23AID203-Software-defined Communication System Unit-2

Sirisha Tadepalli

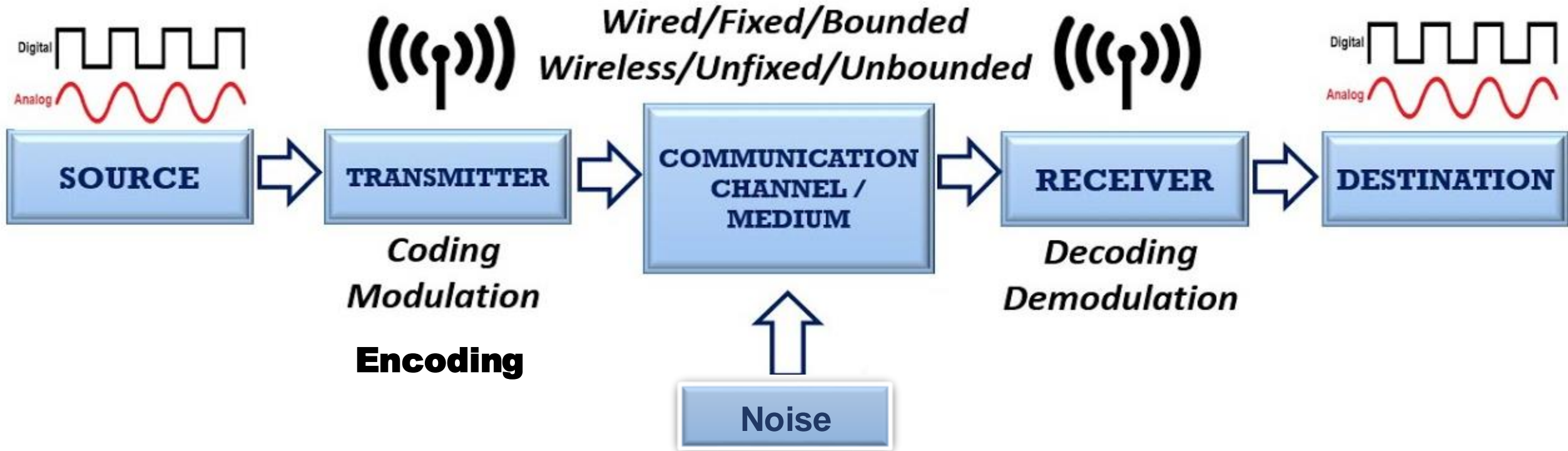# How is the information exchanged between two points?

Fig. Basic block diagram of Communication system [5]

Modulation- (Tx) alters a high-frequency carrier signal's properties with a lower-frequency information signal, enabling long-distance transmission through various media and channels.

Necessity of Modulation

- To reduce the size of the antenna

$$L \propto \lambda \ and \ \lambda = \frac{c}{f}$$

- To reduce the interference

- To improve SNR

SNR= power of signal/power of noise signal

Ex: Without modulation 1mW/0.1mW =10

With modulation 1mW/0.01mW= 100
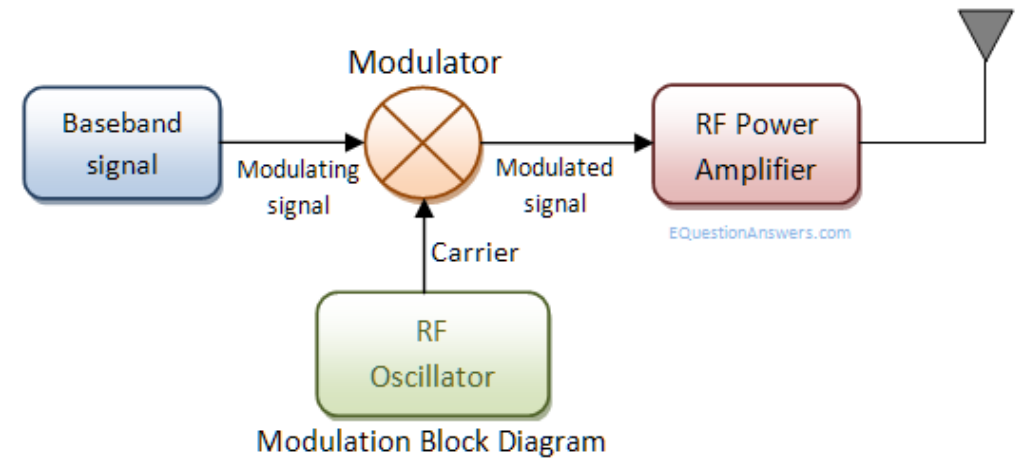
- To allow multiplexing of the signals

- Optimizes bandwidth utilization
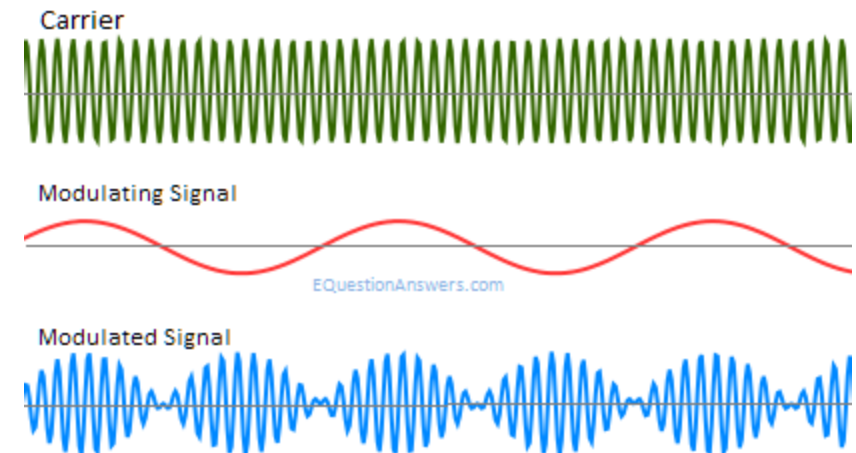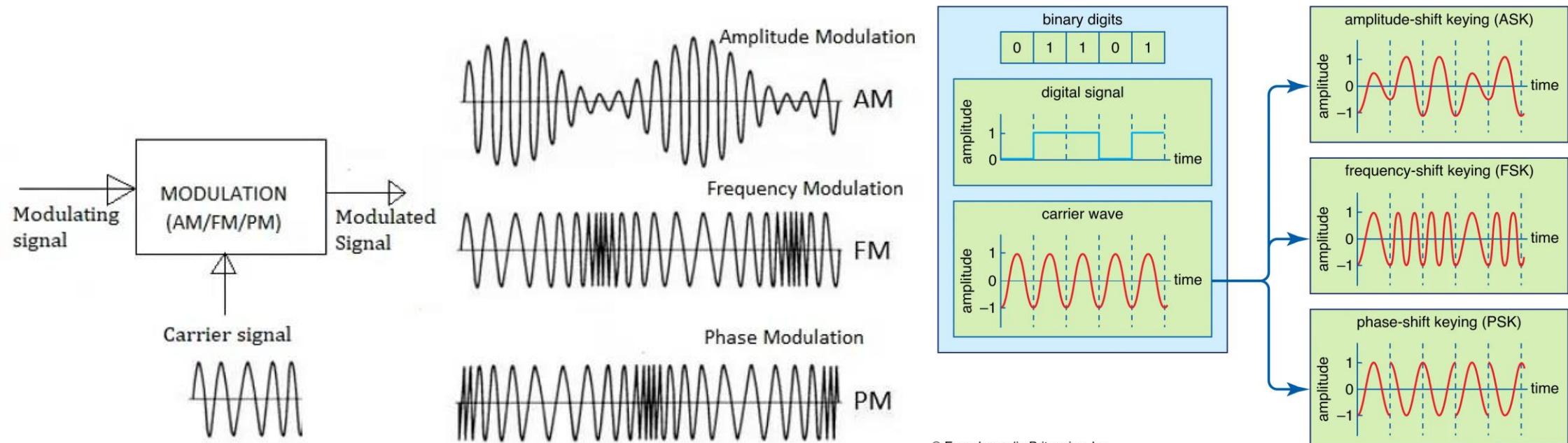

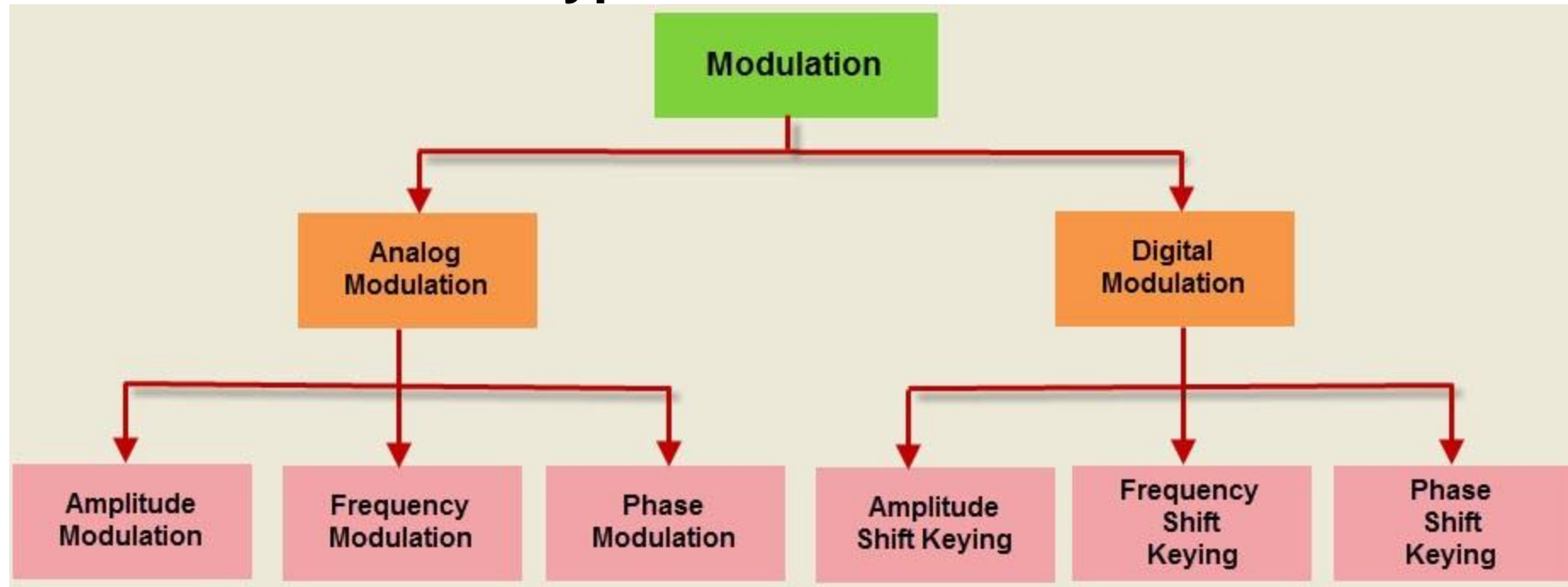Fig. 4 Basic block diagram of modulation [6]

Fig. 5 Amplitude modulation [6]

Types of Modulation

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Analog Modulation (AM)

- AM, the information signal varies the amplitude of the carrier sine wave
- The instantaneous value of the carrier amplitude changes in accordance with the amplitude and frequency variations of the modulating signal
- The carrier frequency remains constant during the modulation process, but its amplitude varies in accordance with the modulating signal

$s_i(t)$

$s_c(t)$

information envelope

$s_{am-dsb-tc}(t)$

Carrier peak is zero reference for modulating signal
v_c

AM wave

Envelope

$v_c \to V_c\uparrow$

V_c

0

Time

Unmodulated carrier wave

Sinusoidal modulating wave

$v_m\uparrow V_m$

0

Time

(a)

(b)

Amplitude Modulation Fundamentals
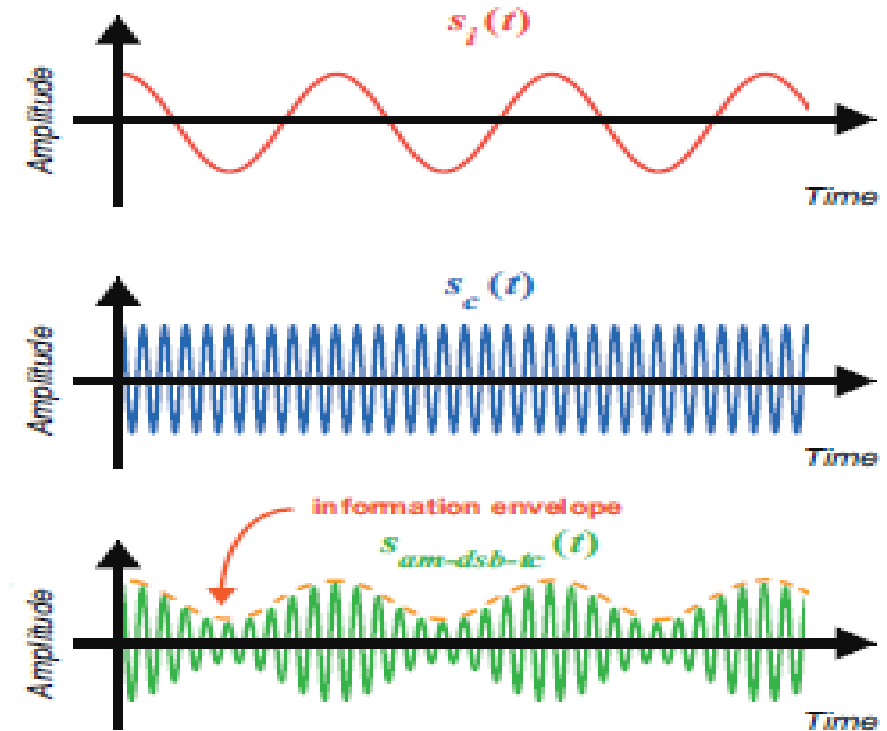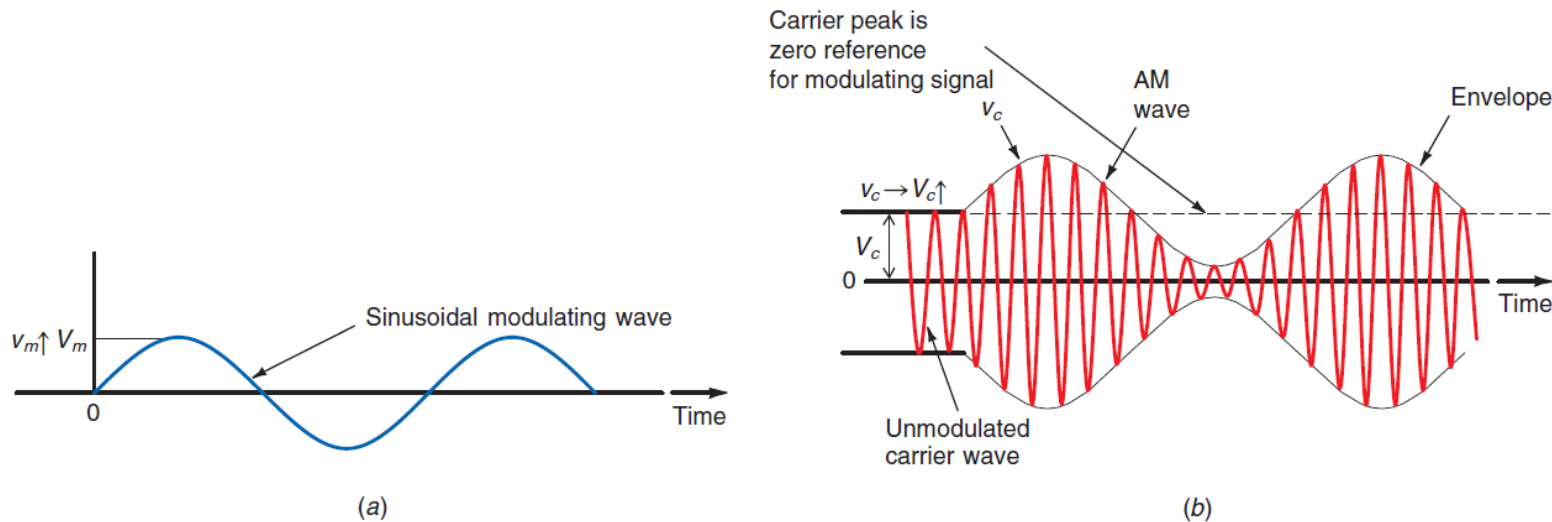
- The sine wave carrier signal is expressed as:

$$v_c = V_c \sin 2\pi f_c t$$

  v_c represents the instantaneous value of the carrier sine wave voltage,

  V_c represents the peak value of the constant unmodulated carrier sine wave

  fc is the frequency of the carrier sine wave; and t is a particular point in time during the carrier cycle

