* Disadvantages of Algebric method of simplification :

→ In this method, we need to write lengthy equations, find the common terms, manipulate the exp. etc. So it is a time consuming work.

→ On the top of that, sometimes, we are not sure whether the further simplification is really possible or not.

* Karnaugh-Map simplification

→ simplification technique to reduce the Boolean equation.

→ overcomes all the disadvantages of the algebric simplification technique.

→ It is a graphical method of simplifying a Boolean exp.

→ It is a graphical chart which contains boxes

→ information contained in truth table or available in the SOP or POS form is represented on a k-map.

→ It can be written for 2, 3, 4 ... upto 6 variables.

→ K-map structure

→ 2 variable K-map

B \ A	$\bar{A}$ 0	A 1
$\bar{B}$ 0		
B 1		

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0		
A 1		

- A and B are i/ps
- 0 and 1 are value of A or B
- Inside 4 boxes we have to enter the values of Y i.e. O/P.

→ 3 variable K-map.

sequence follows gray code

A \ BC	00	01	11	10
$\bar{A}$ 0				
A 1				

BC \ A	0	1
00		
01		
11		
10		

C \ AB	00	01	11	10
$\bar{C}$ 0				
C 1				

- It consists of 8 boxes
- The positions of variables A, B, C are interchangable.

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→ 4 variable K-map

CD \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00				
$\bar{C}D$ 01				
CD 11				
$C\bar{D}$ 10				

AB \ CD	00	01	11	10
00				
01				
11				
10				

⇒ K-map Boxes and Associated product Terms :

Two variables

A \ B	$\bar{B}$ 0	B 1
0 $\bar{A}$	m_0 $\bar{A}\bar{B}_0$	m_1 $\bar{A}B_1$
1 A	m_2 $A\bar{B}_2$	m_3 AB_3

Three variables

A \ BC	00 $\bar{B}\bar{C}$	01 $\bar{B}C$	11 BC	10 $B\bar{C}$
$\bar{A}$ 0	m_0 $\bar{A}\bar{B}\bar{C}_0$	m_1 $\bar{A}\bar{B}C_1$	m_3 $\bar{A}BC_3$	m_2 $\bar{A}B\bar{C}_2$
A 1	m_4 $A\bar{B}\bar{C}_4$	m_5 $A\bar{B}C_5$	m_7 ABC_7	m_6 $AB\bar{C}_6$

Inside each box we substitute a product term

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	m_0 $\bar{A}\bar{B}\bar{C}\bar{D}_0$	m_1 $\bar{A}\bar{B}\bar{C}D_1$	m_3 $\bar{A}\bar{B}CD_3$	m_2 $\bar{A}\bar{B}C\bar{D}_2$
$\bar{A}B$ 01	m_4 $\bar{A}B\bar{C}\bar{D}_4$	m_5 $\bar{A}B\bar{C}D_5$	m_7 $\bar{A}BCD_7$	m_6 $\bar{A}BC\bar{D}_6$
AB 11	m_{12} $AB\bar{C}\bar{D}_{12}$	m_{13} $AB\bar{C}D_{13}$	m_{15} $ABCD_{15}$	m_{14} $ABC\bar{D}_{14}$
$A\bar{B}$ 10	m_8 $A\bar{B}\bar{C}\bar{D}_8$	m_9 $A\bar{B}\bar{C}D_9$	m_{11} $A\bar{B}CD_{11}$	m_{10} $A\bar{B}C\bar{D}_{10}$

m_0 to m_{15} are minterms of a 4 variable equation

⇒ Representation of canonical SOP ~~plan~~ form on K-map.

- It can be represented on a K-map by simply entering 1's in the cells of the K-map corresponding to each minterm present in the equation.
- The remaining ~~fields~~ cells filled with zeros.

ex. Represent the equation given below on karnaugh map.

$Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + ABC$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1 (0)	1 (1)	0 (3)	0 (2)
A 1	1 (4)	0 (5)	1 (7)	1 (6)

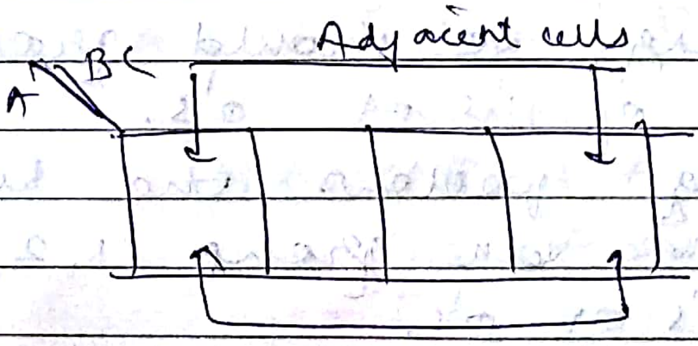
$\bar{A}\bar{B} + A\bar{B}\bar{C} + AB$

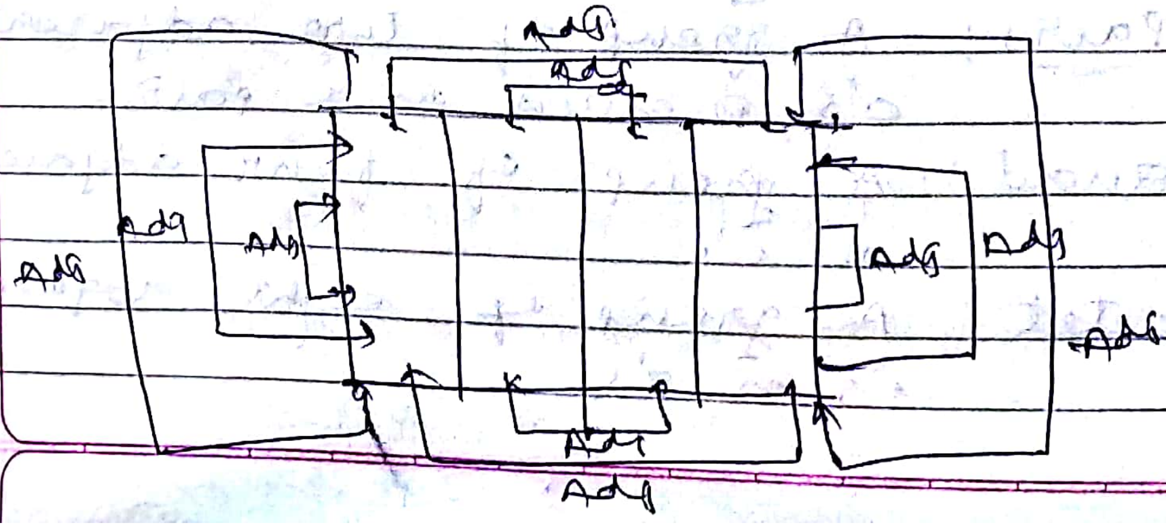
ex. Plot the foll.ⁿ exp. on k-map.

$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}BC\bar{D} + AB\bar{C}\bar{D}.$

→ Simplification of Boolean exp. using k-map.

- It is based on combining or grouping the terms in the adjacent cells of k-map.
- Two cells of a k-map are said to be adjacent if they differ in only one variable.





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How does simplification take place?

→ Once we plot the logic function or truth table on a Karnaugh map, we have to use the grouping technique for simplifying the logic funⁿ.

→ Grouping means combining the terms in the adjacent cells.

→ The grouping of either adjacent 1's or adjacent 0's results in the simplification of Boolean exp.

→ If we group the adjacent 1's, then the result of simplification is SOP form.

→ If the adjacent 0's are grouped, the result of simplification is POS form.

→ Way of grouping:

→ while grouping, we should group most number of 1's or 0's.

