Solving / overcoming the problem of a degenerate solution:

	D_1	D_2	D_3	Supply
S_1	0	5	4	30
S_2	2	9	8	20
S_3	5	11	10	10
Demand	10	30	20	

→ first solving using least cost method

	D_1	D_2	D_3	Supply
S_1	[0] 10	[5]	[4] 20	30/20/0
S_2	[2]	[9] 20	[8]	20/0
S_3	[5]	[11] 10	[10]	10
Demand	10/0	30/10/0	20/0	

$z = 10 \cdot 0 + 4 \cdot 20 + 9 \cdot 20 + 11 \cdot 10$

$z = 370$

now,

$m+n-1 \neq$ no. of allocations the solution is degenerate

→ optimizing using MODI method

$V_1 = 0$ $V_2 =$ $V_3 = 4$

	D_1	D_2	D_3	Supply
$U_1 = 0$ S_1	[0] 10	[5]	[4] 20	30/20/0
$U_2 =$ S_2	[2]	[9] 20	[8]	20/0
$U_3 =$ S_3	[5]	[11] 10	[10]	10
Demand	10/0	30/10/0	20/0	

→ here we can't optimize it properly as the number of allocations are less.

↳ degeneracy is present

→ to remove this degeneracy we add an allocation of a very small value 'ε' which has no physical interpretation. we just add this dummy allocation so that we can continue with the algorithm

we will allocate ε to the cell nearest to the cell because of which degeneracy was created → cell $S_1 D_3$

↳ cause when we allocated 20 to the cell simultaneously the row and col^n (demand & supply) got exhausted

$v_1=0$ $v_2=5$ $v_3=4$

		D_1	D_2	D_3	Supply
$u_1=0$	S_1	[0] 10	[5] ϵ	[4] 20	$\cancel{30}$ $\cancel{20}$ 0
$u_2=4$	S_2	[2]	[9] 20	[8]	$\cancel{20}$ 0
$u_3=6$	S_3	[5]	[11] 10	[10]	10
	Demand	$\cancel{10}$ 0	$\cancel{30}$ $\cancel{10}$ 0	$\cancel{20}$ 0	

⇓ optimizing

$V_1=0$ $V_2=5$ $V_3=4$

		D_1	D_2	D_3	Supply
$U_1=0$	S_1	[0] 10 (-)	[5] ϵ (+)	[4] 20	$\cancel{30}$ $\cancel{20}$ 0
$U_2=4$	S_2	[2] -2	[9] 20 (-)	[8] 0	$\cancel{20}$ 0
$U_3=6$	S_3	[5] -1	[11] 10	[10] 0	10
	Demand	$\cancel{10}$ 0	$\cancel{30}$ $\cancel{10}$ 0	$\cancel{20}$ 0	

⇓

	D_1	D_2	D_3	Supply
S_1	[0]	[5] 10	[4] 20	30
S_2	[2] 10	[9] 10	[8]	20
S_3	[5]	[11] 10	[10]	10
Demand	10	30	20	

$Z = 350$

$m+n-1 = 5$ allocation

Solution is non-degenerate

→ now we will again check for optimality :

$v_1 = -2$ $v_2 = 5$ $v_3 = 4$

	D_1	D_2	D_3	Supply
$u_1 = 0$ S_1	0 +	5 10	4 20	30
$u_2 = 4$ S_2	2 10	9 10	8 0	20
$u_3 = 6$ S_3	5 +	11 10	10 0	10
Demand	10	30	20	

∴ Since all cell improvement values are either zero or positive the solution is optimal.

Q) Solve the following problem using transportation algo

	D_1	D_2	D_3	Supply
S_1	3	6	8	80
S_2	6	1	2	40
S_3	4	7	3	30
Demand	50	40	10	

→ Solution :-

→ getting the basic feasible solution using least cost method

	D_1	D_2	D_3	Supply
S_1	3 / 50	6 / 30	8	80/30/0
S_2	6	1 / 40	2	40/0
S_3	4	7 / 20	3 / 10	30/20/0
Demand	50/0	40/50/20/0	10/0	

$z = 540$

$m+n-1=5$ allocation

non-degenerate.

→ optimizing

$v_1 = 3 \quad v_2 = 6 \quad v_3 = 2$

		D_1	D_2	D_3	Supply
$u_1 = 0$	S_1	3 / 50	6 / 30	8 / +	80
$u_2 = -5$	S_2	6 / +	1 / 40	2 / +	40
$u_3 = 1$	S_3	4 / 0	7 / 20	3 / 10	30
	Demand	50	40	10	

Solution is already optimal.

⇓

optimising S_3D_1

with cell improvement index = 0

	D_1	D_2	D_3	Supply
S_1	3 30	6 50	8	80
S_2	6	1 40	2	40
S_3	4 20	7	3 10	30
Demand	50	40	10	

→ again checking for optimality

$v_1 = 3 \quad v_2 = 6 \quad v_3 = 2$

		D_1	D_2	D_3	Supply
$u_1 = 0$	S_1	3 30	6 50	8 (+)	80
$u_2 = -5$	S_2	6 (+)	1 40	2 (+)	40
$u_3 = 1$	S_3	4 20	7 (0)	3 10	30
	Demand	50	40	10	

$z = 540$

$m+n-1 = 5$

non degenerate

$\therefore$ optimal

Significance → there are multiple solutions for those problem where cell improvement index $= 0$

→ if one road is blocked supply can come from other sources

Mon
19/2/24

Solve the following transportation problem using least cost method and modified distribution method

Power Plant

Coal mine

	1	2	3	Supply
I	1	2	1	25
II	0	4	5	35
III	2	3	3	30
Demand	30	20	20	90 / 70

Case i) Supply > Demand (Unbalanced problem)

→ Balance the problem by creating a dummy destination with fictitious demand of 20 tonnes of coal

		1 (−2)	2 (2)	3	Dummy	Supply
0	I	1	2 (5)	1 (20)	0	25
2	II	0 (30)	4 (5)	5	0	35
	III	2	3 (10)	3	0 (20)	30
	Demand	30	20	20	20	90

The allocations in the dummy cells represent the excess supply available at each source.