- A sine wave-modulating signal can be expressed with a similar formula

$$v_m = V_m \sin 2\pi f_m t$$

  where v_m = instantaneous value of information signal

  Vm = peak amplitude of information signal

  fm = frequency of modulating signal

- The envelope of the modulating signal varies above and below the peak carrier amplitude

- That is, the zero reference line of the modulating

signal coincides with the peak value of the unmodulated carrier

Carrier peak is
zero reference
for modulating signal

V_c

AM
wave

Envelope

$V_c \rightarrow V_c\uparrow$

V_c

0

Time

Unmodulated
carrier wave
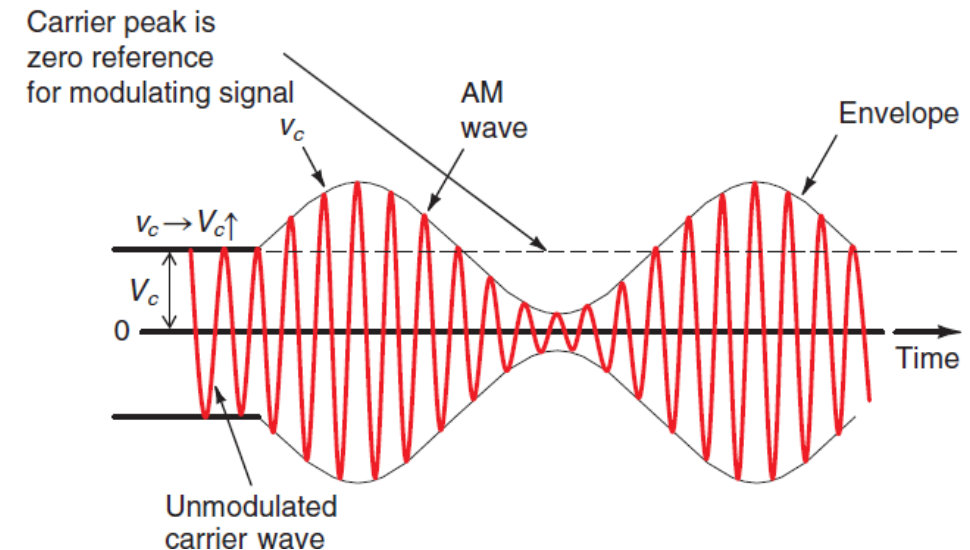
- In amplitude modulation, it is particularly important that the peak value of the modulating signal be less than the peak value of the carrier
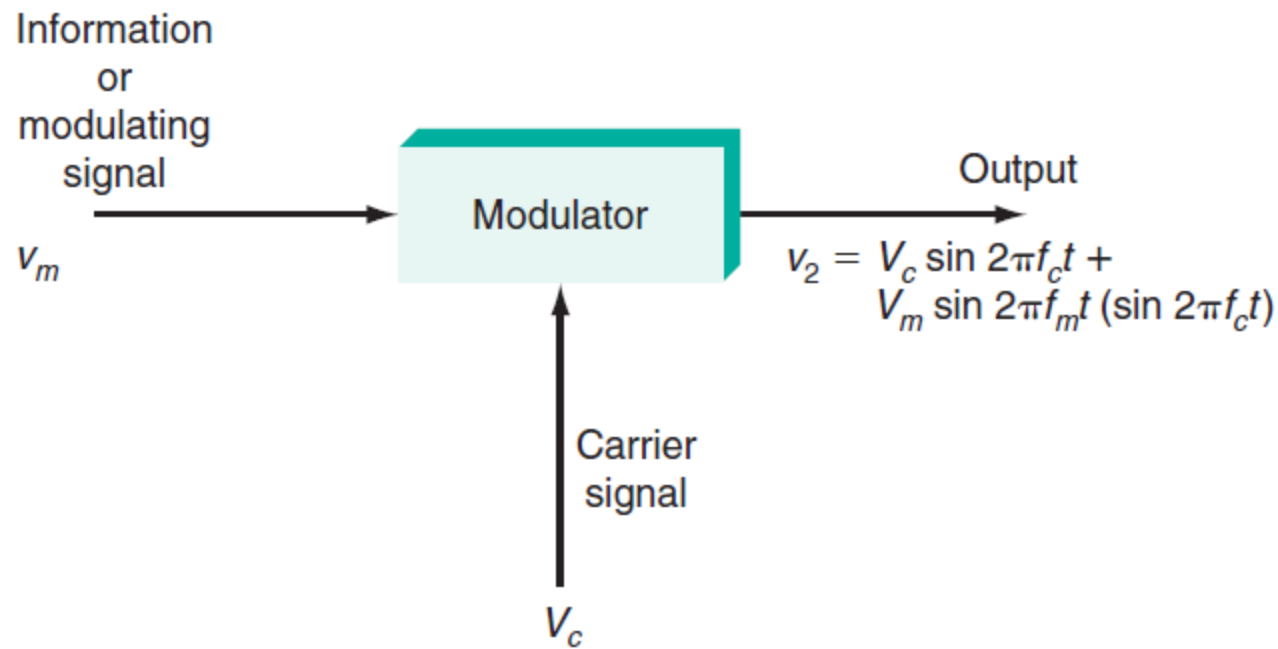
$$V_m < V_c$$

- The value of the modulating signal is added to or subtracted from the peak value of the carrier:

$$v_1 = V_c + v_m = V_c + V_m \sin 2\pi f_m t$$

- Which expresses the fact that the instantaneous value of the modulating signal algebraically adds to the peak value of the carrier
- The instantaneous value of the complete modulated wave v_2 by substituting v_1 for the peak value of carrier voltage Vc as follows:

$$v_2 = v_1 \sin 2\pi f_c t$$

$$v_2 = (V_c + V_m \sin 2\pi f_m t) \sin 2\pi f_c t = V_c \sin 2\pi f_c t + (V_m \sin 2\pi f_m t)(\sin 2\pi f_c t)$$

Information
or
modulating
signal

Modulator

Output

V_m

$v_2 = V_c \sin 2\pi f_c t +$
$V_m \sin 2\pi f_m t (\sin 2\pi f_c t)$

Carrier
signal

V_c

where v_2 is the instantaneous value of the AM wave (or v_{AM}), $V_c \sin 2\pi f_c t$ is the carrier waveform, and $(V_m \sin 2\pi f_m t)$ $(\sin 2\pi f_c t)$ is the carrier waveform multiplied by the modulating signal waveform. It is the second part of the expression that is characteristic of AM

Modulation index m (also called the modulating factor or coeficient, or the degree of modulation)

$$m = \frac{V_m}{V_c}$$

Multiplying the modulation index by 100 gives the *percentage of modulation*

Envelope is no longer the same shape as original modulating signal

Clipping of negative peaks occurs
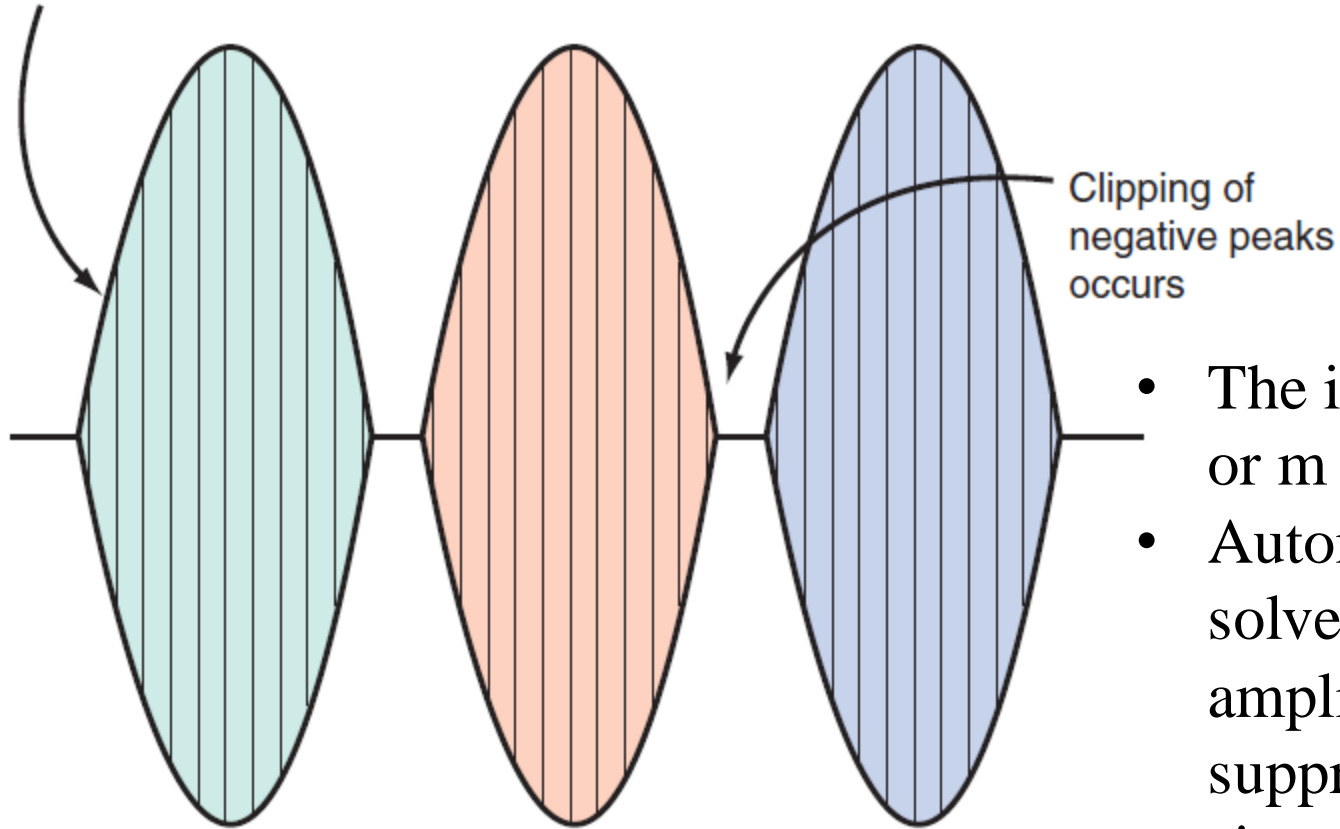
Over modulation → Vm > Vc

- During over-modulation the received signal will produce an output waveform in the shape of the envelope, which in this case is a sine wave whose negative peaks have been clipped off

- The ideal condition for AM is when Vm = Vc, or m = 1, which gives 100 percent modulation
- Automatic circuits called ***compression*** circuits solve this problem (over-modulation) by amplifying the lower-level signals and suppressing or compressing the higher-level signals.
- The result is a higher average power output level without over-modulation.

Sideband Calculations

- When only a single-frequency sine wave modulating signal is used, the modulation process generates two sidebands.
- If the modulating signal is a complex wave, such as voice or video, a whole range of frequencies modulate the carrier, and thus a whole range of sidebands are generated.
- The existence of sidebands can be demonstrated mathematically, starting with the equation for an AM signal described previously:

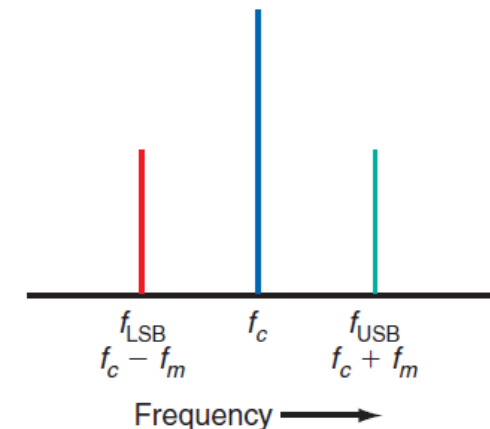$$v_{AM} = V_c \sin 2\pi f_c t + (V_m \sin 2\pi f_m t)(\sin 2\pi f_c t)$$

$$\sin A \sin B = \frac{\cos (A - B)}{2} - \frac{\cos (A + B)}{2}$$

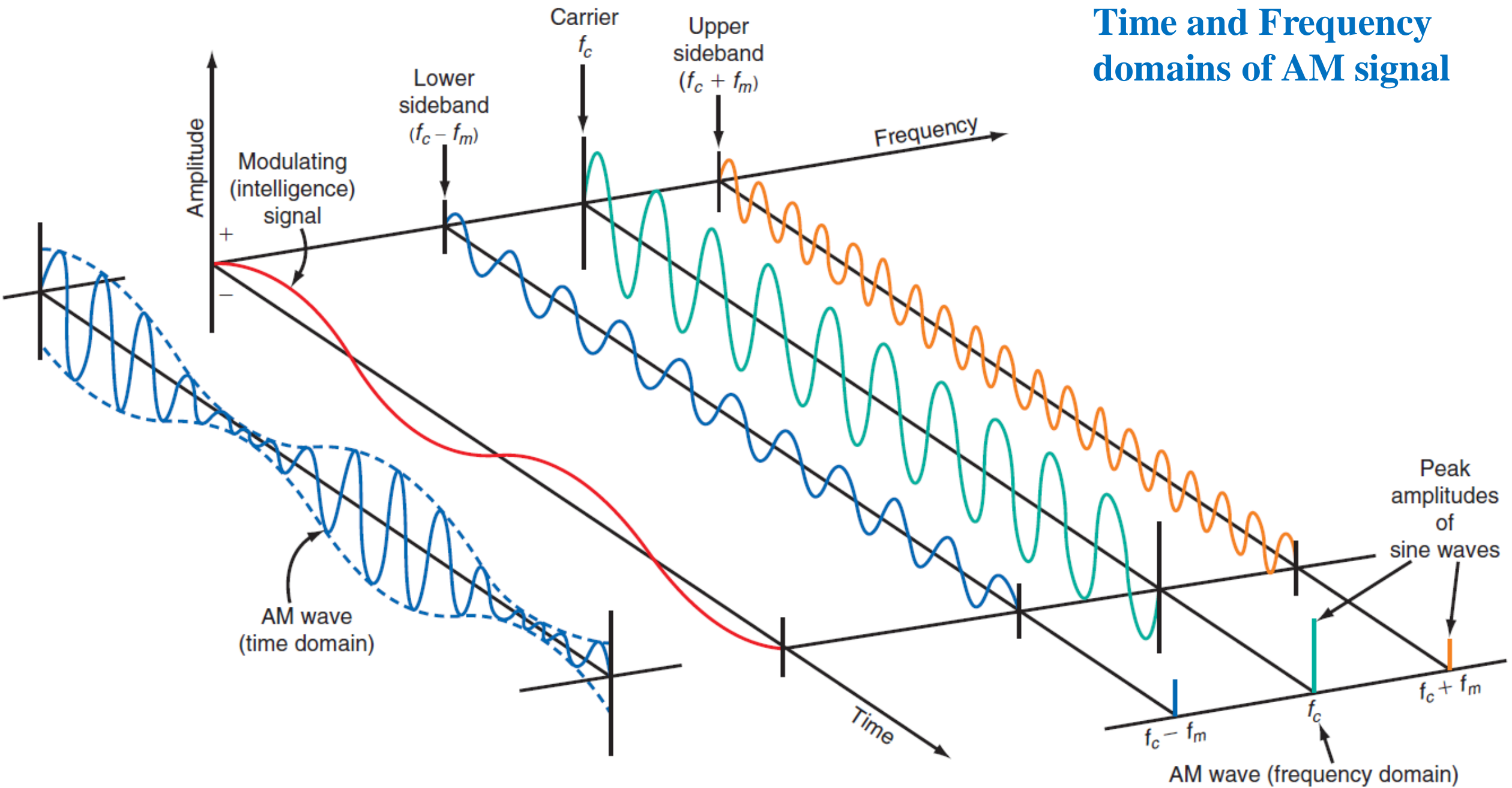$$v_{AM} = V_c \sin 2\pi f_c t + \frac{V_m}{2} \cos 2\pi t (f_c - f_m) - \frac{V_m}{2} \cos 2\pi t (f_c + f_m)$$

A frequency-domain display of an AM signal (voltage).

