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→ Mathematical Programming Problem (MPP)

- deals with determining optimal (max. or min.) allocation of limited resources to meet given objectives.

- resources may be - materials, machine, workforce etc.
- objective - optimize total profit or total cost using limited resources subject to various constraints.

→ Linear Programming Problem (LPP)

- set of simultaneous linear eq^n which represent constraints related to limited resources & a linear function which express objective representing total profit or loss.

- solution shows how much should be produced / sold / purchased to optimize objective function & satisfy constraints.

- Variables are called "decision variables"

→ Formulation of LPP

Q. A furniture manufacturer makes 2 products - chairs and tables. Processing these products is done on 2 machines A and B. A chair requires 2hrs on 'A' and 6hrs on 'B'. A table requires 5hrs on 'A' and no time on 'B'. There are 16 hrs per day available on 'A' and 30 hrs on 'B'. Profits gained by manufacturer from chair is ₹150 & from table is ₹500. Formulate LPP in order to maximize his profit

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Q. A company produces 3 types of clothes - A, B, C. 3 kinds of cotton were required - red, green, blue. 1 unit length of type 'A' need 2m of red & 3m of blue.

1 unit length of type 'B' requires 3m red, 2m green, 2m blue.

1 unit length of 'C' needs 5m green, 4m blue. The company has a stock of 8m red, 10m green & 15m blue.

Income obtained from 1 unit length of A, B, C are ₹3, ₹5, ₹4. Formulate mathematical model.

x - length of ~~red~~ A

y - " " ~~green~~ B

z - " " ~~blue~~ C

Max $Z = 3x + 5y + 4z$

subject to.

$x, y, z \geq 0$

$2x + 3y \leq 8$

$2y + 5z \leq 10$

$3x + 2y + 4z \leq 15$

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Q. An animal feeding company must produce atleast 200kg of products consisting of 2 products - P_1 & P_2, daily. P_1 cost ₹3/kg & P_2 costs ₹8/kg. No more than 80 kg of P_1 can be used & atleast 60kg of P_2 must be used.

x - units of P_1
y - units of P_2.

Min $Z = 3x + 8y$

$x \leq 80$

$y \geq 60$

$x + y \geq 200$

$x \geq 0$

Q: (A A B) - Production planner soft drink machine. He has 2 machines to design bottles.
'A' designed for 8-ounce bottle.
'B' ~~for~~ designed for 16-ounce bottle
Each can be used on both types with some loss of efficiency.

Machine	8 ounce	16 ounce
A	100/min	40/min
B	60/min	75/min.

The machines can be run 8 hrs/day, 5 days/week.
Profit on 8-ounce bottle = ₹1.5, and on 16-ounce bottle = ₹2.5.
Weekly production can't exceed 3 lakh ounces &

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market can absorb 25000 8-ounce Bottle & 7000 16-ounce bottle.

Planner wants to maximize profits subject to all production & marketing restriction.

$x \rightarrow$ no. of 8-ounce
$y \rightarrow$ " " 16-ounce.

Time per machines $= \frac{8 \times 5 \times 60}{}$ ... $= \frac{8\times 5\times}{60}$

$= 2400$ min

Max $Z = 1.5x + 2.5y$

Subject to

$$x, y \geq 0$$

$$\frac{x}{100} + \frac{y}{40} \leq 2400 \quad \text{(time restriction on A)}$$

$$\frac{x}{60} + \frac{y}{75} \leq 2400$$

$$8x + 16y \leq 300000$$

$$x \leq 25000$$

$$y \leq 7000$$

Q. A company buys castings of P & Q type of parts and sells them as finished products after machining, boring & polishing. The purchasing costs are ₹3 & ₹4 and selling costs are ₹8 & ₹10.

The per hour capacity for the 3 processes are given below:

Capacity.	Parts P	Parts Q
M	30	50
B	30	45
P	45	30

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The running costs of M, B, P are ₹30, ₹22.5 & ₹22.5. Determine product mix at max profit.

x - no. of P parts
y - no. of Q parts. } per hour

~~Max $Z = 5x + 6y$~~
~~Subject to.~~
~~$x, y \geq 0$~~

Production cost.

Capacity		P (Parts)	Q (Parts)
₹30	M	$30/30 =$ ₹1	₹0.6
₹22.5	B	$22.5/30 =$ ₹0.75	₹0.5
₹22.5	P	₹0.5	₹0.75
		₹2.25	₹1.85
+ Purchase		+ 3	+ 4
		₹5.25	₹5.85

∴ profit ₹2.75 ₹4.15

objective func: $2.75x + 4.15y$

$$\left.\begin{aligned} \frac{x}{30} + \frac{y}{50} &\leq 1 \\ \frac{x}{30} + \frac{y}{45} &\leq 1 \\ \frac{x}{45} + \frac{y}{30} &\leq 1 \end{aligned}\right\} \text{Time constraints}$$

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Q. a company produced sauces A & B. They are made by blending ingredients X & Y. A certain level of flexibility is permitted in the formula of these products. The restrictions are that:

i) B must contain no more than 75% of X &

ii) A " " " less " 25% of X and

" " " 50% of Y.

upto 400 kg of X & 300 kg of Y could be purchased. The company can sell as much of these sauces as it produces at a price of ₹18 for A & ₹17 for B. X & Y cost ₹1.6 & ₹2.05 per kg. Maximize profit.

⇒ x_1, x_2 → amount of ingredient X in kg

y_1, y_2 → amount of " Y " "

a, b → amount of sauces A & B

cost of purchasing = $1.6x + 2.05y$

selling price = $18a + 17b$

Max. $Z = 18a + 17b - 1.6(x_1 + x_2) - 2.05(y_1 + y_2)$

Subject to.

$$x_1 + y_1 = a$$

$$x_2 + y_2 = b$$

$$x_1 + x_2 \leq 400$$

$$y_1 + y_2 \leq 300$$

$$a, b, x_1, x_2, y_1, y_2 \geq 0$$

$$x_2 \leq 0.75b$$

$$x_1 \geq 0.25a$$

$$y_1 \geq 0.5a$$

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Q. A firm has 2 grades of inspectors - 1 & 2, the members of which are to be assigned for quality control inspection. At least 2000 pieces be inspected per 8hr day.

