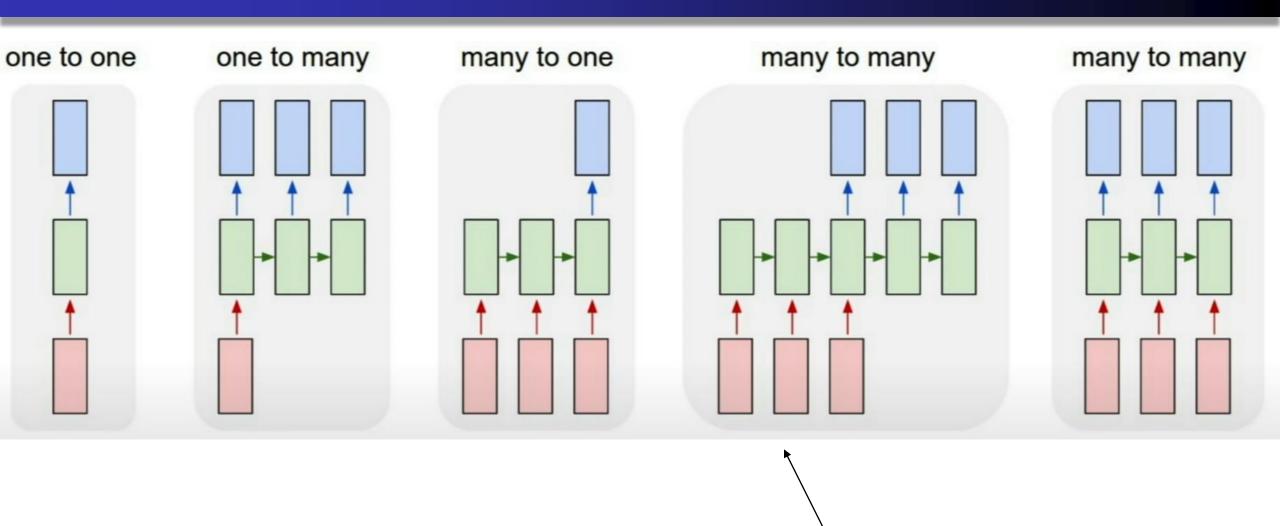
Vanilla RNNs



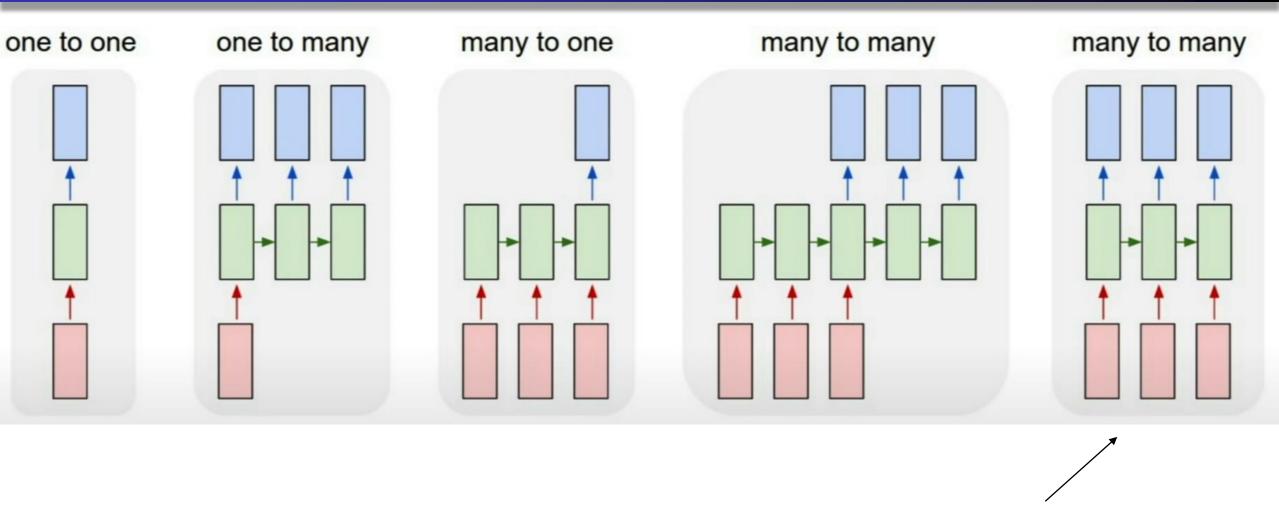
Eg: Image Captioning
Image -> Sequence of words