The upper sideband f_{USB} and lower sideband f_{LSB} are computed as

$$f_{USB} = f_c + f_m \quad \text{and} \quad f_{LSB} = f_c - f_m$$

f_{LSB} f_c f_{USB}
$f_c - f_m$ $f_c + f_m$

Frequency ⟶

Time and Frequency domains of AM signal

AM-Power

- The AM signal is a composite of several signal voltages, namely, the carrier and the two sidebands, and each of these signals produces power in the antenna
- The total transmitted power P_T is simply the sum of the carrier power P_c and the power in the two sidebands P_{USB} and P_{LSB} :

$$P_T = P_c + P_{USB} + P_{LSB}$$

- Original AM signal

$$v_{AM} = V_c \sin 2\pi f_c t + \frac{V_m}{2} \cos 2\pi t\,(f_c - f_m) - \frac{V_m}{2} \cos 2\pi t\,(f_c + f_m)$$

- For power calculations, rms values must be used for the voltages
- Convert the voltages from peak to rms by dividing the peak value by √2 or multiplying by 0.707

$$v_{AM} = \frac{V_c}{\sqrt{2}} \sin 2\pi f_c t + \frac{V_m}{2\sqrt{2}} \cos 2\pi t\,(f_c - f_m) - \frac{V_m}{2\sqrt{2}} \cos 2\pi t\,(f_c + f_m)$$

$$P_T = \frac{(V_c/\sqrt{2})^2}{R} + \frac{(V_m/2\sqrt{2})^2}{R} + \frac{(V_m/2\sqrt{2})^2}{R} = \frac{V_c^2}{2R} + \frac{V_m^2}{8R} + \frac{V_m^2}{8R} \quad \text{since} \quad P = V^2/R$$

- Since m= Vm/Vc , the modulating signal be expressed as follows:

$$V_m = mV_c$$

- Therefore,

$$P_T = \frac{V_c^2}{2R}\left(1 + \frac{m^2}{4} + \frac{m^2}{4}\right) \quad \gg \quad P_T = P_c\left(1 + \frac{m^2}{2}\right)$$

For example, if the carrier of an AM transmitter is 1000 W and it is modulated 100 percent ($m = 1$), the total AM power is

$$P_T = 1000\left(1 + \frac{1^2}{2}\right) = 1500 \text{ W}$$

Problem-1

An AM transmitter has a carrier power of 30 W. The percentage of modulation is 85 percent. Calculate (a) the total power and (b) the power in one sideband.

Types of AM

- Double Sideband (AM-DSB)

- Single Sideband (AM-SSB)
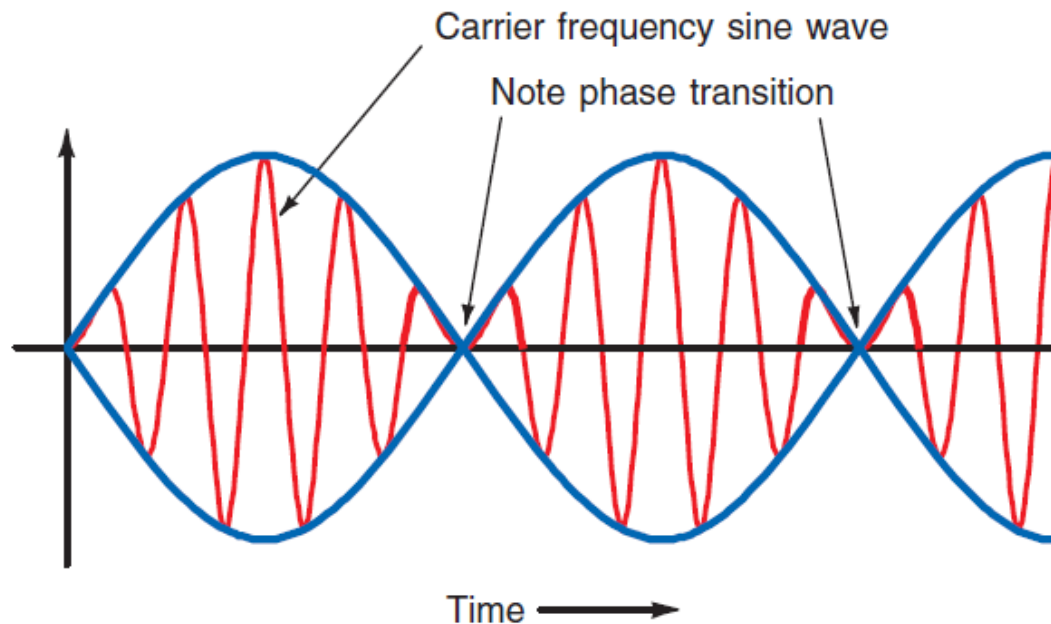
- Vestigial Sideband (AM-VSB

Types of AM-DSB

- Suppressed Carrier (AM-DSB-SC)
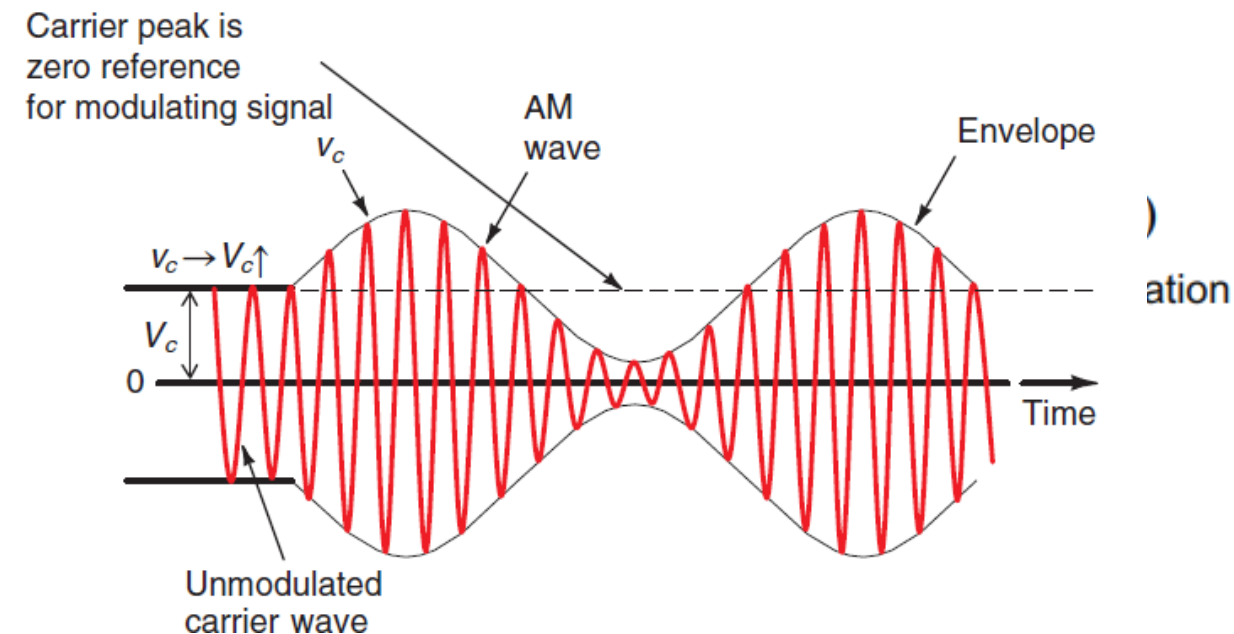
- Transmitted Carrier (AM-DSB-TC)

AM-DSB-SC

- In amplitude modulation, two-thirds of the transmitted power is in the carrier, which conveys no information

- The benefit is that no power is wasted on the carrier

- DSB signal is a sine wave at the carrier frequency, varying in amplitude

- The information signal is mixed with a carrier wave to produce the AM-DSB-SC signal
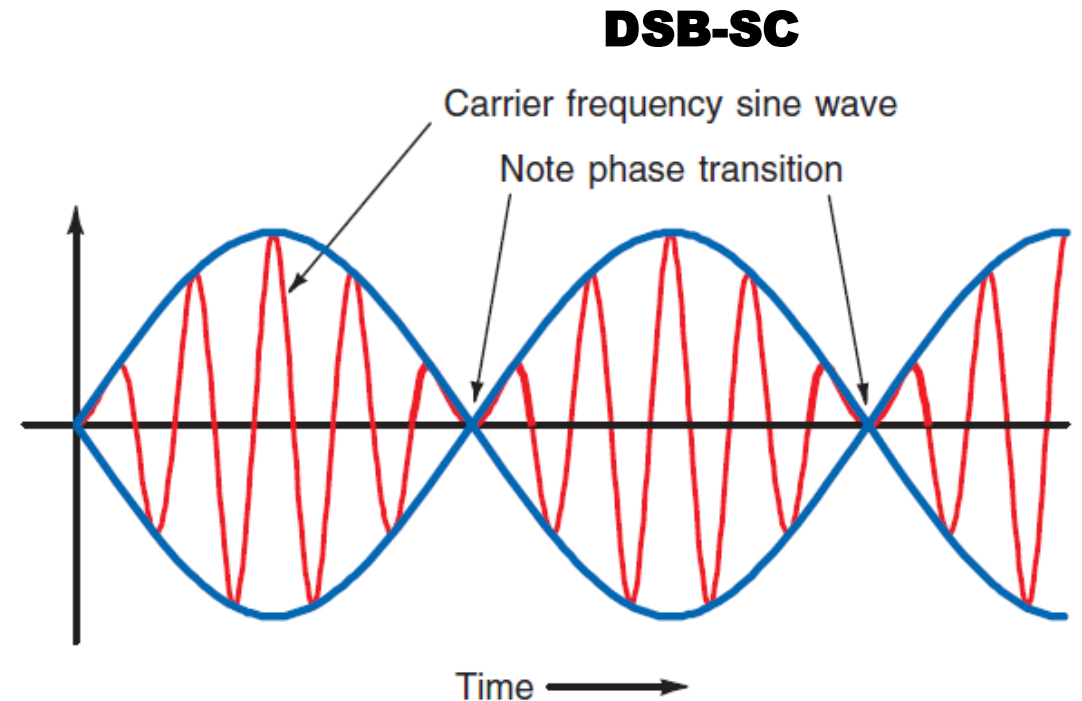
DSB-SC

DSB-FC

Carrier frequency sine wave

Note phase transition

Time

Carrier peak is
zero reference
for modulating signal

V_c

AM
wave

Envelope

$v_c \rightarrow V_c\uparrow$

V_c

0

ation

Time

Unmodulated
carrier wave

- A unique characteristic of the DSB signal is the phase transitions that occur at the lower-amplitude portions of the wave
- DSB-SC is an amplitude modulation wave without the carrier, therefore reducing power waste, giving it a 50% efficiency
- One important application for DSB, however, is the transmission of the color information in a TV signal
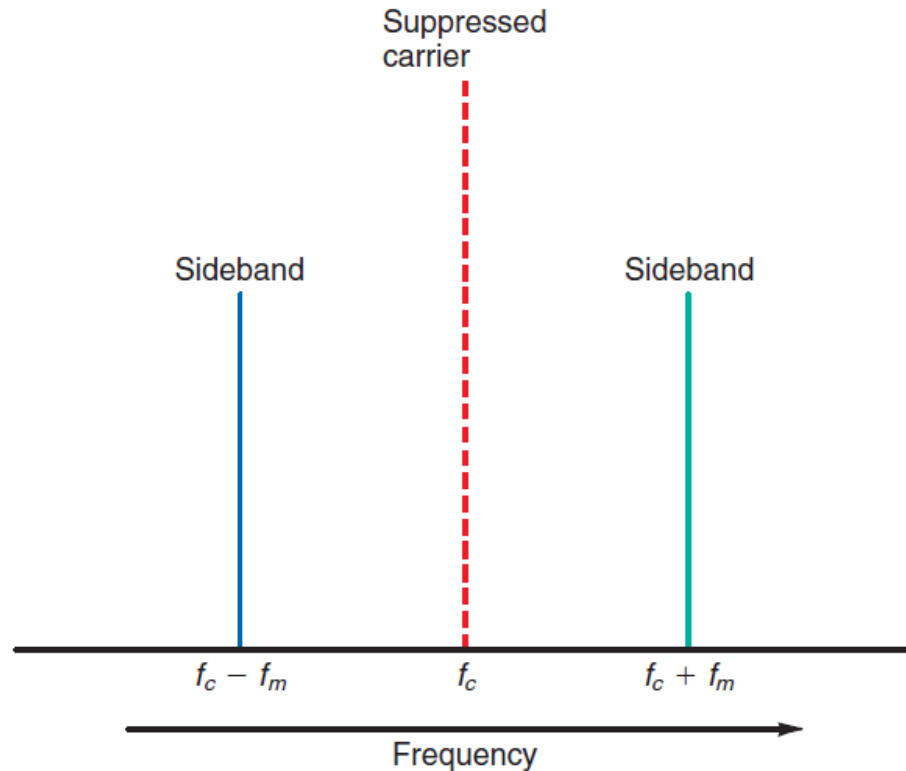
AM-DSB-SC: Modulating baseband signals