Grade 1 insp. can check pieces at the rate 40/hr & with an accuracy of 97%.

Grade 2 check at the rate 30/hr with an accuracy of 95%.

The wage of grade 1 is ₹5/hr & grade 2 is ₹4/hr.

An error made costs ₹3 to the company. There are 9 grade 1 & 11 grade 2 insp.

Company wishes to assign work to available insp. to minimize total cost of inspection.

⇒ no. of pieces inspected by grade 1 $= x$

" " " " " 2 $= y$

Min Z =

Subject to.

$$x + y \geq 2000$$

$$\frac{x}{40} + \frac{y}{30} \leq 8 \quad \text{(Time constraint)}$$

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→ Solution of LPP

- finding value of decision variables such that it is in general form and optimized.
- constraints must be satisfied.
- any sol^n which satisfies non-negativity constraint is called feasible solution.
- any feasible sol^n which optimize object func. is optimal solution.

SOLUTION BY GRAPHICAL METHOD [only 2 variable]

Step 1: formulate problem as LPP model

Step 2: plot constraints as inequalities on x-y plane & determine convex region formed by them.

Step 3: determine vertices of convex region & find value of objective function at each vertex.

Q. Solve the LPP graphically.

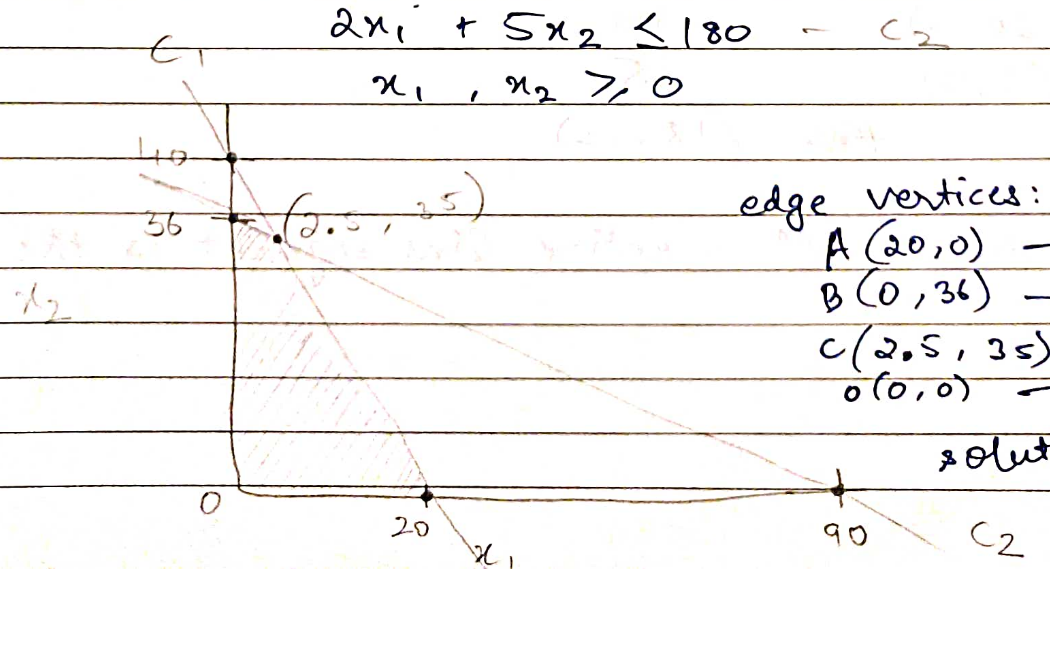
Max $Z = 3x_1 + 4x_2$

subject to

$4x_1 + 2x_2 \leq 80$ — C_1

$2x_1 + 5x_2 \leq 180$ — C_2

$x_1, x_2 \geq 0$



edge vertices:

A (20, 0) ⟶ 60

B (0, 36) ⟶ 144

C (2.5, 35) ⟶ 147.5

O (0, 0) ⟶ 0

solution: (2.5, 35)

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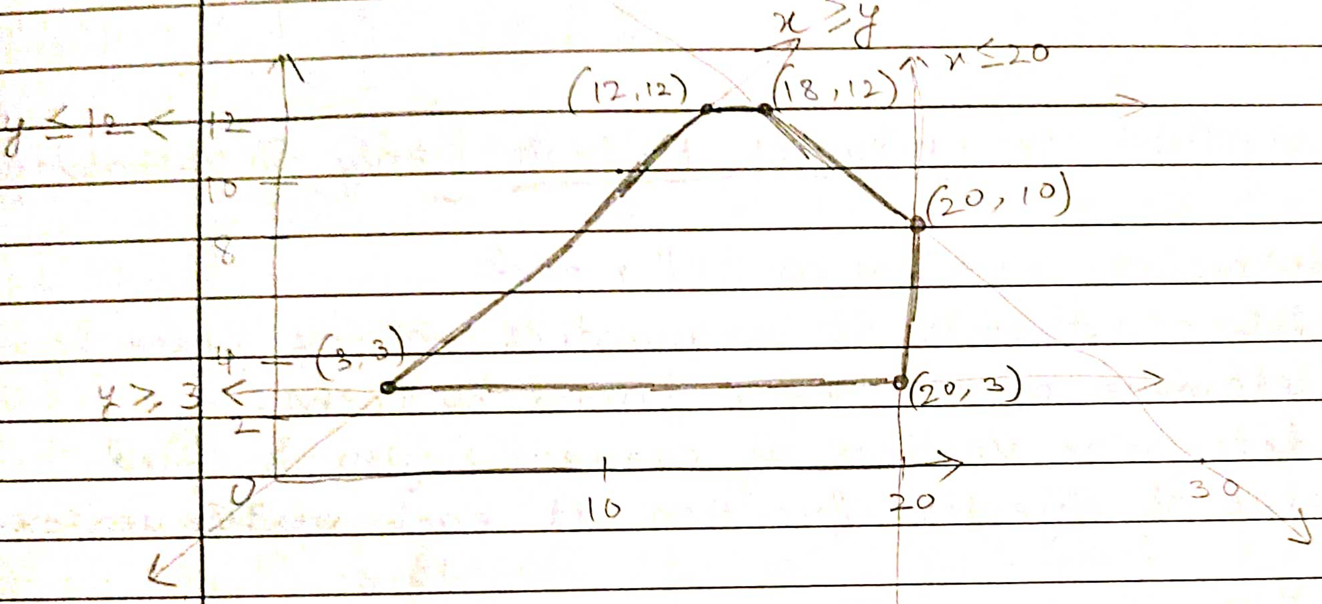
Q. Max. $Z = 2x + 3y$
subject to.