→ the grouping follows the binary rule. i.e. we can group 1, 2, 4, 8, 16, 32 number of 1's or 0's.

→ we cannot group 3, 5, 7 .. number of 1's or 0's

1. Pairs: A group of two adjacent 1's or 0's is called as a pair

2. Quad: A group of four adjacent 1's or 0's

3. Octet: A group of eight adjacent 1's or 0's.

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→ Grouping of Two Adjacent one's (Pairs):

→ If we group Two adjacent 1's on a K-map to form a pair, then the resulting term will have one less literal than the original.

→ **ex.** $Y = \bar{A}BC + \bar{A}B\bar{C}$

A \ BC	00 $\bar{B}\bar{C}$	01 $\bar{B}C$	11 BC	10 $B\bar{C}$
$\bar{A}$	0	0	1	1
A	0	0	0	0

$$Y = \bar{A}BC + \bar{A}B\bar{C}$$
$$= \bar{A}B(C + \bar{C})$$
$$= \bar{A}B$$

ex.

$\bar{A}\bar{B}C$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	1	0	0
A	0	1	0	0

$A\bar{B}C$

$$Y = \bar{A}\bar{B}C + A\bar{B}C$$
$$= \bar{B}C(A + \bar{A})$$
$$= \bar{B}C$$

ex.

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	0	0	0
A	1	0	0	1

$$Y = A\bar{B}\bar{C} + AB\bar{C}$$
$$= A\bar{C}(\bar{B} + B)$$
$$= A\bar{C}$$

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→ ex.

A \ B	$\bar{B}$	B
$\bar{A}$	1	1
A	1	0

$$Y = \bar{A}\bar{B} + \bar{A}B + A\bar{B}$$
$$= \bar{A}(\bar{B}+B) + A\bar{B}$$
$$= \bar{A} + A\bar{B}$$

Pair 1: $Y_1 = \bar{A}\bar{B} + \bar{A}B$
$$= \bar{A}(\bar{B}+B) = \bar{A}$$

Pair 2: $Y_2 = \bar{A}\bar{B} + A\bar{B}$
$$= \bar{B}(\bar{A}+A) = \bar{B}$$

$$\therefore\ Y = Y_1 + Y_2 = \bar{A} + \bar{B}$$

→ ex For the K-map shown in the figure, write a simplified Boolean exp.

C \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$ 0	0	1	1	1
C 1	0	0	1	0

$$Y_1 = \bar{A}B\bar{C} + AB\bar{C}$$
$$= B\bar{C}(A+\bar{A}) = B\bar{C}$$

$$Y_2 = AB\bar{C} + A\bar{B}\bar{C}$$
$$= A\bar{C}(B+\bar{B}) = A\bar{C}$$

$$Y_3 = AB\bar{C} + ABC$$
$$= AB(C+\bar{C}) = AB$$

$$\therefore\ Y = B\bar{C} + A\bar{C} + AB$$

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→ Minimization of SOP exp. (K-map)

procedure:

1. Prepare the K-map and place 1's according to the truth Table or logical exp.
2. Locate the isolated 1's which cannot be combined with any other 1.
3. Identify 1's which can be combined to form a pair in only one way and encircle them.
4. Identify 1's which can form a quad in only one way and encircle them.
5. Identify 1's which can form an octet in only way, and encircle them
6. After identifying the pairs, quads and octets, check if any 1 is yet to be encircled. If yes, then encircle them with eachother or with the already encircled 1's.

ex. A logical expression in the canonical SOP form is as follows:

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C$$

minimize it using the K-map technique

→

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1		1	1
A 1		1		

$\cancel{Y = \bar{A}\bar{C} + BC + \bar{A}B}$

$$Y = \bar{A}\bar{C} + \bar{A}B + A\bar{B}C$$

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* The logical exp. representing a logic circuit is $Y = \Sigma m(0, 1, 2, 5, 13, 15)$. Draw the K-map and find the minimized logical exp.