The baseband information signal $s_i(t)$ had a bandwidth of f_h Hz (where f_h is the highest frequency component contained within the signal), then AM-DSB-SC modulating will results in a signal with a bandwidth of $2f_h$ Hz . The modulated signal, which is centred around f_c as shown in figure below. The sidebands are symmetric around f_c
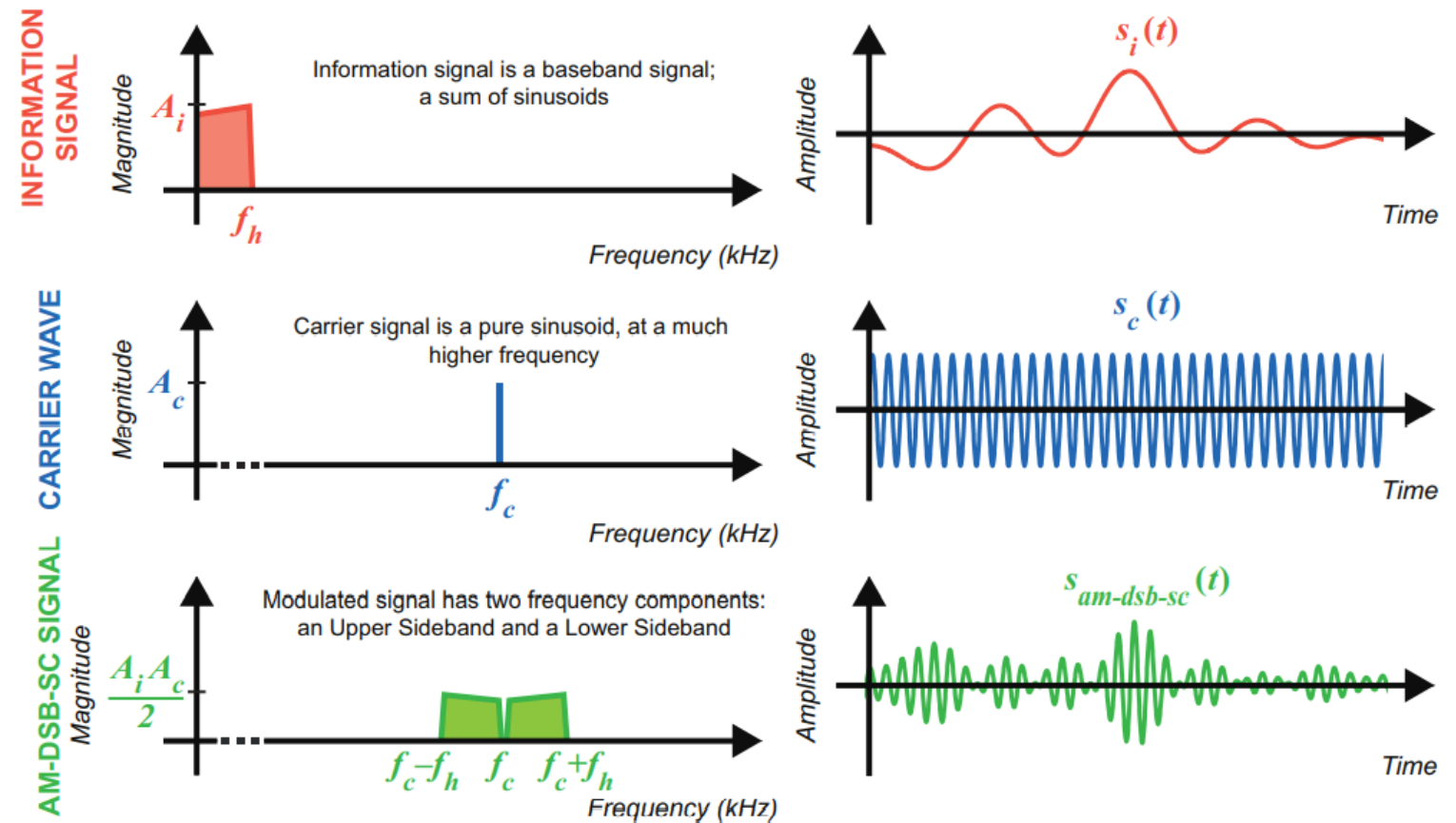
AM-SSB: Single Sideband AM

- Though mathematically simple, AM-DSB modulation is spectrally inefficient because the process creates a pair of identical sidebands

- This means the signals have a wider bandwidth than is strictly necessary to carry the information

- Therefore, if only a finite amount of bandwidth is available in a communication channel, this would not be the best choice

- The main advantage of AM-SSB modulation is the bandwidth occupied by the modulated signal is the same as the bandwidth of the baseband signal

Transmission efficiency of AM wave

- Transmission efficiency is defined as the ratio of the power carried by the sidebands to the total transmitted power

$$\eta = \frac{P_{USB} + P_{LSB}}{P_t}$$

$$\eta = \frac{\mu^2}{\mu^2 + 2}$$

Types AM-SSB

- Single Upper Sideband (AM-SUSB)
- Single Lower Sideband (AM-SLSB)

- The upper and lower sidebands of AM-SSB signals have very similar envelopes

- AM-SSB modulation results in the modulated signal of same bandwidth as that of the baseband information signal

- One sideband can be suppressed; the remaining sideband is called a *single sideband suppressed carrier (SSSC* or *SSB)* signal.
- SSB signals offer four major benefits
  1. The primary benefit of an SSB signal is that the spectrum space it occupies is only one-half that of AM and DSB signals. This greatly conserves spectrum space and allows more signals to be transmitted in the same frequency range.
  2. The single sideband channeled all previously devoted power to the carrier and other sidebands, resulting in a stronger signal that can be reliably received at greater distances.
  3. Alternatively, SSB transmitters can be made smaller and lighter than an equivalent AM or DSB transmitter because less circuitry and power are used.
  4. Because SSB signals occupy a narrower bandwidth, the signal's noise is reduced.
  5. There is less selective fading of an SSB signal over long distances

Comparison of AM-DSB-SC, AM-SUSB, AM-SLSB signals
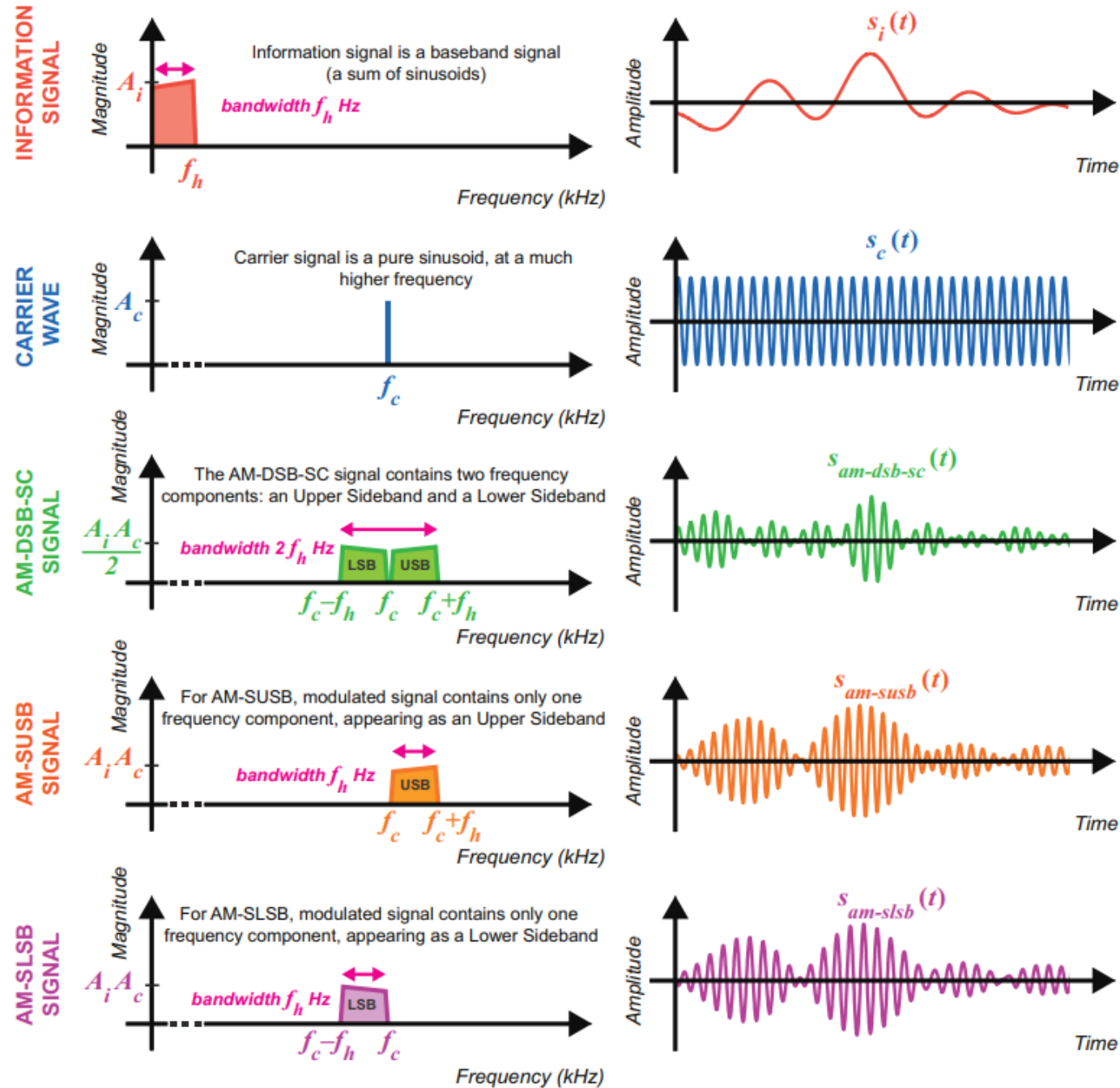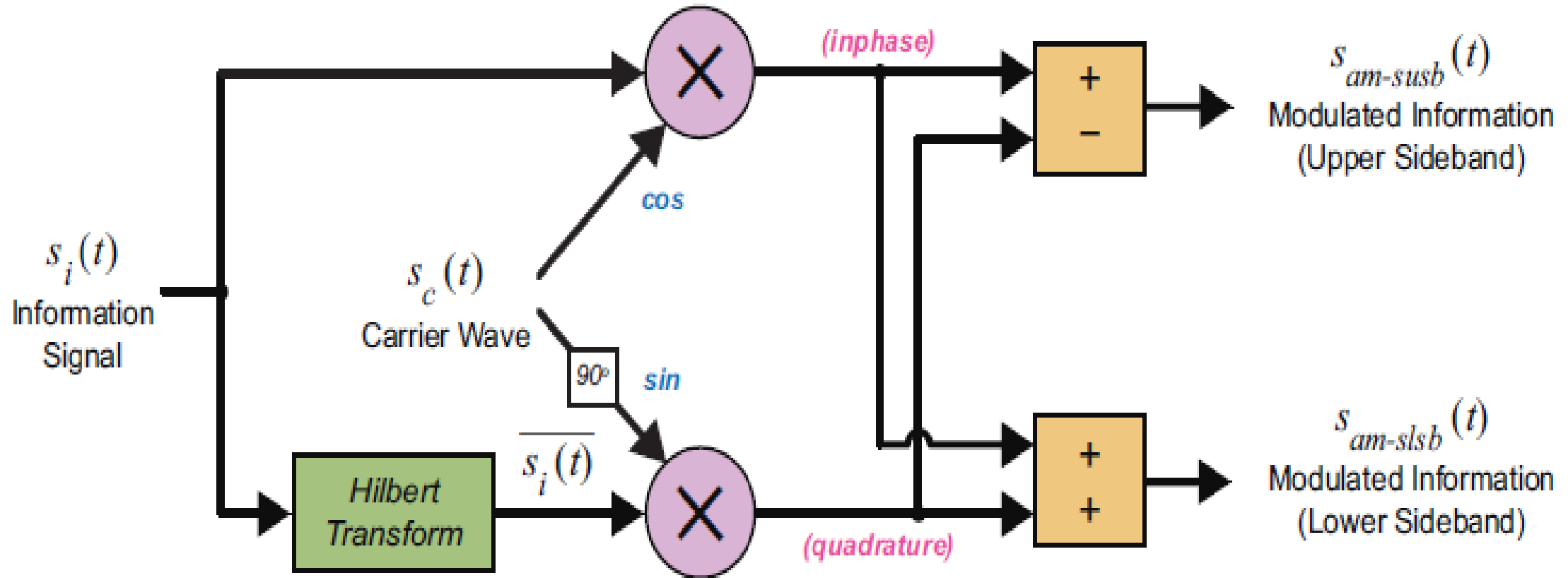
Generation of AM-SSB signals

Figure: Block diagram of AM-SSB modulator

The general equation for this modulator is as follows:

$$s_{am-ssb}(t) = s_i(t) \, \Re e\left[s_c(t) \right] \mp \overline{s_i(t)} \, \Im m\left[s_c(t) \right]$$

where: $\overline{s_i(t)}$ *is the Hilbert transform* $H(s_i)(t)$ *of the information signal* $s_i(t)$,

$\Re e\left[s_c(t) \right]$ *and* $\Im m\left[s_c(t) \right]$ *are the real and imaginary components of the quadrature carrier,*

and the $\mp$ *relates to whether the modulator is configured in AM-SUSB or AM-SLSB mode.*

AM-SSB: Modulating a sine wave

Information signal $\quad s_i(t) = A_i \cos(2\pi f_i t) = A_i \cos(\omega_i t)$

HT of message signal $\quad s_i(t) \xrightarrow{\ HT\ } \overline{s_i(t)} \Rightarrow A_i \cos(\omega_i t) \xrightarrow{\ HT\ } A_i \sin(\omega_i t)$

The quadrature carrier wave has the form

$$s_c(t) = A_c \cos(2\pi f_c t) + A_c \sin(2\pi f_c t) = A_c \cos(\omega_c t) + A_c \sin(\omega_c t)$$
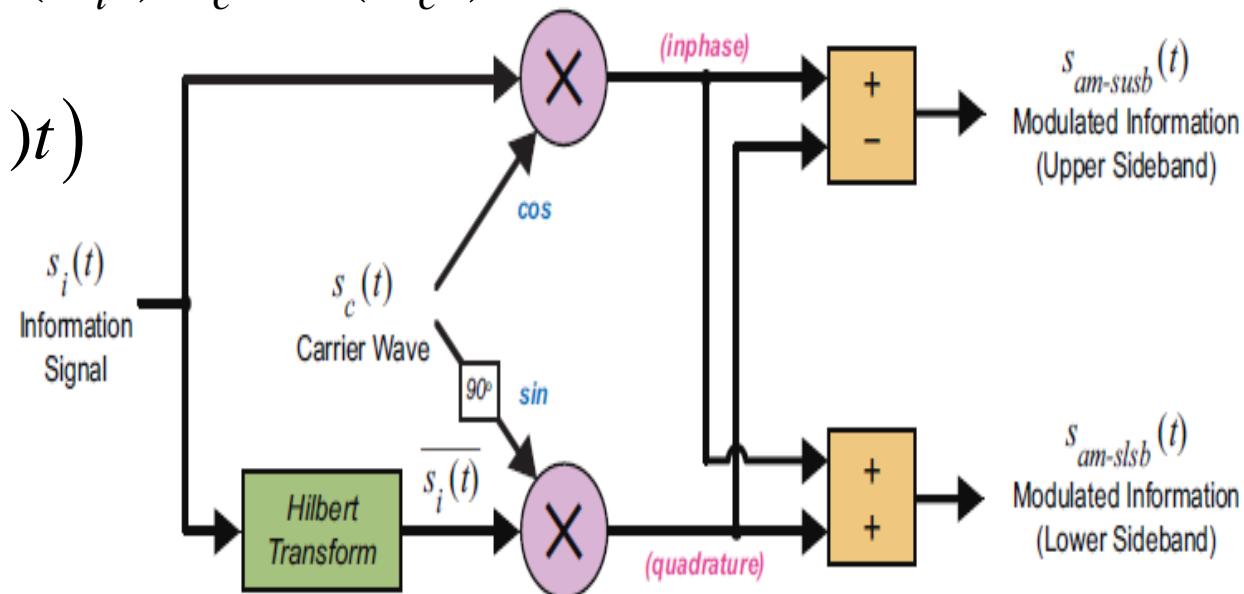
The AM-SSB modulated signal is obtained as

$$\cos a . \cos b = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$$

$$\sin a . \sin b = -\frac{1}{2}\left[\cos(a+b) - \cos(a-b)\right]$$

$$s_{am-ssb}(t) = A_i \cos(\omega_i t) A_c \cos(\omega_c t) \ \mathrm{m} A_i \sin(\omega_i t) A_c \sin(\omega_c t)$$

$$s_{am-ssb}(t) = \frac{A_i A_c}{2}\left(\cos(\omega_c - \omega_i)t + \cos(\omega_c + \omega_i)t\right)$$

$$\mathrm{m}\frac{A_i A_c}{2}\left(\cos(\omega_c - \omega_i)t - \cos(\omega_c + \omega_i)t\right)$$

