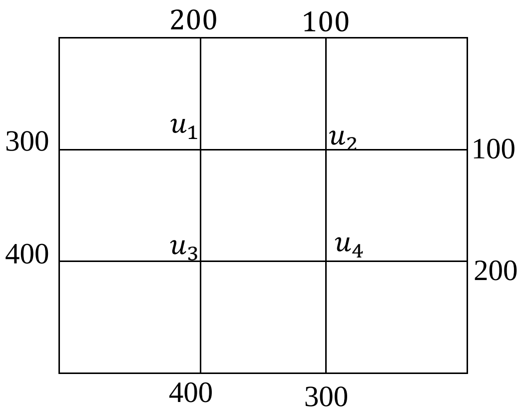
23MAT204 – Mathematics for Computing - 3

Practise Sheet-12

(Numerical Solutions to Partial Differential Equations)

1. Solve $u_{xx} + u_{yy} = 0$, for the following square mesh numerically. The values of u along the boundary are given in the figure. Carry out iterations with excel until you get solutions correct up to 5 decimal places. Assume the initial values as:

   $u_1^{(0)} = 200, u_2^{(0)} = 100, u_3^{(0)} = 300,\ u_4^{(0)} = 200.$



2. Solve $u_{xx} + u_{yy} = 0, 0 \le x \le 1, 0 \le y \le 1,$ given that $u(0, y) = 0, u(x, 0) = 0,\ u(1, y) = 100$ and $u(x, 1) = 100$. Choose the mesh length as 0.25 .
3. Consider the partial differential equation: $u_{xx} + u_{yy} = 0$ with boundary conditions,

   $$U(0, y) = 0, y = 0,1,2,3,4; u(4, y) = 2y + 8, y = 0,1,2,3,4; u(0,0) = 0.5;\ u(1,0) = 0.5;\ u(2,0) = 2; u(3,0) = 4.5; u(4,0) = 8; u(x, 4) = x^2, x = 0,1,2,3,4;$$

   Construct a square mesh with h=k=1 and write the iterative equations that would be obtained while solving the given partial differential equation and thus find the values for u(x,y) for x=1,2,3 and y=1,2,3.
4. Consider the partial differential equation:

   $u_{xx} + u_{yy} = 0$ with boundary conditions,

   $$u(0, y) = 0, 0 \le y \le 4; u(4, y) = 12 + y, 0 \le y \le 4;$$
   $$u(x, 0) = 3x, 0 \le x \le 4; u(x, 4) = x^2, 0 \le x \le 4$$

(a) Construct a square mesh with h=1 and write the iterative equations that would be obtained while solving the given partial differential equation.

(b) Taking the initial approximation as zero for the u values at all the mesh points, find the solution (up to 6 decimal places) at all mesh points.

(c) Under the same boundary conditions, solve the Poisson equation, $u_{xx} + u_{yy} = 5 - 36xy$ correct up to 6 decimal places, assuming the initial values to be zero.

5. Solve $u_{xx} + u_{yy} = -81xy, 0 \le x \le 1, 0 \le y \le 1$, given that $u(0, y) = 0, u(x, 0) = 0,\ u(1, y) = 100$ and $u(x, 1) = 100$. Choose the mesh length as $\frac{1}{3}$.
6. Solve $u_{xx} + u_{yy} = -5(x^2 + y^2 + 3)$ over the square mesh with sides x=0,y=0,x=3 and y=3 under the boundary conditions u=0 along all the four boundaries and mesh length one. Choose initial values as zero.
7. Solve $u_{xx} + u_{yy} = 3 - 5xy$ over the square mesh with sides x=0,y=0,x=3 and y=3 under the boundary conditions u=0 along all the four boundaries and mesh length one. Choose the mesh length as one.
8. Solve $u_{xx} + u_{yy} = 10 - xy$ over the square mesh with sides x=0,y=0,x=2.5 and y=2.5 under the boundary conditions u=0 along all the four boundaries. Choose the mesh length as 0.5.

9. Solve $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial t} = 0$, given u(0,t)=0, u(4,t)=0, u(x,0)=x(4-x), taking h=k=1. Also find the values of u up to t=5.
10. Solve $u_t = u_{xx}$ under the conditions, u(0,t)=u(5,t)=0, u(x,0)=x(25-x^2) choosing h=1, k=0.5. Find u(x,t) up to t=2.
11. Solve $u_t = u_{xx}$ under the conditions, u(0,t)=u(5,t)=0, u(x,0)=x^2(25-x^2) choosing h=1, k=0.5. Find u(x,t) up to t=2.5.
12. Solve $u_t = u_{xx}$ under the conditions, u(0,t)=u(1,t)=0, $u(x,0) = \sin(\pi x)$, $0 \le x \le 1$ choosing h=0.2, k=0.02.
13. Solve $u_{xx} = 16u_t$, $0 < x < 1$, $t > 0$, given u(x,0)=0, u(0,t)=0, u(1,t)=100t.

(a) Compute u for one step in t direction taking Δx=0.25 and Δt=0.25

(b) Compute u in t direction choosing Δx=0.25 and Δt=0.05.

14. (a) Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$, taking h = 1, k=0.25 up to t = 1.25. The boundary conditions are u(0,t) = u(5,t) = 0, u_t(x,o) = 0 and u(x,0) = x^2(5-x).

    (b) For the same PDE in 14(a) with same boundary conditions, choose h=1 and k=0.05 and find u(x,t) up to t=1.25 using excel. Compare the values you get in 14(a).
15. Solve $25u_{xx} = u_{tt}$ at the pivotal points if $u(0,t) = u(5,t) = 0$; $u(x,0) = \begin{Bmatrix} 20x, 0 \le x \le 1 \\ 5(5-x), 1 \le x \le 5 \end{Bmatrix}$ and $u_t(x,0) = 0$. Compute $u(x,t)\ for\ 0 \le t \le 1$ choosing k=0.2 and h=1.

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