Eg: Sentiment classification
Sequence of words -> Sentiment

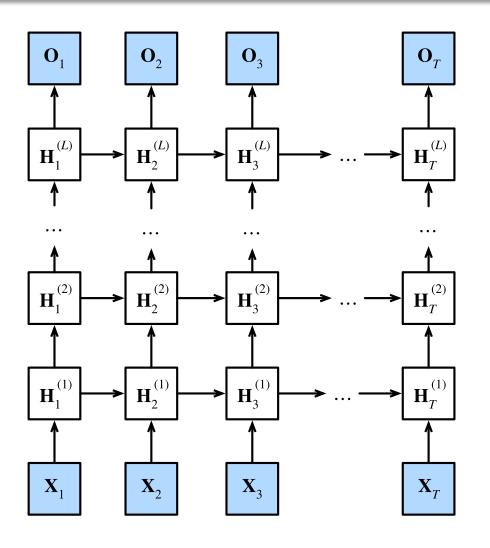


Eg: Machine Translation
Sequence of words -> Sequence of words



Eg: Video Classification on frame level

Deep RNNs



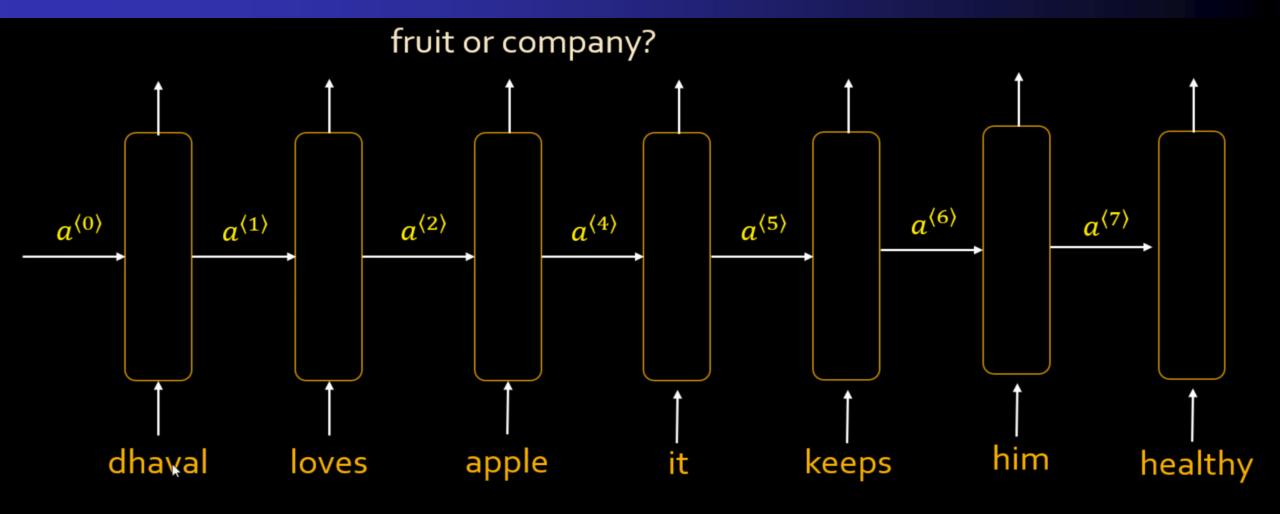
RNNs that are deep not only in the time direction but also in the input-to-output direction.

$$\mathbf{H}_{t}^{(l)} = \phi_{l}(\mathbf{H}_{t}^{(l-1)}\mathbf{W}_{xh}^{(l)} + \mathbf{H}_{t-1}^{(l)}\mathbf{W}_{hh}^{(l)} + \mathbf{b}_{h}^{(l)})$$

