4.12 HALL EFFECT

If a specimen (metal or semiconductor) carrying a current I is placed in a transverse magnetic field B, an electric field E is induced in the direction perpendicular to both I and B. This phenomenon is known as Halleffect and the generated voltage is called the Hallvoltage.

Theory

Consider a rectangular slab of an n-type semiconductor material which carries current I along the positive X-direction, as shown in Figure 4.33. In an n-type semiconductor electrons are the majority carries.

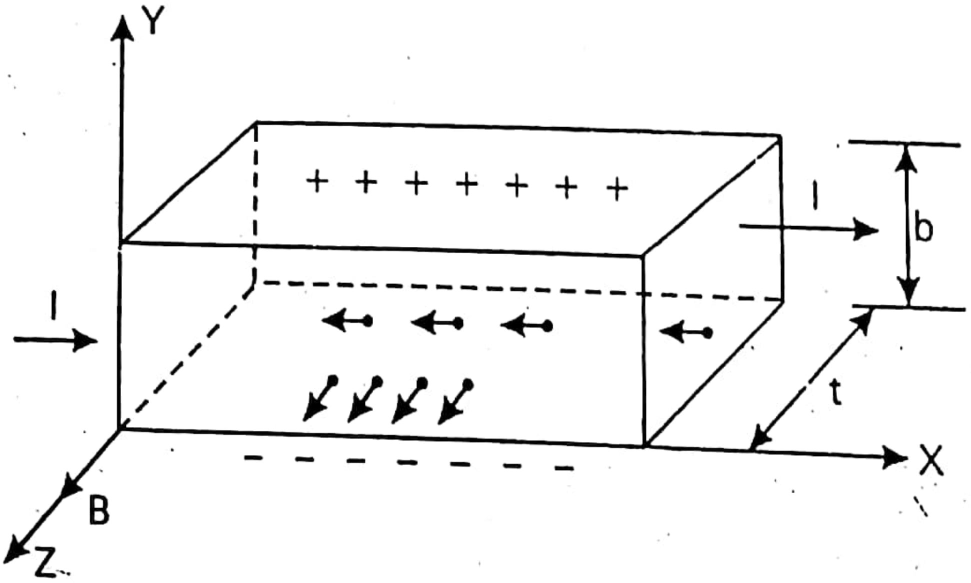

Figure 4.33: Hall effect

Let a magnetic field B is applied along the positive Z-direction. Under the influence of this magnetic field, the electrons experience a force called Lorentz force given by

$$F_L = -Bev_d \quad ...(1)$$

where e is the magnitude of charge of the electrons and v_d is the drift velocity.

This Lorentz force is exerted on the electrons in the negative Y-direction. The direction of this force is given by Fleming's left-hand rule. Thus, the electrons are, therefore, deflected downwards and collect at the bottom surface of the specimen.

On the other hand, the top edge of the specimen becomes positively charged due to the loss of electrons. Hence, a potential called the Hallvoltage V_H is developed between the upper and lower surfaces of the specimen, which establishes an electric field E_H called the Hallfield across the specimen in the negative Y-direction.

This electric field exerts an upward force on the electron and is given by

$$F_E = -eE_H \quad ...(2)$$

At equilibrium, the Lorentz force and the electric force gets balanced. Hence,

$$F_E = F_L$$

Therefore, from equations (1) and (2)

$$-eE_H = -Bev_d$$

or

$$E_H = Bv_d \quad ...(3)$$

If b is the width (i.e., the distance between the top and bottom surface) of the specimen, then

$$E_H = \frac{V_H}{b} \quad ...(4)$$

or

$$V_H = E_H b \quad ...(5)$$

and

$$V_H = Bv_d b \quad ...(6)$$

Let t be the thickness of the specimen along the Z direction. Therefore, its area of cross-section normal to the direction of current is bt.