The configuration required to generate AM-SUSB is

$$s_{am-susb}(t) = \frac{A_i A_c}{2}\left(\cos(\omega_c - \omega_i)t + \cos(\omega_c + \omega_i)t\right)$$

$$-\frac{A_i A_c}{2}\left(\cos(\omega_c - \omega_i)t - \cos(\omega_c + \omega_i)t\right)$$

$$= A_i A_c \cos(\omega_c + \omega_i)t$$

The configuration required to generate AM-SLSB is

$$s_{am-slsb}(t) = \frac{A_i A_c}{2}\left(\cos(\omega_c - \omega_i)t + \cos(\omega_c + \omega_i)t\right)$$

$$+\frac{A_i A_c}{2}\left(\cos(\omega_c - \omega_i)t - \cos(\omega_c + \omega_i)t\right)$$

$$= A_i A_c \cos(\omega_c - \omega_i)t$$

Revisit: Hilbert Transform

Hilbert transform convert cosine function into sine function (90 degree phase shift) to help us to create a complex exponential signal of the form

$$f_A(t) = \cos(\phi(t)) + j\sin(\phi(t)) = e^{j\phi(t)}$$ ⟵ Analytical signal

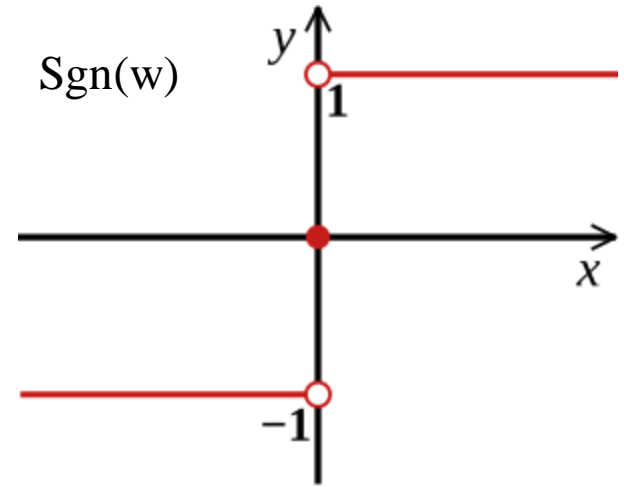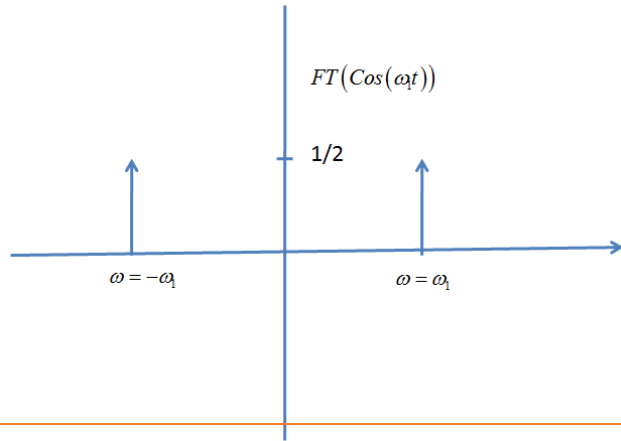How do we generate $\sin(\phi(t))$ a given $\cos(\phi(t))$

This is done by convolution of $\cos(\phi(t))$ with $\dfrac{1}{\pi t}$

So, the analytical signal corresponding to $f(t)$

$$f_A(t) = \left(\delta(t) + \frac{j}{\pi t} \right) * f(t)$$

We know, $$Cos(\omega_1 t) = \frac{1}{2}e^{j\omega_1 t} + \frac{1}{2}e^{-j\omega_1 t}$$

$$FT\left(\frac{1}{\pi t}\right) \text{ is } \frac{1}{j}\times \text{sgn}(\omega); \quad j = \sqrt{-1}$$

$$FT\left(Cos(\omega_1 t)\right) = \frac{1}{2}\delta(\omega - \omega_1) + \frac{1}{2}\delta(\omega + \omega_1)$$

Sgn(w)

$FT(Cos(\omega_1 t))$

1/2

$\omega = -\omega_1$ $\omega = \omega_1$

Convolution in the time domain is equivalent to multiplication in the frequency domain

$$FT\left(\frac{1}{\pi t} * Cos(\omega_1 t)\right) = \left(\frac{1}{j}\times \text{sgn}(\omega)\right) \times \left(\frac{1}{2}\delta(\omega-\omega_1) + \frac{1}{2}\delta(\omega+\omega_1)\right)$$

$$= \left(\frac{1}{2j}\delta(\omega-\omega_1) - \frac{1}{2j}\delta(\omega+\omega_1)\right) = FT^{-1}\left(Sin(\omega_1 t)\right)$$

$$\frac{1}{\pi t} * Cos(\omega_1 t) = Sin(\omega_1 t) = Cos(\omega_1 t - \pi/2)$$

$$\delta(\omega+\omega_1) = 1 \text{ if } \omega = -\omega_1 \text{ else } 0$$

$$\text{sgn}(\omega)\times \delta(\omega+\omega_1) = -\delta(\omega+\omega_1)$$

$$\text{sgn}(\omega)\times \delta(\omega-\omega_1) = \delta(\omega-\omega_1)$$

Pros and Cons. of SSB

Advantages
- Half the bandwidth is required compared to AM-DSB signals

- Due to suppression of carrier and one sideband power is saved

- Reduced noise interference due to the reduced bandwidth

Disadvantages
- The generation and reception of SSB signals are complex

- SSB transmitter and receiver need to have an excellent frequency stability

- SSB modulation is highly complex and expensive to implement

Applications of SSB

- Used at HF segment of the spectrum

  When single-sideband is used in amateur radio voice communications, it is common practice that for frequencies below 10 MHz, lower sideband (LSB) is used and for frequencies of 10 MHz and above, upper sideband (USB) is used

- SSB modulation is used in applications where power saving is required

- SSB is also used in low bandwidth required applications (eg: point-to-point communication, telemetry, military applications, radio navigation etc)

A 400W, 1MHz carrier is amplitude-modulated with a sinusoidal signal of 2500Hz. The depth of modulation is 75%. Calculate the sideband frequencies, bandwidth and power in sidebands and the total power in modulated wave?

Phase Alternating Line (**PAL**)- Video transmission

- PAL is a composite video because luminance (luma, monochrome image) and chrominance (chroma, color applied to the monochrome image) are transmitted as one signal.
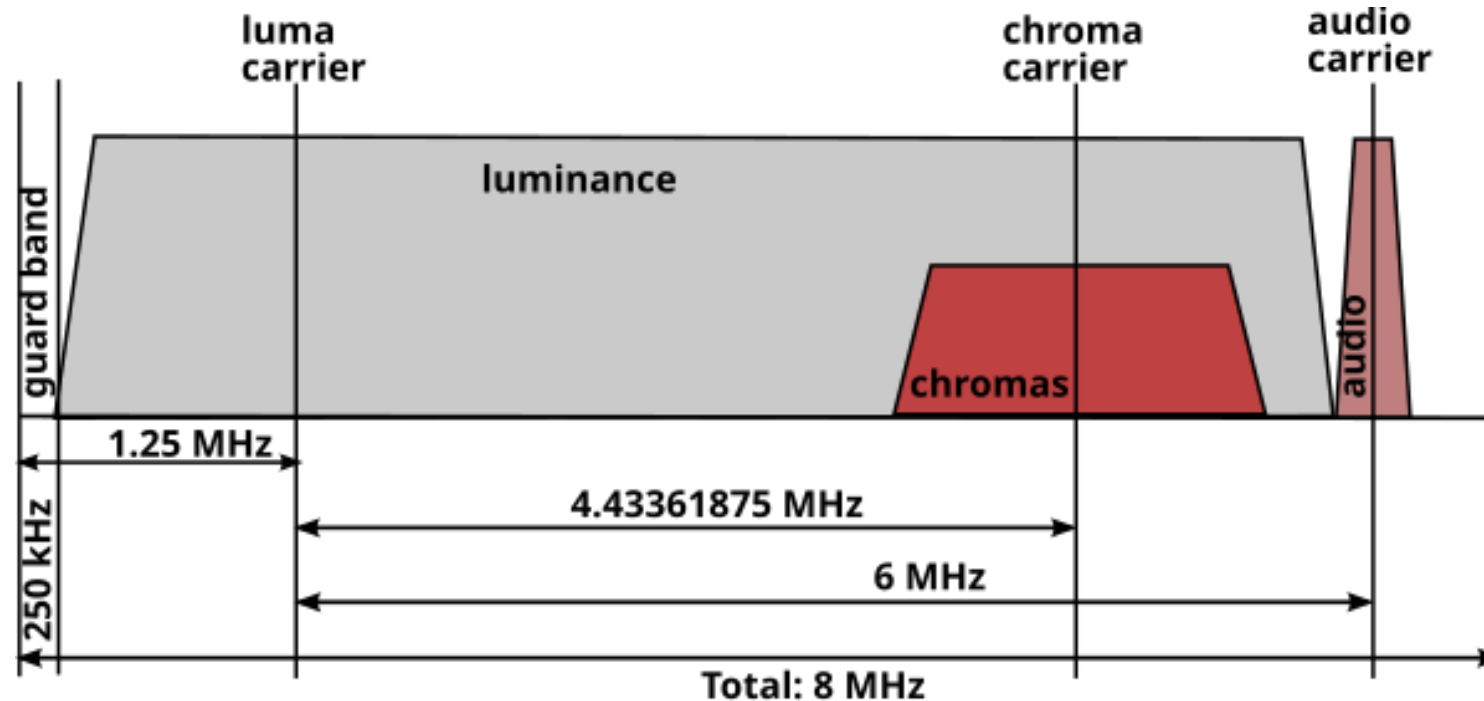
AM-VSB: Vestigial sideband AM

- Created to reduce the spectral requirements of analog TV

- Apply BPF to AM-DSB-TC signals to suppress most of the sidebands to reduce bandwidth

- This partial sideband is called the vestigial sideband

- In VSB, one sideband and a part of the other sideband called vestige is transmitted

- The bandwidth requirement is slightly higher than SSB

- VSB is used in the transmission of TV signals

- Easy to implement than SSB
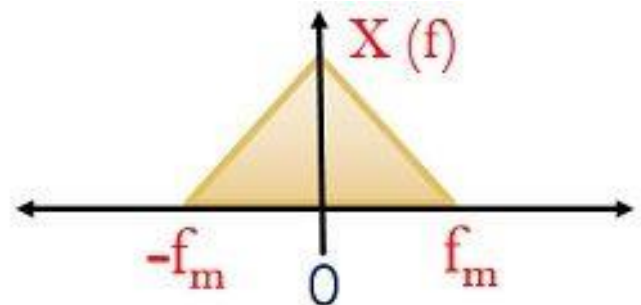
Spectrum of message signal
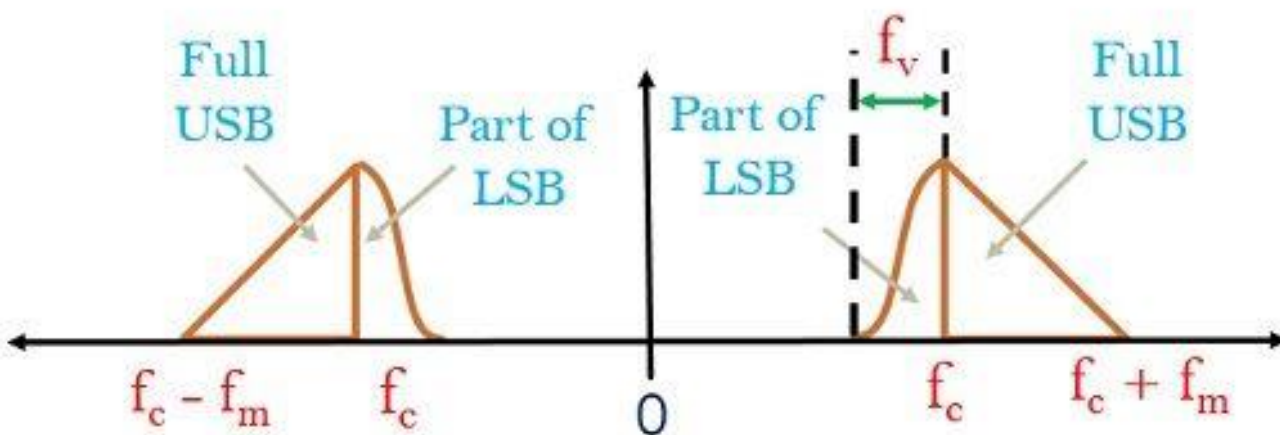
Spectrum of VSB signal
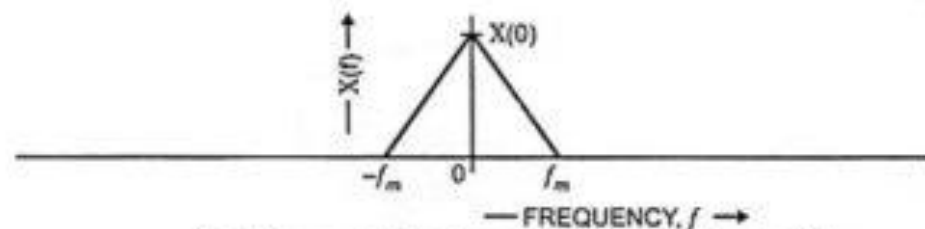
(a) Frequency Spectrum of Baseband Signal x(t)

(b) Frequency Spectrum of DSBSC Signal

(c) Frequency Spectrum of SSB Signal

(d) Frequency Spectrum of VSB Signal

Fig. 22.24 Illustration of Frequency Spectrum of VSB Signal

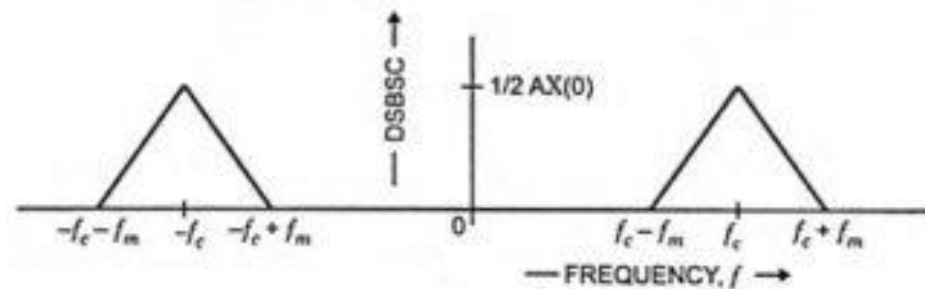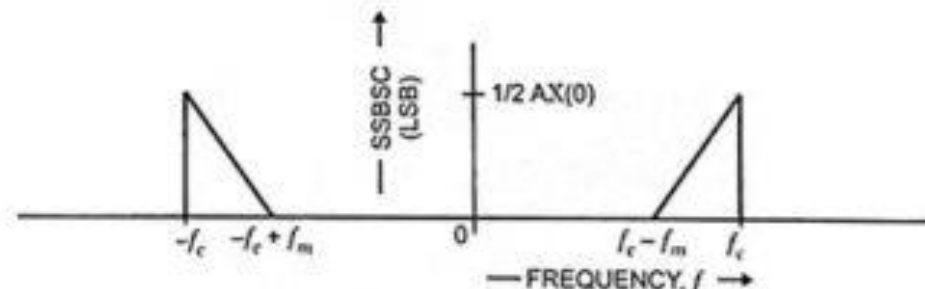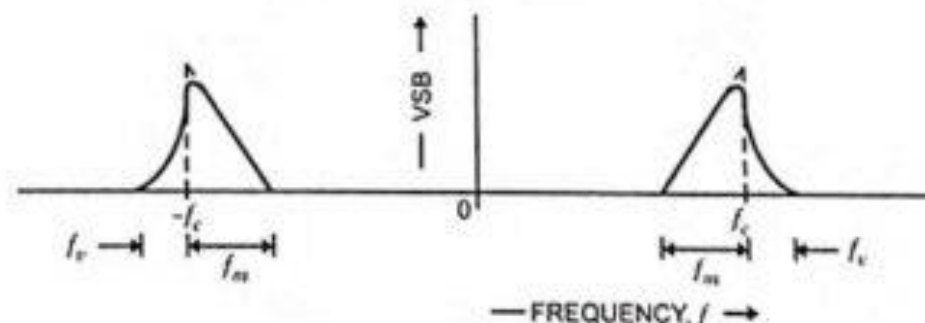**BASEBAND PAL TV SIGNAL**

Magnitude

bandwidth f_h Hz = 6.75MHz

Luminance Chromas Audio Guard Band

f_h = 6.75MHz

Baseband PAL, SECAM and NTSC signals are multiplexes containing video and audio information. The spectrum shown here is for PAL and SECAM.

