Modelling, Simulation And Analysis Notes

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Part I

Notes

Chapter 1

Course Overview

1. Introduction to Modelling
2. Bond graph modelling
3. Basic System Models

The textbooks to be followed is: “System Dynamics - Modeling, Simulation And Control Systems”, “Modern Control Engineering, Katsuhiko Ogata”

What is modelling? It’s a mathematical representation of a system. These representations allow us to evaluate the results of that given system. This course is focused on equipping us with the working knowledge to develop such mathematical models for our systems.

Upon modelling the system, we can start controlling this system. With the equations we have made, we can now add our own inputs to those equations. This in a nutshell is what the course is about.

Chapter 2

Unit 1

2.1 What Is Modelling?

- Model - Models of the systems are simplified, abstracted constructs used to predict their behaviour.
- Mathematical Modelling - Predicts only a certain aspect of the behaviour

2.2 Uses Of Modelling

Modelling consists of input variables U, which performs certain operations on a dynamic system S, with state variables X to give some output variables U. Analysis means that given an input for the present, and the system itself.

We can predict the outputs for the future. To exert some form of control, the system S are used. Another term used is identification, identification means that you already U, and Y. We know the inputs and the outputs, and we map U to Y with the system S. We **identify** the system that would map the inputs to the outputs. This is done by experimenting and figuring out what mapping works best to generate the outputs from the inputs. The way to tell whether a model is good is to see if it is consistent with large sets of input and outputs.

2.3 Types Of Modeling

Principle of superposition - it means that there's a linear correlation of the inputs to the outputs.

Type Of System	Description	Differential Equations
Linear	Principle of superposition applies	Linear differential equations.
Distributed	Dependent variables are functions of space and time	Partial differential equations
Lumped	Dependent variables are independent of spatial coordinates	Ordinary differential equations
Time - varying	Model parameters vary in time	Differential equations with time-var
Stationary	Model parameters are constant in time	Differential equations with constant
Continuous	Dependent variables over continuous range	Differential equations
Discrete	dependent variables defined only for distinct values	Time-difference equations

2.4 System Nomenclature

Definition 2.4.1

A Dynamical system is a particle or ensemble of particles whose state varies over time and thus obeys differential equations involving time derivatives.

Definition 2.4.2

The state of a Dynamic system is the smallest set of variables(called state variables) such that the knowledge of these variables at $t = t_0$, together with the knowledge of the input for $t \geq t_0$, completely determines the behaviour of the system for any time $t \geq t_0$. The number of state variables is the same as the number of inital conditions needed to completely solve the system models

Definition 2.4.3

If n state variables are needed to completely describe the behaviour of a given system, then the n state variables can be considered the components of a vector X. Such a vector is called a state vector.

Definition 2.4.4

The state space is the n -dimensional space whose coordinate axes consist of the x_1 axis, $x_2 axis$ where $x_1, \cdots, x_n$

Suppose you started observing your model at point t_0. You can read any point in the state space and immediately figure out exactly how the system was behaving at any given point.

Input Variables: $U_{n\times 1} = [U_1, U_2, \cdots, U_r]^T$ State Variables: $X_{n\times 1} = [X_1, X_2, \cdots, X_r]^T$ Output Variables: $Y_{n\times 1} = [Y_1, Y_2, \cdots, Y_r]^T$

We can write the derivatives of the state variables as functions of the inputs and the state variables and time.

We can then write these functions as one vector.

$$\dot{X}(t) = F(U, x, t)$$

We can similarly write the derivatives of the outputs as functions of the inputs and state variables.

$$\dot{Y}(t) = G(U, X, t)$$

We write the states' derivative as a linear combination, which can then be written as a matrix.

$$\dot{X}(t) = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{1n} \\ \vdots & \ddots & & \vdots \\ a_{n1} & a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \begin{bmatrix} b_{11} & b_{21} & \cdots & b_{1n} \\ \vdots & \ddots & & \vdots \\ b_{n1} & b_{n1} & \cdots & b_{nn} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$$