If J is the current density, then,

$$J = \frac{I}{bt} \quad ...(7)$$

But J can also be expressed as

$$J = -n_e ev_d \quad \text{where } n_e \text{ is the density of electrons.} \quad ...(8)$$

$$\therefore \quad v_d = \frac{J}{n_e e} \quad ...(9)$$

Hence, substituting equation (9) in (6)

$$V_H = -Bb\frac{J}{n_e e} \quad ...(10)$$

But V_H is also equal to $E_H b$

$J = n e v_d$

$= (n e \mu) E_H$

$= \sigma E_H$

$$\therefore \quad E_H b = -Bb\frac{J}{n_e e}$$

$$\text{or} \quad E_H = \frac{BJ}{n_e e} \qquad ...(11)$$

Note:

The polarity of Hallvoltage for an n-type semiconductor is positive at the top surface. For a p-type semiconductor the polarity of Hallvoltage is positive at the bottom surface. The polarity of the Hallvoltage developed at the top and bottom surface of the specimen can be identified by using probes.

Hall Coefficient R_H

The Halleffect is described by means of Hallcoefficient R_H. It is given by

$$R_H = \frac{1}{ne}$$

where n is, in general, the carrier concentration.

R_H for n-type and p-type Material

A negative sign is used while denoting the Hall coefficient for an n-type material, i.e., it is given by $R_H = -\frac{1}{n_e e}$ ...(12)

where n_e is the density of electrons.

But, for a p-type material a positive sign is used to denote the Hall coefficient, i.e., it is given by $R_H = \frac{1}{n_h e}$ ...(13)

where n_h is the density of holes.

Therefore, equation (11) can be written as

$$E_H = BJ R_H,$$

$$\text{i.e.,} \quad R_H = \frac{E_H}{JB} \qquad ...(14)$$

But, we know $E_H = \frac{V_H}{b}$ and $J = \frac{I}{bt}$. Hence equation (14) becomes,

$$R_H = \frac{V_H bt}{IBb}$$

$$\therefore \quad R_H = \frac{V_H t}{IB} \qquad ...(15)$$

Since the quantities V_H, t, I and B are measurable, the Hall coefficient R_H can be determined.

4.12.1 Experimental Determination of Hall Coefficient

A rectangular slab of thickness t and width b is placed at right angles to a magnetic field B. A known current I is passed through the material along the X-axis by connecting it to a dc battery, key, a rheostat and a milliammeter, as shown in Figure 4.34.

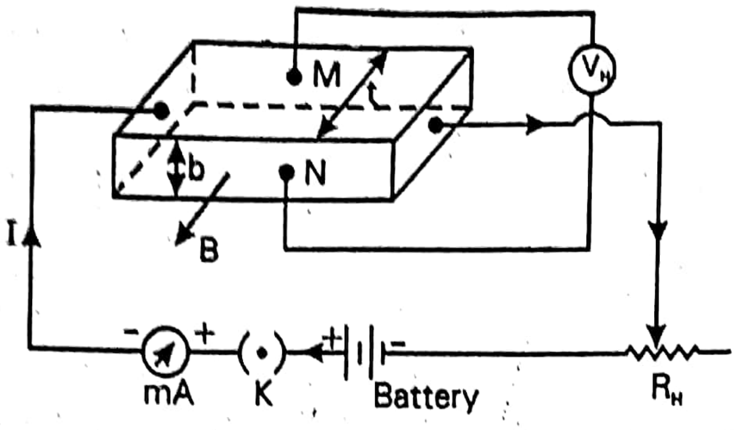

Figure 4.34: Experimental setup to determine Hall coefficient

A calibrated voltmeter connected between the opposite ends of the slab measures the Hall voltage V_H. Thus, the Hallfield is given by

$$E_H = \frac{V_H}{b}$$

The current density is given by

$$J = \frac{I}{bt}$$

Thus, R_H can be calculated using the formula

$$R_H = \frac{V_H t}{IB}$$

4.12.2 Mobility Determination

For an n-type material the conductivity is given by

$$\sigma_e = n_e e \mu_e$$

where μ_e is the mobility of electrons.

$$\therefore \quad \mu_e = \frac{\sigma_e}{n_e e} \qquad ...(16)$$

or

$$\mu_e = -\sigma_e R_H \qquad ...(17)$$

Similarly, for a p-type material, the conductivity is given by

$$\sigma_h = n_h e \mu_h$$

where μ_h is the mobility of holes.

$$\therefore \quad \mu_h = \frac{\sigma_h}{n_h e} \qquad ...(18)$$

or

$$\mu_h = \sigma_h R_H \qquad ...(19)$$

In the above discussion, it is assumed that all the charge carriers travel with average velocity. But actually, the charge carriers have a random thermal distribution in velocity.