	1	2	3	Supply
I	3	8	6	5
II	4	5	2	4
III	1	10	9	6
Demand	3	9	7	15 / 19

Case ii) Demand > Supply

Since the demand > supply, to balance the problem we add a dummy source

	1	2	3	Supply
I	3	8 (2)	6 (3)	25
II	4	5	3 (4)	35
III	1	10 (6)	9	30
Dummy	0 (3)	0 (1)	0	4
Demand	3	9	7	19

Allocate dummy to column with highest cost

Any allocations made at the dummy reservoir, represents shortage of given resources in the current problem city 2, has 4 million gallons of shortage

Wed (Sat's tt)
21/2/24

Maximization Case:

Q) Solve the following transportation problem with an objective to maximize the total sales revenue.

profit per unit product in the market

Market

Product

	1	2	3	Supply
I	20	15	13	24
II	14	12	21	11
III	17	13	18	15
Demand	16	20	14	50

Same product having different prices in different markets & maximize the sales

→ Generating the Basic feasible solution using the (I) maximum profit cell method:

	1	2	3	Supply
I	20 / 16	15 / 8	13	24/8/0
II	14	12	21 / 11	11/0
III	17	13 / 12	18 / 3	15/12
Demand	16/0	20/0	14/3/0	50

$z = 881$

$m+n-1 = 5$ allocation

∴ the solution is non-degenerate

→ now calculate $u_1, v_2 \ldots$ & the cell improvement indexes

$v_1 = 20$ $v_2 = 15$ $v_3 = 20$

		1	2	3	Supply
$u_1 = 0$	I	20 / 16	15 / 8	13	24/8/0
$u_2 = 1$	II	14	12	21 / 11	11/0
$u_3 = -2$	III	17	13 / 12	18 / 3	15/12
	Demand	16/0	20/0	14/3/0	50

⇓

$v_1 = 20$ $v_2 = 15$ $v_3 = 20$

		1	2	3	Supply
$u_1 = 0$	I	(20) 16	(15) 8	(13)	24/8/0
$u_2 = 1$	II	(14)	(12)	(21) 11	11/0
$u_3 = -2$	III	(17)	(13) 12	(18) 3	15/12
	Demand	16/0	20/0	14/3/0	50

As all cell improvement indexes are either 0 or negative the solution is optimal.

Q2 Solve the following transportation problem with objective of minimizing total cost of transportation.

Destination

		A	B	C	Supply
Sources	1	(2)	(2)	(3)	10
	2	(4)	(1)	(2)	15
	3	(1)	(3)	(X)	40
	Demand	20	15	30	65

product can't reach from Source 3 to Destination C probably because of road block or something

We block this path by assuming the value of cost coefficient as M

for easy calculation we assume that M will be a very high positive value preferrably +100.

	A	B	C	Supply
1	2	2	3 · 10	10/0
2	4	1 · 15	2	15/0
3	1 · 20	3	100 · 20	40/20
Demand	20/0	15/0	30/20/0	65

$z = 2335$

$n+m-1 = 5$

∴ non degenerate solution

→ now we'll calculate the cell improvement indexes.

→ final sol^n is not optimal → error in formulation → final cost 625

23/2/24
FRI

Q3 3 warehouses supply 5 stores the table indicates the cost of shipment per unit b/w the warehouses and stores. However a major bridge has been damaged preventing delivery from warehouse A to store 5. and from warehouse B to store 2 and from warehouse C to store 4. within these limitations determine the optimal delivery schedule to minimize the cost

Store

Warehouse

	1	2	3	4	5	Supply
A	2	3	4	4	2	850
B	4	8	3	6	6	300
C	6	7	8	3	5	450
Demand	75	345	180	40	210	1600 / 900

Soln →

We will start by adding a dummy destination as Supply > Demand

	1	2	3	4	5	Dummy	Supply
A	2	3	4	4	M	0	850
B	4	M	3	6	6	0	300
C	6	7	8	M	5	0	450
Demand	75	345	180	40	210	700	1600 / 900

∴ blocking paths
warehouse A → store 5
warehouse B → 2
warehouse C → 4

⇓

	1	2	3	4	5	Dummy	Supply
A	2 / 75	3 / 75	4	4	100	0 / 700	850 / 150/75/0
B	4	100 / 30	3 / 180	6 / 40	6	0	300 / 120/30
C	6	7 / 240	8	100	5 / 210	0	450 / 240/0
Demand	75 / 0	345 / 270/30/0	180 / 0	40 / 0	210 / 0	700 / 0	1600 / 900

$\Rightarrow Z = 7185$

(8)
$m+n-1 = 8$ allocations
$\therefore$ non degenerate

$v_1 = 2$ $v_2 = 3$ $v_3 = -94$ $v_4 = -91$ $v_5 = 1$ $v_6 = 0$

	1	2	3	4	5	Dummy	Supply
$u_1 = 0$ A	2 / 75	3 / 75	4 (+)	4 (+)	100 (+)	0 / 700	850
$u_2 = 97$ B	4 (−95)	100 / 30	3 / 180	6 / 40	6 (−92)	0 (−97)	300
$u_3 = 4$ C	6 (0)	7 / 240	8 (+)	100 (+)	5 / 210	0 (−4)	450
Demand	75	345	180	40	210	700	1600 / 900

→ not optimal
→ most negative cell improvement index B, Dummy

→ ((+) BD → (−) AD (700) → (+) A2 (75) → (−) B2 (30))

$v_1 = 2$ $v_2 = 3$ $v_3 = -94$ $v_4 = -91$ $v_5 = 1$ $v_6 = 0$

		1	2	3	4	5	Dummy	Supply
$u_1 = 0$	A	2 / 75	3 / 105	4 / +	4 / +	100 / +	0 / 670	850
$u_2 = 97$	B	4 / -95	100	3 / 180	6 / 40	6 / -92	0 / 30	300
$u_3 = 4$	C	6 / 0	7 / 240	8 / +	100 / +	5 / 210	0 / -4	400
	Demand	75	345	180	40	210	700	1600 / 900

$Z = 4275$

24/2/24

Application of Transportation Algo

1) Production - Scheduling Problem

Q) A company has to draw a schedule for weekly production of a certain item for the next three weeks. The production cost of the item in the first two weeks is Rs 10 per unit & Rs 15 per unit in the 3rd week. The Demands of 300, 700, 900 units for 1st, 2nd, 3rd week respectively must be met. The plant can produce a maximum of 700 units each week. In Addition the company can employ overtime during the Second week only. This increases the weekly production by 200 units but the production cost increases by Rs 5 per unit. Any excess production can be stored at a unit storage cost of Rs 3 per unit per week. How should the production be scheduled, so that the total cost of production is Minimized. Use transportation Algorithm.