$$x + y \le 30$$
$$y \ge 3$$
$$0 \le y \le 12$$
$$x - y \ge 0$$
$$0 \le x \le 20$$
$$x, y \ge 0$$



Edge vertices	value Z
(12,12)	60
(18,12)	72
(20,10)	70
(20,3)	49
(3,3)	15

Ans: (18,12)

* 2 optimal solⁿ : entire line segment is the solⁿ.

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Q. A company has 2 cold drink bottling machines at cities C_1 & C_2. each plant produces 3 drinks - A, B, C. The production capacity per day is given below:

	Plant	
Drink	C_1	C_2
A	6000	2000
B	1000	2500
C	3000	3000

The marketing dept. forecasts a demand of 80000 bottles of A 22000 bottles of B 40000 bottles of C during the month of June. The per day operating costs of plants at C_1 & C_2 are ₹6000 & ₹4000. Formulate as LPP to find the no. of days for which each plant must be run so as to minimize operating cost.

$\Rightarrow$ $x, y \rightarrow$ no. of operating days of C_1 & C_2

Min. $Z = 6000x + 4000y$.

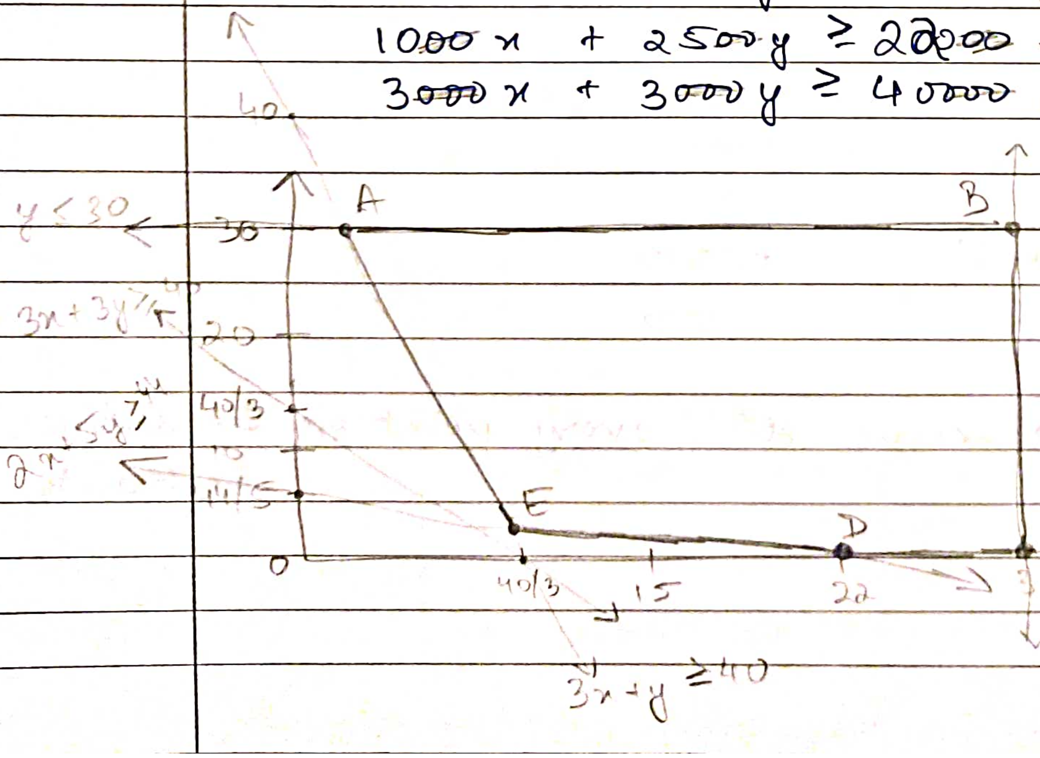
Subject to.

$$0 \le x, y \le 30$$

$$6000x + 2000y \ge 80000 \Rightarrow 3x + y \ge 40$$

$$1000x + 2500y \ge 22000 \Rightarrow 2x + 5y \ge 44$$

$$3000x + 3000y \ge 40000 \Rightarrow 3x + 3y \ge 40$$



	Z
A $(\frac{10}{3}, 30)$	140000
B $(30, 30)$	300000
C $(30, 0)$	180000
D $(22, 0)$	132000
E $(12, 4)$	88000