With this distribution taken into consideration, R_H is defined in general as

$$R_H = \frac{3\pi}{8ne} = \frac{1.18}{ne} \qquad ...(20)$$

Therefore, equations (16) and (18) can be written as

$$\mu_e = \frac{-\sigma_e R_H}{1.18} \quad \text{(for n-type material)} \qquad ...(21)$$

and

$$\mu_h = \frac{\sigma_h R_H}{1.18} \quad \text{(for p-type material)} \qquad ...(22)$$

Applications of Hall Effect

The Hall effect can be used for:

1. Determining whether a semiconductor is n-type or p-type.
2. Determining the carrier concentration and mobility.
3. Determining the magnetic field B in terms of Hallvoltage V_H.
4. Designing the gauss meter and electronic meters based on Hallvoltage.

Example 4.3

The conductivity and the Hall coefficient of an n-type silicon specimen are 112 $\Omega^{-1} m^{-1}$ and $1.25 \times 10^{-4}\ m^3 C^{-1}$, respectively. Calculate the charge carrier density and electron mobility.

Solution:

Given, $\sigma_e = 112\ \Omega^{-1} m^{-1}$; $R_H = 1.25 \times 10^{-4}\ m^3 C^{-1}$; n_e = ? and μ_e = ?

Formula: $\mu_e = \sigma_e R_H$

$= 112 \times 1.25 \times 10^{-4} = 0.014\ m^2V^{-1}s^{-1}$.

$\Omega^{-1} m^{-1} \cdot m$

$\Omega^{-1} m^{2}$

We know $\mu_e = \dfrac{\sigma_e}{n_e e}$.

$$\therefore \quad n_e = \frac{\sigma_e}{\mu_e e} = \frac{112}{0.014 \times 1.6 \times 10^{-19}}$$

$$= \frac{112}{2.24 \times 10^{-21}} = 5 \times 10^{22}\ \text{electrons}/m^3.$$

Example 4.4

A semiconducting crystal 12 mm long, 1 mm wide and 1 mm thick has a magnetic flux density of 0.5 Wb/m^2 applied from front to back, and is perpendicular to largest faces. When a current of 20 mA flows lengthwise through the specimen, the voltage measured across its width is found to be 37 μV. What is the Hall coefficient of semiconductor and the density of charge carrier?

Solution:

Given,

$l = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}; t = 1 \text{ mm} = 1 \times 10^{-3} \text{ m};$

$b = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}, I = 20 \text{ m A} = 20 \times 10^{-3} \text{A};$

$v_H = 37; \mu V = 37 \times 10^{-6} \text{ V}; B = 0.5 \text{ Wb/m}^2;$

$R_H = ?$ and $n = ?$

We know

$$R_H = \frac{V_H t}{IB}$$

$$= \frac{37 \times 10^{-6} \times 1 \times 10^{-3}}{20 \times 10^{-3} \times 0.5} = \frac{3.7 \times 10^{-8}}{0.01}$$

$$\mathbf{R_H = 3.7 \times 10^{-6}\ C^{-1}\ m^3.}$$

Example 4.5

A silicon plate of thickness 1 mm, breadth 10 mm and length 100 mm is placed in a magnetic field of 0.5 Wb/m² acting perpendicular to its thickness. If 10^{-2} A current flows along its length, calculate the Hall coefficient, if Hallvoltage developed is 1.83 mV.