→ $Y = m_0 + m_1 + m_2 + m_5 + m_{13} + m_{15}$

CD \ AB	$\bar A\bar B$ 00	$\bar A B$ 01	AB 11	$A\bar B$ 10
$\bar C\bar D$ 00	1_0	1_1	0_3	1_2
$\bar C D$ 01	0_4	1_5	0_7	0_6
CD 11	0_{12}	1_{13}	1_{15}	0_{14}
$C\bar D$ 10	0_8	0_9	0_{11}	0_{10}

$$Y = \bar B\bar C\bar D + \bar A\bar C\bar D + A\bar B\bar C + \bar A B D + BCD$$

CD \ AB	$\bar A\bar B$	$\bar A B$	AB	$A\bar B$
$\bar C\bar D$	1_0	1_1	0_3	1_2
$\bar C D$	0_4	1_5	0_7	0_6
CD	0_{12}	1_{13}	1_{15}	0_{14}
$C\bar D$	0_8	0_9	0_{11}	0_{10}

$$\cancel{Y = \bar B\bar C\bar D}$$

AB \ CD	$\bar C\bar D$ 00	$\bar C D$ 01	CD 11	$C\bar D$ 10
$\bar A\bar B$	1_0	1_1	0_3	1_2
$\bar A B$	0_4	1_5	0_7	0_6
AB	0_{12}	1_{13}	1_{15}	0_{14}
$A\bar B$	0_8	0_9	0_{11}	0_{10}

$$Y = \bar A\bar B\bar D + \bar A\bar C D + ABD$$

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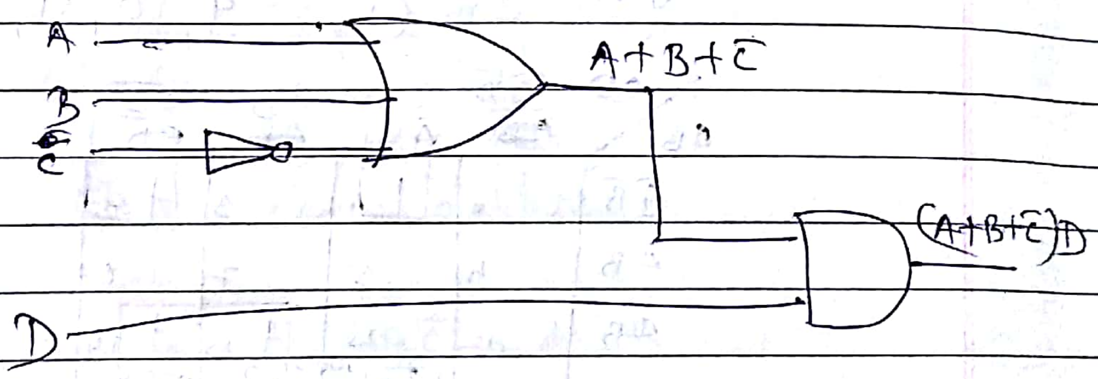
* For the logical exp. given below draw the K-map and obtain the simplified logical exp.

$Y = \sum m(1, 5, 7, 9, 11, 13, 15)$

Realize the minimized exp. using basic gates.

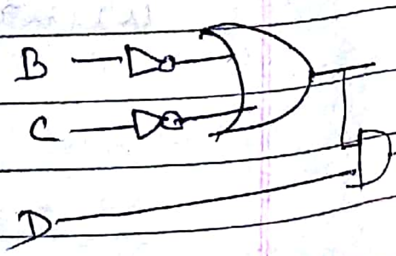
AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1 1	3	2
$\bar{A}B$	4	1 5	1 7	6
AB	12	1 13	1 15	14
$A\bar{B}$	8	1 9	1 11	10

$$Y = \bar{C}D + BD + AD$$
$$= D(A + B + \bar{C})$$



Minimize the foll.ⁿ Boolean exp. using K-map and realize it using the basic gates. $Y = \sum m(1, 3, 5, 9, 11, 13)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1 1	1 3	2
$\bar{A}B$	4	1 5	7	6
AB	12	1 13	15	14
$A\bar{B}$	8	1 9	1 11	10



$$Y = \bar{C}D + \bar{B}D$$
$$= (\bar{C} + \bar{B})D$$

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* Using K-map realize the foll.ⁿ exp. using minimum no. of gates.