Ans: $(12, 4)$

12 days C_1

4 days C_2

₹88000

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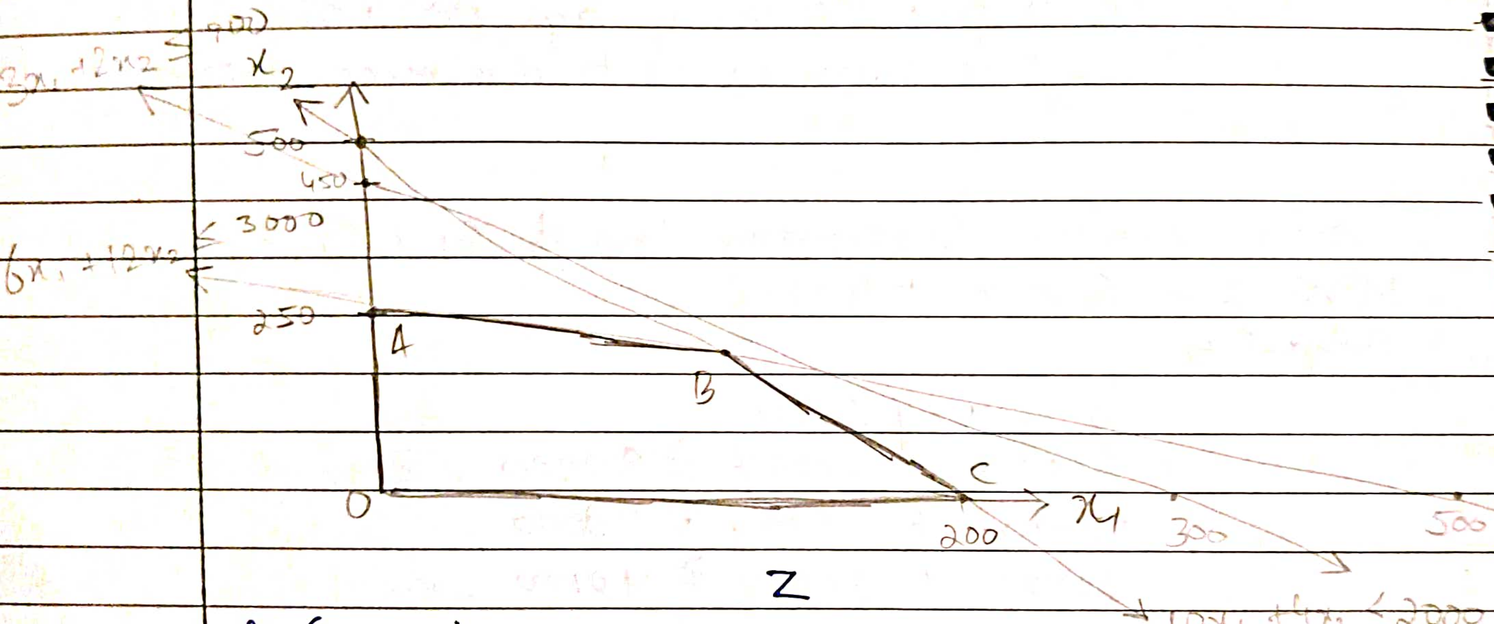
→ Exceptional cases :

i) infinite no. of soln.
ii) unbounded soln.
iii) no. soln.

Q. Max. $Z = 100x_1 + 40x_2$
subject to

$$10x_1 + 4x_2 \leq 2000$$
$$3x_1 + 2x_2 \leq 900$$
$$6x_1 + 12x_2 \leq 3000$$

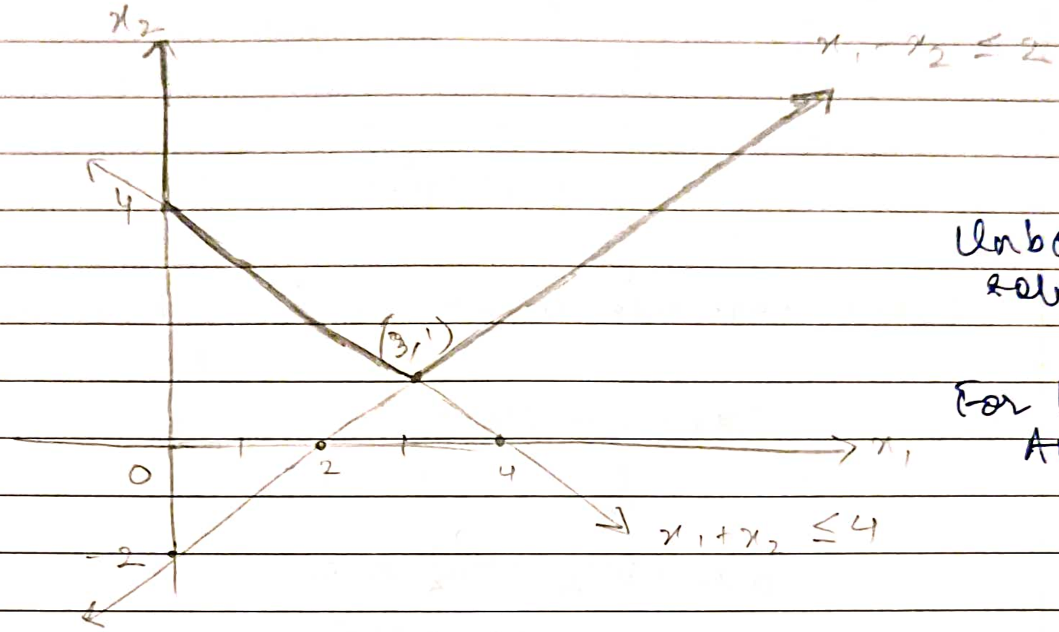


	Z
A (0, 250)	10000
O (0, 0)	0
B (125, 187.5)	20000
C (200, 0)	20000

Ans: No unique sol^n. every point on BC satisfies constraints.