We get the equation,

$$\dot{X}(t) = A_{n\times n} X_{n\times 1} + B_{n\times r} U_{r\times 1}$$

A is the 'state' matrix, and B is the 'input' matrix

Similarly we can write,

$$\dot{Y}(t) = C_{m\times n}X_{n\times 1} + D_{m\times r}U_{r\times 1}$$

We apply the Laplace transform to convert integrals and derivatives into algebraic operations.

$$U(t) \rightarrow G(s) \rightarrow Y(t)$$

We get the transfer function $G(s)$ by the formula

$$G(s) = \frac{Y(s)}{U(s)}$$

So we can get our outputs, by

$$Y(s) = G(s)U(s)$$

Definition 2.4.5

A block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals. Such a diagram depicts the interrelationships that exist among the various components. Differing from a purely abstract mathematical representation, a block diagram has the advantage of indicating more realistically the signal flows of the actual system.

2.5 Modeling Of Dynamical Systems.

Consider the system shown in the figure. The main body of mass M is propelled along a horizontal track by a traction force f. The main body contains an actuator for rotating the pendulum. The actuator applies a torque T to the arm. The pendulum has a total mass m and a moment of inertia I relative to its mass center at point C. Katsuhiko Ogata - Modern Control Engineering-Prentice Hal (2010).pdf: Page 78 Systems example

2.6 Bond Graph Modelling

What we did earlier is a classical approach to modelling. We find the differential equations and generate matrix equations with inputs, state variables and output.

What we do here is,

$$\begin{gathered}\text{PhysicalSystem}\\ \downarrow\\ \text{EngineeringModel}\\ \downarrow\\ \text{BondGraphs}\\ \downarrow\\ \text{Software generateddifferential equations}\\ \downarrow\\ \text{SimulationLanguage}\\ \downarrow\\ \text{Output}\end{gathered}$$

2.7 Generalizd Variables

- Power Variables
  - Effort $e(t)$
  - Flow $f(t)$

Power $= e(t) \times f(t)$

- Energy variables
  - Momentum $p(t)$
  - Displacement $q(t)$

$p(t) = \int e(t)dt \; q(t) = \int f(t)dt$

	Effort	Flow
Electrical	Voltage	Current
Translational	Force	Velocity
Rotational	Torque	Angular velocity
Hydraulic	Pressure	Volumetric Flow
Chemical	Chemical Potential	Molar Flow
Thermodynamics	Temperature	Entropy Flow

1. Bond Graphs A bond graph is a labelled directed graph, each link has an assigned orientation.
2. Multiports Places at which subsystems can be interconnected, power can flow through ports between subsystems.

```
global m b k;
m = 1;
b =  1; %Damping constant
k = 1; % Spring constant

tspan = [0 10];
IC = [0 0];

[time, X_state] = ode45(@f,tspan , IC);

function x_dot = f(t,X)
  global m b k
  u = sin(t);
  x_dot = [X(2);-(b/m)*X(2) - (k/m)*X(1) + (u/m)];
end
figure('units', 'normalized', 'outerposition', [0 0 1 1])
subplot(2,2,1)
grid on;
xlabel('Time(s)')
```

```
ylabel('Displacement')
plot(time, X_state(:,1))

subplot(2,2,3)
grid on;
xlabel('Time(s)')
ylabel('Velocity')
plot(time, X_state(:,2))


subplot(2,2,[2 4])
grid on;
xlabel('Displacement(m)')
ylabel('Velocity(m/s)')
plot(X_state(:,1), X_state(:,2))
s output
```

MIT COURSE NOTES ON BOND GRAPH MODELLING

Chapter 3

Unit 2

3.1 Transformers

We have bond graphs now, and we can say certain things about the efforts and flows using bond graphs.

What we can do next is write functions known as transformers that convert the input effort into the output effort, or the input flow into the output flow, based on the causal stroke.

3.2 Gyrators

What we can do with gyrators convert the input of one form into the output of another form, based on the causal stroke.

