DSE 2256 DESIGN & ANALYSIS OF ALGORITHMS

Lecture 10 & 11

Brute force Techniques:

Exhaustive Search

Recap of L10 & L11

- Brute Force techniques
  - Definition
  - Brute Force Sorting
    - Algorithm : Selection sort
    - Algorithm : Bubble sort
  - Brute Force Searching
    - Sequential search
  - Brute Force Sting Matching

Exhaustive search

- ***Exhaustive search*** **is simply a brute-force approach to combinatorial problems.**
- **It suggests generating each and every element (combinatorial object) of the problem domain, selecting those of them that satisfy all the constraints, and then finding a desired element (object).**

Exhaustive search

- **A brute force solution to a problem involving search for an element with a special property, usually among combinatorial objects such as permutations, combinations, or subsets of a set.**

Method:

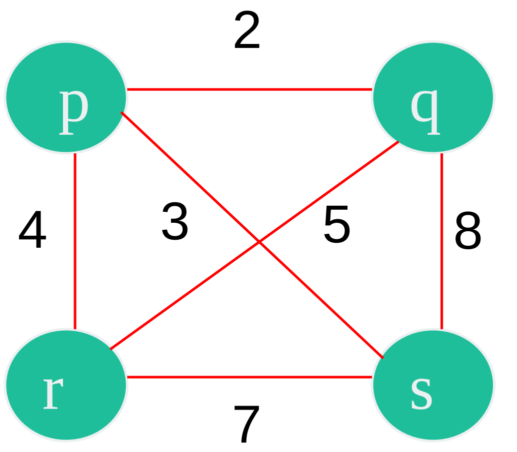
1. **Generate a list of all potential solutions to the problem in a systematic manner.**
2. **Evaluate potential solutions one by one, disqualifying infeasible ones and, for an optimization problem, keeping track of the best one found so far.**
3. **When the search ends, announce the solution(s) found.**

Traveling Salesman Problem using Exhaustive Search

The Travelling Salesman Problem:

- **Given *n* cities with known distances between each pair, find the shortest tour that passes through all the cities exactly once before returning to the starting city.**
- **Alternatively: Find shortest Hamiltonian circuit in a weighted connected graph.**

Example:



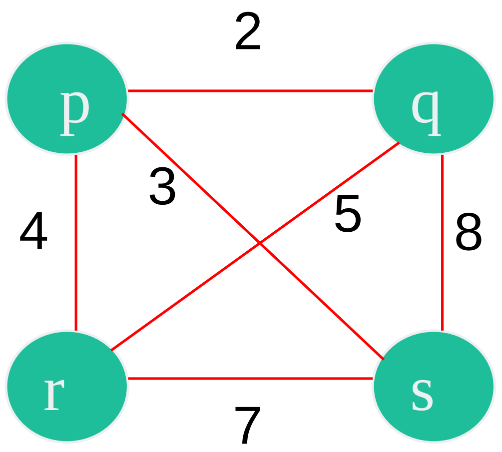
Which are the Hamiltonian Circuits ?

- If the tour starts with "p"
- If the tour starts with "r"

Traveling Salesman Problem using Exhaustive Search

<u>Example</u>: Let the start city be "q"

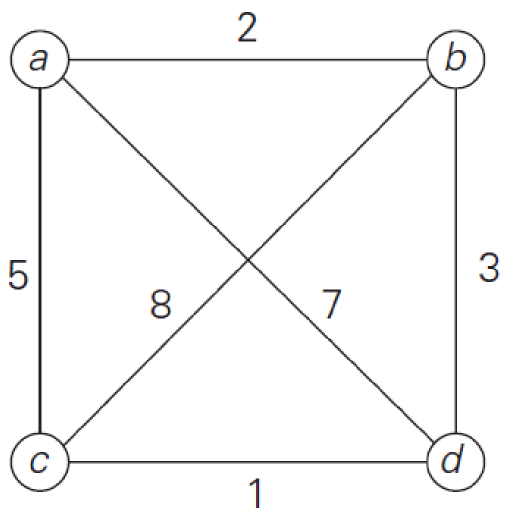
Tour	**Cost**
q→p→s→r→q	2+3+7+5 = 17
q→p→r→s→q	2+4+7+8 = 21
q→s→p→r→q	8+3+4+5 = 20
q→s→r→p→q	8+7+4+2 = 21
q→r→p→s→q	5+4+3+8 = 20
q→r→s→p→q	5+7+3+2 = 17



Traveling Salesman Problem using Exhaustive Search

Class exercise:

- Solve the TSP for the following graph, where start city = "a"