Solⁿ →

Demand per week

Supply produced per week

U can't meet the demand of previous week (week 1) this week (Second week)

Overtime

	1	2	3	Supply
1W RT	10	13	16	700
2W RT	M	10	13	700
2W OT	M	15	18	200
3W RT	M	M	15	700
Demand	300	700	900	

2) Finance / Investment / Portfolio Management :

Q) A consulting firm is faced with the problem of its source and application of funds. funds are available from the following.

1) $ 30,000 from bonds at 4%. per annum
2) $ 1,20,000 from Common stocks at 9%. per annum
3) $ 3,00,000 from preferred stock at 7%. per annum
4) $ 2,00,000 from a bank loan from 8%. per annum
5) $ 1,80,000 from a private mortage at 10%. per annum.

The money can be invested in 1 or more of the following projects.

1) Modernisation of plant at a gross profit of 10%. and upto $ 29000.
2) Investment in R&D at a gross profit of 15%. and upto $ 1,00,000.
3) purchase new patent right at a gross profit of 13%. & upto $ 90,000
4) Purchase another company at a gross profit of 12%. & upto $ 1,10,000.
5) Retain employees at a gross profit of 11%. and upto $ 2,00,000..

How should the company maximise it's ROI. Use transportation algorithm.

Soln →

	modernisation (10%)	R&D (15%)	Patent (13%)	Company (12%)	Retain employee (11%)	Supply
(4%) Bond	6	11	9	8	7	30
(9%) C stock	1	6	4	3	2	120
(7%) P stock	3	8	6	5	4	300
(8%) Loan	2	7	5	4	3	200
(10%) mortage	0	5	3	2	1	180
Demand	29	100	90	110	200	

3) <u>Logistics</u> :

8) A trucking firm servicing 8 cities has trucks located as shown. It now desires to relocate them in view of a shift in its customer demands

City

	A	B	C	D	E	F	G	H
Current No's	1	11	3	16	20	5	7	18
Required No's	10	7	11	10	9	12	14	8

The distance b/w the cities is as shown below

FROM \ TO	A	B	C	D	E	F	G	H
A	0	13	3	11	12	5	8	7
B		0	11	11	25	17	23	9
C			0	19	10	16	7	15
D				0	31	26	16	3
E					0	4	9	12
F						0	8	9
G							0	14
H								0

Cost α Distance

The route b/w E & F involves barges & tolls that triple's the normal cost of movement. Because of flood damages to roads & bridges. Truck movement b/w A & B should not be attempted at this time. Recommend an optimal movement plan to minimize the total dist travelled

Q) 3 Refineries with max daily capacity of 6 million, 5 mil & 8 mil gallons of gasoline. Supply 3 distribution areas with daily demand of 4 mil, 8 mil & 7 mil gallons. Gasoline is transported to the 3 distribution areas through a network of pipelines. The transportation cost is estimated based on the length of pipeline. At abt 1 cent per 100 gallon per mile. The mileage table summarized below shows that refinery 1 is not connected to dist area 3. (a) formulate the problem as a transportation model.

		Distribution area 1	2	3
Refinery	1	120	180	-
	2	300	100	80
	3	200	250	120

→ dist b/w refinery & area

(b) In the above suppose that the capacity of refinery 3 is reduced to 6 mil gallon also dist area 1 must recieve all its demand & any shortage at 2 & 3 will result in a penalty of 5 cents per gallon. Formulate the problem as a transportation problem

(c) In the above problem suppose that the daily demand at area 3 drops to 4 mil gallons & any surplus production at refineries 1 & 2 must be diverted to other distribution areas by trucks. The resulting avg transportation cost per 100 gallons are \$1.5 from ref 1 & \$2.2 from ref 2. Ref 3 can direct its surplus gasoline to other chemical process within the plant. Formulate the problem as a transportation model.

Soln →

(a)

	1	2	3	Supply (mgd)
1	12	18	M	6
2	30	10	8	5
3	20	25	12	8
Demand	4	8	7	19

Cost = 1 cent / 100 gallon / mile

$= \frac{\$0.01}{100 \text{ gallon / mile}}$

$= \$\frac{0.01}{100} \times 10^6 \times 120$

$= 12000$

$= 12$ ('000s)

(b)

	1	2	3	Supply (mgd)
1				6
2				5
3				6
Demand	4	8	7	

→ Supply is reduced here
∴ unbalanced problem
will add dummy
Supply // refinery

⇓

	1	2	3	Supply (mgd)
1	12	18	M	6
2	30	10	8	5
3	20	25	12	6
Dummy	M	50	50	2
Demand	4	8	7	

first const is
refinery 1 should
get all it's
demand.
supply is more the demand.
the Dummy should
not getting
cause in reality its not there

(c)

	1	2	3	Supply (mgd)
1				6
2				5
3				6
Demand	4	8	4	

Supply > demand

⇓

	1	2	3	dummy	Supply (mgd)
1	12	18	14	15	6
2	30	10	8	22	5
3	20	25	12	0	8
Demand	4	8	4	3	

no additional money to spend as

10) 3 orchids supply crates of oranges to 4 retailers. The daily demand, daily supply & the transportation cost per crate are given below.

		Retailer				
		1	2	3	4	Supply
orchid	1	1	2	3	2	150
	2	2	4	1	2	200
	3	1	3	5	3	250
Demand		150	150	400	100	

Both orchid 1 & 2 here indicated that they could supply more crates if necessary by using overtime labour. orchard 3 doesn't offer this option, determine the optimal schedule using transportation algorithm.

