Laws of Form Reference

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This is an equation reference for other *Laws of Form* articles. For axioms and consequences, we use Spencer-Brown's original labels. For most other equations, we use variations on the following transformations (where we simplify every $\overline{\overline{x}\rceil}\rceil$ to x):

1. Let the *complement* of a form p be the crossed form $\overline{p}\rceil$. Then the complement of an equation E, denoted by En, equates the complements of the LHS and RHS of E. For example, if E is $p = q$, then En is $\overline{p}\rceil = \overline{q}\rceil$. Since $Enn = E$, $E \leftrightarrow En$.

2. The *contradual* of a form results from crossing every variable. For example, the contradual of $\overline{p\,\overline{q}\rceil}\rceil$ is $\overline{\overline{p}\rceil\, q}\rceil$. The contradual of an equation E, denoted by Ec, equates the contraduals of the LHS and RHS of the equation. Since we are effectively substituting the same value for each instance of a given variable, $E \leftrightarrow Ec$.

3. The *dual* of a form results from crossing every variable *and* the entire form (i.e., a combination of contradual and complement). The dual of an equation E is denoted by Ed. As above, $E \leftrightarrow Ed$.

Remark. All three transformations are involutions: whether $\theta = n, c$, or d, $E\theta\theta = E$. Together with the identity transformation, these form a Klein 4-group. Among the transformations, duals are particularly significant — interpreted logically, Ed means the same as E with the truth values reversed. Occasionaly, the same equation can be the result of more than one transformation. In such cases, we make an arbitrary choice for the label.

We may also resort to *selective* duals and contraduals, where not every variable is crossed. An equation will typically have multiple selective [contra]duals.

Axioms

$$\overline{\overline{a|}\,a|} = \tag{Position J1}$$

$$\overline{\overline{a\,c|}\;\overline{b\,c|}|} = \overline{\overline{a|}\;\overline{b|}|}\,c \tag{Transposition J2}$$

Consequences

$$\overline{\overline{a|}|} = a \tag{Reflexion C1}$$

$$\overline{a\,b|}\,b = \overline{a|}\,b \tag{Generation C2}$$

$$\overline{\ |}\,a = \overline{\ |} \tag{Integration C3}$$

$$\overline{\overline{a|}\,b|}\,a = a \tag{Occultation C4}$$

$$a\,a = a \tag{Iteration C5}$$

$$\overline{\overline{a|}\;\overline{b|}|}\;\overline{\overline{a|}\,b|} = a \tag{Extension C6}$$

$$\overline{\overline{\overline{a|}\,b|}\,c|} = \overline{a\,c|}\;\overline{\overline{b|}\,c|} \tag{Echelon C7}$$

$$\overline{\overline{a|}\;\overline{b\,r|}\;\overline{c\,r|}|} = \overline{\overline{a|}\;\overline{b|}\;\overline{c|}|}\;\overline{\overline{a|}\;\overline{r|}|} \tag{Modified transposition C8}$$

$$\overline{\overline{\overline{a|}\;\overline{r|}|}\;\overline{\overline{b|}\;\overline{r|}|}\;\overline{\overline{x|}\,r|}\;\overline{\overline{y|}\,r|}|} = \overline{\overline{r|}\,a\,b|}\;\overline{r\,x\,y|} \tag{Crosstransposition C9}$$

Corollaries

$$\overline{a|}\,a = \overline{\ |} \tag{J1n}$$

$$\overline{\overline{a|}\,a\,b|} = \tag{J1.1}$$

$$\overline{a\,c|}\;\overline{b\,c|} = \overline{\overline{\overline{a|}\;\overline{b|}|}\,c|} \tag{J2n}$$

$$\overline{\overline{a|}\,c|}\;\overline{\overline{b|}\,c|} = \overline{\overline{a\,b|}\,c|} \tag{Combination K5}$$

$$\overline{\overline{a|}\;\overline{b|}|}\;\overline{\overline{c|}\;\overline{d|}|} = \overline{\overline{a\,c|}\;\overline{a\,d|}\;\overline{b\,c|}\;\overline{b\,d|}|} \tag{Distribution K9}$$

