

Module - 3

Sets

$$A = \{2, 4, 6, 8, 10, 12, \dots\}$$

$$B = \{1, 3, 5, 7, 9, 11, \dots\}$$

1. Union Set

$$A = \{1, 2, 3\}$$

$$B = \{4, 5, 6\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

The union of two sets A & B is the set of all those elements x such that x belongs to at least one of the two sets A & B. Denoted $A \cup B$.

2. Intersection

$$A = \{1, 2, 3, 4\}$$

$$B = \{1, 2\}$$

$$A \cap B = \{1, 2\}$$

The intersection of two sets A & B is the set of all those elements x such that x belongs to both A & B. Denoted by $A \cap B$.

Q. Prove $A \cup B = B \cup A$

$A \cup B$

x be an any element of $A \cup B$.

$x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in B \cup A$.

finite and infinite Set :-

A set with finite number of elements in it, is called a finite set.

- ex - @) The set vowels in english alphabets.
 b) The set of students in a class.

An infinite set is a set which contains infinite number of elements is called infinite set

- ex- @ A = a set of Integers
= {0, 1, 2, ..., }

Null Set

A set who is contains no elements at all is called Null Set. (also known as empty set or Void set). It is denoted by the symbol \emptyset .

~~#~~ Singleton Set

A set which has only one element is called singleton set.

$C_1 = S = \{a\}$ is a Singleton Set.

operations of sets

$$\begin{array}{l} \textcircled{1} \\ \textcircled{2} \end{array} \quad A \cup B = B \cup A \quad \begin{array}{l} \text{from diagram} \\ \text{commutative law} \end{array}$$

exam

\rightarrow Prove $A \cup B = B \cup A$

$$\begin{aligned} &= x \in A \cup B \Rightarrow x \in B \cup A \Rightarrow x \in B \text{ or } x \in A \\ &= x \in A \text{ or } x \in B \Rightarrow x \in B \text{ or } x \in A \\ &= x \in B \cup A \Rightarrow x \in A \cup B \end{aligned}$$

$$A \cup B \subseteq B \cup A \quad \text{or} \quad A \cup B = B \cup A$$

(2) Prove $A \cap B = B \cap A$

$$\begin{aligned} &= x \in A \cap B \\ &= x \in A \text{ and } x \in B \end{aligned}$$

E

Ex 2 Multi-set

$$A \cap B \subseteq B \cap A$$

$$\begin{aligned} \text{Ans} \quad &x \in B \cap A \Rightarrow x \in B \text{ and } x \in A \\ &\Rightarrow x \in A \text{ and } x \in B \\ &\Rightarrow x \in A \cap B \end{aligned}$$

$$B \cap A \subseteq A \cap B \rightarrow \textcircled{2}$$

from (1) and (2), we get

$$A \cap B = B \cap A$$

Proved

ex :-

$$(2) A \cup (B \cup C) = (A \cup B) \cup (A \cup C) \quad \rightarrow \text{Distributive law}$$

$$(3) A \cap (B \cap C) = (A \cap B) \cap (A \cap C) \quad \rightarrow \text{Distributive law}$$

(1) $A \cup (B \cup C) = (A \cup B) \cup (A \cup C)$

$$\begin{aligned} &= x \in A \cup (B \cup C) \\ &\Rightarrow x \in A \cup B \text{ or } x \in A \cup C \\ &\Rightarrow x \in A \text{ or } (x \in B \text{ or } x \in C) \\ &= (x \in A \text{ or } B) \text{ or } (x \in A \text{ or } C) \end{aligned}$$

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Subset :-

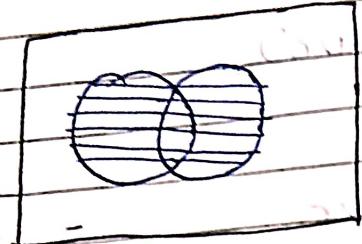
If A and B are sets such that every element of A is also an element of B; then A is said to be a subset of B (or it is contained in B); and is denoted by $A \subseteq B$.

ex:- The set A = {1, 2, 3, 4, 5} is a subset of B = {1, 2, 3, 4, 5, 6}.