Soln →

	1	2	3	4	Dummy Demand	Supply
1	1	2	3	2		150
2	2	4	1	2		200
3	1	3	5	3		250
Dummy1						200
Dummy2						200
Demand	150	150	400	100	200	

orchid 1 & orchid 2

1/3/24
FRI

Sensitivity Analysis of Transportation Problems

Q) 3 warehouses supply a particular product to 4 retailers, the max supply from each warehouse, the demand of each retailer & the unit cost of transportation from each warehouse to retailer is as shown in the table below. Solve the problem using transportation algo. with an objective to minimize the total transportation cost.

	1	2	3	4	Supply
1	11	5	6	5	24
2	2	10	5	9	23
3	7	4	2	7	13
Demand	12	16	17	15	

Sol^n →

	1	2	3	4	Supply
1	11	5 16	6	5 8	24/8/0
2	2 12	10	5 4	9 7	23/11/7/0
3	7	4	2 13	7	13/0
Demand	12/0	16/0	17/4/0	15/7/0	60

$m+n-1 = 6$

∴ non degenerate sol^n

$z = 253$

→ checking the for optimality

$v_1 = -2$ $v_2 = 5$ $v_3 = 1$ $v_4 = 5$

		1	2	3	4	Supply
$u_1 = 0$	1	11 +	5 (−) 16	6 +	5 (+) 8	24
$u_2 = 4$	2	2 12	10 +	5 (+) 4	9 (−) 7	23
$u_3 = 1$	3	7 +	4 (+) −2	2 (−) 13	7 +	13
	Demand	12	16	17	15	60

$v_1 = 0 \quad v_2 = 5 \quad v_3 = 3 \quad v_4 = 5$

		1	2	3	4	Supply
$u_1 = 0$	1	(11) +	(5) 9	(6) +	(5) 15	24
$u_2 = 2$	2	(2) 12	(10) +	(5) 11	(9)	23
$u_3 = -1$	3	(7) +	(4) 7	(2) 6	(7) +	13
	Demand	12	16	17	15	60

As cell improvement index is positive for all $\therefore$ optimal solⁿ

$z = 239$

2 Marks set

Sensitivity Analysis for Balanced Changes in Demand and Supply

→ a) In the above problem, Suppose that Customer 4 wants 1 additional unit and warehouse 2 has it. what is the extra cost of meeting this demand. also show the revised allocation. (Shown with this color)

$v_1 = 0 \quad v_2 = 5 \quad v_3 = 3 \quad v_4 = 5$

		1	2	3	4	Supply
$u_1 = 0$	1	(11)	(5) 9	(6)	(5) 15	24
$u_2 = 2$	2	(2) 12	(10)	(5) 11	(9) 1	23+1
$u_3 = -1$	3	(7)	(4) 7	(2) 6	(7)	13
	Demand	12	16	17	15+1	61

for this cell
$u_2 + v_4 = 2+5$
$= 7$
but we are paying Rs 9
$\therefore 9-7 = 2$ Rs
U can still save 2 Rs

$\therefore$ new cost $= 239 + 9 = 248$

⇓

To save this we don't add the 1 unit

in 2, 4 box.
we'll add in 1 in one of the already allocated box

	$v_1 = 0$ 1	$v_2 = 5$ 2	$v_3 = 3$ 3	$v_4 = 5$ 4	Supply
$u_1 = 0$ 1	11	5 9 −1	6	5 15+1	24
$u_2 = 2$ 2	2 12	10	5 11	9	23
$u_3 = -1$ 3	7	4 7+1	2 6−1	7	13
Demand	12	16	17	15+1	61

$\therefore z = 239 + 7$
$= 246$

b) suppose customer 1 wanted 1 extra unit & warehouse 3 had it. what is the cost of meeting this extra demand show the revised allocation.

we will use the optimized table not the above one

	$v_1 = 0$ 1	$v_2 = 5$ 2	$v_3 = 3$ 3	$v_4 = 5$ 4	Supply
$u_1 = 0$ 1	11	5 9	6	5 15	24
$u_2 = 2$ 2	2 12+1	10	5 11−1	9	23
$u_3 = -1$ 3	7	4 7−1	2 6+1	7	13 +1
Demand	12+1	16	17	15	61

Instead of directly allocating here we will supply from warehouse 2

$z = 239 - 1 = 238$

Assignment Algorithm

Pre requisite ⟶ problem is balanced → no. of projects = no. of workers
no. of rows = no. of columns

Hungarian Method to solve Assignment Algo

Step 1: Develop the cost table from the given problem. Check if the problem is balanced. i.e no. of rows = no of columns. if the no. of rows is not equal to the no. of columns & vice versa. add a dummy row or dummy column with zero assignment cost.

Step 2: find the opportunity cost table:

row opportunity cost matrix ← 1) identify the smallest element in each row & then subtract that no. from every other number of that row.

col^n opportunity cost matrix ← 2) In the reduced matrix obtained in the above step. locate the smallest element in each column & subtract this element from each element of the col^n. Each row & col^n now has atleast 1 zero value

Step 3: Make assignments in the opportunity cost matrix as below

1) Examine rows successively until a row with exactly one unmarked zero is obtained. make an assignment to this zero by marking a square around it.
2) for each zero that has been assigned eliminate all the other zeroes in that row or column
3) Repeat this process for each col^n with exactly one zero that has not been eliminated or assigned
4) if a row or a col^n has two or more unmarked zeroes then choose the assigned zero arbitrarily
5) Continue this process until all the zeroes in rows or col^n are either assigned or eliminated.

Step 4: If the no. of assignments are equal to the no. of rows & col^n then the sol^n is optimal
The total cost associated with the assignment is obtained by adding the original cost figures in the occupied cells.
If no optimal sol^n is obtained, revise the sol^n

Step 5: Revise the opportunity cost table

1) Draw a set of vertical & horizontal lines to cover all the zeroes in the revised cost table
2) from among the cells not covered by any line identify the smallest element
3) subtract this no. from every element in the cell that are not covered by any lines
4) Add this no. to the cells, at the intersection of two lines
5) elements in the cells covered by lines remain unchanged

Step 6: Repeat steps 3 to 5 until an optimal sol^n is obtained.