3.3 Dynamical Response Of 1st Order Systems

A transfer function is the ratio of the output function by the input function, provided that the initial conditions are zero.

A first order system is a system such that the highest power is 1.

$$\frac{\mathcal{L}Output}{\mathcal{L}Input}$$

For a first order system we get,

$$\frac{1}{T_s + 1}$$

3.4 S-Plane

3.5 Maximum Overshoot

$$M_p = \frac{c(t_p) - c(\infty)}{c(\infty)}$$

3.6 Frequency Response

We write $s = \sigma + j\omega$

If we have a block diagram where X(s) through some linear transformation G(s) becomes Y(s)

We can write that

$$
\begin{aligned}
y(t) &= Y sin(\omega t + \phi) \\
&= X|G(j\omega)|sin(\omega t + \phi)
\end{aligned}
$$

$$
\begin{aligned}
Y &= X|G(j\omega) \\
\phi &= tan^{-1}\frac{Im(G(j\omega))}{Re(G(j\omega))} & = G(s)\frac{\omega X}{s^2 + \omega^2}
\end{aligned}
$$

We can then write it out as a polynomial.

$$
\begin{aligned}
Y(s) &= \frac{p(s)}{q(s)}\frac{\omega X}{s^2 + \omega^2} \\
Y(s) = \frac{a}{s + j\omega} + \frac{\overline{a}}{s - j\omega} &+ \frac{b_1}{s + s_1} + \frac{b_2}{s + s_2} \\
y(t) = ae^{-j\omega t} + \overline{a}e^{j\omega t} &+ b_1 e^{-s_1 t} + b_2 e^{-s_2 t}
\end{aligned}
$$

Thus the steady state,

$$
y(t) = ae^{-j\omega t} + \overline{a}e^{j\omega t}
$$

Ogata Chapter 7, first three pages.

Chapter 4

Unit 3

4.1 Frequency Response

A stable system is such that the output has a frequency with a negative real part.

A system where the input is a sinusoid.

A system is linear, then the output's frequency must be the same as the input frequency

The output's amplitude is,

$$Y = X * |G(j\omega)|$$

The output is shifted by a phase ϕ

$$\phi = tan^{-1}(\frac{Im(G(j\omega))}{Re(G(j\omega))})$$

$$G(j\omega) = \frac{K}{T(j\omega + 1)} = \frac{K}{1 + j(\omega T)}$$

The phase is defined for the transfer function as,

$$ang(G(j\omega)) = tan^{-1}\frac{Im(numerator)}{Re(num)} - tan^{-1}\frac{Im(denominator)}{Re(denominator)}$$

Thus the steady state becomes,

$$y_{ss}(t) = \frac{XK}{\sqrt{1 + \omega^2 T^2}} sin(\omega t - tan^{-1}\omega T)$$

We express the magnitude of the transfer function $G(j\omega)$ in terms of decibels

$$G(j\omega)\text{in Db} = 20log_{10}|G(j\omega)$$

The plot is a 'semi-log' plot where one of the axes are plotted on a logarithmic scale instead of linear. The decibel values act as the y-axis and the angular frequency is equally spaced in multiples of 10.

4.2 Bode Plots

Part II

Syllabus

- Mathematical Modelling
  - Mechanical and Electrical(FBD, dynamics)
- State-Space Representation
- Bond Graph Modelling
  - Mechanical And Electrical
  - Source Of Flow, Source Of Effort, Inductance, Capacitance, Resistance, Transformer, Gyrator

Modern Control Engineering 5th Edition.pdf: Page 172 Dynamical Response of 2nd Order System in under - damping conditions with Step Input. In pt 3 above, get yourself comfortable with

- Dynamic Response Of 1st and 2nd Order Systems
  - Unit step, ramp and impulse.
  - Underdamped Unit Step
  - theoretical and graphical meaning of rise time, peak time, overshoot, settling time, etc.

Part III

Project Notes

- Simscape, or a matlab simulink model
- Provision is made for solidworks CAD models.