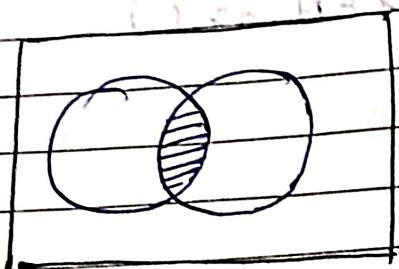
- Every set A is a subset itself.

Venn-Diagram

1. AUB



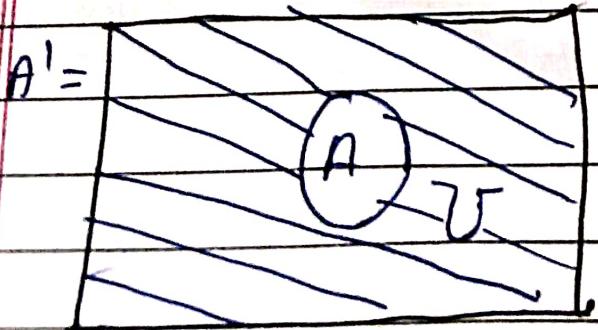
2. ANB



Complements: universal set and absolute complement

Let U be the universal set and A be any subset of U . The absolute complement of A or simply, complement of A , denoted by A' or A^c is the set of elements which belong to U but which do not belong to A .

$$A' = \{x : x \in U \text{ and } x \notin A\}$$



Cartesian Product:-

Let A and B be sets. Cartesian Product of A and B , denoted by $A \times B$ is defined as

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

If $A = \{a, b\}$ and $B = \{1\}$, Then
 $A \times B = \{(a, 1), (b, 1)\}$

$$B = \{1\}$$

$$A = \{a, b\}$$

$$B \times A = \{(1, a), (1, b)\}$$

exam

Q1. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let (x, y) be any element of $A \times (B \cap C)$.

$$\begin{aligned} &\text{Then } (x, y) \in A \times (B \cap C) \Rightarrow x \in A \text{ and } y \in (B \cap C) \\ &\Rightarrow x \in A \text{ and } y \in B \cap C \Rightarrow x \in A \text{ and } (y \in B \text{ and } y \in C) \\ &\Rightarrow x \in A \text{ and } (y \in B \text{ and } y \in C) \Rightarrow y \in B \cap C \\ &\Rightarrow (x \in A \text{ and } y \in B) \text{ and } (x \in A \text{ and } y \in C) \\ &\Rightarrow (x, y) \in A \times B \text{ and } (x, y) \in A \times C \\ &\Rightarrow (x, y) \in (A \times B) \cap (A \times C) \end{aligned}$$

$$A \times (B \cap C) \subseteq (A \times B) \cap (A \times C) \quad \text{--- (1)}$$

again $\cdot (x, y) \in (A \times B) \cap (A \times C)$

$$\begin{aligned} &\Rightarrow (x, y) \in A \times B \text{ and } (x, y) \in A \times C \\ &\Rightarrow (x \in A, y \in B) \text{ and } (x \in A, y \in C) \\ &\Rightarrow x \in A \text{ and } (y \in B \text{ and } y \in C) \\ &\Rightarrow x \in A \text{ and } y \in B \cap C \\ &\Rightarrow (x, y) \in A \times (B \cap C) \end{aligned}$$

$$\therefore (A \times B) \cap (A \times C) \subseteq A \times (B \cap C) - \textcircled{2}$$

From \textcircled{1} & \textcircled{2}, we get that
 $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Q.

$$A \cup B$$

$$A = \{(1, 2, 3, 4, 5, 6)\} : (d, o) \in A \times A$$

$$B = \{3, 6, 8, 12, 17, 18\}$$

$$\Rightarrow A \cup B = \{1, 2, 3, 4, 5, 6, 8, 12, 17, 18\} : (d, o) \in A \times A$$

$$\Rightarrow A \cap B = \{3, 6\}$$

$$\{d, o\} \in A$$

$$\{(3, 6), (6, 12)\} \in A \times A$$

$$(A \times A) \cap (B \times A) = (A \cap B) \times A$$

$$(A \cap B) \times A = (A \cap B) \times \{x \mid x \in A\}$$

$$(A \cap B) \times \{x \mid x \in A\} = (A \cap B) \times A$$

$$(A \cap B) \times A = A \cap (B \times A)$$