Q1) A computer center has 3 expert programmers the center wants 3 app programs to be developed. The head of the centre after studying the programs to be developed estimates the time in min's required by the experts for each app program. use assignment algo to assign the app's to experts with an objective to minimize the total time consumed

Experts

Application		A	B	C
	1	120	100	80
	2	80	90	110
	3	110	140	120

Solⁿ →

row opportunity cost matrix :

	A	B	C
1	40	20	0
2	0	10	30
3	0	30	20

⇒ colⁿ opportunity cost matrix / total opportunity cost matrix :

	A	B	C
1	40	10	0
2	0	0	30
3	0	20	10

now,

	A	B	C
1	40	10	$\boxed{0}$
2	~~0~~	$\boxed{0}$	30
3	$\boxed{0}$	20	10

∵ The solution is optimal as the no of assignments is equal to the no of rows or colⁿ

⇒

App	Expert	time
1	C	80
2	B	90
3	A	110
		280

Q2) A department has 5 employees with 5 jobs to be performed. The time each employee will take to perform each job is given in the effectiveness matrix below. Use assignment algo to allocate jobs to employees.

Employees

Jobs	1	2	3	4	5
A	10	5	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

Solⁿ → row oppⁿ cost matrix :

	1	2	3	4	5
A	5	0	8	10	11
B	0	6	15	10	3
C	8	5	0	0	0
D	0	4	2	0	5
E	3	5	6	0	8

⇒ no need for colⁿ oppⁿ cost matrix as each row has one or more zero

now,

	1	2	3	4	5
A	5	$\boxed{0}$	8	10	11
B	$\boxed{0}$	6	15	10	3
C	8	5	$\boxed{0}$	~~0~~	~~0~~
D	~~0~~	4	2	$\boxed{0}$	5
E	3	5	6	~~0~~	8

⇒ the solⁿ is not optimal as the no. of assignments is not equal to the no. of rows or colⁿ

∴ Revise

→ Revising.

	1	2	3	4	5
A	5	$\boxed{0}$	8	10	11
B	$\boxed{0}$	6	15	10	3
C	8	5	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$
D	$\cancel{0}$	4	2	$\boxed{0}$	5
E	3	5	6	$\cancel{0}$	8

if in this iteration the no of lines come out to be equal to the no of rows or no of coln this could mean:

1) u drew the lines in the wrong way
2) there was optimal soln is the last step u made a mistake.

now,

	1	2	3	4	5
A	7	0	8	12	11
B	0	4	13	10	1
C	10	5	0	2	0
D	0	2	0	0	3
E	3	3	4	0	6

now, will again make again make the assignments

	1	2	3	4	5
A	7	$\boxed{0}$	8	12	11
B	$\boxed{0}$	4	13	10	1
C	10	5	$\cancel{0}$	2	$\boxed{0}$
D	$\cancel{0}$	2	$\boxed{0}$	$\cancel{0}$	3
E	3	3	4	$\boxed{0}$	6

∵ The no of assignments are equal no of rows or no of coln.
∴ the soln is optimal

⇒

Job	Emp	Assign
A	2	5
B	1	3
C	5	2
D	3	9
E	4	4
		23

Q3) Solve the following Assignment problem using the hungarian method.

	1	2	3	4	5
A	10	5	9	8	11
B	13	19	6	12	14
C	3	2	4	4	5
D	18	9	12	17	15
E	11	6	14	19	10

Sol^n →

	1	2	3	4	5
A	5	0	4	3	6
B	7	13	0	6	8
C	1	0	2	2	3
D	9	0	3	8	6
E	5	0	8	13	4

⇒

	1	2	3	4	5
A	4	0	4	1	3
B	6	13	0	4	5
C	0	0	2	0	0
D	8	0	3	6	3
E	4	0	8	11	1

now,

	1	2	3	4	5
A	4	$\boxed{0}$	4	1	3
B	6	13	$\boxed{0}$	4	5
C	$\boxed{0}$	~~0~~	2	~~0~~	~~0~~
D	8	~~0~~	3	6	3
E	4	~~0~~	8	11	1

→ the sol^n is not optimal

∴ Revise

Revise →

	1	2	3	4	5
A	4	$\boxed{0}$	4	1	3
B	6	13	$\boxed{0}$	4	5
C	$\boxed{0}$	~~0~~	2	~~0~~	~~0~~
D	8	~~0~~	3	6	3
E	4	~~0~~	8	11	1

⇒

	1	2	3	4	5
A					
B					
C					
D					
E					

optimal = 36

Special Cases

Q) In the modification of a plant layout of a factory. 4 new machines m_1, m_2, m_3 & m_4 are to be installed at a machine shop. There are 5 vacant spaces A, B, C, D & E that are available. Because of the limited space machine m_2 cannot be placed at C & machine m_3 cannot be placed at A. The cost of placing ith machine jth place is as shown below. Find the optimal machine placement. Use assignment algorithm.

Location

Machine	A	B	C	D	E
M_1	9	11	15	10	9
M_2	12	9	–	10	9
M_3	–	11	14	11	7
M_4	14	8	12	7	8

Sol^n → Unbalanced problem → no of rows ≠ no of col^n → ∴ we add a dummy machine

	A	B	C	D	E
M_1	9	11	15	10	9
M_2	12	9	~~M~~ 100	10	9
M_3	~~M~~ 100	11	14	11	7
M_4	14	8	12	7	8
Dummy	0	0	0	0	0

very high positive value

first row opp^n table

	A	B	C	D	E
M_1	0	2	6	1	0
M_2	3	0	91	1	0
M_3	93	4	7	4	0
M_4	7	1	5	0	1
Dummy	0	0	0	0	0

no need for col^n as each row & col^n has 1 zero

⇓

	A	B	C	D	E
M_1	$\boxed{0}$	2	6	1	~~0~~
M_2	3	$\boxed{0}$	91	1	~~0~~
M_3	93	4	7	4	$\boxed{0}$
M_4	7	1	5	$\boxed{0}$	1
Dummy	~~0~~	~~0~~	$\boxed{0}$	~~0~~	~~0~~

⇒ $M_1 \to A \to 9$
$M_2 \to B \to 9$
$M_3 \to E \to 7$
$M_4 \to D \to 7$
Dummy → C → 0
32

∴ optimal → 32

8) A company has a team of 4 salesmen & there are 4 districts where the company wants to start the business, after taking into account the capabilities of the salesmen. The company estimates that the profit per day in Rs. each salesman will generate in each district is given below find the assignment of salesmen to various district to yield the maximum profit.

