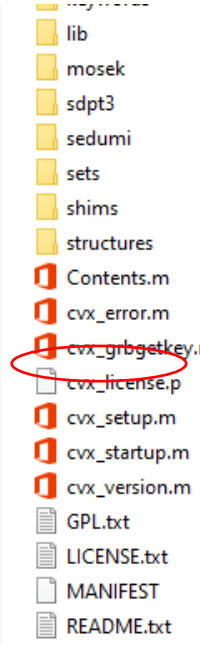
22MAT204 – Mathematics for Intelligent Systems - 3
Practise Sheet-11
(CVX and SVM)

- CVX – Convex Optimization Programming
  Download CVX from: http://cvxr.com/cvx/download/

OS	mexext	Download links	
Standard bundles, including Gurobi and/or MOSEK			
Linux	mexa64	cvx-a64.zip	cvx-a64.tar.gz
Mac	mexmaci64	cvx-maci64.zip	cvx-maci64.tar.gz
Windows	mexw64	cvx-w64.zip	cvx-w64.tar.gz
Redistributable: free solvers only			
All platforms		cvx-rd.zip	cvx-rd.tar.gz
All platforms (v1.22)		cvx-1.22.zip	cvx-1.22.tar.gz
Commercial solvers only			
Linux	mexa64	cvx-a64-co.zip	cvx-a64-co.tar.gz
Mac	mexmaci64	cvx-maci64-co.zip	cvx-maci64-co.tar.gz
Windows	mexw64	cvx-w64-co.zip	cvx-w64-co.tar.gz



Run this file

Automatically Set the path. Matlab should be 'On'

Example1: Solve using CVX

Maximise $Z = 4x + y$

subject to the constraints:

$$x + y \leq 50$$
$$3x + y \leq 90$$
$$x \geq 0, y \geq 0$$

```
% Example 1
cvx_begin quiet
variables x y
maximize 4*x+y
subject to
x+y<=50
3*x+y<=90
x>=0
y>=0
cvx_end
sprintf('x=%0.2f  y=%0.2f maxvalue=%0.2f',x,y,4*x+y)
```

Output:

ans = 'x=30.00 y=0.00 maxvalue=120.00'

```
% Example 1 after introducing the slack variables and writing as
matrices and vectors
A=[1 1 1 0; 3 1 0 1]; b=[50;90];
c=[4 1 0 0];
cvx_begin quiet
variables x(4)
maximize c*x
subject to
A*x==b;
x>=0
cvx_end
% display
x
Z=c*x
```

Maximize $Z = c^T x$

subject to

$Ax = b;\ x \geq 0;$

where

$$A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{pmatrix}; b = \begin{pmatrix} 50 \\ 90 \end{pmatrix}; x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}; c = \begin{pmatrix} 4 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Output:

```
x =   30.00
      0.00
     20.00
      0.00
```

Z = 120.00

Example 2: Solve using CVX

Minimise $Z = 200x + 500y$

subject to the constraints:

$$x + 2y \geq 10$$
$$3x + 4y \leq 24$$
$$x \geq 0, y \geq 0$$

```
%Example2:
cvx_begin quiet
variables x y
minimize 200*x+500*y
subject to
x+2*y>=10
3*x+4*y<=24
x>=0
y>=0
cvx_end
sprintf('x=%0.2f  y=%0.2f minvalue=%0.2f',x,y,200*x+500*y)
```

Output:

ans = 'x=4.000000 y=3.000000 minvalue=2300.000024'

```
% Example 1 using matrix and vector representations
A=[1 2 -1 0; 3 4 0 1]; b=[10;24];
c=[200 500 0 0];
cvx_begin quiet
variables x(4)
maximize c*x
subject to
A*x==b;
x>=0
cvx_end
% display
x
Z=c*x
```

Output:

x=4.00
3.00
0.00
0.00

Z=2300

Example 3: Solve using CVX

Maximize z=3x+9y

subject to the constraints:

$$x + 3y \le 60$$
$$x + y \ge 10$$
$$x \le y$$
$$x \ge 0, y \ge 0$$

```
cvx_begin quiet
variables x y
maximize 3*x+9*y
subject to
x+3*y<=60
x+y>=10
x<=y
x>=0
y>=0
cvx_end
sprintf('x=%0.2f  y=%0.2f maxvalue=%0.2f',x,y,3*x+9*y)
```

'x=8.23 y=17.26 maxvalue=180.00'

- The solution for maximization problem is actually, 'infinite solutions-all pts between (15,15) and (0,20) with maximum value 180. But CVX will give you one point on this line segment.
- If we change the problem to minimize we will get the answer as (5,5) with minimum value as 60.