Frequency (MHz)

AM-DSB-TC PAL TV SIGNAL

Magnitude

bandwidth 2f_h Hz = 13.5MHz

Guard Band Audio Chromas Luminance Luminance Chromas Audio Guard Band

$f_c - f_h$ f_c $f_c + f_h$

Frequency (MHz)

AM-VSB PAL TV SIGNAL

Magnitude

bandwidth f_h +1.25MHz = 8MHz

When AM-VSB modulated, the signal has a lower bandwidth than would have resulted from AM-DSB modulation, and critically has an intact carrier.

Guard Band Luminance Chromas Audio Guard Band

$f_c - f_{vsb}$ f_c $f_c + f_h$

Frequency (MHz)

Frequency domain plots showing PAL/SECAM TV signal first
Being AM-DSB-TC modulated then AM-VSB modulated
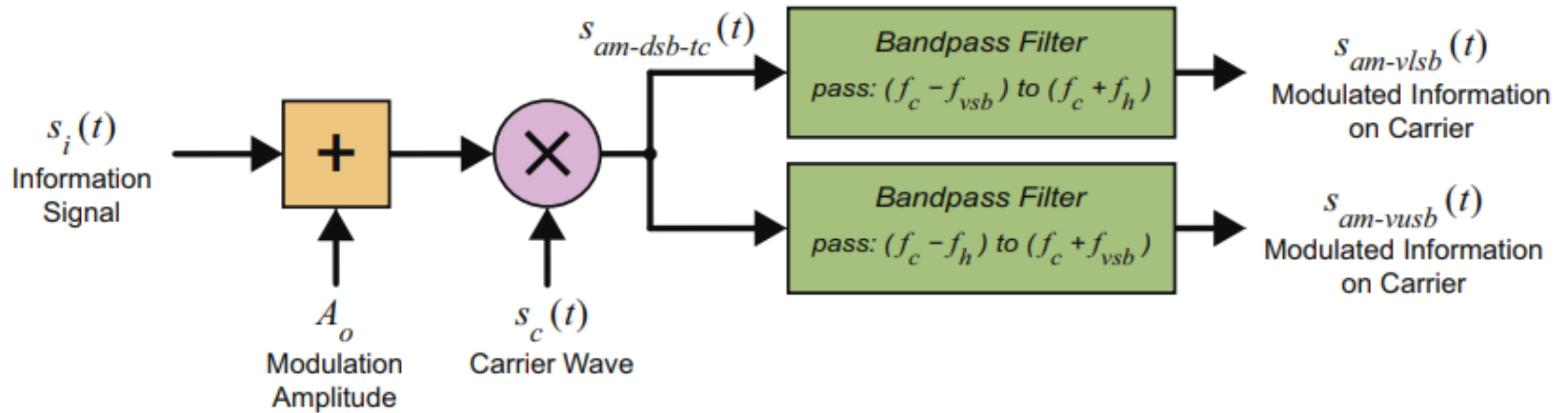
Block diagram of the AM-VSB modulator, showing configurations for both AM-VLSB and AM-VUSB

AM-VLSB SIGNAL

Magnitude

bandwidth $f_h + f_{vsb}$ Hz

There are two variations of AM-VSB, as depicted here.

$s_{am\text{-}vlsb}(t)$

$f_c - f_{vsb}$ f_c $f_c + f_h$

Frequency (MHz)

AM-VUSB SIGNAL

Magnitude

bandwidth $f_h + f_{vsb}$ Hz

$s_{am\text{-}vusb}(t)$

$f_c - f_h$ f_c $f_c + f_{vsb}$
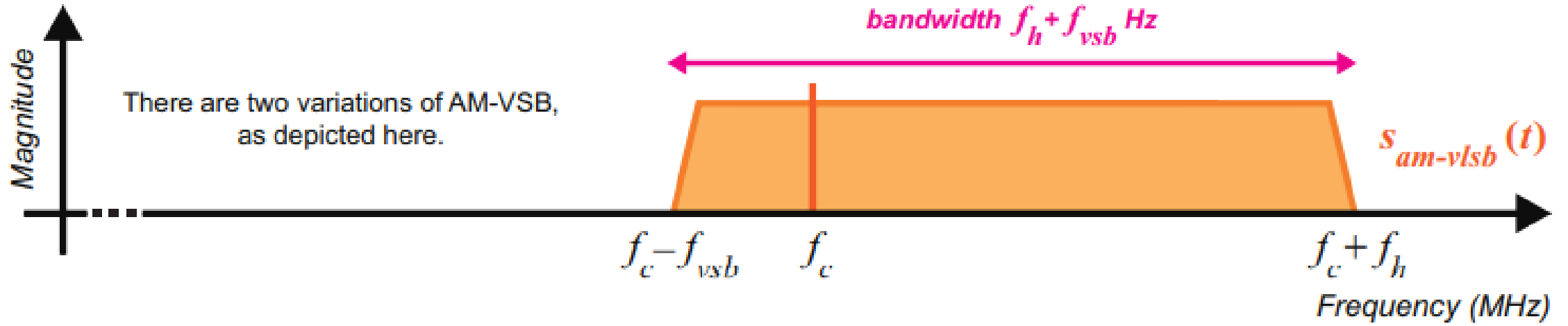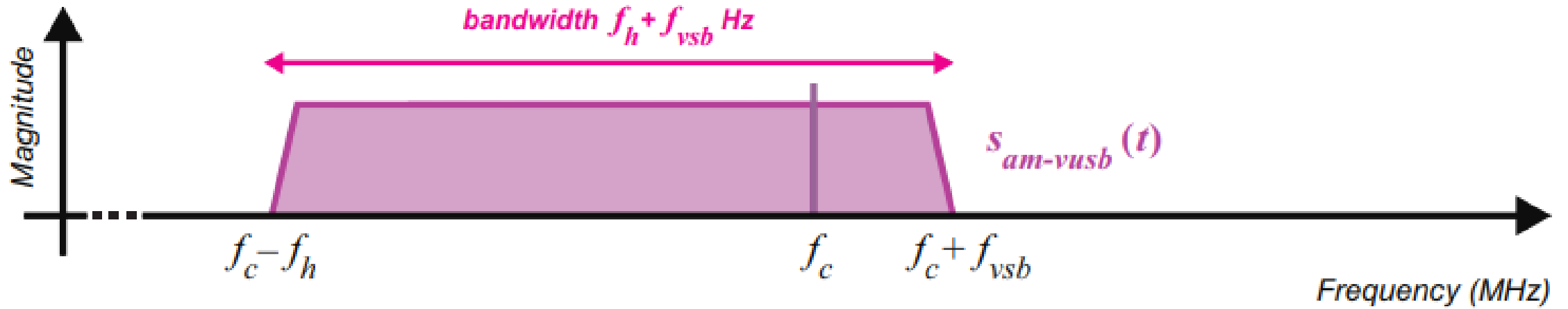
Frequency (MHz)

AM Demodulation

- **Coherent demodulation** –
  - which involves synthesizing a sine wave with the same frequency and phase as the carrier of the received signal and using it to demodulate the received signal

- **Non-coherent demodulation** –
  - The receiver carrier need not be in phase locked with the transmitter carrier

Coherent AM Demodulation

- To demodulate an AM signal, a recover must multiply the received signal with a sine wave that has exactly the same frequency and phase as the carrier embedded within it

- The mixing operation shifts the modulated information from being centred around carrier frequency, back to baseband

- This approach → synchronous demodulation

Demodulation of single-tone AM-DSB-SC signals

DSB-SC signal

$$s_{am-dsb-sc}(t) = \frac{A_i A_c}{2}\left(\cos(\omega_c - \omega_i)t + \cos(\omega_c + \omega_i)t\right)$$

For demodulation, mix the signal with $\cos(\omega_c t)$

$$s_d(t) = \frac{A_i A_c}{2}\left(\cos(\omega_c - \omega_i)t + \cos(\omega_c + \omega_i)t\right)\cos(\omega_c t)$$

$$= \frac{A_i A_c}{2}\left[\cos\left((\omega_c - \omega_i)t\right)\cos(\omega_c t) + \cos\left((\omega_c + \omega_i)t\right)\cos(\omega_c t)\right]$$

$$= \frac{A_i A_c}{2}\left[\begin{array}{l} \dfrac{1}{2}\cos(-\omega_i t) + \cancel{\dfrac{1}{2}\cos\left((2\omega_c - \omega_i)t\right)} \\[2em] +\dfrac{1}{2}\cos(\omega_i t) + \cancel{\dfrac{1}{2}\cos\left((2\omega_c + \omega_i)t\right)} \end{array}\right](lowpassfilterd)$$

- LPF removes the high frequency components and resulting in the scaled information signal as

$$s_d(t) = \frac{A_i A_c}{4}\cos(-\omega_i t) + \frac{A_i A_c}{4}\cos(\omega_i t) = \frac{A_i A_c}{2}\cos(\omega_i t)$$

- The frequency and phase of the modulating and demodulating sines will not always match exactly when there is offset. Thus, this demodulation scheme becomes unsuccessful

Non-coherent AM demodulation-
The envelope detector

- Simplest form of non-coherent demodulator in the analog domain

- 3 components : diode, resistor and capacitor
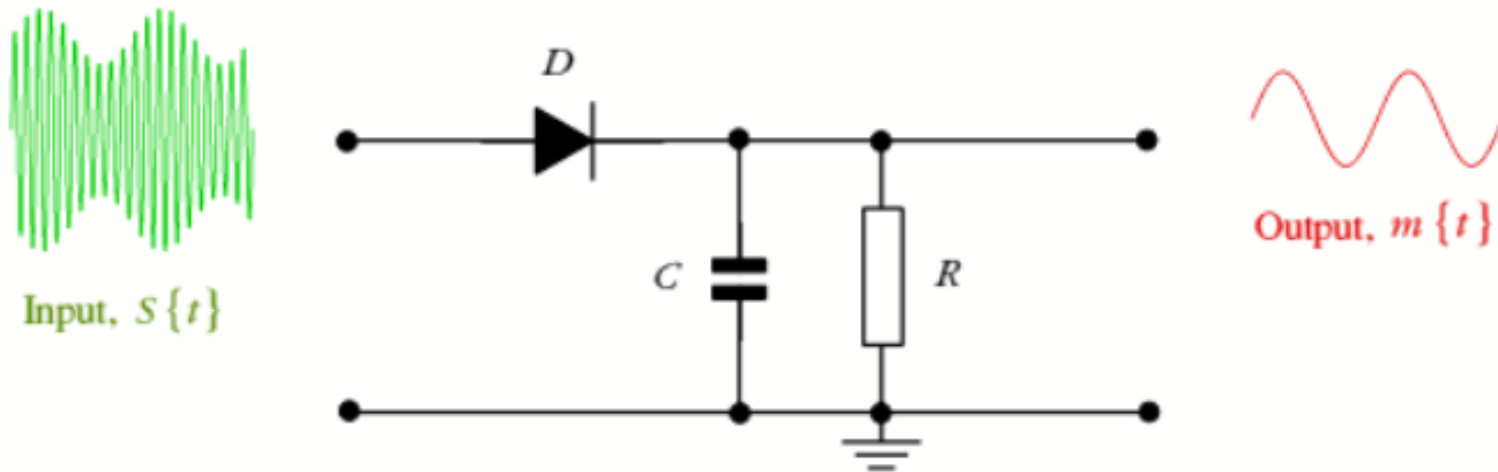
- Can be used only to demodulate AM-DSB-TC