District

Salesmen		1	2	3	4
	A	16	10	14	11
	B	14	11	15	15
	C	15	15	13	12
	D	13	12	14	15

sol^n → we'll have to convert this maximization into minimization. → we will develop the opportunity loss / cost matrix
↓
Subtract all the numbers from the highest element (in this table 16)
↓
objective is to minimize the opportunity loss or cost by applying the hungarian method

opportunity loss / cost →

District

	1	2	3	4
A	0	6	2	5
B	2	5	1	1
C	1	1	3	4
D	3	4	2	1

now, we will apply hungarian method :

→ row opp^n cost matrix

	1	2	3	4
A	0	6	2	5
B	1	4	0	0
C	0	0	2	3
D	2	3	1	0

no need for coln opp^n cost ⇒

	1	2	3	4
A	$\boxed{0}$	6	2	5
B	1	4	$\boxed{0}$	$\cancel{0}$
C	$\cancel{0}$	$\boxed{0}$	2	3
D	2	3	1	$\boxed{0}$

optimal

⇒ A → 1 → 16
B → 3 → 15
C → 2 → 15
D → 4 → 15

61

Application of Assignment Algo

1) Iternary planning :

Q) A business executive must make 4 round trips as listed below b/w his head office in Delhi & branch office in Bangalore.

Departure from Delhi	Departure from Bangalore
Monday, June 3	Friday, June 7
Monday, June 10	Wednesday, June 12
Monday, June 17	Friday, June 21
Tuesday, June 25	Friday, June 28

The price of a round trip ticket in either direcⁿ is 4k, the discount of 25% is granted if the dates of tickets span a weekend. if the stay at bangalore lasts for more than 21 days the discount is increased to 30%. A one way ticket b/w delhi & Bangalore (in either direcⁿ cost Rs 2,500) how should the executive purchase the ticket so as to minimize the total cost incurred. Use assignment algo.

Solⁿ → cost of one way ticket = 2500 * 8 = 20,000

Cost of 4 round ticket = 4000 * 4 = 16,000

BLR → DEL

DEL → BLR

Departure from BLR

Departure from Delhi	FRI Jun 7	Wed Jun 12	FRI, Jun 21	FRI Jun 28
Mon June 3	4000	3000	3000	2800
Mon, June 10	3000	4000	3000	3000
Mon, June 17	3000	3000	4000	3000
Tue, June 25	3000	3000	3000	4000

applying hungarian method :

row oppⁿ cost →

1200	200	200	0
0	1000	0	0
0	0	1000	0
0	0	0	1000

⇒

1200	200	200	$\boxed{0}$
$\boxed{0}$	1000	$\cancel{0}$	$\cancel{0}$
$\cancel{0}$	$\boxed{0}$	1000	$\cancel{0}$
$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	1000

June 3 → June 28 → 2800
June 10 → June 5 → 3000
June 17 → June 12 → 3000
June 25 → June 12 → 3000

11,800

Define question unseen

2) Crew Scheduling Using Assignment Algorithm

2 condⁿ

- rest period for crew members b/w their journey's of min 12–15 hours
- rest " should not be too long or it will be an expense to the company. (Company will have to provide with stay ... etc)

Q) A Trip from Chandigarh to Delhi takes 6 hrs by bus. A typical time table of bus service in both the directions is given below.

Departure from Chandigarh	Route No	Arrival @ Delhi
6:00	a	12:00
7:30	b	13:30
11:30	c	17:30
19:00	d	01:00
00:30	e	6:30

Departure from Delhi	Route No	Arrival @ Chandigarh
5:30	1	11:30
9:00	2	15:00
15:00	3	21:00
18:30	4	00:30
00:00	5	6:00

The cost of providing this service by the transport company depends upon the time spend by the crew members away from their places in addition to their service time. There are 5 crew members. There is a constraint that every group should be provided with more than 4 hrs of rest before their return & should not wait for more than 24 hrs for the return trip. The company has residential facilities for the crew members at Delhi as well as in Chandigarh. Find the optimal service line connections. Use Assignment Algo.

26 March TUE
FRI TT

Solⁿ →

In first case,
We'll assume all crew members are natives of Chandigarh they will start from Chandigarh go Delhi use the facilities of company in delhi & return back

Waiting time for crew members residing at Chandigarh at the Delhi's accomodation facility

Chandigarh → Delhi

rest less than 4 hrs

		5:30	9:00	15:00	18:30	00:00
		1	2	3	4	5
6:00	a	17.5	21	M	6.5	12
7:30	b	16	19.5	M	5	10.5
11:30	c	12	15.5	21.5	M	6.5
19:00	d	4.5	8	14	17.5	23
00:30	e	23	M	8.5	12	17.5

Waiting time for crew members of Delhi at the company's accomodation at Chandigarh

Delhi → Chandigarh

		5:30	9:00	15:00	18:30	00:00
		1	2	3	4	5
6:00	a	18.5	15	9	5.5	M
7:30	b	20	16.5	10.5	7	M
11:30	c	M	20.5	14.5	11	5.5
19:00	d	7.5	M	22	18.5	13
00:30	e	13	9.5	M	M	18.5

Final table taking minimum

		5:30	9:00	15:00	18:30	00:00
		1	2	3	4	5
6:00	a	17.5	15	9	5.5	12
7:30	b	16	16.5	10.5	5	10.5
11:30	c	12	15.5	14.5	11	5.5
19:00	d	4.5	8	14	17.5	13
00:30	e	13	9.5	8.5	12	17.5

