Amrita Vishwa Vidyapeetham

School of Artificial Intelligence
Course Code: 23AIDS202 Introduction to Robotics
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Practice Sheet 01
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Batch: B.TECH. AI&DS; Sem-III Sec-F **Campus:** Bengaluru
Duration: - **Full Marks:** -

(Use this as a practice to clear and apply concepts relevant to the course and taught during lecture hours.)

1. For the RPP - robotic manipulator shown below, using Denvit-Hartenberg convention draw the directions of coordinate axes and find the DH-parameters. This robotic manipulator has 3 joints - revolute, prismatic, and prismatic, hence the name RPP-robotic manipulator. Using DH-Parameters, construct the following homogeneous transformation matrices (order 4 x 4): ${}^0_3T$, ${}^0_1A$, ${}^1_2A$, and ${}^2_3A$.

   Hint: ${}^{i-1}_{i}A = Rot(Z, \theta_i) * Trans(Z, d_i) * Trans(X, a_i) * Rot(X, \alpha_i)$ and, ${}^0_3T = {}^0_1A\,{}^1_2A\,{}^2_3A$

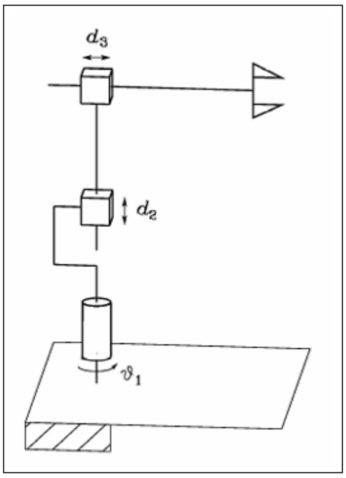

Figure 1: RPP Robotic Manipulator (Q1)

2. Repeat the above question for the SCARA Robotic Manipulator in Figure 2 below.
3. Prove the relation ${}^0_2\vec{\omega} = {}^0_1\vec{\omega} + {}^0_1R\,{}^1_2\vec{\omega}$; where $\vec{\omega}$ is the angular velocity, R is the rotation matrix, and $\{0\}$, $\{1\}$, $\{2\}$ are the three coordinate frames.
4. Compute the homogeneous transformation representing a translation of 3 units along the x-axis followed by a rotation of $\frac{\pi}{2}$ about the current z-axis followed by a translation of 1 unit along the fixed y-axis. Sketch the new frame's pose relative to the fixed frame.
5. Consider the diagram shown in Figure 3. Find the homogeneous transformations ${}^0_2H$, ${}^0_1H$, and ${}^1_2H$ from the geometry of the pose. Show that ${}^0_2H = {}^0_1H\,{}^1_2H$.
6. The rotation matrix between a fixed frame $\{A\}$ and a moving frame $\{B\}$ is given as:

$$R = \begin{bmatrix} 1.000 & 0.000 & 0.000 \\ 0.000 & 0.866 & -0.500 \\ 0.000 & 0.500 & 0.866 \end{bmatrix}$$

   Find the equivalent x-y-z Roll-Pitch-Yaw angles associated with this rotation matrix.

7. A vector must be mapped through three rotation matrices:

$${}^{A}P = {}^{A}_{B}R\,{}^{B}_{C}R\,{}^{C}_{D}R\,{}^{D}P$$

One choice is to first multiply the three rotation matrices together to form ${}^{A}_{D}R$ giving:

$${}^{A}P = {}^{A}_{D}R\,{}^{D}P$$

Another choice is to transform the vector through the matrices one at a time - that is:

$$\begin{aligned}
{}^{A}P &= {}^{A}_{B}R\,{}^{B}_{C}R\,{}^{C}_{D}R\,{}^{D}P \\
{}^{A}P &= {}^{A}_{B}R\,{}^{B}_{C}R\,{}^{C}P \\
{}^{A}P &= {}^{A}_{B}R\,{}^{B}P \\
{}^{A}P &= {}^{A}P
\end{aligned}$$

Answer the following: (i) Which method is computationally effective? (ii) If ${}^{D}P$ is changing at 100 Hz, we would have to recalculate ${}^{A}P$ at the same rate. However, the three rotation matrices (${}^{A}_{B}R$, ${}^{B}_{C}R$, and ${}^{C}_{D}R$) are also changing at 30 Hz. What is the best way to organize the computation to minimize the calculation effort (multiplications and additions)?

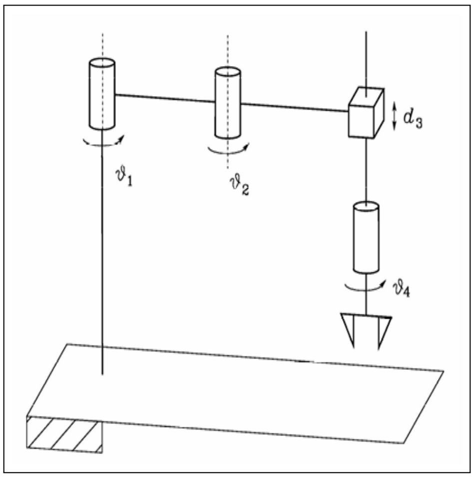

Figure 2: SCARA Robotic Manipulator (Q2)

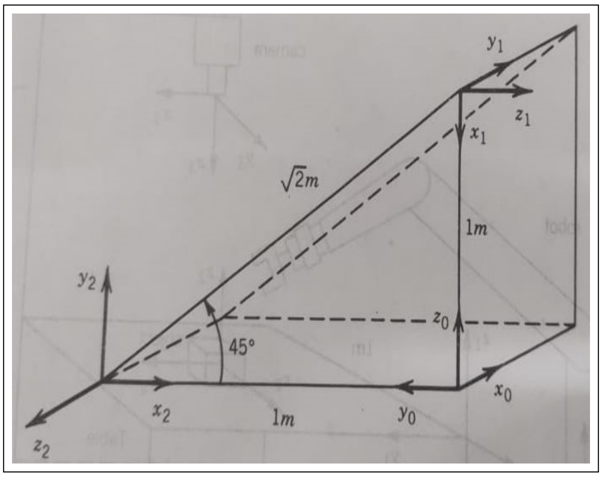

Figure 3: Pose of different coordinate frames

8. Two frames A and B are related by the following homogeneous transformation matrix:

$$
{}^{A}_{B}T = \begin{bmatrix} 0 & 1 & 0 & 15 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & -9 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

a) Where is the origin of the frame B located when seen in frame A and vice-versa?
b) If the orientation vector of a parameter is given in frame B as $V_B = [10, 10, 10]^T$. What is the orientation in frame A?
c) If there exists a point P whose coordinate in frame A are given as ${}^{A}P = [10, 10, 10]^T$, find its coordinates in frame B?

9. Under what condition do two rotation matrices representing finite rotations commute?

10. Find the axis of rotation (unit vector and the angle of rotation) associated with the rotation matrix given below. Show the steps.

$$
{}^{A}_{B}R = \begin{bmatrix} a_{11} & a_{12} & a13 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}
$$

====== Best of Luck ======