Input, $S\{t\}$
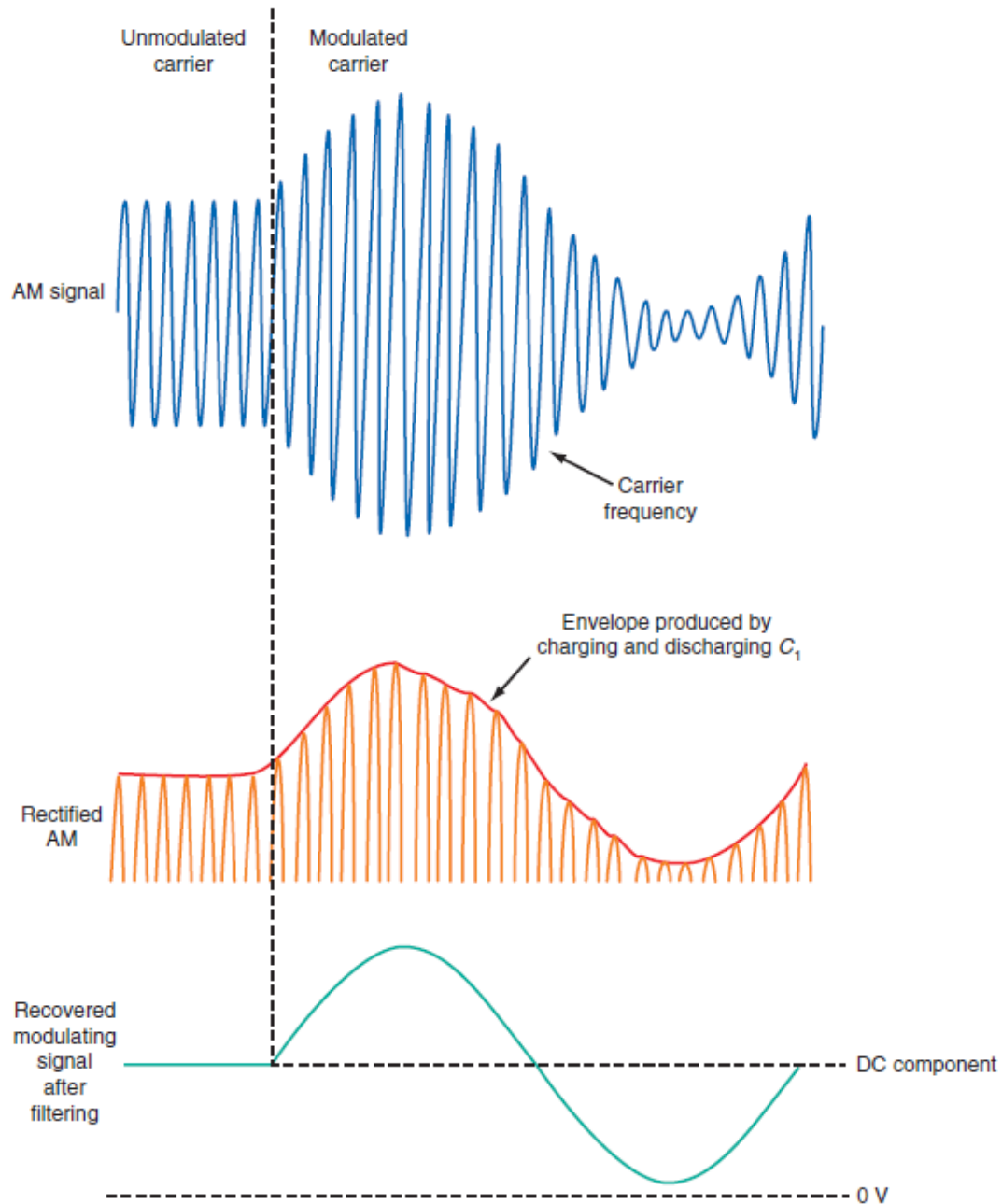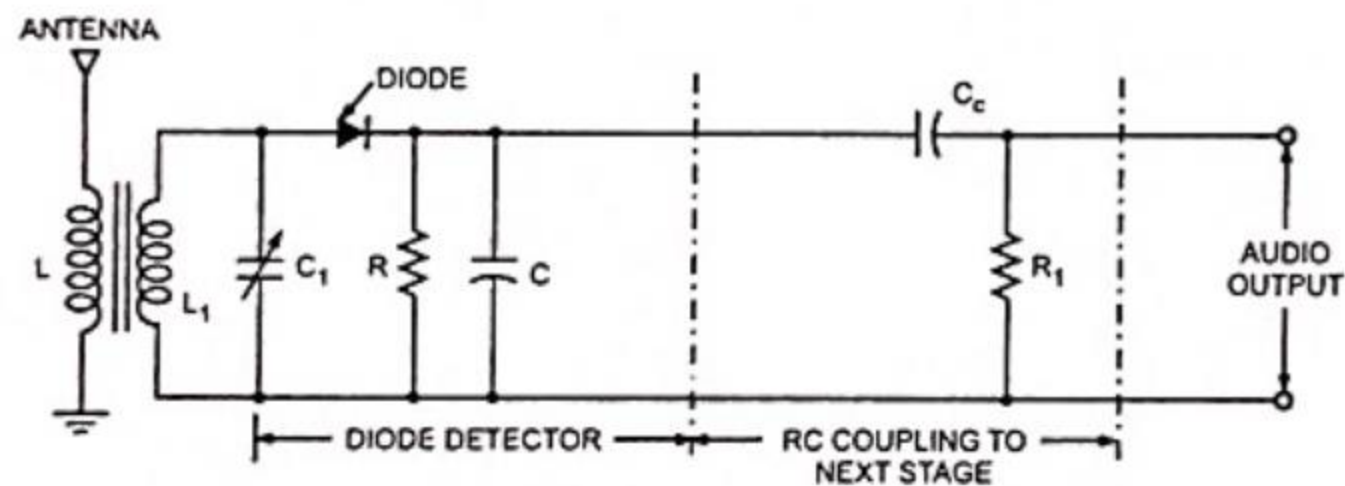
D

C R

Output, $m\{t\}$

Figure 4-16 Diode detector waveforms.

- During the +ve half cycle diode conducts and the capacitor charges to the peak voltage
- During –ve half cycle capacitor discharges
- $1/f_c \ll RC \ll 1/f_m$
- Because the capacitor charges and discharges, the recovered signal has a small amount of ripple on it, distorting of the modulating signal
- Distortion of the original signal can occur if the time constant of the load resistor R1 and the shunt filter capacitor C1 is too long or too short
- If the time constant is too long, the capacitor discharge will be too slow to follow the faster changes in the modulating signal. This is referred to as diagonal distortion.

ANTENNA

DIODE

C_c

L

L_1

C_1 R C

R_1

AUDIO OUTPUT

DIODE DETECTOR — RC COUPLING TO NEXT STAGE

(a) Diode Detector and Coupling Circuit

MODULATED SELECTED SIGNAL

TIME

RC TOO LARGE   OUTPUT WAVE SHAPE
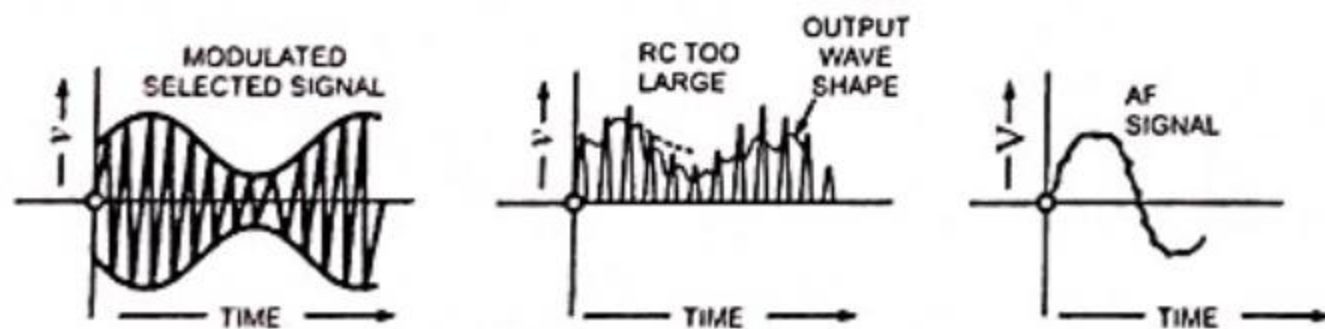
TIME

AF SIGNAL

TIME

(b) Waveshapes At Various Points In The Circuit of (a)

ion envelope

$\Re e\,[\,s_{RTL\text{-}SDR}(t)\,]$

AM-DSB-TC SIGNAL

Amplitude

Time

saturated signal

$s_d(t)$

DEMODULATED SIGNAL

Amplitude

The filter smoothes the gaps between the peaks to detect the envelope
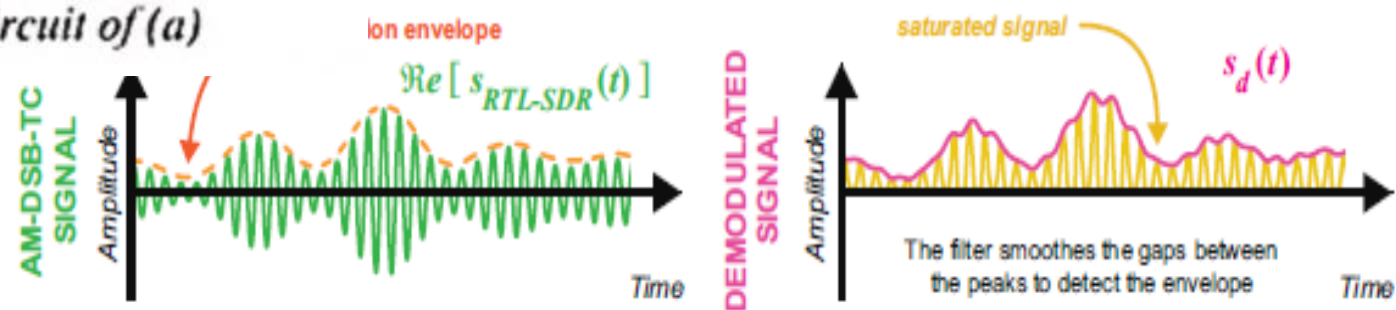
Time

Sketch of an AM-DSB-TC signal in the time domain before (left) and after (right)
Traditional Envelope Detection

Frequency Modulation (FM)

- In FM, the carrier amplitude remains constant and the carrier frequency is changed by the modulating signal

- As the modulating signal amplitude increases, the carrier frequency increases.

- If the amplitude of the modulating signal decreases, the carrier frequency decreases

- The amount of change in carrier frequency produced by the modulating signal is known as the frequency deviation f_d.

- Maximum frequency deviation occurs at the maximum amplitude of the modulating signal.

- FM scheme is used for commercial radio stations due to its high resilience to additive noise
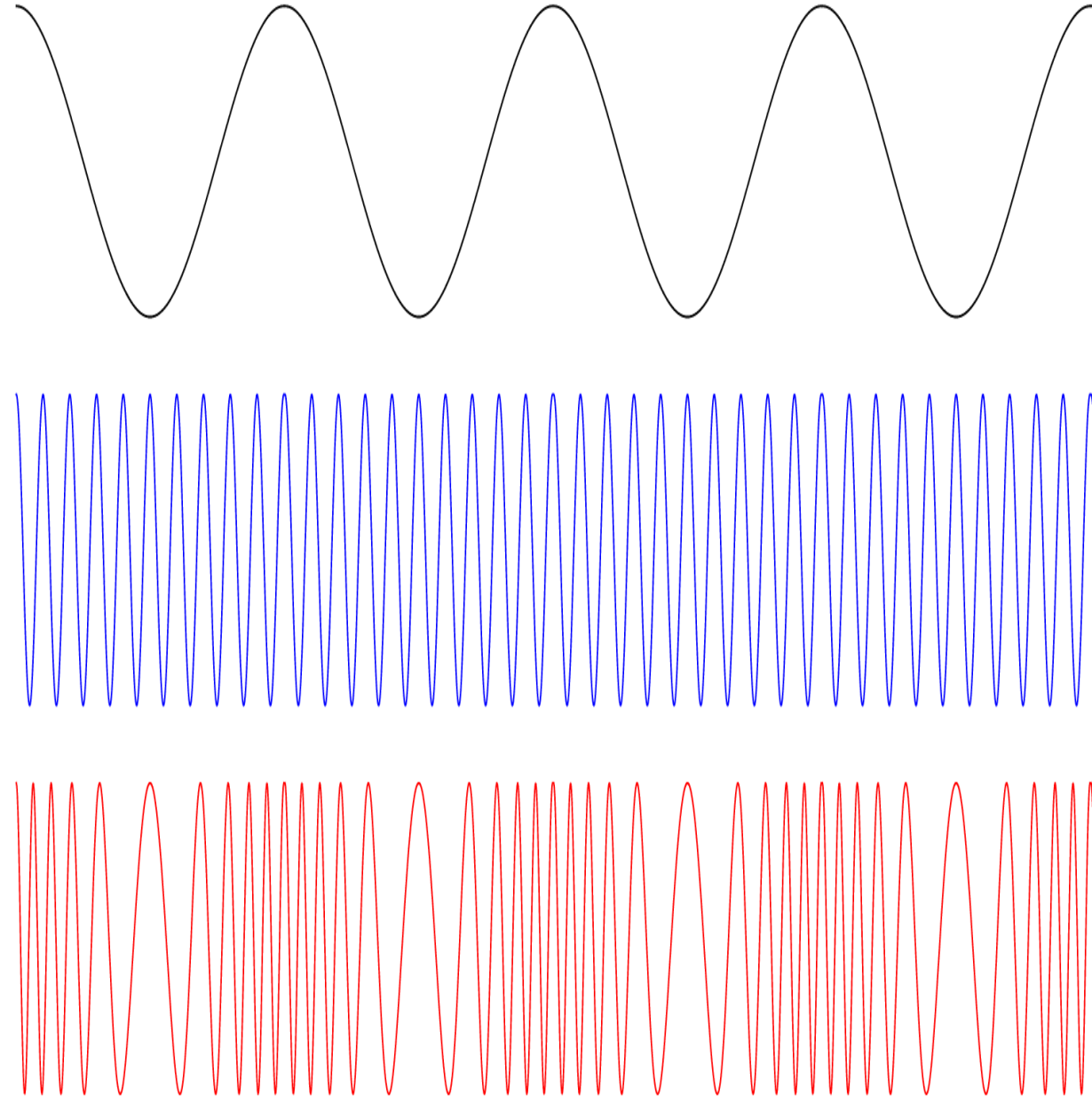
- The frequency of the modulating signal determines the frequency deviation rate, or how many times per second the carrier frequency deviates above and below its center frequency.
- If the modulating signal is a 500-Hz sine wave, the carrier frequency shifts above and below the center frequency 500 times per second.
- Assume a carrier frequency of 150 MHz. If the peak amplitude of the modulating signal causes a maximum frequency shift of 30 kHz.
- The carrier frequency will deviate up to 150.03 MHz and down to 149.97 MHz.
- The total frequency deviation is 150.03 −149.97 5 0.06 MHz = 60 kHz.

Example:
- A transmitter operates on a frequency of 915 MHz. The maximum FM deviation is ±12.5 kHz. What are the maximum and minimum frequencies that occur during modulation?

Voltage Controlled Oscillator (VCO)

VCO is an oscillator with a standard or quiescent frequency (f0) and a control input that can adjust the output frequency upwards or downwards from the quiescent value

When the control signal is input to the VCO, it is multiplied by k_o, a constant representing the 'voltage to frequency gain ratio' of the device (measured in Hz/V). The product of $v(t)$ and k_o is then integrated (changing its phase by 90 degrees). The integrated signal is denoted $\hat{\theta}(t)$:

$$\hat{\theta}(t) = k_o \int_{-\infty}^{t} v(t)\, dt$$

Eq 1.1

The sinusoid generated by the VCO is configured to have quiescent frequency f_o and amplitude A_o. The phase of the sinusoid is determined by the instantaneous value of $\hat{\theta}(t)$. The signal output from the VCO takes the following form:

$$c(t) = A_o \cos\left(2\pi f_o t + \hat{\theta}(t)\right)$$

$$= A_o \cos\left(\omega_o t + k_o \int_{-\infty}^{t} v(t)\, dt\right)$$

Eq 1.2

When an information signal is input to the VCO's control port, and k_o is substituted with the FM modulation constant, $k_o = 2\pi K_{fm}$, Eq 1.1 becomes

$$\hat{\theta}(t) = \theta_{fm}(t) = 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)\, dt$$

Eq 1.3

Substituting Eq 1.3 into Eq 1.2 , and replacing A_0 and ω_0 with parameters of an FM carrier signal, we get the FM signal as follows

$$s_{fm}(t) = A_c \cos\left(\omega_c t + \theta_{fm}(t) \right) = A_c \cos\left(\omega_c t + 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)\, dt \right)$$

Eq 1.4

FM: Modulating a sine wave

Information signal $\boxed{s_i(t) = A_i \cos(2\pi f_i t) = A_i \cos(\omega_i t)}$

Inputting this signal into the VCO's control port, the phase θ at time t becomes,

$$\theta_{fm}(t) = 2\pi K_{fm} A_i \times \int_{-\infty}^{t} \cos(\omega_i t)\, dt$$

$$= 2\pi K_{fm} A_i \times \frac{\sin(\omega_i t)}{\omega_i}$$

$$= \frac{K_{fm} A_i}{f_i} \sin(\omega_i t)$$

$$= \frac{\Delta f}{f_i} \sin(\omega_i t)$$

$$= \beta_{fm} \sin(\omega_i t)$$

$W_i = 2\pi fi$

The frequency-modulated signal is

$$s_{fm}(t) = A_c \cos\left(\omega_c t + \beta_{fm} \sin(\omega_i t)\right), \qquad\qquad (1)$$

$$where : \omega_c = 2\pi f_c, \omega_i = 2\pi f_i, \beta_{fm} \sin(\omega_i t) = \frac{\Delta f}{f_i} \sin(\omega_i t)$$

$\Delta f \rightarrow$ frequency deviation

$\beta_{fm} \rightarrow$ modulation index

Δf represents the maximum frequency deviation of the FM carrier. We can note that the highest and lowest frequencies (found by differentiating its instantaneous phase) will be $f_c \pm \Delta f$.