Applying hungarian method

row

		5.30	9:00	15:00	18:30	00:00
		1	2	3	4	5
6:00	a	12	9.5	3.5	0	6.5
7:30	b	11	11.5	5.5	0	5.5
11:30	c	6.5	10	4	5.5	0
19:00	d	0	3.5	4.5	13	8.5
.30	e	4.5	1	0	3.5	9

colⁿ

		5.30	9:00	15:00	18:30	00:00
		1	2	3	4	5
6:00	a	12	8.5	3.5	0	6.5
7:30	b	11	10.5	5.5	0	5.5
11:30	c	6.5	9	4	5.5	0
19:00	d	0	2.5	4.5	13	8.5
.30	e	4.5	0	0	3.5	9

		5.30	9:00	15:00	18:30	00:00
		1	2	3	4	5
6:00	a	12	8.5	3.5	$\boxed{0}$	6.5
7:30	b	11	10.5	5.5	~~0~~	5.5
11:30	c	6.5	9	4	5.5	$\boxed{0}$
19:00	d	$\boxed{0}$	2.5	4.5	13	8.5
.30	e	4.5	$\boxed{0}$	~~0~~	3.5	9

Revise -

		1 (5.30)	2 (9:00)	3 (15:00)	4 (18:30)	5 (00:00)
6:00	a	12	8.5	3.5	0	6.5
7:30	b	11	10.5	5.5	0	5.5
11:30	c	6.5	9	9	5.5	0
19:00	d	0	2.5	4.5	13	8.5
.30	e	4.5	0	0	3.5	9

		1 (5.30)	2 (9:00)	3 (15:00)	4 (18:30)	5 (00:00)
6:00	a	8.5	5	$\boxed{0}$	$\cancel{0}$	6.5
7:30	b	7.5	7	2	$\boxed{0}$	5.5
11:30	c	3	5.5	5.5	5.5	$\boxed{0}$
19:00	d	$\boxed{0}$	2.5	4.5	16.5	12
.30	e	4.5	$\boxed{0}$	$\cancel{0}$	7	12.5

Routes paired	Waiting time	Crew Base
a → 3	9	Delhi
b → 4	5	Delhi
c → 5	5.5	Chandigarh
d → 1	4.5	Delhi
e → 2	2.5	Chandigarh

27 March Wed

SAT TT

Travelling Salesman Problem

Q) A salesman has been assigned 5 territories, the distance b/w the cities has been tabulated as below. Use the travelling salesman problem algo to come up with a sequence of cities that the salesman has to follow, such that the total travelled distance is the minimum

	A	B	C	D	E
A	∞	10	8	9	7
B	10	∞	10	5	6
C	8	10	∞	8	9
D	9	5	8	∞	6
E	7	6	9	6	∞

Soln using hungarian method :

row opn :

	A	B	C	D	E
A	∞	3	1	2	0
B	5	∞	5	0	1
C	0	2	∞	0	1
D	4	0	3	∞	1
E	1	0	3	0	∞

col opn wit :

	A	B	C	D	E
A	∞	3	0	2	0
B	5	∞	4	0	1
C	0	2	∞	0	1
D	4	0	2	∞	1
E	1	0	2	0	∞

$\Rightarrow$

	A	B	C	D	E
A	∞	3	$\boxed{0}$	2	~~0~~
B	5	∞	4	$\boxed{0}$	1
C	$\boxed{0}$	2	∞	~~0~~	1
D	4	$\boxed{0}$	2	∞	1
E	1	~~0~~	2	~~0~~	∞

Revise

$\Rightarrow$

	A	B	C	D	E
A	∞	3	0	2	0
B	5	∞	4	0	1
C	0	2	∞	0	1
D	4	0	2	∞	1
E	1	0	2	0	∞

	A	B	C	D	E
A	∞	4	$\boxed{0}$	3	$\cancel{0}$
B	4	∞	3	$\cancel{0}$	$\boxed{0}$
C	$\boxed{0}$	3	∞	1	1
D	3	$\boxed{0}$	1	∞	$\cancel{0}$
E	$\cancel{0}$	$\cancel{0}$	1	$\boxed{0}$	∞

here

$A \to C$

$C \to A$

$B \to E$

$E \to D$

$D \to B$

here, the solution is optimal as there are 5 assignments against a 5×5 matrix however it is not cyclical or sequential.
Hence, revise the solution using the PENALTY COST METHOD

$\Rightarrow$

	A	B	C	D	E
A	∞	4	0^{0+1}	3	0^{0+0}
B	4	∞	3	0^{0+0}	0^{0+0}
C	0^{0+1}	3	∞	1	1
D	3	0^{0+0}	1	∞	0^{0+0}
E	0^{0+0}	0^{0+0}	1	0^{0+0}	∞

We'll calculate the penalty for each 0 in each row and column
for penalty we take the 0 and add the minimum value in the row and the column of that zero (excluding that zero in min value)

→ we now select the zero with maximum penalty.
if there is a tie we choose arbitarily.

→ here we select $A \to C$

$\therefore$ we make the path $C \to A$ infinite

→ cancel row A & col C

→ & develop the reduced matrix.

$\Rightarrow$

	A	B	D	E
B	4	∞	0^{0+0}	0^{0+0}
C	∞	3	1	1
D	3	0^{0+0}	∞	0^{0+0}
E	0^{0+3}	0^{0+0}	0^{0+0}	∞

& we again calculate the penalties

→ We choose $E \to A$

now, $A \to E$ is already cancelled

→ Cancel row E & col A

$\Rightarrow$

$$\begin{array}{c|ccc} & B & D & E \\ \hline B & \infty & 0^{0+1} & 0^{0+0} \\ C & 3 & 1 & 1 \\ D & 0^{0+3} & \infty & 0^{0+0} \end{array}$$

Again calculate penalty,

→ we choose D → B

→ B → D as ∞

→ cancel row D & column B

$\Rightarrow$

$$\begin{array}{c|cc} & D & E \\ \hline B & \infty & 0^{1+\infty} \\ C & 1 & 1 \end{array}$$

