Exam Date & Time: 16-May-2022 (04:30 PM - 05:30 PM)

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

MATHEMATICAL FOUNDATION FOR DATA SCIENCE-II [MAT 2213]

Marks: 15 **Duration: 60 mins.**

MCQ

Answer all the questions. Section Duration: 20 mins

1) If $X \sim N_p(\mu, \Sigma)$, then the linear combination of all X_j , $j = 1, 2, \ldots, p$ is (0.5)

1)	Univariate normal		2)	Bivariate normal		3)	Multivariate normal		4)	Not a normal	

2) Consider the two-state Markov chain with transition probability matrix $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then which of the following statement is true (0.5)

1)	Stationary distribution exists and unique		2)	Stationary distribution does not exists		3)	Infinitely many stationary distributions exist		4)	There exists two stationary distributions.	

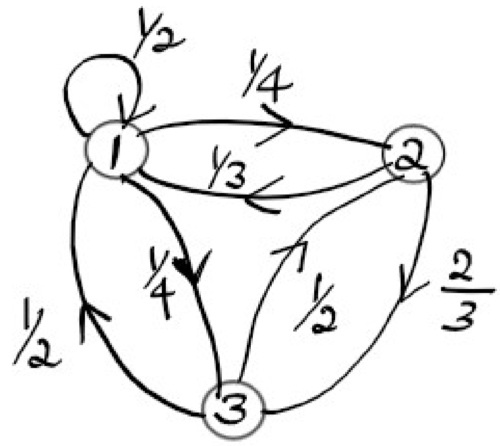
3) If r is the coefficient of correlation between two variables, which of the following indicates the weakest relationship? (0.5)

1)	$r = -0.5$		2)	$r = 0.9$		3)	$r = 0.1$		4)	$r = -0.6$	

4) Suppose $\Sigma_{p\times p}$ is the population covariance matrix, where all the variables are independent of each other. The off-diagonal elements of $\Sigma_{p\times p}$ are (0.5)

1)	1s		2)	0s		3)	Combination of 1s and 0s		4)	Can take any value	

5) (0.5)



For the Markov chain with state space $S = \{1, 2, 3\}$ and the state transition diagram given below, the value of $P\{X_3 = 1 | X_2 = 1\}$

1)			2)			3)	0.5		4)	0.33	

	$\frac{2}{3}$			$\frac{3}{5}$								

6) If $Y \sim N_4(\mu, \Sigma)$ where $\mu = (1, 2, 3, -2)$ and $\Sigma = \begin{bmatrix} 4 & 2 & -1 & 2 \\ 2 & 6 & 3 & -2 \\ -1 & 3 & 5 & -4 \\ 2 & -2 & -4 & 4 \end{bmatrix}$ The correlation coefficient ρ_{13} is (0.5)

1)	-1		2)	$\frac{-1}{2\sqrt{5}}$		3)	$\frac{1}{2\sqrt{5}}$		4)	20	

7) Suppose $V \sim N_p(\mu, \Sigma), V = (Y, X)$, then which of the following is true? (0.5)

1)	$Var(Y\|X) \geq Var(Y)$		2)	$Var(Y\|X) \leq Var(Y)$		3)	$Var(Y\|X) < Var(Y)$		4)	$Var(Y\|X) > Var(Y)$	

8) Let $Y = (Y_1, Y_2, Y_3)^T$ with $E(Y) = (1, -1, 3)^T$. Then $E(Z)$, where $Z = 2Y_1 - 3Y_2 + Y_3$, is (0.5)

1)	8		2)	3		3)	0		4)	-8	

9) The univariate distribution corresponding to Wishart distribution is (0.5)

1) Univariate normal 2) chi-squared 3) Student's t 4) Snedecor's F

10) The variance-covariance matrix is always (0.5)

1) Positive semidefinite 2) Positive definite 3) Negative definite 4) Negative semidefinite

DES

Answer all the questions. Section Duration: 40 mins

11) Let $X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$ be a random vector with correlation matrix $R = \begin{bmatrix} 1 & 0.60 & 0.70 \\ 0.60 & 1 & 0.65 \\ 0.70 & 0.65 & 1 \end{bmatrix}$.

Find (a) the multiple correlation coefficient, $R_{1.23}$ (2)

(b) the partial correlation between X_1 and X_2 , $r_{12.3}$.

12) Let $\boldsymbol{Z_1 = Y_1 + Y_2 + Y_3}$, $\boldsymbol{Z_2 = 3Y_1 + Y_2 - 2Y_3}$ with $\boldsymbol{EY = (1 \quad -1 \quad 3)^T}$ and $\mathbf{Var}(\boldsymbol{Y}) = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{0} & \mathbf{3} & \mathbf{10} \end{bmatrix}$. Find $\boldsymbol{Covariance(Z), where\ Z = (Z_1 \quad Z_2)^T}$. (2)

13) For the pdf $f(x_1, x_2, x_3) = 2x_2(x_1 + x_3), 0 < x_1, x_2, x_3 < 1$, find the mean vector and the dispersion matrix.. (3)

14) Let $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ be a bivariate normal random vector and let $X \sim N_2\left(\begin{bmatrix} 5 \\ 10 \end{bmatrix}, \begin{bmatrix} 16 & 12 \\ 12 & 36 \end{bmatrix}\right)$. (a) Find the distribution of $X_1 + X_2$ and $X_1 - X_2$. (b) Are $X_1 + X_2$ and $X_1 - X_2$ independent? (3) Justify your answer..

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