23MAT204
Class Notes

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Chapter 1

Revision Linear Algebra

Theorem 1.0.1. Every matrix satisfies its own characteristic matrix.

What this means is

$$\lambda^3 - 2\lambda^2 + \lambda - 4 = 0$$
$$A^3 - 2A^2 + A - 4I = 0$$
$$A^2 - 2A + I - 4A^{-1} = 0$$
$$A^{-1} = \frac{1}{4}[A^2 - 2A + I]X \qquad = A^{-1}b = \frac{1}{4}[A^2 - 2A + I]b$$

There are some implications behind this.

Here's a problem, generate a random 3x3 matrix, find the ranks of $(A - \lambda_1 I)$ and then $(A - \lambda_1 I)(A - \lambda_2 I$

What you will see is that the rank reduces upon multiplying roots of the characteristic equation

1.1 Ways To Calculate Whether A Matrix Is Positive Definite

1. The minors are positive then negative alternating.
2. The eigenvalues are positive.

1.2 Large Eigenvalue Computation

Large eigenvalues are computed by using a matrix multiplication method. This method involves:

1.3 Specral Decomposition

$$S = Q\Lambda Q^T = \lambda_1 q_1 q_1^T + \lambda_2 q_2 q_2^T + \cdots + \lambda_n q_n q_n^T$$

This allows us to represent a matrix by the sum of n rank 1 matrices. This is used in square symmetric matrices.

This is the basis behind Principal Component Analysis.

1.4 Singular Value Decomposition

For rectangular matrices.

$$A = U\Sigma V^T = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \cdots + \sigma_r u_r v_r^T$$

This is mainly used for getting useful properties about the matrix without needing to perform significant operations on this matrix, such as the orthogonality.

Note:-

The trace of A^TA equal to the sum of all a_{ij}^2 The trace is the sum of all eigenvalues of A^TA, and For $A_{m\times n}$ that's the sum of the eigenvalues squared.
This is known as the **Frobenius Norm**