23MAT204
Mathematics for Intelligent Systems-3
Practice Sheet -1
Creation of Matrices with Given Properties

- rand(1) — Generation of a random number between 0 and 1
- rand(2)
- rand(2,3) — Generation of a random matrix(square/rectangular) with elements
- randi(5) — Generates an integer random number between 0 and 5
- randi([3,15]) — Generates an integer random number between 3 and 15
- randi([0,5],2) — Generates random square matrix of order 2, with elements between 0 and 5
- randi([2,6],3,4) — Generates random 3×4 matrix with elements between 2 and 6
- magic(5) — Generates a magic matrix that has row sum, column sum to be same

1. **Generation of an integer valued symmetric matrix**

```
>> A=randi([1 9],3,3)
>> S=A+A'
```

2. **Generation of an integer valued skew-symmetric matrix**

```
>> A=randi([-9 9],5,5)
>> S=A-A'
```

3. **Generation of an orthogonal matrix**

Result used – QR decomposition of a square matrix A as a product of an orthogonal matrix Q and an upper triangular matrix, R using the MATLAB command qr(A).

```
>> A= randi([1 9],3,3)
>> [Q,R]=qr(A)
>> Q
% Q is the orthogonal matrix
```

4. **Generation of random integer matrices with given rank**

- Results used –
  1. Maximum Rank of an m×n matrix is min(m,n)
  2. Rank(AB)≤min(Rank(A),Rank(B))

```
>> A=randi([0 9],3,1)*randi([0 9],1,3)
        % Generates a random 3×3 matrix of rank 1.
```

```
>>rank(A)

>>B=randi([0 9],5,2)*randi([0 9],2,7)
        % Generates a random 5×7 matrix of rank 2.
>>rank(B)
>>C=randi([0 9],6,3)*randi([0 9],3,4)
        % Generates a random 6×4 matrix of rank 3.
>>rank(C)
```

5. (a) Generating an upper triangular matrix (extracting the upper triangular elements of a given matrix)

```
A=randi([0,9],4)
for i=2:n      % start from second row
               for j =1:i-1 % upto diagonal
A(i,j)=0;
    end
    end
U=A
```

(b) Generating an upper triangular matrix using triu command

```
>> A=randi([0,9],4,4)
>> L=triu(A)
```

6. (a) Generating a lower triangular matrix (extracting the lower triangular elements of a given matrix)

```
A=randi([0,9],4)
for i=2:n      % start from second row
   for j =1:i-1 % upto diagonal
A(j,i)=0;
end
end
L=A
```

(c) Generating a lower triangular matrix using tril command

```
>> A=randi([0,9],4,4)
>> L=tril(A)
```

7. Generating an integer valued square matrix with integer valued inverse

```
A=randi([0,9],3,3);
B= A-diag(diag(A)); % make diagonal elements as zero
B=B+eye(3) ; % Make a matrix with all diagonal elements as 1
A1=triu(B);      % det(A1)=1
A2= tril(B); % det(A2)=1
AA=A1*A2 ; % det(A1*A2)= det(A1)*det(A2)=1
% Therefore its inverse will also be integer matrix
% det(AA)
detAA=det(AA)
inverseAA=inv(AA)
%AA is the integer mtrix which has inverse also as an integer matrix
```

8. Generating an integer valued square matrix with given eigenvalues and eigenvectors

```
Ld1=1;
Ld2=2;
Ld3=3;
Ev1=[1;1;1];
Ev2=[2;1;-1];
Ev3=[1;2;-1];
V=[Ev1,Ev2,Ev3]
D=diag([Ld1,Ld2,Ld3])
A=V*D*inv(V)
```

9. Generating an integer valued square matrix with given eigenvalues

```
% First create a matrix with determinant is 1
    A=randi([-3 3], 3,3);
    B= A-diag(diag(A));
    B=B+eye(3) ;
    A1=triu(B);
    A2= tril(B);
    AA=A1*A2 ;
% Create a diagonal matrix with eigenvalues 3,4,5
    S=diag([3,4,5]);
    BB=AA*S*inv(AA)
% BB is the required matrix
```

10. Generating an integer valued square matrix with given determinant

```
% First create a matrix with determinant as 1
        A=randi([-3 3], 3,3);
        B= A-diag(diag(A));
        B=B+eye(3) ;
        A1=triu(B);
        A2= tril(B);
        AA=A1*A2 ;

% Create the diagonal matrix with required eigenvalues as diagonal elements
detM=5; % Given determinant is 5
        S=diag([1,1,detM]); % product of eigenvalues of S = determinant of S
% determinant of similar matrices(matrices with same eigenvalues) are same
% determinant of M and S are same
        M=AA*S*inv(AA)
        Det(M)  % Determinant M
```

11. Generation of two square matrices A and B such that AB=0

Concept used: Every square matrix satisfies it's own characteristic equation.

i.e., For 3 by 3 matrix, $(A-\lambda_1 I)(A-\lambda_2 I)(A-\lambda_3 I)=0$

```
% First create a matrix M with determinant 1
```

```
A=randi([-3 3], 3,3);
B= A-diag(diag(A));
B=B+eye(3) ;
A1=triu(B);
A2= tril(B);
M=A1*A2 ;

% Create the diagonal matrix with known eigenvalues
L1=1;L2=2;L3=3;
S=diag([L1,L2,L3]);
% Create a matrix with known eigenvalues
D=M*S*inv(M);
I3=eye(3);
A=(D-L1*I3);
B=(D-L2*I3)*(D-L3*I3);
A*B
```

Practice questions

1) Create a matrix A whose eigenvalues are 24, 10, 5 with corresponding eigenvectors as $[1,1,0]^T$, $[1,-1,0]^T$ and $[0,0,1]^T$ Also use diagonalization to find A^5.

2) Create a 5x5 non-diagonal matrix with eigenvalues as 9,8,7,6,5.

3) Create an integer valued 4x4 matrix with determinant 1.

4) Create an integer valued 7x7 matrix which has an integer valued inverse.

5) Generate an integer valued 15 by 15 matrix with determinant 3.

6) Consider the matrix, $A = \begin{bmatrix} -4 & 1 & 1 \\ 1 & -6 & 7 \\ 1 & 7 & -9 \end{bmatrix}$. Find two matrices B and C that are similar to matrix A and verify their eigenvalues are same.

7) Using MATLAB generate a 10×5 matrix A of rank 3. Use this matrix to generate a 5×5 symmetric matrix and a 10×10 symmetric matrix.

8) Create an integer valued 15×15 matrix with determinant 3.

9) Create three 3×3 matrices A, B and C such that their product A*B*C gives a zero matrix.

10) Create three 4×4 matrices A, B and C such that their product A*B*C gives a zero matrix.