Tour	Length	
a —> *b* —> *c* —> *d* —> *a*	*l* = 2 + 8 + 1 + 7 = 18	
a —> *b* —> *d* —> *c* —> *a*	*l* = 2 + 3 + 1 + 5 = 11	optimal
a —> *c* —> *b* —> *d* —> *a*	*l* = 5 + 8 + 3 + 7 = 23	
a —> *c* —> *d* —> *b* —> *a*	*l* = 5 + 1 + 3 + 2 = 11	optimal
a —> *d* —> *b* —> *c* —> *a*	*l* = 7 + 3 + 8 + 5 = 23	
a —> *d* —> *c* —> *b* —> *a*	*l* = 7 + 1 + 8 + 2 = 18	

Efficiency : **$\Theta((n-1)!)$**

Knapsack Problem using Exhaustive Search

<u>The Knapsack Problem:</u>

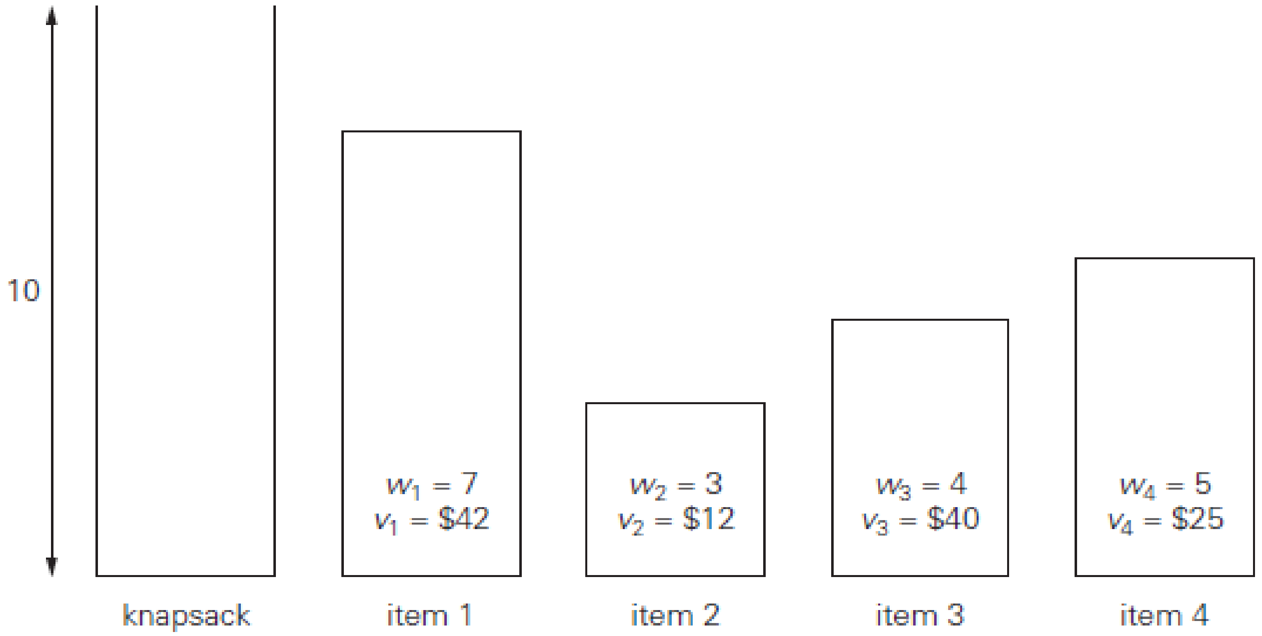
Given *n* items:

weights: $w_1 \quad w_2 \ \ldots \ w_n$

values: $v_1 \quad v_2 \ \ldots \ v_n$

&

a Knapsack of capacity *W*

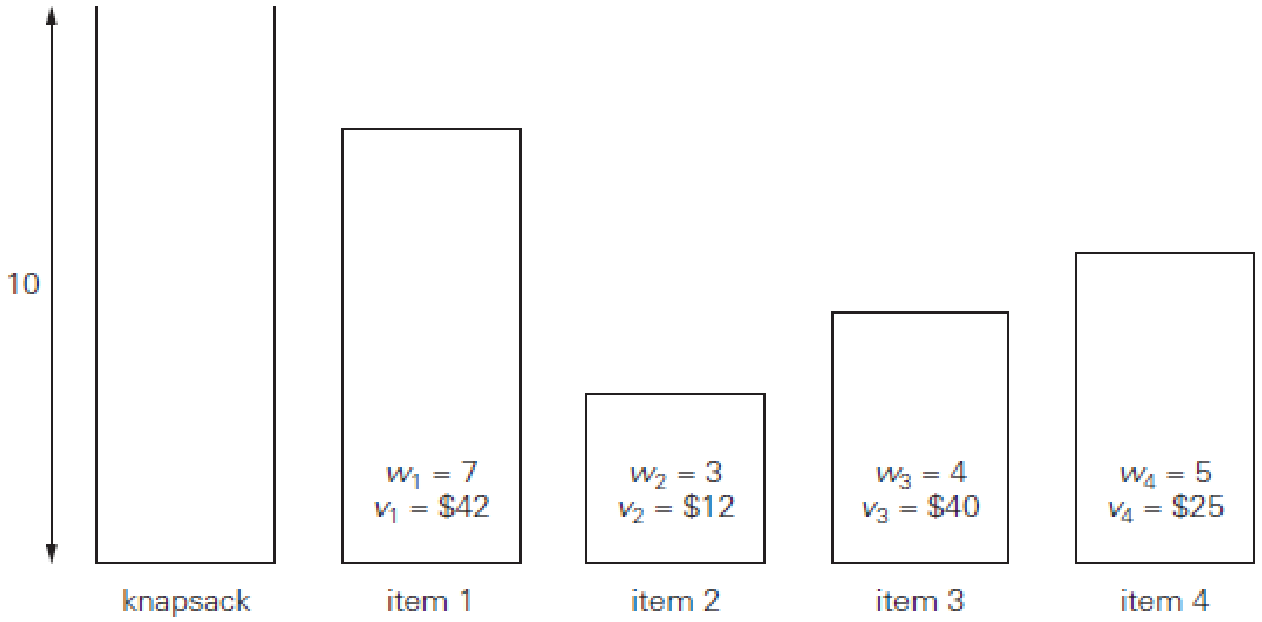


Find most valuable subset of the items that fit into the knapsack.

Knapsack Problem using Exhaustive Search

Subset	Total weight	Total value
∅	0	$ 0
{1}	7	$42
{2}	3	$12
{3}	4	$40
{4}	5	$25
{1, 2}	10	$54
{1, 3}	11	not feasible
{1, 4}	12	not feasible
{2, 3}	7	$52
{2, 4}	8	$37
{3, 4}	**9**	**$65**
{1, 2, 3}	14	not feasible
{1, 2, 4}	15	not feasible
{1, 3, 4}	16	not feasible
{2, 3, 4}	12	not feasible
{1, 2, 3, 4}	19	not feasible

Knapsack of capacity = *10*



Knapsack Problem using Exhaustive Search

Class exercise:

- Given a knapsack of capacity = 16,
  Solve the knapsack problem for the following set of items.

Item	Weight	Value
1	2	$20
2	5	$30
3	10	$50
4	5	$10

Efficiency: Θ(2^n)

Subset	Total weight	Total value
{1}	2	$20
{2}	5	$30
{3}	10	$50
{4}	5	$10
{1,2}	7	$50
{1,3}	12	$70
{1,4}	7	$30
{2,3}	15	$80
{2,4}	10	$40
{3,4}	15	$60
{1,2,3}	17	Not feasible
{1,2,4}	12	$60
{1,3,4}	17	Not feasible
{2,3,4}	20	Not feasible
{1,2,3,4}	22	Not feasible

Assignment Problem using Exhaustive Search

The Asignment Problem:

- There are *n* people who need to be assigned to *n* jobs, one person per job.
- The cost of assigning person ***i*** to job ***j*** is **C[i, j].**
- Find an assignment that minimizes the total cost.

	Job 1	Job2	Job3	Job 4
Person 1	9	2	7	8
Person 2	6	4	3	7
Person 3	5	8	1	8
Person 4	7	6	9	4

Algorithmic Plan: Generate all legitimate assignments, compute their costs, and select the cheapest one.

How many assignments are there?

Assignment Problem using Exhaustive Search

Assignment (col.#s)	**Cost**		**Total Cost**	
<1, 2, 3, 4>	**9+4+1+4**	**=**	**18**	
<1, 2, 4, 3>	**9+4+8+9**	**=**	**30**	
<1, 3, 2, 4>	**9+3+8+4**	**=**	**24**	
<1, 3, 4, 2>	**9+3+8+6**	**=**	**26**	
<1, 4, 2, 3>	**9+7+8+9**	**=**	**33**	
<1, 4, 3, 2>	**9+7+1+6**	**=**	**23**	**etc.**

$$C = \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix}$$

For this particular instance, the optimal assignment can be found by exploiting the specific features of the number given. It is: **<2,1,3,4>**

Final Comments on Exhaustive Search

- Exhaustive-search algorithms run in a realistic amount of time only on very small instances

- In some cases, there are much better alternatives!
  - shortest paths
  - minimum spanning tree
  - assignment problem

- In many cases, exhaustive search or its variation is the only known way to get exact solution.

Thank you!

Any queries?