Amrita School of Engineering, Bengluru-35
23MAT204
Mathematics for Intelligent Systems – 3
Lab Practice Sheet-7
(Probability – pdf, cdf, icdf)

- Evaluation of pmf (f(x)=P(X=x)) and cmf (F(x)=P(X≤x)) for standard discrete distributions:

  Binomial:

  >> pdf('bino', x, n,p)

  %finds f(x) =P(X=x) in binomial distribution with n trials and probability of success, p.

  >> cdf('bino', x, n,p)

  %finds F(x) =P(X≤x) in binomial distribution with n trials and probability of success, p.

  Poisson:

  >> pdf('pois', x, lambda) or

  %finds f(x) =P(X=x) in Poisson distribution with parameter lambda.

  >> cdf('pois', x, lambda)

  %finds F(x) =P(X≤x) in Poisson distribution with parameter lambda.

- Evaluation of cdf (F(x)=P(X≤x)) for standard continuous distributions:

  Continuous Uniform:

  >> cdf('unif', x, a,b)

  or

  >>unifcdf(x, a,b)

  %finds F(x) =P(X≤x) for uniform rv X with pdf, f(x)=1/(b-a), a<x<b

  Exponential

  >> cdf('exp', x, mean)

  Or

  >> expcdf(x, mean)

  %finds F(x) =P(X≤x) for Poisson rv X with given mean.

  Normal:

  >> cdf('norm', x, μ, σ)

  Or

  >> normcdf(x, μ, σ)

  %finds F(x) =P(X≤x) for normal rv X with mean μ and standard deviation, σ.

Examples:

```
%Binomial pdf
>> pdf('bino',2,4,0.1) % f(2) for n=4, p=0.1
ans =
   0.0486
```

```
%Binomial cdf
>> cdf('bino',2,4,0.1) % F(2) for n=4, p=0.1
ans =
   0.9963
```

```
%Poisson pdf
>> pdf('pois',5,5) % f(5) for λ=5
ans =
   0.1755
```

```
%Poisson cdf
>> cdf('pois',2,5) % F(2) for λ=5
ans =
   0.1247
```

```
%Exponential cdf
>> cdf('exp',30000,25000) % Evaluates F(30000) for exponential distrn with mean 25000
ans =
   0.6988
```

```
% Normal cdf
>> cdf('norm',290,300,50) % Evaluates F(290) for normal distrn with mean 300 and s.d. 50
ans =
   0.4207
```

- **Use of 'icdf' to find 'a' if cumulative function at a, F(a) is given**

>> **a** = icdf('D', F(a), P1, P2)

is used to find the value of 'a' for the distribution 'D', given the cdf at a is the probability value, F(a) and P1 and P2 are the parameters of the distribution D.

Examples:

1. Given $P(X<a)=0.85$, where X is normally distributed with mean 9 and standard deviation 2. Find 'a'.

   >>a = icdf('norm',0.85,9,2)

   a = 11.0729

2. Given $P(X>b)=0.35$, where X is an exponential random variable mean 5. Find b.

   $P(X>b)=1-F(b)=0.35$

   So $F(b)=0.65$

   >>b = icdf('exp',0.65,5)

   b = 5.2491

3. Given $P(-c < X < c) = 0.8732$ where X is a standard normal random variable. Find c.

   $P(-c < X < c) = 0.8732 \rightarrow 2*P(0<X<c) = 0.8732$ (since std normal is symmetric about the origin)

   $\rightarrow 2 * (F(c) - F(0)) = 0.8732$

   $\rightarrow 2 * (F(c) – 0.5) = 0.8732$

   $\rightarrow 2 * F(c) = 1.8732 \rightarrow F(c) = 0.9366$

   >>c = icdf('exp',0.9366,0,1)

   c = 1.5268

- **Plot of pdf and cdf of probability distributions:**

```
%Plot of pdf of exponential distribution with lambda=0.5 (mean=2)
clf;clc;
x=0:0.1:10;
mean=2;
f=exppdf(x,mean);
plot(x,f)
hold on
F=expcdf(x,mean);
plot(x,F,'r')
```

```
%Plot of pdf of uniform distribution with a=2, b=7
clf;clc;
x=0:0.1:15;
a=2;
b=10;
f=unifpdf(x,a,b);
plot(x,f)
F=unifcdf(x,a,b);
plot(x,F)
```

```
%Plot of pdf of normal distribution with mu=3, sigma=1
clf;clc;
x=0:0.1:10;
Mu=3;
Sigma=1;
f=normpdf(x,Mu,Sigma);
plot(x,f)
F=normcdf(x,Mu,Sigma)
plot(x,F)
```

```
%Plot of pdf of standard normal distribution in [-5,5]
clf;clc;
x=-5:0.1:5;
Mu=0;
Sigma=1;
f=normpdf(x,Mu,Sigma);
plot(x,f)
F=normcdf(x,Mu,Sigma)
plot(x,F)
```

Practice Problems:

1. If X is a Binomial random variable with n=10, probability of success, p=0.3, find P(X=5), P(X=10), P(X≤2), P(X>8).
2. If X is a Poisson random variable with mean 5, find (P(X=0), P(X=2), P(X≤11), P(X>25).
3. The thickness of a flange on an aircraft component is uniformly distributed between 0.96 and 1.06 millimeters.

   (a) Determine the cumulative distribution function of flange thickness.

(b) Determine the proportion of flanges that exceeds 1.02 millimeters.

(c) What thickness is exceeded by 90% of the flanges?

4. Suppose X has a uniform distribution over the interval [1.5, 5.5]. Determine (i) the mean and variance (ii) $P(X < 2.5)$ (iii) $P(X > 2 / X < 2.5)$ and (iv) $P(X > 1.5 / X < 2.5)$
5. Suppose X has an exponential distribution with mean 0.75, determine the following:
   (a) $P(X < 1.5)$ (b) $P(X \geq 1.5)$ (c) $P(-1<X<2)$
6. Suppose X has an exponential distribution with mean equal to 10. Determine :
   (a) $P(X > 10)$ b) $P(X > 20)$ c) $P(X > 30)$ d) Find a such that $P(X < a) = 0.95$.
7. Given that X has a normal distribution with $\mu = 200$ and $\sigma = 75$, find the probability that X assumes a value (a) greater than 300 (b) lesser than 170 (c) between 180 and 240
8. Assume X is a standard normal RV. Find the value of 'b' if $P(X<b) = 0.75$
9. Assume X is a standard normal RV. Find the value of 'c' if $P(X>c) = 0.002$
10. Given that X has a normal distribution with mean $\mu = 10$ and standard deviation $\sigma=3$.
    (a) If the probability that X assumes a value greater than b is 0.6, find b.
    (b) Also find $P(7<X<11)$.
11. Assume X is a normal RV with mean 15 and variance 4. Find the value of 'b' if $P(X<b) = 0.9$.