$$Y = \Sigma m(1,3,4,5,7,9,11,13,15)$$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1_{1}	1_{3}	2
$\bar{A}B$	1_{4}	1_{5}	1_{7}	6
AB	12	1_{13}	1_{15}	14
$A\bar{B}$	8	1_{9}	1_{11}	10

$$Y = D + \bar{A}B\bar{C}$$

* Minimize the foll.ⁿ exp. using K-map and realize using the basic gates.

$$Y = \Sigma m(1,2,9,10,11,14,15)$$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1_{1}	3	1_{2}
$\bar{A}B$	4	5	7	6
AB	12	13	1_{15}	1_{14}
$A\bar{B}$	8	1_{9}	1_{11}	1_{10}

$$Y = \bar{B}\bar{C}D + \cancel{A\bar{B}D} + AC + \bar{B}C\bar{D}$$
$$= \bar{B}(C\bar{D} + \bar{C}D) + AC$$
$$= \bar{B}(C \oplus D) + AC$$

* Minimize the foll.ⁿ exp. and realize using basic gates.

$$Y = \Sigma m(0,2,5,6,7,8,10,13,15)$$

pos / practical

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 (0)	(1)	(3)	1 (2)
$\bar{A}B$	(4)	1 (5)	1 (7)	1 (6)
AB	(12)	1 (13)	1 (15)	(14)
$A\bar{B}$	1 (8)	(9)	(11)	1 (10)

$$Y = \bar{B}\bar{D} + BD + \bar{A}BC$$

* Minimize the foll^n exp.

$$Y = \Sigma m(1, 5, 6, 7, 11, 12, 13, 15)$$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	(0)	1 (1)	(3)	(2)
$\bar{A}B$	(4)	1 (5)	1 (7)	1 (6)
AB	1 (12)	1 (13)	1 (15)	(14)
$A\bar{B}$	(8)	(9)	1 (11)	(10)

$$Y = \underbrace{BD}_{\text{Redundant Quad}} + AB\bar{C} + \bar{A}\bar{C}D + \bar{A}BC + ACD$$

→ If all the 1's in a group are already involved in some other groups, then the group is called as redundant group.

→ A redundant group has to be eliminated, because it increases the no. of gates.

So here BD can be eliminated.

$$\therefore Y = AB\bar{C} + \bar{A}\bar{C}D + \bar{A}BC + ACD$$

$$= D\underbrace{(\bar{A}\bar{C} + AC)}_{\text{EX-NOR}} + B\underbrace{(\bar{A}C + A\bar{C})}_{\text{X-OR}}$$

$$\therefore Y = D(\overline{A \oplus C}) + B(A \oplus C)$$

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minimize the foll^n exp. using k-map.

$Y = \Sigma m(0, 1, 2, 5, 13, 15)$

Ans. $= \bar A \bar B \bar D + ABD + \bar A \bar C D$

Minimization of Logic Functions not specified in canonical SOP form;

ex $Y = \bar A \bar B CD + \bar A BC\bar D + \bar A \bar B \bar C + \bar A \bar B \bar D + A\bar C + \bar B$

steps to:

1. Enter 1 for minterms (i.e. $\bar A \bar B CD$ and $\bar A BC\bar D$)
2. Enter a pair of 1's for each term with one less variable than total (i.e. $\bar A \bar B \bar C$, $\bar A \bar B \bar D$)
3. Enter four adjacent 1's for each term with two less variables than total (i.e. $A\bar C$)
4. Repeat for the other terms in a similar way.
5. Once K-map is obtained, the minimization is same as the the one discussed earlier.

