DSE 2256 DESIGN & ANALYSIS OF ALGORITHMS

Lecture 10 & 11

Brute force Techniques:
Selection sort, Bubble sort,
Sequential Search,
String Matching

Recap of L8 & L9

- Mathematical analysis of recursive algorithms
  - Recurrence relations
  - Method of backward substitution
  - Algorithm : Factorial of a number
  - Algorithm : Towers of Hanoi

Brute force

- **A straightforward approach, usually based directly on the problem's statement and definitions of the concepts involved.**
- **Easiest to apply.**
- **Applicable to a wide variety of problems.**

Example:

1. **Problem: Cracking a 4-digit PIN.**

   What could be the solution using brute force strategy ?

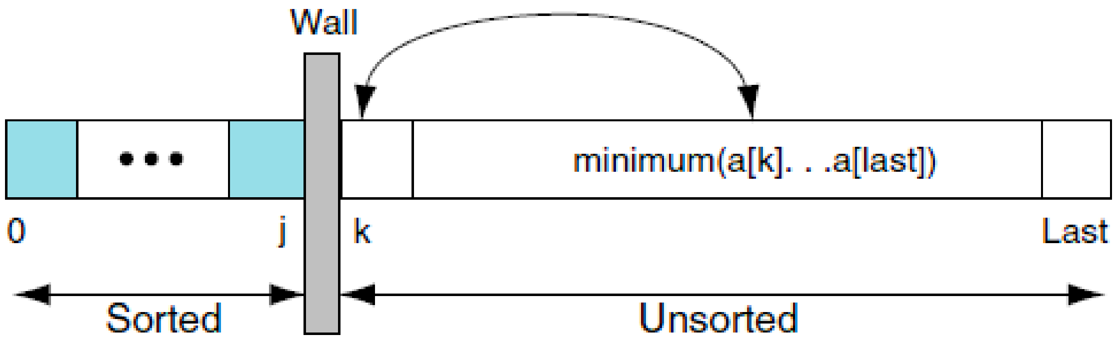
2. **Problem: GCD of 2 non-negative integers.**

   What could be the solution using brute force strategy ?

Brute force Sorting algorithm I

Selection Sort

- **Scan the array to find its smallest element and swap it with the first element.**
- **Then, starting with the second element, scan the elements to the right of it to find the smallest among them and swap it with the second element.**
- **Continue this process for 0 ≤ *i* ≤ *n*-2.**



5 3 4 1 2

| 89 45 68 90 29 34 **17**

17 | 45 68 90 **29** 34 89

17 29 | 68 90 45 **34** 89

17 29 34 | 90 **45** 68 89

17 29 34 45 | 90 **68** 89

17 29 34 45 68 | 90 **89**

17 29 34 45 68 89 | 90

Brute force Sorting algorithm I

ALGORITHM *SelectionSort*($A[0..n-1]$)
    //Sorts a given array by selection sort
    //Input: An array $A[0..n-1]$ of orderable elements
    //Output: Array $A[0..n-1]$ sorted in nondecreasing order
    for $i \leftarrow 0$ **to** $n-2$ **do**
        $min \leftarrow i$
        for $j \leftarrow i+1$ **to** $n-1$ **do**
            if $A[j] < A[min]$ $min \leftarrow j$
        swap $A[i]$ and $A[min]$

$$C(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1$$

$$= \sum_{i=0}^{n-2} [(n-1) - (i+1) + 1]$$

$$= \sum_{i=0}^{n-2} (n-1-i)$$

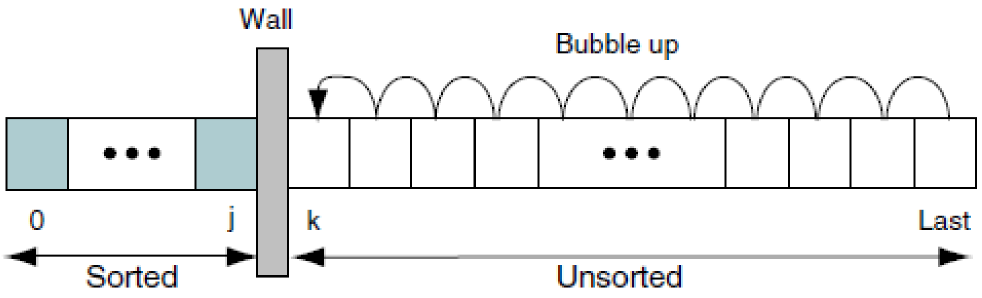
$$\boxed{C(n) = \frac{(n-1)n}{2}}$$

$$\Theta(n^2)$$

Brute force Sorting algorithm II

Bubble Sort

- **Compare adjacent elements of the list and exchange them if they are out of order.**
- **By doing it repeatedly, we end up "bubbling up" the largest element to the last position on the list.**
- **The next pass bubbles up the second largest element, and so on, until after n − 1 passes the list is sorted.**



8 5 3 1 4 7 9

Brute force Sorting algorithm II

ALGORITHM *BubbleSort*$(A[0..n-1])$

//Sorts a given array by bubble sort
//Input: An array $A[0..n-1]$ of orderable elements
//Output: Array $A[0..n-1]$ sorted in nondecreasing order
for $i \leftarrow 0$ **to** $n-2$ **do**
  for $j \leftarrow 0$ **to** $n-2-i$ **do**
    if $A[j+1] < A[j]$ swap $A[j]$ and $A[j+1]$