Q. Max. $Z = 2x_1 + 3x_2$
subject to.

$$x_1 - x_2 \le 2$$
$$x_1 + x_2 \ge 4$$
$$x_1, x_2 \ge 0$$

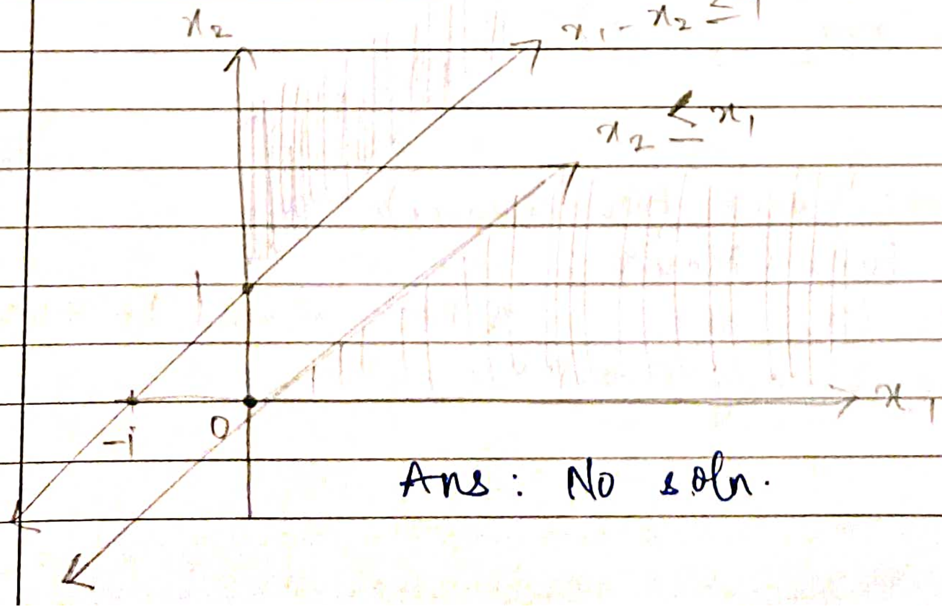


Unbounded soln (Max)

For Min.
Ans: (4, 0)

Q. Max $Z = 4x_1 + 3x_2$
subject to

$$x_1 - x_2 \le -1$$
$$-x_1 + x_2 \le 0$$
$$x_1, x_2 \ge 0$$



Ans: No soln.

★ Min z = Max$(-z)$ (reverse sign of variables)

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→ Simplex Method

- convert inequality to equality by ~~ind~~ introducing $\boxed{\text{slack variable } (<=)}$ to LHS of inequality

$$ax_1 + bx_2 \leq c$$
$$\downarrow$$
$$ax_1 + bx_2 + \underbrace{S_1}_{\text{(slack)}} = c$$

- supplus variable in case of >= inequality.

$$ax_1 + bx_2 \geq c$$
$$\downarrow$$
$$ax_1 + bx_2 - \underbrace{S_2}_{\text{surplus}} = c$$

→ Canonical & Standard LPP

CANONICAL

- objective func. Max type
- constraints <= type
- variables non-negative

STANDARD (used for Simplex)

- objective func. Max
- constraints = type ★ RHS should be +ve
- variables non-negative

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Q. Express LPP in standard form.

$\text{Min } Z = -x_1 + 2x_2 + 7x_3$

subject to

$$x_1 - x_2 + 3x_3 \le 5$$
$$2x_1 + 4x_3 \ge -7$$
$$x_1 + x_2 \le -4$$
$$x_3, x_1, x_2 \ge 0$$

$\Rightarrow$ $\text{Max } Z' = x_1 - 2x_2 - 7x_3 + 0S_1 + 0S_2 + 0S_3$

subject to

$$x_1 - x_2 + 3x_3 + S_1 + 0S_2 + 0S_3 = 5$$
$$0S_1 + 0S_3 \quad -2x_1 - 4x_3 + S_2 = 7$$
$$0S_1 + 0S_2 \quad -x_1 - x_2 - S_3 = 4$$
$$x_1, x_2, x_3 \ge 0$$
$$S_1, S_2, S_3 \ge 0$$

Q. Express as standard form

$\text{Max. } Z = 3x_1 + 5x_2 + 7x_3$

Subj. to.

$$6x_1 - 4x_2 \le 5$$
$$3x_1 + 2x_2 + 5x_3 \ge 11$$
$$x_1, x_2 \ge 0$$

$\Rightarrow$ x_3 is unrestricted in sign

$\therefore x_3 = x_3' - x_3''$

$x_3', x_3'' \ge 0$

$\text{Max } Z = 3x_1 + 5x_2 + 7x_3 + 0S_1 + 0S_2$

Subj to.

$$6x_1 - 4x_2 + S_1 + 0S_2 = 5$$
$$x_3' - x_3'' = x_3 \qquad 3x_1 + 2x_2 + 5x_3 + 0S_1 - S_2 = 11$$
$$x_1, x_2, x_3', x_3'' \ge 0$$

In simplex method, an infinite no. of soln is reduced to finite no. of promising sol^n by using following facts:

$(m < n)$

- when there are n decision variables & m constraints the starting soln is found by setting $n-m$ variables to 0, then solving remaining m equations

- The $n-m$ variables are called non-basic variables while the remaining m variables are called basic variables & they form Basic Solution.

- no. of basic sol^n = * ${}^nC_m$

- if basic variables are -ve, then solution is called basic infeasible solution

- if all basic variables are non-zero, then it is called non-degenerate solution.

- if some variables are zero, then it is degenerate solution.

- a new basic solution is obtained from the previous one by equating one of the basic variables to zero & replacing it by a new non-basic variable.

  Eliminated variable → outgoing / leaving

  New variable → Incoming.

* Repeat procedure till no further improvement

Resulting soln → optimal basic feasible sol^n.

Q. Find all basic soln. of following system of equations & investigate whether degenerate or not. Find optimal soln.

Max $Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$

subj. to

$$2x_1 + 3x_2 - x_3 + 4x_4 = 8$$
$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

variables $= 4$

$eq^n = 2$

No. of soln. $= {}^4C_2 = 6$

Sl. No.	Basic variable	Non-basic	feasible	Non-degen	Value of Z
1.	$2x_1 + 3x_2 = 8$ $x_1 - 2x_2 = -3$ $(x_1 = 1 \quad x_2 = 2)$	$x_3 = x_4 = 0$	Yes	Yes	8
2.	$2x_1 - x_3 = 8$ $x_1 + 6x_3 = -3$ $\left(x_1 = \frac{45}{13}, x_3 = -\frac{14}{13}\right)$	$x_2 = 0, x_4 = 0$	X	—	—
3.	$3x_2 - x_3 = 8$ $-2x_2 + 6x_3 = -3$ $\left(x_2 = \frac{45}{16}, x_3 = \frac{7}{16}\right)$	$(x_1 = 0, x_4 = 0)$	✓	✓	10.2
4.	$2x_1 + 4x_4 = 8$ $x_1 - 7x_4 = -3$ $x_1 = \frac{22}{9}, x_4 = \frac{1}{9}$	$x_2 = 0, x_3 = 0$	✓	✓	10.3
5.	$3x_2 - 4x_4 = 8$ $-2x_2 - 7x_4 = -3$ $x_2 = 132/39 \quad x_4 = -7/13$	$x_1 = x_3 = 0$	X	—	—