$$\overline{\overline{\ |}|}\,a = a \tag{Meguire B2}$$

$$\overline{\overline{\ |}\,a|} = \tag{C3n}$$

$$\overline{a\,b|}\;\overline{a|} = \overline{a|} \tag{C4c}$$

$$\overline{\overline{a|}\,b|}\;\overline{a\,b|} = \overline{b|} \tag{C6c}$$

$$\overline{\overline{\overline{a|}\,b|}\;\overline{a\,b|}|} = b \tag{Robbins C6d}$$

$$\overline{\overline{a|}\;\overline{b\,r|}|} = \overline{\overline{a|}\;\overline{b|}|}\;\overline{\overline{a|}\;\overline{r|}|} \tag{C8.1}$$

$$\overline{\overline{\overline{a|}\;\overline{r|}|}\;\overline{\overline{x|}\;\overline{r|}|}|} = \overline{\overline{r|}\,a|}\;\overline{r\,x|} \tag{C9.1}$$

Metatheorems

$$\overline{\overline{a_1 r\rvert}\ \overline{a_2 r\rvert}\ \dots\ \overline{a_n r\rvert}\rvert} = \overline{\overline{a_1\rvert}\ \overline{a_2\rvert}\ \dots\ \overline{a_n\rvert}\rvert}\, r \tag{J2*}$$

$$\overline{a_1 r\rvert}\ \overline{a_2 r\rvert}\ \dots\ \overline{a_n r\rvert} = \overline{\overline{\overline{a_1\rvert}\ \overline{a_2\rvert}\ \dots\ \overline{a_n\rvert}\rvert}\, r\rvert} \tag{J2n*}$$

$$\overline{\overline{\overline{\overline{a_n b\rvert}\ \dots\rvert}\ a_2\rvert}\ a_1\rvert}\ b = \overline{\overline{\overline{\overline{a_n\rvert}\ \dots\rvert}\ a_2\rvert}\ a_1\rvert}\ b \tag{C2*}$$

$$\overline{\overline{a\rvert}\ \overline{b_1 r\rvert}\ \overline{b_2 r\rvert}\ \dots\ \overline{b_n r\rvert}\rvert} = \overline{\overline{a\rvert}\ \overline{b_1\rvert}\ \overline{b_2\rvert}\ \dots\ \overline{b_n\rvert}\rvert}\ \overline{\overline{a\rvert}\ \overline{r\rvert}\rvert} \tag{C8*}$$

$$\begin{aligned}&\overline{\overline{\overline{a_1\rvert}\ \overline{r\rvert}\rvert}\ \overline{\overline{a_2\rvert}\ \overline{r\rvert}\rvert}\ \dots\ \overline{\overline{a_n\rvert}\ \overline{r\rvert}\rvert}\ \ \overline{\overline{x_1\rvert}\, r\rvert}\ \overline{\overline{x_2\rvert}\, r\rvert}\ \dots\ \overline{\overline{x_m\rvert}\, r\rvert}\rvert}\\ &= \overline{\overline{r\rvert}\, a_1 a_2 \dots\ a_n\rvert}\ \overline{r\, x_1 x_2 \dots\ x_m\rvert}\end{aligned} \tag{C9*}$$

For all even $n \geq 2$:

$$\overline{\overline{\overline{\overline{a_n\rvert}\ \dots\rvert}\ a_2\rvert}\ a_1\rvert} = \overline{\overline{a_n\rvert}\, a_{n-1} \dots\ a_3 a_1\rvert}\ \dots\ \overline{\overline{a_4\rvert}\ a_3 a_1\rvert}\ \overline{\overline{a_2\rvert}\ a_1\rvert} \tag{C7.1*}$$

and

$$\begin{aligned}&\overline{\overline{\overline{\overline{\overline{a_{n+1}\rvert}\ a_n\rvert}\ \dots\rvert}\ a_2\rvert}\ a_1\rvert}\\ &= \overline{a_{n+1} a_{n-1} \dots\ a_3 a_1\rvert}\ \overline{\overline{a_n\rvert}\, a_{n-1} \dots\ a_3 a_1\rvert}\ \dots\ \overline{\overline{a_4\rvert}\ a_3 a_1\rvert}\ \overline{\overline{a_2\rvert}\ a_1\rvert}\end{aligned} \tag{C7.2*}$$