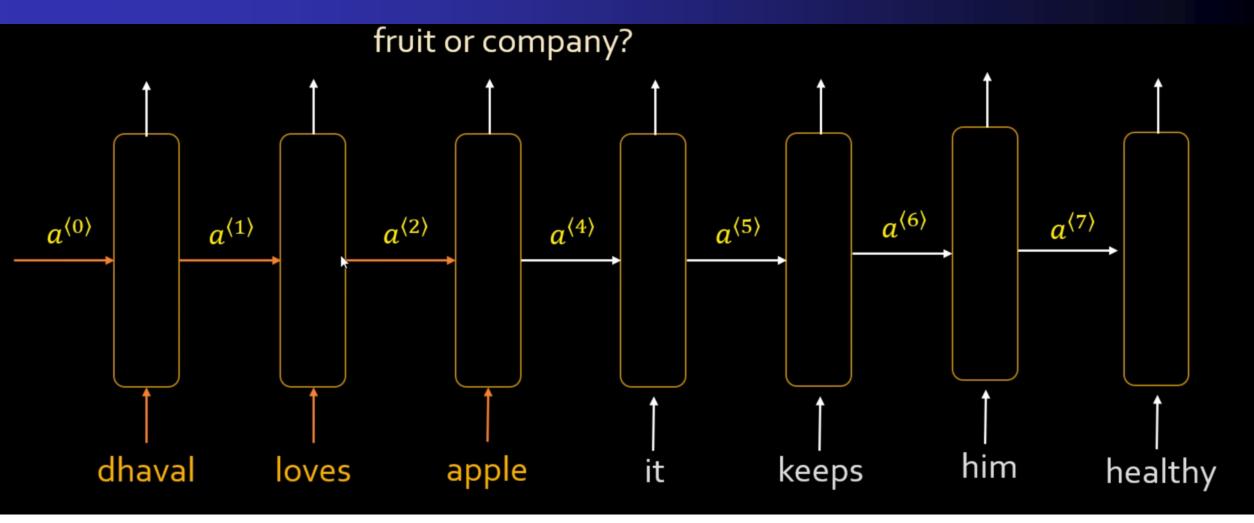
$$\mathbf{O}_t = \mathbf{H}_t^{(L)} \mathbf{W}_{hq} + \mathbf{b}_q$$

Source: Deep Recurrent Neural Networks — Dive into Deep Learning 1.0.0-alpha1.post0 documentation (d2l.ai)

Bi-Directional RNNs: Intuition



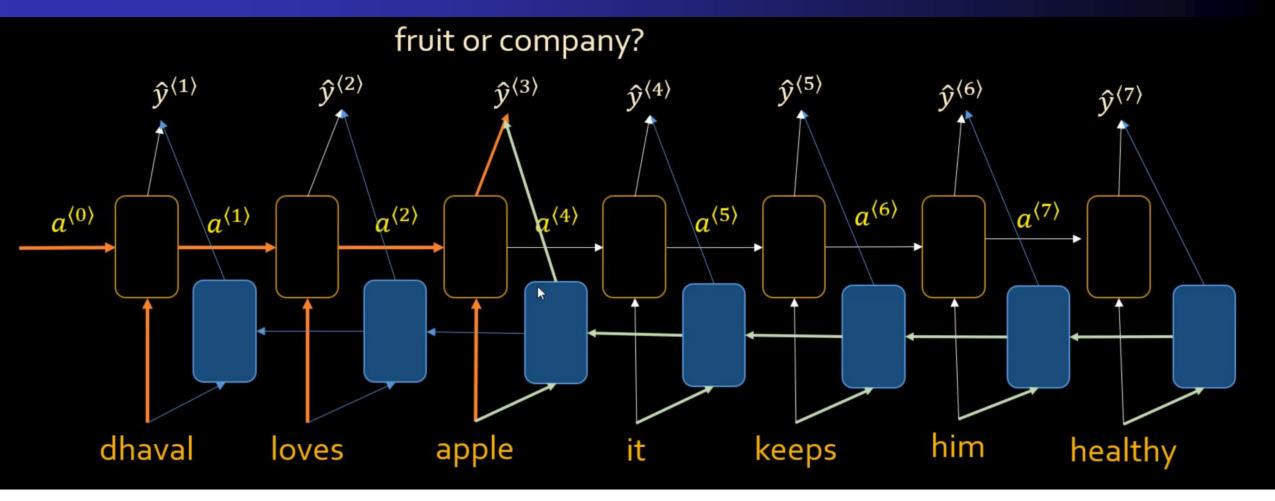
Bi-Directional RNNs: Intuition



• The o/p at the third time step (where input is the string "apple") depends on only previous two i/ps