89	$\overset{?}{\leftrightarrow}$	45		68		90		29		34		17
45		89	$\overset{?}{\leftrightarrow}$	68		90		29		34		17
45		68		89	$\overset{?}{\leftrightarrow}$	90	$\overset{?}{\leftrightarrow}$	29		34		17
45		68		89		29		90	$\overset{?}{\leftrightarrow}$	34		17
45		68		89		29		34		90	$\overset{?}{\leftrightarrow}$	17
45		68		89		29		34		17	$\vert$	90
45	$\overset{?}{\leftrightarrow}$	68	$\overset{?}{\leftrightarrow}$	89	$\overset{?}{\leftrightarrow}$	29		34		17	$\vert$	90
45		68		29		89	$\overset{?}{\leftrightarrow}$	34		17	$\vert$	90
45		68		29		34		89	$\overset{?}{\leftrightarrow}$	17	$\vert$	90
45		68		29		34		17	$\vert$	89		90

$$C(n) = \sum_{i=0}^{n-2}\sum_{j=0}^{n-2-i} 1 = \sum_{i=0}^{n-2}[(n-2-i)-0+1] = \sum_{i=0}^{n-2}(n-1-i) = \frac{(n-1)n}{2} \boxed{\in \Theta(n^2)}$$

Brute force Sequential search

ALGORITHM *SequentialSearch*$(A[0..n-1], K)$

//Searches for a given value in a given array by sequential search
//Input: An array $A[0..n-1]$ and a search key K
//Output: The index of the first element in A that matches K
// or -1 if there are no matching elements
$i \leftarrow 0$
while $i \cancel{<} n$ **and** $A[i] \neq K$ **do**
    $i \leftarrow i + 1$
if $i < n$ **return** i
else return -1

ALGORITHM *SequentialSearch2*$(A[0..n], K)$

//Implements sequential search with a search key as a sentinel
//Input: An array A of n elements and a search key K
//Output: The index of the first element in $A[0..n-1]$ whose value is
// equal to K or -1 if no such element is found
$A[n] \leftarrow K$
$i \leftarrow 0$
while $A[i] \neq K$ **do**
    $i \leftarrow i + 1$
if $i < n$ **return** i
else return -1

Brute force String Matching

- **Problem: find a substring in the text that matches the pattern**

- **Pattern:** a string of m characters to search for.
- **Text:** a (longer) string of n characters to search in.

Brute-force algorithm

- **Step 1** Align pattern at beginning of text.
- **Step 2** Moving from left to right, compare each character of pattern to the corresponding character in text until all characters are found to match (successful search); or a mismatch is detected.
- **Step 3** While pattern is not found and the text is not yet exhausted, realign pattern one position to the right and repeat Step 2.

Brute force String Matching

- **Problem: find a substring in the text that matches the pattern**

- **Pattern: a string of *m* characters to search for.**
- **Text: a (longer) string of *n* characters to search in.**

Brute-force algorithm

- **Step 1** **Align pattern at beginning of text.**
- **Step 2** **Moving from left to right, compare each character of pattern to the corresponding character in text until all characters are found to match (successful search); or a mismatch is detected.**
- **Step 3** **While pattern is not found and the text is not yet exhausted, realign pattern one position to the right and repeat Step 2.**

Example 1:

Text:
10010101101001100101111010

Pattern: 001011

Example 2:

Text: It is never too late to have a happy childhood.

Pattern: happy

Brute force String Matching

ALGORITHM *BruteForceStringMatch*$(T[0..n-1], P[0..m-1])$

```
//Implements brute-force string matching
//Input: An array T[0..n − 1] of n characters representing a text and
//        an array P[0..m − 1] of m characters representing a pattern
//Output: The index of the first character in the text that starts a
//        matching substring or −1 if the search is unsuccessful
for i ← 0 to n − m do
    j ← 0
    while j < m and P[j] = T[i + j] do
        j ← j + 1
    if j = m return i
return −1
```

```
N O B O D Y _ N O T I C E D _ H I M
N O T
  N O T
    N O T
      N O T
        N O T
          N O T
            N O T
              N O T
```

Brute force: Strengths and Weaknesses

Strengths

- Wide applicability
- Simplicity
- Yields reasonable algorithms for some important problems (e.g., matrix multiplication, sorting, searching, string matching)

Weaknesses

- Rarely yields efficient algorithms
- Some brute-force algorithms are unacceptably slow

Matrix Multiplication

- Brute force approach for matrix multiplication

$$\begin{bmatrix} c_{00} & c_{01} \\ c_{10} & c_{11} \end{bmatrix} = \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} * \begin{bmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \end{bmatrix}$$

$$= \begin{bmatrix} a_{00} * b_{00} + a_{01} * b_{10} & a_{00} * b_{01} + a_{01} * b_{11} \\ a_{10} * b_{00} + a_{11} * b_{10} & a_{10} * b_{01} + a_{11} * b_{11} \end{bmatrix}$$

- Time complexity = $O(n^3)$

Thank you!

Any queries?