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6. $x_3 = \frac{44}{17}$, $x_4 = \frac{45}{17}$ $x_1 = x_2 = 0$ ✓ ✓ 28.9

$\therefore$ optimal solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = \frac{44}{17}$

$x_4 = \frac{45}{17}$

and optimal value = 28.9

→ **Working procedure : Simplex Method**

Consider LPP

$\text{Max } Z = 4x_1 + 3x_2 + 6x_3$

Subj. to

$2x_1 + 3x_2 + 2x_3 \le 440$

$4x_1 + 3x_3 \le 470$

$2x_1 + 5x_2 \le 430$

Step 1: • check whether objective function is to be max or min. If min. , convert to max.

• convert RHS to +ve

Step 2: Express problem in standard form.

Step 3: • Find an initial basic feasible soln.

• Express using Simplex Table

Step 4: • apply optimality test

• compute $C_j = c_j - z_j$

$z_j = \sum c_B a_{ij}$

If all $C_j \le 0$, then soln. is optimal

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* Simplex works only for <= constraints.

Max $Z = 4x_1 + 3x_2 + 6x_3 + 0S_1 + 0S_2 + 0S_3$

S. to

$$2x_1 + 3x_2 + 2x_3 + S_1 + 0S_2 + 0S_3 = 440$$
$$4x_1 + 3x_3 + 0S_1 + S_2 + 0S_3 = 470$$
$$2x_1 + 5x_2 + 0S_1 + 0S_2 + S_3 = 430$$
$$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$$

var = 6
eqn = 3
no. of basic soln = 20

coefficient of S in Z

Key element

outgoing variable ratio

		4	3	6	0	0	0		
C_B	Basis C_j	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
0	S_1	2	3	2	1	0	0	440	440/2 = 220
0	S_2	4	0	[3]	0	1	0	470	470/3 = 150 ←
0	S_3	2	5	0	0	0	1	430	430/0 = ∞

key row

Body matrix   key column   Unitary Matrix

outgoing variable

Z_j	0	0	0	0	0	0	0
C_j	4	3	6	0	0	0	

↑

incoming variable [highest C_j value]

Step 5: Identify incoming (highest C_j) & outgoing (b/key col) variable. (pick min ratio)

* If all θ are ≤ 0, soln is unbounded & no further iteration is required.

Step 6: Drop outgoing & introduce incoming variable along with its C_B value. Convert key ~~row~~ element to unity by dividing key row by key element & convert key col. to 0 by using elementary row operation.

REPEAT

	C_j	4	3	6	0	0	0		
C_B	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
0	S_1	$-2/3$	$\boxed{3}$	0	1	$-2/3$	0	$380/3$	$380/9$ ←
6	x_3	$4/3$	0	1	0	$1/3$	0	$470/3$	∞
0	S_3	2	5	0	0	0	1	430	$430/2$
	Z_j	8	0	6	0	2	0	940	
	C_j	-4	3 ↑	0	0	-2	0		

3	x_2	$-2/9$	1	0	$1/3$	$-2/9$	0	$380/9$
6	x_3	$4/3$	0	1	0	$1/3$	0	$470/3$
0	S_3	$16/3$	0	0	-5	$10/3$	1	$-610/3$
	Z_j	$22/3$	3	6	1	$4/3$	0	$3200/3$
	C_j	$-10/3$	0	0	-1	$-4/3$	0	

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Q. Min $Z = x_1 - 3x_2 + 3x_3$ using simplex Method

S.to

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$2x_1 + 4x_2 \geq -12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

Max $Z' = -x_1 + 3x_2 - 3x_3 + 0S_1 + 0S_2 + 0S_3$

S. to

$$3x_1 - x_2 + 2x_3 + S_1 = 7$$
$$-2x_1 - 4x_2 + S_2 = 12$$
$$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$$
$$x_1, x_2, x_3 \geq 0$$

	C_j	-1	3	-3	0	0	0		
C_B	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
0	S_1	3	-1	2	1	0	0	7	$7/-1 = -7$
0	S_2	-2	-4	0	0	1	0	12	$12/-4 = -3$
0	S_3	-4	[3]	8	0	0	1	10	$10/3 \leftarrow$ +ve
	Z_j	0	0	0	0	0	0	0	
	C_j	-1	3 ↑	-3	0	0	0		

$R_1 \to R_1 + R_3$

$R_2 \to R_2 + 4R_3$

C_B	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
0	S_1	[5/3]	0	14/3	1	0	1/3	31/3	$\leftarrow$ +ve
0	S_2	-22/3	0	32/3	0	1	4/3	76/3	
3	x_2	-4/3	1	8/3	0	0	1/3	10/3	
	Z_j	-4	3	8	0	0	1	10	
	C_j	3 ↑	0	-11	0	0	-1		

$R_2 \to R_2 + \frac{22}{3} R_1$

$R_3 \to R_3 + \frac{4}{3} R_1$

		-1	3	-3	0	0	0	
C_b	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b
-1	x_1	1	0	14/5	3/5	0	1/5	31/5
0	S_2	0	0	156/5	22/5	1	14/5	364/5
3	x_2	0	1	32/5	4/5	0	3/5	58/5
	Z_j	-1	3	82/5	9/5	0	8/5	143/5
	C_j	0	0	(-)	(-)	0	(-)	