Solution:

Given,

$t = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}, b = 10 \text{ mm} = 10 \times 10^{-3} \text{m};$

$l = 100 \text{ mm} = 100 \times 10^{-3} \text{ m}; B = 0.5 \text{ Wb/m}^2;$

$I = 10^{-2} \text{A}; V_H = 1.83 \text{ mV} = 1.83 \times 10^{-3} \text{V};$

$R_H = ?$

Formula:

$$R_H = \frac{V_H t}{IB} = \frac{1.83 \times 10^{-3} \times 1 \times 10^{-3}}{10^{-2} \times 0.5}$$

$$\therefore \quad \mathbf{R_H = 3.66 \times 10^{-4}\ m^3C^{-1}.}$$

Example 4.6

The Hallcoefficient of certain silicon specimens was found to be -7.35×10^{-5} m²C⁻¹ from 100 to 400 K. Determine the nature of the semiconductor if the conductivity was found to be 200 Ω^{-1}m^{-1}. Calculate the density and mobility of the charge carrier.

Solution:

Given, $R_H = -7.35 \times 10^{-5} \text{m}^3\text{C}^{-1}$; $\sigma = 200\ \Omega^{-1}\text{m}^{-1}$; $n_e = ?$ and $\mu_e = ?$

The negative sign of the Hall coefficient indicates that the nature of the semiconductor is n-type.

Formula: $$n_e = \frac{1}{R_H e} = \frac{1}{7.35\times10^{-5}\times1.6\times10^{-19}}$$

∴ $$n_e = 8.503 \times 10^{22} \text{ electrons / m}^3.$$

Mobility $$\mu_e = \frac{\sigma}{n_e e} = \frac{200}{8.503\times10^{22}\times1.6\times10^{-19}}$$

∴ $$\mu_e = 14.7006 \times 10^{-3} m^2V^{-1}s^{-1}.$$

or $$\mu_e = \sigma R_H$$
$$= 200 \times 7.35 \times 10^{-5}$$

∴ $$\mu_e = 14.7 \times 10^{-3} m^2V^{-1}s^{-1}.$$

Example 4.7

An n-type semiconductor has Hall coefficient = 4.16×10^{-4} m^3C^{-1}. The conductivity is 108 $\Omega^{-1}m^{-1}$. Calculate its charge carrier density n_e and electron mobility at room temperature.

Solution:

Given, $R_H = 4.16 \times 10^{-4}$ m^3C^{-1}; $\sigma = 180$ $\Omega^{-1}m^{-1}$; $n_e = ?$ and $\mu_\varepsilon = ?$

$$R_H = \frac{1.18}{n_e e} \quad \text{(with correction factor for } R_H\text{)}$$

∴ $$n_e = \frac{1.18}{R_H e} = \frac{1.18}{4.16\times10^{-4}\times1.6\times10^{-19}}$$

∴ $$n_e = 1.772 \times 10^{22} / m^3.$$

We know $$\sigma = n_e e \mu_e$$

∴ $$\mu_e = \frac{\sigma}{n_e e} = \frac{180}{1.772\times10^{22}\times1.6\times10^{-19}}$$

∴ $$\mu_e = 0.06348 \ m^2V^{-1}s^{-1}.$$

Example 4.8

A rectangular plane sheet of doped silicon has dimensions of 1 cm along Y-direction, and 0.5 mm along Z- direction. Hallprobes are attached on its two surfaces parallel to X-Z plane and a magnetic field of flux density 0.7 Wb/m² is applied along Z-direction. A current of 1 mA is flowing in it in X-direction. Calculate the Hallvoltage measured by the probes if the Hallcoefficient of the material is 1.25×10^{-3} m^3C^{-1}.

Solution:

Given, $B = 0.7$ Wb/m²; $I = 1 \times 10^{-3}$ A; $R_H = 1.25 \times 10^{-3}$ m^3C^{-1}; $b = 1$ cm $= 1 \times 10^{-2}$m;

$t = 0.5$ mm $= 0.5 \times 10^{-3}$m ; $V_H = ?$

$$V_H = \frac{R_H IB}{t} = \frac{1.25\times10^{-3}\times1\times10^{-3}\times0.7}{0.5\times10^{-3}}$$
$$= 1.75 \times 10^{-3} V$$

or $$V_H = 1.75 \text{ mV}.$$