Source: codebasics - YouTube

Bi-Directional RNNs

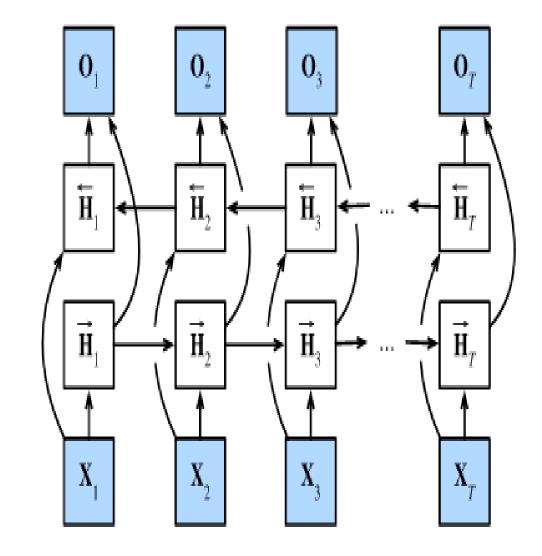


• Adding an additional backward layer with connections as shown above makes the o/p at a time step depend on both previous as well as future i/ps.

Source: codebasics - YouTube

Bi-directional RNNs

- Example speech detection
- I am ____.
- I am hungry.
- I am ____ hungry, and I can eat half a cake.
- Regular RNNs are causal
 - look at past and present inputs to generate output.
- Use 2 recurrent layers on the same inputs
 - One reading words from left to right
 - Another reading words from right to left
- Combine their outputs at each time step



Source: Bidirectional Recurrent Neural Networks — Dive into Deep Learning 1.0.0-alpha1.post0 documentation (d2l.ai)

Bi-directional RNN computation

$$H_{L}(Forward) = A(X_{L} * W_{XH}(forward) + H_{L}(Forward) * W_{HH}(Forward) + b_{H}(Forward))$$

$$H_{t}$$
 (Backward) = $A(X_{t} * W_{xH} (Backward) + H_{t+1} (Backward) * W_{HH} (Backward) + b_{H} (Backward))$

where,

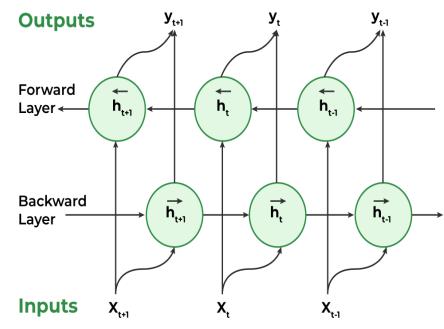
A = activation function,

W = weight matrix

b = bias

The output at any given hidden state is :

 $Y_t = H_t * W_{AY} + b_y$, where H_t is a concatenation of H_t (Forward) and H_t (Backward)



Bidirectional Recurrent Neural Network - GeeksforGeeks

Generating Shakespearan Text using a Character RNN

- "The unreasonable Effectiveness of Recurrent Neural Networks" – Andrej Karpathy (2015)
- 3-layer RNN with 512 hidden nodes on each layer
- Char-RNN was trained on Shakpeare's work used to generate novel text- one character at a time
- PANDARUS:

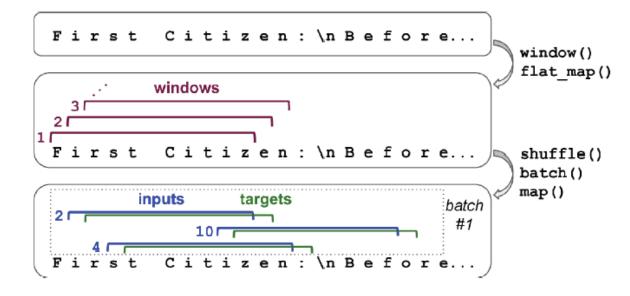
Alas, I think he shall be come approached and the day

When little srain would be attain'd into being never fed.

And who is but a chain and subjects of his death,

I should not sleep.

Chop the Sequential dataset into multiple windows



Stateful RNN

Stateless RNNs

- at each training iteration the model starts with a hidden state full of 0s
- Update this state at each time step
- Discards the output at the final state when moving onto next training batch

Stateful RNN

- Uses sequential nonoverlapping input sequences
- Preserves the final state after processing one training batch
- use it as initial state for next training batch
- Model will learn long-term patterns despite only backpropagating through short sequences