$\therefore x_1 = 31/5 \qquad S_1 = 0$

$x_2 = 58/5 \qquad S_2 = 364/5 \quad , \; Z_{min} = -143/5$

$x_3 = 0 \qquad S_3 = 0$

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Q. Max $z = 107x_1 + x_2 + 2x_3$

S. to

$$14x_1 + x_2 - 6x_3 + 3x_4 = 7$$
$$16x_1 + x_2 - 6x_3 \le 5$$
$$3x_1 - x_2 - x_3 \le 0$$
$$x_1, x_2, x_3, x_4 \ge 0$$

$\Rightarrow$ Max $z = 107x_1 + x_2 + 2x_3 + 0x_4 + 0S_1 + 0S_2$

S. to

$$14x_1 + x_2 - 6x_3 + 3x_4 = 7$$
$$16x_1 + x_2 - 6x_3 + S_1 = 5$$
$$3x_1 - x_2 - x_3 + S_2 = 0$$
$$x_1, x_2, x_3, x_4, S_1, S_2 \ge 0$$

	~~A~~	107	1	2	0	0	0		
C_B	Basis	x_1	x_2	x_3	x_4	S_1	S_2	b	
0	~~S_1~~ x_4	14/3	1/3	~~-6~~ -2	~~3~~ 1	0	0	7/3	
0	~~S_2~~ S_1	16	1	-6	0	1	0	5	
0	~~S_3~~ S_2	[3]	-1	-1	0	0	1	0	$\leftarrow$
	Z_j	0	0	0	0	0	0	0	
	C_j	107	1	2	0	0	0		
		$\uparrow$							
0	x_4	0	17/9	-4/9	1	0	-14/9	7/3	
0	S_1	0	19/3	-2/3	0	1	16/3	5	
107	x_1	1	-1/3	-1/3	0	0	1/3	0	
	Z_j	107	-107/3	-107/3	0	0	107/3	0	
	C_j	0	(-)	(-)	0	0	(-)		

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→ ## Artificial Variable Technique

- to solve LPP of $\geq$ & $=$ type where slack variable fails to give soln.

i) Big M Method / Method of penalty
ii) Two-phase Method

Big M Method

Step 1: Express LPP in standard form
Step 2: Introduce an artificial variable ($A_1, A_2, \dots A_n$) for those constraints of $\geq$ or $=$ type, to obtain initial basic feasible soln.
Subtract a large quantity of A (MA) from the objective function as penalty.

Step 3: solve modified LPP by simplex method. One of the following cases may arrive:

i) No artificial variable in basis: if all $C_j \leq 0$, current soln. is optimal.

ii) atleast one variable in basis at zero level ($x_B^b = 0$) and optimality condition ($C_j \leq 0$) is satisfied, then current soln is degenerate soln.

iii) atleast one artificial variable in basis at non-zero level and optimality condition is satisfied, then no feasible soln.
Since objective function contains non-zero large value of M & doesn't give an optimal soln, it is $\therefore$ called pseudo optimal soln.

Step 4: Repeat

★ artificial variables are only a computational device for getting a starting soln. Once artificial variable leaves the basis, the column for this variable can be omitted from the next simplex table

Q. Min $Z = 2x_1 + x_2$

s. to

$$3x_1 + x_2 = 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Max $z' = -2x_1 - x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$

S. to

$$3x_1 + x_2 + A_1 = 3$$

$$4x_1 + 3x_2 - S_1 + A_2 = 6$$

$$x_1 + 2x_2 + S_2 = 3$$

$$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$$

	c_j	-2	-1	0	0	$-M$	$-M$		
C_B	Basis	x_1	x_2	S_1	S_2	A_1	A_2	b	θ
$-M$	A_1	[3]	1	0	0	1	0	3	1 ←
$-M$	A_2	4	3	-1	0	0	1	6	1.5
0	S_2	1	2	0	1	0	0	3	3
	z_j	$-7M$	$-4M$	M	0	$-M$	$-M$	$-9M$	
	C_j	$7M-2$ ↑	$4M-1$	$-M$	0	0	0		

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$R_2 \to R_2 - 4R_1$
$R_3 \to R_3 - R_1$

C_B	Basis	x_1	x_2	S_1	S_2	A_1	A_2	b
-2	x_1	1	$1/3$	0	0	—	0	1
$-M$	A_2	0	$[5/3]$	-1	0	—	1	2 ←
0	S_2	0	$5/3$	0	1	—	0	2
	Z_j	-2	$-\frac{2-5M}{3}$	M	0	—	$-M$	$-2-2M$
	C_j	0	$\left(\frac{5M}{3}-\frac{1}{3}\right)$ ↑	$-M$	0	—	0	

$R_1 \to R_1 - \frac{1}{3}R_2$
$R_3 \to R_3 - \frac{5}{3}R_2$

-2	x_1	1	0	$1/5$	0	—	—	$3/5$
-1	x_2	0	$[1]$	$-3/5$	0	—	—	$6/5$
0	S_2	0	0	1	1	—	—	0
	Z_j	-2	-1	$1/5$	0	—	—	$-12/5$
	C_j	0	0	$-1/5$	0	—	—	

no artificial variable in basis, current soln. is optimal.

$x_1 = \frac{3}{5}$; $x_2 = \frac{6}{5}$; $S_1 = 0$; $S_2 = 0$,

$Z'_{max} = -\frac{12}{5} \Rightarrow Z_{min} = \frac{12}{5}$

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Q. Solve the following LPP using Big M Method

Max $z = 3x_1 + 2x_2$

s. to

$2x_1 + x_2 \le 2$

$3x_1 + 4x_2 \ge 12$

$x_1, x_2 \ge 0$

Max $Z = 3x_1 + 2x_2 + 0S_1 + 0S_2 - MA_1$

S. to

$2x_1 + x_2 + S_1 = 2$

$3x_1 + 4x_2 - S_2 + A_1 = 12$

$x_1, x_2 \ge 0$

		3	2	0	0	$-M$		
C_b	Basis	x_1	x_2	S_1	S_2	A_1	b	θ
0	S_1	2	[1]	1	0	0	2	1 ←
$-M$	A	3	4	0	-1	1	12	4.
	Z_j	$-3M$	$-4M$	0	M	$-M$	$-12M$	4
	C_j	$3+3M$	$2+4M$ ↑	0	$-M$	0		

2	x_2	2	1	1	0	0	2
$-M$	A	-5	0	-4	-1	1	4
	Z_j	$4+5M$	2	$2+4M$	M	$-M$	$4-4M$
	C_j	$-1-5M$	0	$-2-4M$	$-M$	0	

Since A is present in basis with non zero value, there is no feasible soln. using this method.