Practice Questions:

1. Solve the following Convex optimization problems using CVX.

(a) Minimize $6x - 9y$

subject to $x - y \geq 2$

$3x + y \geq 1$

$2x - 3y \geq 3$

(b) Minimize $x^2 + 2y^2$

subject to $x + y \geq 1;\ \ x, y \geq 0$

(c) Maximize, $3 - (x-1)^2 - (y-1)^2$

subject to $2x + x^2 + y^2 \leq 16$

$3x - 7y = 21$

(d) Minimize $x^2 + y^2$

subject to $x + y \leq 4$

$2x + x^2 + y^2 \leq 15$

2. Consider the problem: Minimize x+y

subject to $(x-1)^2 + (y-1)^2 \leq 1$

$x \leq 1, y \leq 1$

(a) Draw the feasible region for the problem and solve graphically.

(b) Solve the problem using CVX

3. Solve all the convex problems that was discussed in the class using CVX tool.

4. Solve the following optimization problems graphically (Manually). Also solve them using CVX.

(a) Maximize $-6x + 9y$

subject to $x - y \leq 2$

$3x + y \leq 1$

$2x - 3y \leq 3$

(b) Maximize $x + y$

subject to $x^2 + y^2 \leq 4$

$x \geq 1$

(c) Minimize $x^2 + y^2$

subject to $(x-1)^2 + y^2 \leq 9$

$x \leq 1, y \leq 1$

(d) Minimize x+y

subject to $x + y \leq 4$

$2x + x^2 + y^2 \leq 15$

Support Vector Machine - SVM

- **Linear SVM Classifier (Hard Margin):**

$$\min_{w,\gamma} \frac{1}{2} w^T w \qquad \text{In matrix form,} \qquad \min_{w,\gamma} \frac{1}{2} w^T w$$
$$s.t\ d_i\left[w^T x_i - \gamma\right] \geq 1; \forall i \qquad s.t\ D\left[Aw - \gamma e\right] \geq e$$

```
clc; clear all; close all;
a = 3;
A = [1 2; 2 1; 2 2; 3 3; 3 4; 4 3]; % data points
d = [-1*ones(a,1);ones(a,1)]; % class labels
n = size(A,2); m = size(A,1); e = ones(m,1);
figure; gscatter(A(:,1),A(:,2),d,'br','*o');hold on

%% Linear SVM Primal form
cvx_begin
   variables w(n) g;
   minimize ((0.5*w'*w))
   subject to
      for i= 1:m
         d(i)*(A(i,:)*w- g) >= 1;
      end
cvx_end
w
g
```

- **Linear SVM Classifier (Soft Margin):**

$$\min_{w,\gamma,\xi} \frac{1}{2} w^T w + c\sum_i \xi_i \qquad \text{In matrix form,} \qquad \min_{w,\gamma,\xi} \frac{1}{2} w^T w + ce^T \xi$$
$$s.t\ d_i\left[w^T x_i - \gamma\right] + \xi_i \geq 1 \qquad s.t\ D\left[Aw - \gamma e\right] + \xi \geq e$$
$$\xi_i \geq 0 \qquad \xi \geq 0$$

```
clc;clear;close;
a = 5;
A = [0 0;2 0; 2 2;0 2;2.5 2.5;2.6 2.4; 3 3;4 3;4 4;3 4];
d = [-1*ones(a,1);ones(a,1)]; % class labels
D = diag(d); % diagonal matrix
figure; gscatter(A(:,1),A(:,2),d,'br','*o');hold on

%% L1 SVM soft margin Primal
n = size(A,2); m = size(A,1); e = ones(m,1);
c = 1.1;
cvx_begin
   variables w(n) g Psi(m)
   minimize ((0.5*w'*w)+(c*e'*Psi))
   subject to
      D*(A*w-e*g)+Psi >= e;
      Psi >= 0;
cvx_end
```