ex. $Y = \bar A \bar B CD + \bar A BC\bar D + \bar A \bar B \bar C + \bar A \bar B \bar D + A\bar C + \bar B$

AB \ CD	$\bar C \bar D$ 00	$\bar C D$ 01	CD 11	$C\bar D$ 10
$\bar A \bar B$ 00	1	1	1	1
$\bar A B$ 01				1
AB 11	1	1		
$A\bar B$ 10	1	1	1	1

$Y = \cancel{\bar A \bar B} \; \bar A C\bar D + A\bar C + \cancel{AB} \; \bar B$

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Don't care conditions:

→ It is not always true that the cells not containing 1's (in SOP) will contain 0's, because some combinations of I/P variable do not occur.

→ Also for some functions, the O/Ps corresponding to certain combinations of I/Ps variables do not matter.

→ In such situation we have a freedom to ~~assume~~ assume a 0 or 1 as o/p for each of these combinations.

→ These conditions are known as the Don't care conditions and in the K-map it is represented as 'X' (cross) mark in the corresponding cell.

ex Simplify the exp. given below using K-map. The don't care conditions are indicated by d( ).

$$Y = \Sigma m(1,3,7,11,15) + d(0,2,5).$$

→ The given eqn is

$$Y = m_1 + m_3 + m_7 + m_{11} + m_{15} + d(0,3,5)$$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	X (0)	1 (1)	1 (3)	X (2)
$\bar{A}B$ 01	(4)	X (5)	1 (7)	(6)
AB 11	(12)	(13)	1 (15)	(14)
$A\bar{B}$ 10	(8)	(9)	1 (11)	(10)

Note: Every don't care mark need not be considered while grouping.

$$Y = \bar{A}\bar{B} + CD$$

or $Y = \bar{A}D + CD$

Simplify the foll^n exp- once by considering the don't care conditions and once by ignoring it.

$Y = \Sigma m(1,4,8,12,13,15) + d(3,14)$

1) without don't care

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0 (0)	1 (1)	0 (3)	0 (2)
$\bar{A}B$ 01	1 (4)	0 (5)	0 (7)	0 (6)
AB 11	1 (12)	1 (13)	1 (15)	0 (14)
$A\bar{B}$ 10	1 (8)	0 (9)	0 (11)	0 (10)

$Y = \bar{A}\bar{B}\bar{C}D + B\bar{C}\bar{D} + A\bar{C}\bar{D} + ABD$

2) using Don't care

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0 (0)	1 (1)	X (3)	0 (2)
$\bar{A}B$ 01	1 (4)	0 (5)	0 (7)	0 (6)
AB 11	1 (12)	1 (13)	1 (15)	X (14)
$A\bar{B}$ 10	1 (8)	0 (9)	0 (11)	0 (10)

$Y = \bar{A}\bar{B}D + AB + B\bar{C}\bar{D} + A\bar{C}\bar{D}$

$= \bar{A}\bar{B}D + AB + \bar{C}\bar{D}(A+B)$

Note: The don't care condition reduces the no. of gates required for implementation.

Q. A) Minimize the foll.^{n} exp. using K-map.

$$Y = \Sigma m(0, 1, 5, 9, 13, 14, 15) + d(3, 4, 7, 10, 11)$$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1 (0)	1 (1)	X (3)	(2)
$\bar{A}B$ 01	X (4)	1 (5)	X (7)	(6)
AB 11	(12)	1 (13)	1 (15)	1 (14)
$A\bar{B}$ 10	(8)	1 (9)	X (11)	X (10)

$$Y = \bar{A}\bar{C} + D + AC$$

$$= \underbrace{(\bar{A}\bar{C} + AC)}_{\text{EX-NOR}} + D$$

$$= D + (A \odot C)$$

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