When the amplitude of the information signal input to the VCO is 0, $s_{fm}(t)$ has frequency f_c. In all other situations, the instantaneous frequency of $s_{fm}(t)$ can be calculated with

$$f_{fm\ inst}(t) = f_c + K_{fm} s_i(t) \ \text{Hz}$$

$K_{fm} \rightarrow$ Frequency sensitivity (Hz/volt). It indicates the change in the

carrier frequency per 1 volt of the message signal

- In FM a large number of pairs of upper and lower sidebands are generated.
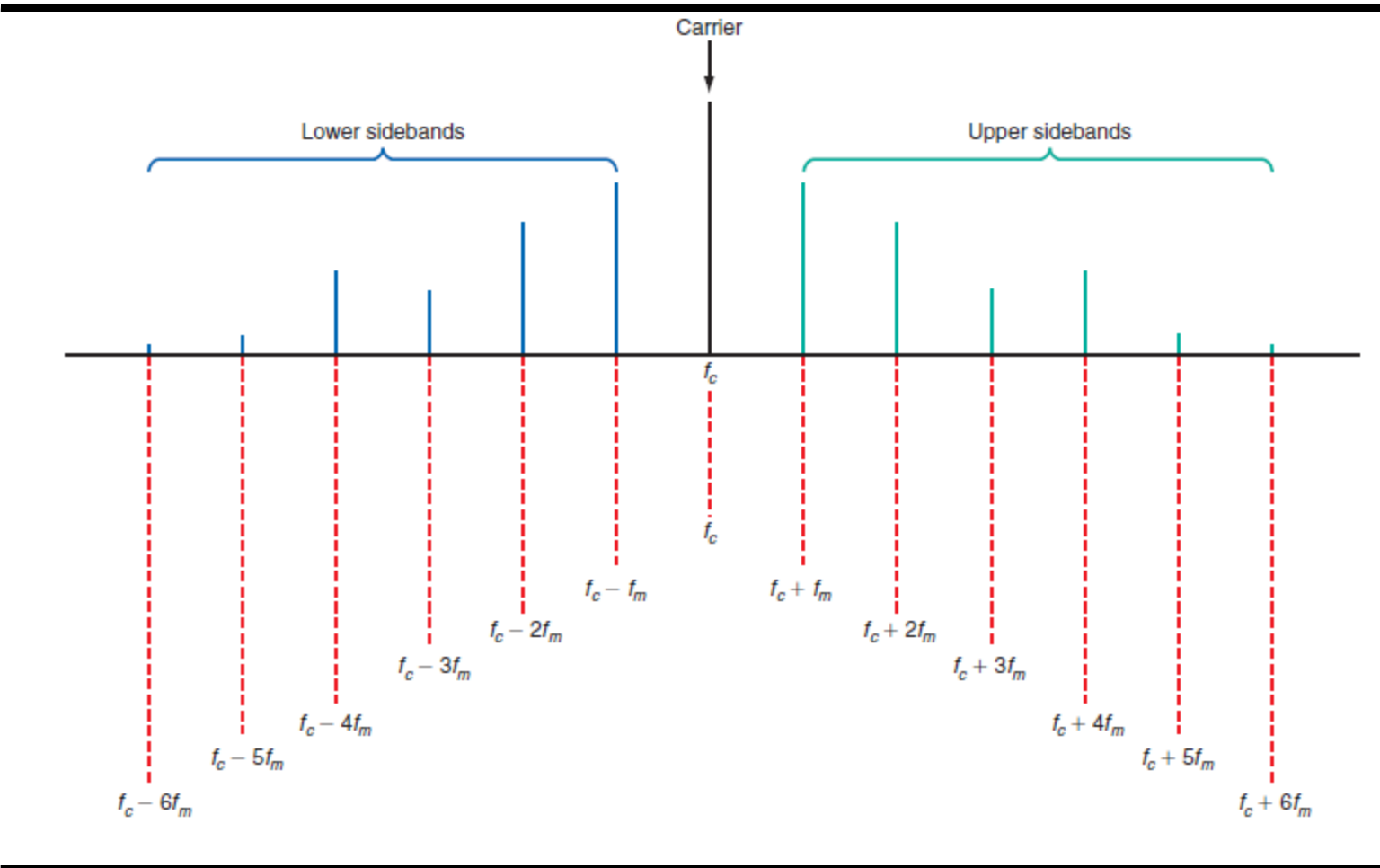
Frequency spectrum of FM signal

- As a result, the spectrum of an FM or a PM signal is usually wider than that of an equivalent AM signal
- Note that the sidebands are spaced from the carrier fc and one another by a frequency equal to the modulating frequency fm.

- Theoretically, the FM process produces an infinite number of upper and lower sidebands and a theoretically infinitely large bandwidth.
- The ratio of the frequency deviation to the modulating frequency is known as the modulation index mf or β_{fm} :

$$\beta_{fm} = f_d/f_m$$

- where f_d is the frequency deviation and f_m is the modulating frequency.
- For example, if the maximum frequency deviation of the carrier is ±12 kHz and the maximum modulating frequency is 2.5 kHz, the modulating index is $\beta_{fm} = f_d/f_m = 12/2.5 = 4.8$

Ex: What is the deviation ratio of TV sound if the maximum deviation is 25 kHz and the maximum modulating frequency is 15 kHz?

Types of FM

✓Narrowband FM

✓Wideband FM

- The classification is based on the modulation index

- Narrowband FM (NFM) $\beta_{fm} << \pi/2$ or 1

- Wideband FM (WFM) $\beta_{fm} >> \pi/2$ or 1

Narrowband FM (NFM)

- We know,

$$s_{fm}(t) = A_c \cos\left(\omega_c t + \beta_{fm} \sin(\omega_i t)\right),$$

- By using sum-to-difference trigonometric identity,

$$s_{fm}(t) = A_c \cos(\omega_c t) \cos(\beta_{fm} \sin(\omega_i t)) - A_c \sin(\omega_c t) \sin(\beta_{fm} \sin(\omega_i t))$$
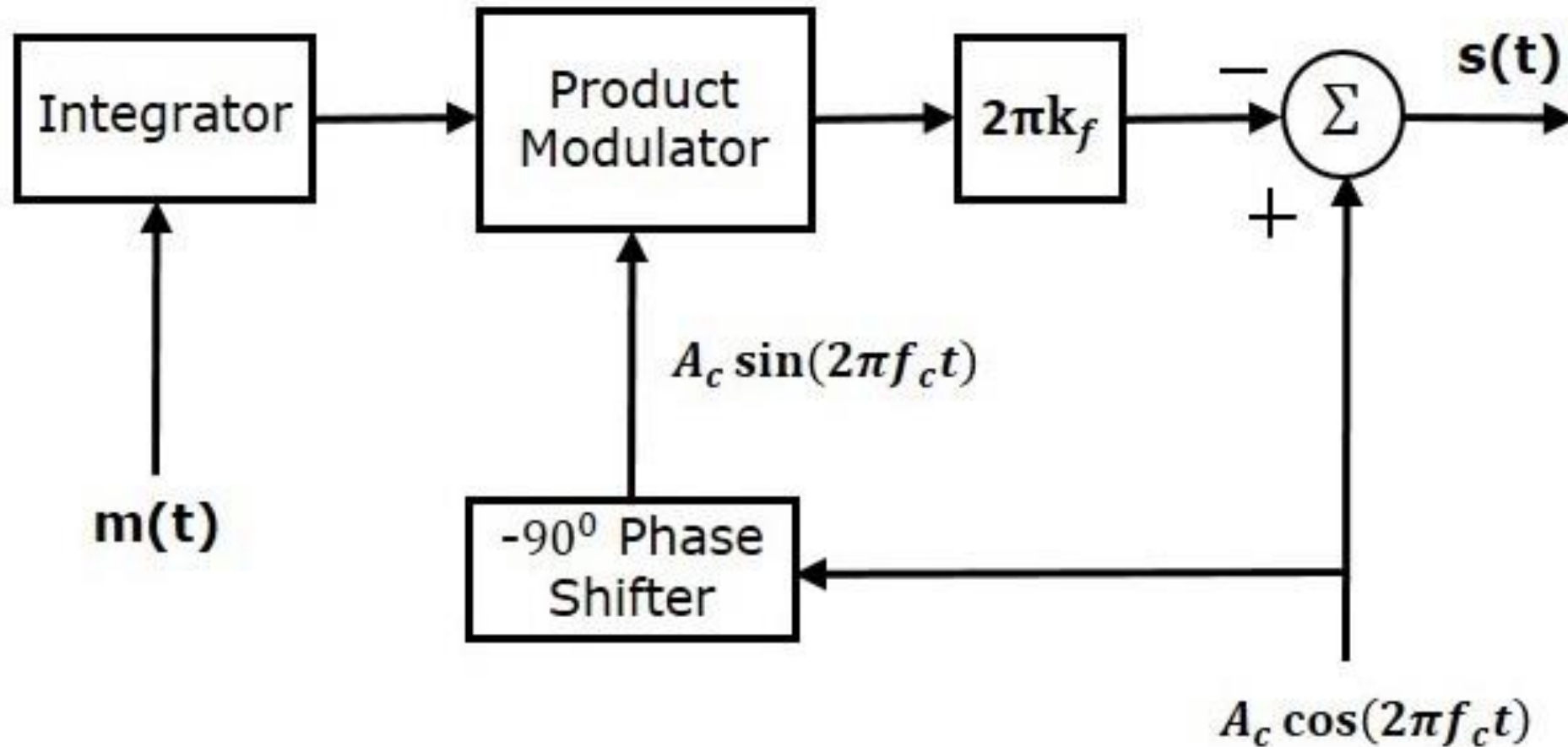
- For minimal values of β_{fm} or m_{fm}, we assume

$$\cos(\beta_{fm} \sin(\omega_i t)) \cong 1, \quad and \quad \sin(\beta_{fm} \sin(\omega_i t)) \cong \beta_{fm} \sin(\omega_i t)$$

- Thus,

$$s_{fm-nfm}(t) = A_c \cos(\omega_c t) - A_c \sin(\omega_c t)\beta_{fm} \sin(\omega_i t)$$

$$= A_c \left[\cos(\omega_c t) + \frac{\beta_{fm}}{2} \cos(\omega_c + \omega_i)t - \frac{\beta_{fm}}{2} \cos(\omega_c - \omega_i)t \right]$$

- This equation closely resembles the equation of the AM-DSB-TC signal.

Narrowband FM (NBFM) Generation

- NFM signal is essentially the same as the AM-DSB-TC signal, with the one difference that its lower sideband (LSB) is inverted.

- That is the LSB of the NFM signal is 180 degrees out of phase with the lower sideband of the AM-DSB-TC signal

- The spectra of NFM and AM-DSB-TC are similar meaning that the bandwidths of these two signals are approximately the same.
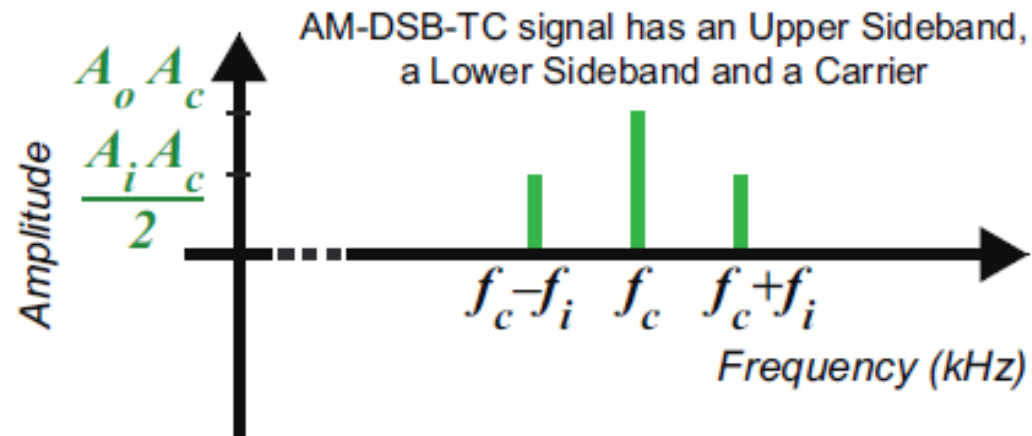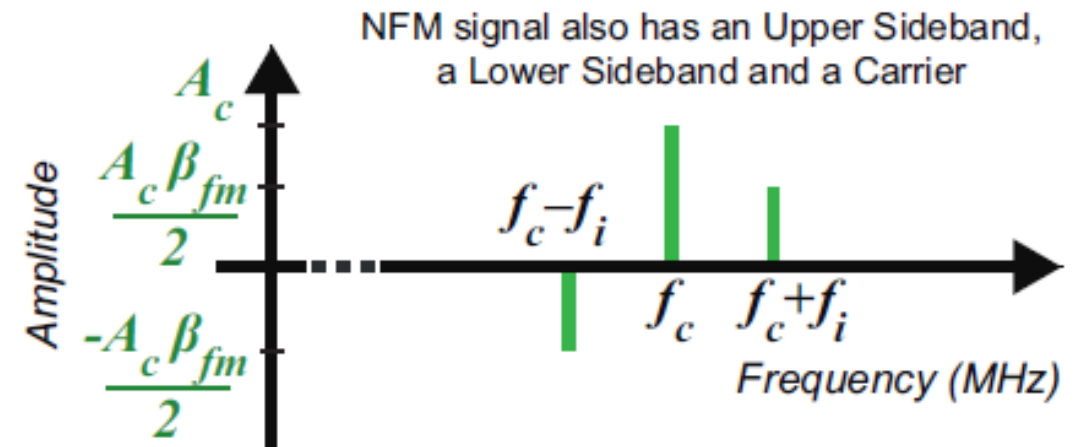
Advantages of FM over AM

- **Improved Signal-to-Noise Ratio (SNR)** FM is less susceptible to noise compared to AM. In FM, the information is carried by the frequency variations, whereas in AM, it is carried by the amplitude. Since most noise in communication systems affects amplitude, FM signals are less affected, leading to better sound quality and a higher signal-to-noise ratio. FM is beneficial in environments with interference or background noise, such as urban areas.
- **Better Sound Quality:** FM generally provides better sound quality than AM. The wider bandwidth used by FM allows for higher fidelity and more accurate reproduction of audio signals. This is why FM is commonly used for high-fidelity music broadcasts. FM can carry a greater range of frequencies, resulting in a better listening experience for music and speech.
- **Reduced Distortion** FM is less prone to signal distortion than AM. Since amplitude variations in AM can be easily distorted by noise or fading, this leads to distortions in the received signal. FM avoids this issue as the amplitude remains constant, and only frequency changes carry information.

- **Capture Effect** FM receivers experience a phenomenon called the capture effect, where they tend to lock onto the strongest signal and ignore weaker ones. This makes FM less prone to interference from other stations broadcasting at nearby frequencies, unlike AM, where interference from adjacent stations can be a problem.
- **Constant Transmitted Power** In FM, the power of the transmitted signal is constant, regardless of the modulation index or the complexity of the modulating signal. In AM, the power of the transmitted signal varies with the amplitude of the modulating signal, which means that AM transmission can be less power-efficient.
- **Wider Bandwidth:** Although FM requires a wider bandwidth compared to AM, this is an advantage in certain situations. Wider bandwidth in FM allows for the transmission of additional information, such as stereo sound in radio broadcasts and higher-quality audio.
- **Resilience to Fading:** FM is more resistant to fading, which is caused by multiple path propagation of signals (when radio waves reflect off objects like buildings or mountains). In AM, such effects cause variations in signal strength, leading to a distorted signal, while FM's constant amplitude makes it more immune to such issues.
- **Better for Long-Distance Broadcasting:** FM is less affected by changes in atmospheric conditions than AM, leading to more reliable long-distance communication, especially when using repeaters.