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Q. Max. $Z = 3x_1 + 2x_2 + 3x_3$

s. to

$2x_1 + x_2 + x_3 \le 2$

$3x_1 + 4x_2 + 2x_3 \ge 8$

$x_1, x_2, x_3 \ge 0$

Max. $Z = 3x_1 + 2x_2 + 3x_3 + 0S_1 + 0S_2 - MA$

s. to

$2x_1 + x_2 + x_3 + S_1 = 2$

$3x_1 + 4x_2 + 2x_3 - S_2 + A_1 = 8$

$x_1, x_2, x_3, S_1, S_2, A \ge 0$

		3	2	3	0	0	-M		
C_B	Basis	x_1	x_2	x_3	S_1	S_2	A	b	θ
		~~3~~	~~2~~	~~3~~	~~0~~	~~0~~	~~-M~~		
0	S_1	2	[1]	1	1	0	0	2	2 ←
-M	A	3	4	2	0	-1	1	8	2
	Z_j	-3M	-4M	-2M	0	M	-M	-8M	
	C_j	3+3M	2+4M ↑	3+2M	0	-M	0		

$R_2 \to R_2 - 4R_1$

									θ
2	x_2	2	1	1	1	0	0	2	2
-M	A	-5	0	[-2]	-4	-1	1	0	0
	Z_j	4+5M	2	2+2M	2+4M	M	-M	4	
	C_j	-1-5M	0	1-2M	(-)	(-)	0		

~~$R_1 \to R_1 - R_2$~~

2	~~x_2~~	~~-1/2~~	~~1~~	~~0~~	~~-1~~	~~-1/2~~	~~1/2~~	~~2~~
3	~~x_3~~	~~5/2~~	~~0~~	~~1~~	~~2~~	~~1/2~~	~~-1/2~~	~~0~~
	~~Z_j~~	~~13/2~~	~~2~~	~~3~~	~~4~~	~~1/2~~	~~-1/2~~	~~4~~
	~~C_j~~	~~(-)~~	~~0~~	~~0~~	~~(-)~~	~~(-)~~	~~(-)~~	

solution is optimal

$x_1 = 0$

$x_2 = 2$ Max $Z = 4$

$x_3 = 0$

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Q. Min. $Z = 4x_1 + 3x_2 + x_3$

s. to

$$x_1 + 2x_2 + 4x_3 \geq 12$$
$$3x_1 + 2x_1 + x_3 \geq 8$$
$$x_1, x_2, x_3 \geq 0$$

Max $Z' = -4x_1 - 3x_2 - x_3 + 0S_1 + 0S_2 - MA_1 - MA_2$

S. to

$$x_1 + 2x_2 + 4x_3 - S_1 + A_1 = 12$$
$$3x_1 + 2x_2 + x_3 - S_2 + A_2 = 8$$
$$x_1, x_2, x_3, S_1, S_2, A_1, A_2 \geq 0$$

		-4	-3	-1	0	0	-M	-M		
C_B	Basis	x_1	x_2	x_3	S_1	S_2	A_1	A_2	b	θ
-M	A_1	1	2	[4]	-1	0	1	0	12	3 ←
-M	A_2	3	2	1	0	-1	0	1	8	8
	Z_j	-4M	-4M	-5M	M	M	-M	-M	-20M	
	C_j	4M-4	4M-3	5M-1 ↑	(-)	(-)	0	0		

$R_2 \to R_2 - R_1$

										θ
-1	x_3	1/4	1/2	1	-1/4	0	1/4	0	3	12
-M	A_2	[11/4]	3/2	0	1/4	-1	-1/4	1	5	20/11 ←
	Z_j	$\frac{-1}{4} - \frac{11M}{4}$	$\frac{-1-3M}{2}$	-1	$\frac{1-M}{4}$	M	$\frac{-1+M}{4}$	-M	-3-5M	
	C_j	$\frac{11M}{4} - \frac{15}{4}$ ↑	$\frac{3M}{2} - \frac{5}{2}$	0	$\frac{M}{4} - \frac{1}{4}$	(-)	(-)	0		

$R_1 \to R_1 - \frac{R_2}{4}$

		x_1	x_2	x_3	S_1	S_2	A_1	A_2	b	θ
-1	x_3	0	4/11	1	-3/11	1/11	3/11	-1/11	28/11	
-4	x_1	1	6/11	0	[1/11]	-4/11	-1/11	4/11	20/11	←
	Z_j	-4	-28/11	-1	-1/11	15/11	1/11	-15/11	-108/11	
	C_j	0	(-)	0	1/11 ↑	(-)	(-)	(-)		

~~soln is optimal.~~

~~$x_1 = \frac{20}{11}$, $x_2 = 0$, $x_3 = \frac{28}{11}$~~

~~$\text{Max } Z' = \frac{-108}{11}$~~

~~$\therefore \text{Min } Z = \frac{108}{11}$~~

		-4	-3	-1	0	0			
		x_1	x_2	x_3	S_1	S_2	A_1	A_2	b
-1	x_3	3	2	1	0	-1	0	1	8
0	S_1	11	6	0	[1]	-4	-1	4	20
	z_j	-3	-2	-1	0	1	-	-	-8
	C_j	(-)	(-)	0	0	(-)	-	-	

Optimal

$x_1 = x_2 = 0$

$x_3 = 8$

$\text{Max } Z' = -8 \Rightarrow \text{Min } Z = 8$