→ B → E

→ C → D

$\therefore$ Total $\Rightarrow$ A → C → D → B → E → A

A → C – 8

C → D – 8

D → B – 5

B → E – 6

E → A – 7

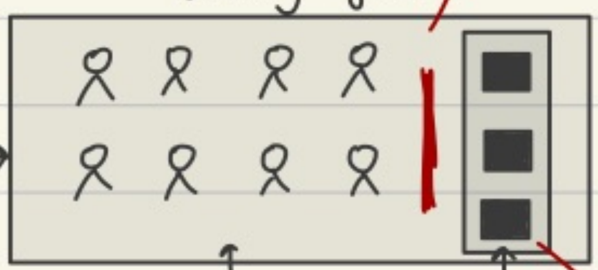
Queuing Theory

① Analytical Method
↳ consists of several established mathematical model
② Simulation Model

The Queuing System Characteristics :

Selection
- Limited needs first
- FIFO (first in - first out)
- LIFO (Last in - last out)
- SIRO (Service in random order)
- SPT (Shortest processing time)
- Priority

Queuing System



Source Population → Queuing System

Source Population
· finite
· infinite

Arrival
· individual · pre-scheduled
· batches · unscheduled

· controllable · constant arrival
· uncontrolled · variable arrival

· patient customer
· impatient customer
↳ Balking (sees the long queue & leaves)
↳ Reneging (sees the long queue, joins the queue but leaves)
↳ Jockeying (Switching / hopping between queues)

Queue
No. of lines
✓ Single Server Model
✓ Multiple Server Model

Servers / Counters → providing some service

Whether u will get your service at one server or through multiple servers.

Configuration of Servers

① Single channel Single Phase
② Single channel Multi Phase → bank
③ Multi Channel Single Phase → male & female channel but one server counter
④ Multi Channel Multi Phase → passport

* Channel → no. of queues
Phase → no. of counters

for example : McDonalds drive - thru is Single Channel Multi Phase
↳ (there is a single queue for cars, but two counters, one for ordering, another for payment)

total time user spends from entry to exit (arrival → standing in queue → time in recieving of service after in front of the queue + exit)

Mathematical analysis of a queuing system

User oriented Statistics

Waiting time in the system ← W_s = time spent in the queue + service time

waiting time in the queue ← W_q = time spent waiting for the service.

System oriented statistics

length of system ← L_s = no. of ppl waiting in the queue + no of ppl recieving service

length of queue ← L_q = no of ppl waiting in the queue

ρ = % of the time the server is busy

MODEL - 1 — Single channel Single phase ; Infinite popn ; Poission Distribution ; FCFS

Arrival rate / mean arrival rate (λ) = no of ppl visiting the facility per unit time

mean service rate (μ) = no. of ppl getting served per unit time.

formulas:

W_s = time spent in the queue + service time $= \dfrac{1}{\mu-\lambda}$

W_q = time spent waiting for the service. $= \dfrac{\lambda}{\mu(\mu-\lambda)}$

L_s = no. of ppl waiting in the queue + no of ppl recieving service $= \dfrac{\lambda}{\mu-\lambda}$

L_q = no of ppl waiting in the queue $= \dfrac{\lambda^2}{\mu(\mu-\lambda)}$

ρ = % of the time the server is busy $= \dfrac{\lambda}{\mu} * 100$

Q) Patrons arrive at a small post office at a rate of 30 per hour, service by the clerk on duty takes an average of 1 minute per customer. Calculate

i) mean customer time spent waiting in the line

ii) " " " " recieving or waiting for service

iii) no. of customers in the line

iv) no. of customers recieving or waiting for service

v) traffic density (utilization)

Soln :

Arrival rate (λ) = 30 per hour $= \frac{30}{hour} = \frac{30}{60} = \frac{1}{2}$ customer / min

mean service rate (μ) = 1 min per customer. $= \frac{1}{1}$ (reciprocal) = 1 (customer) min

i) $W_s = \frac{1}{1-0.5} = 2$ min

ii) $W_q = \frac{0.5}{1(1-0.5)} = 1$ min

iii) $L_s = \frac{0.5}{1-0.5} = 1$ customer

iv) $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = 0.5$

v) $\rho = \frac{0.5}{1} \times 100 = 50\%$

Monte Carlo Simulation

is used when λ & μ are not known (arrival rate & service rate are unknown)

8) Patients arrive at a dentists clinic every 30 min for some treatment, the dentist offers the following services & the time for each service varies run a monte carlo simulation for 10 patients & determine the avg waiting time for patients. Use the following random numbers.

Random numbers : 40, 82, 11, 34, 25, 66, 17, 79, 90, 44

(consider only first two digits of the random numbers if the number have more than two digits

ex 26 7 1
3 3 1 5)

Service	Service time	Probability of respective service
Filling	40	0.4
Crown	60	0.15
cleaning	15	0.15
Extraction	45	0.1
Checkup	15	0.2

Assume first person arrives at 8 am

mutually inclusive

Sol^n →

Service	(min) service time	Probability of respective service	Cummulative probability	Random No/ range
Filling	40	0.4	0.4	00 – 39
Crown	60	0.15	0.55	40 – 54
Cleaning	15	0.15	0.7	55 – 69
Extraction	45	0.1	0.8	70 – 79
Checkup	15	0.2	1	80 – 99

now,

Patient :	1	2	3	4	5	6	7	8	9	10
arrival :	8:00	8:30	9:00	9:30	10:00	10:30	11:00	11:30	12:00	12:30
Random No's :	40	82	11	34	25	66	17	79	90	44
Type of Service :	Crown	Checkup	Filling	Filling	Filling	Cleaning	Filling	Extraction	Checkup	Crown
Service time :	60	15	40	40	40	15	40	45	15	60

decide based on Random no. range

now,

Simulation worksheet:

Patient No.	Arrival time	Service time	Service start	Service end	waiting time
1	8:00	60	8:00	9:00	0
2	8:30	15	9:00	9:15	30
3	9:00	40	9:15	9:55	15
4	9:30	40	9:55	10:35	25
5	10:00	40	10:35	11:15	35
6	10:30	15	11:15	11:30	45
7	11:00	40	11:30	12:10	30
8	11:30	45	12:10	12:55	40
9	12:00	15	12:55	1:10	55
10	12:30	60	1:10	2:10	40
					315

avg waiting time $= \frac{315}{10} = 31.5$ min.