Mathematics for Intelligent Systems – 3
Practice Sheet - 3
Contour plots, Gradient, Hessian

- **Contour curves/ level curves**
  x = linspace(-2*pi,2*pi);
  y = linspace(0,4*pi);
  [X,Y] = meshgrid(x,y);
  Z = sin(X)+cos(Y);
  contour(X,Y,Z)   % contour curves for the function f(x,y)=sinx+cosy

- **Contour curves/ level curves with value of the function written on the curves**
  x = linspace(-2*pi,2*pi);
  y = linspace(0,4*pi);
  [X,Y] = meshgrid(x,y);
  Z = sin(X)+cos(Y);
  contour(X,Y,Z,'ShowText','on')

  ➢ contour(x,y,z,50) can be used to get fixed number of contour curves/ level curves
  ➢ contour(X,Y,Z,'--') can be used to get contour curves in -- style

- **Surface plots with contour curves**
  x = linspace(-2*pi,2*pi);
  y = linspace(0,4*pi);
  [X,Y] = meshgrid(x,y);
  Z = sin(X)+cos(Y);
  surfc(X,Y,Z); shading interp

- **Gradient using MATLAB**
  syms x y z; gradient(x^2 + 3*y^2-5*z, [x, y, z])

- **Plot of surface, level curves and gradient vectors of a scalar function f at different points on the level curves**
  [x,y] = meshgrid(-2:.2:2,-2:.2:2);
  z = x.^2 + y.^2;
  figure
  surf(x,y,z);
  figure
  [px,py] = gradient(z);
  contour(x,y,z,'ShowText','on'), hold on
  quiver(x,y,px,py), hold off, axis image
  % Here surface plot will appear in one figure window and the gradient vectors in another

- **Hessian matrix using MATLAB**
  syms x y z; hessian(x*y + 2*z*x, [x, y, z])

Practice questions
1. Consider the function: $f(x,y) = 1/(|x| + |y| + 0.5)$. Plot 15 level curves for the function $f(x,y)$ in the rectangular domain $-5 \leq x, y \leq 5$.
2. Consider the function: $f(x,y) = 4x^2 + 9y^2 - 72$.
   (a) Plot the level curves for $f(x,y)$ with function values written in each curve in $-10 \leq x, y \leq 10$

(b) Find the gradient and hessian of the function using 'syms'.

(c) Plot the gradient vectors at some of the points on the level curves.

3. Consider the function $f(x,y) = \sin x + \cos y + 3e^{-x^2-y^2}$.

(a) Show the surface plot of the function in one figure and the gradient vectors at some points on level curves in another figure.

(b) Looking at the surface plot and from the figure with function values on level curves, try to find a minimum and a maximum for the function.

4. Consider the function: $f(x,y) = xe^{-(x^2+y^2)}$

a) Plot the surface given for x and y varying from -2 to 2

b) Write the General expression for gradient and Hessian for the function

c) Evaluate Hessian at the points given in the following figures

d) Find eigenvalues of those hessian matrices and decide the nature of curvature at the points where it is evaluated. (curved upward, downward, neither)