Infinite group

→ not Abelian

→ bcz $(ac, bc+d) \neq (\cancel{bc+d}, ac, ad+b)$

$(a, b) * (c, d) \neq (c, d) * (a, b)$

* Groupoid :- Suppose G is non empty set and * is a binary operation then G(*) is called a groupoid if * is closed in G, that is, given any elements.

* Semi group :- A non empty set G together with binary operation *, (G, *) is a semi group if binary operation * is associative.

* Monoid :- A Non empty set G together with a binary operation *, (G *) is called a monoid if it satisfies the following properties:

1) * is closed in (G, *)
2) * is associative in (G, *)
3) There exist an Identity element in (G, *)

Ex-1: $a * b = a + b + 1$

For groupoid, ⇒ clousre ⇒ $a * b = a + b + 1 \in Z$

Ex-1 The set

for Semi groupoid :-

$$a*(b*c) = (a*c)*b$$

$$a*(b+c+1) = (a+c+1)*b$$

$$a+b+c+2 = a+b+c+2$$

LHS = RHS

For Monoid,

$$a*e = a = e*a$$

$$a+e+1 = a = e+a+1$$

$$\boxed{e=-1}$$

$e=-1 \in Z$

for group,

$$\cancel{a*a^{-1} = e} \qquad a \cdot a^{-1} = e$$

$$\cancel{a+a^{-1} = -1} \qquad a^{-1} = \frac{e}{a}$$

$$a^{-1} = \frac{e}{a} = -\frac{1}{a}$$

* order of group and order of element:-

* order of group :- The order of the group is defined as the member of element in the group. It is dented by O(G).

* order of element :- Let G be a group with binary operation

*. By the order of an element $a \in G$ is meant the least possitive integer. n, If one exists, such that $a^n = e$ (the Identity

Page No.
Date :

of a). It is denoted by $o(a)$.

* **Remarks:-**

(i) If there doesnot exist any possitive Integer n such that $a^n = e$, then we say that a is of infinite order.

ii) The order of the Idenity element is always 1.

1 Example: Find the order of each element of the multiplicative group $\{1, -1, i, -i\}$

Soln:- Since 1 is the Identity element there fore $o(1) = 1$.

(-1) $\quad (-1)^1 = 1$

$(-1)^2 = 1$ (i e identity element)

$\therefore o(-1) = 2$

i $\quad (i)^1 = 1$

$(i)^2 = -1$

$(i)^3 = -i$

$(i)^4 = 1$

$o(i) = 4$

$-i$ $\quad (-i)^1 = -i$

$(-i)^2 = -1$

$(-i)^3 = -i$

$(-i)^4 = 1$ $\quad o(-i) = 4.$

Scanned with CamScanner

Example2:- Find the order of each element of the group {0, 1, 2, 3, 4, 5}, the Composition being addition modulo 6.

ના લખવું.

ઓકે

sol^n: Since 0 is the Identity element therefore $o(0) = 1$.

$e = \{0, 1, 2, 3, 4, 5\}$

~~for~~ Here, first Comopitision table

+	0	1	2	3	4	5
0	%6=0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

* closure property:- since all elements of the table lie in the set $e = \{0, 1, 2, 3, 4, 5\}$, we can say that closure property is satisfied.

* Associative property:- For all elements int the table, it can be

Date :

3 Example: Is Multiplicative modulo 6 a group $U_6 = \{0, 1, 2, 3, 4, 5\}$ $*_6$? If not, how to make it a group? Find the order of the all elements.

Solⁿ :-

$*_6$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

Page No.
Date :

* closure Property: Since all elements of the table lie in the set $U_6 = \{0, 1, 2, 3, 4, 5\}$, we Can say that closure property is satisfied.

* Associative property :- For all the elements in the table, it can be verfied that $[a]_6 *([a] *_6 [b]) *_6 [c]$, for all $[a], [b], [c] \in U_6$

* Existence of Idenity:- from the table, it Can be observed that 1 will the identity element as $[a] *_6 [1] = [a]$ for all $[a] \in U_6$

* Existense of Inverse:- for $[0], [2], [3], [4]$ we don't get any of the element from U_6 so that $[a] *_6 [a^{-1}] = e = [1]$ for $[a] = [0], [2], [3], [4]$ is satisfied. Hence, inverse element do not exist for $[0], [2], [3], [4]$.

→ Therefore, this property is not Satishfied and hence we can say that this U_6 is not a group.

→ observe that if we remove $[0], [2], [3], [4]$ from U_6, then $s = \{[1], [5]\}$ will turns out the group.

Page No.
Date :

for $S = \{1, 5\}$

* closure property :- it's present in table.

* Associative property :- we need 3 elements but here 2 elements so 3 element we Assume As. so Associative property also Include in it.

* Existense of Identity :- In table, $a * e = a$
$1 * 1 = 1$
$5 * 1 = 5$

so, Identity element $= 1$.

* Existense of Inverse :- In table, $a \cdot a^{-1} = e$
$1 \cdot 1 = 1$
$5 \cdot 5 = 1$
so $\{1, 5\} \in S$.

* $O(1) = 1$
$O(5) = 2 \Rightarrow (5)^1 = 5$
$(5)^2 = 5 * 6^5 = 1 = e$

$\Rightarrow$ and this case, the order of the group $O(S) = 2$ and $O(S) = 1$ and $O(5) = 2$.