6. Interpolation & Curve Fitting.

Error :

→ In mathematics, error is the difference between a true value (exact value) & approximation of that value.

There are different types of errors but we are going to study only numerical errors.

Numerical Errors :

Numerical errors are introduced during the process of implementation of a numerical method. There are 2 types :

i) Rounding off error &
ii) Truncation error

i) Rounding off errors : It arises from the process of rounding off the numbers during the computation and it also occurs when a fix number of digits are used to represent exact numbers

ii) Truncation error : It is the error made by truncating an infinite sum and approximating it by a finite sum.

Absolute error : It is the numerical difference between the true value of a quantity and its approximate value.
let us suppose that true value of a given data is denoted by 'x_t' and approximate value is denoted by 'x_a'. Then absolute error is denoted by 'e_a' & defined as

$$\boxed{e_a = |x_t - x_a|}$$

Relative error : It is denoted by 'e_r' & defined as $e_r = \frac{e_a}{|x_t|} = \left|\frac{x_t - x_a}{x_t}\right|$

Percentage error : e_p & $e_p = 100 \times e_r$.

15/03/19.

Q.1) If 0.333 is the approximate value of 1/3, then find absolute, relative & percentage error.

Solⁿ: Let us suppose that x_t and x_a denotes the true value & approximate value.

$\therefore$ Here $x_t = \frac{1}{3}$ & $x_a = 0.333$

So The absolute error is

$$e_a = |x_t - x_a|$$
$$= 0.00033$$

Also relative error is

$$e_r = \frac{e_a}{|x_t|} = 0.00033 \times 3$$
$$= 0.00099$$

Percentage error $= e_r \times 100$

$$= 0.00099 \times 100$$
$$= 0.099\ \%$$

Finite difference :

Suppose we are given a funⁿ $y = f(x)$. If $x_0, x_0+h, x_0+2h, \ldots, x_0+nh$ are discrete values of x, then the corresponding values of y are $y_0, y_1, y_2, \ldots, y_n$.

Here the values of the independent variable x is called 'argument' and the corresponding values is called 'entry'.

The arguments & entry are represented in a tabular form as.

Argument x	x_0	$x_1 = x_0 + h$	...	$x_n = x_0 + nh$
Entry y	$y_0 = f(x_0)$	$y_1 = f(x_0 + h)$	...	$y_n = f(x_0 + nh)$.

To determine the values of $f(x)$, $f'(x)$, $f''(x)$ ... etc. for some independent variable we will use following 3 types of differences.

1. Forward difference
2. Backward difference
3. Central difference

Forward difference : (Δ) (des)

We are given the values $y_0, y_1, \ldots, y_n$ of a funⁿ $y = f(x)$.

Then the first forward difference is denoted by Δy_0 and defined as

$$\Delta y_0 = y_1 - y_0$$ → where Δ is known as 1st forward difference operator.

$$y_2 - y_1 = \Delta y_1$$
$$y_3 - y_2 = \Delta y_2.$$

hence $\Delta y_i = y_{i+1} - y_i \quad ; \quad i = 0, 1, 2, \ldots, n.$

→ Second forward difference are defined as differences of first differences ie:

$$\Delta y_1 - \Delta y_0 = \Delta^2 y_0$$
$$\Delta y_2 - \Delta y_1 = \Delta^2 y_1$$

$$\Delta y_{i+1} - \Delta y_i = \Delta^2 y_i \quad ; \; i = 0, 1, 2, \dots, n.$$

Here Δ^2 is known as second forward difference operator.

→ Similarly third forward difference are defined as

$$\Delta^2 y_1 - \Delta^2 y_0 = \Delta^3 y_0$$
$$\Delta^2 y_2 - \Delta^2 y_1 = \Delta^3 y_1$$
$$\vdots$$
$$\Delta^2 y_{i+1} - \Delta^2 y_i = \Delta^3 y_i \quad ; \; i = 0, 1, 2, \dots, n.$$

Hence in general the n^{th} difference are defined as

$$\Delta^{n-1} y_{i+1} - \Delta^{n-1} y_i = \Delta^n y_i$$

The forward differences can be represented in the form of a table as follows:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0				
		Δy_0			
x_1	y_1		$\Delta^2 y_0$		
		Δy_1		$\Delta^3 y_0$	
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$
		Δy_2		$\Delta^3 y_1$	
x_3	y_3		$\Delta^2 y_2$		
		Δy_3			
x_4	y_4				

This is also known as descending difference table.

2 Backward difference: (∇) (asc.)

We are given the values $y_0, y_1, y_2, \ldots, y_n$ of a fun^n $y = f(x)$

Then first backward diff^n are defined as

$$y_1 - y_0 = \nabla y_1$$
$$y_2 - y_1 = \nabla y_2$$
$$y_3 - y_2 = \nabla y_3$$
$$\vdots$$
$$y_{i+1} - y_i = \nabla y_{i+1} \quad ; \; i = 0, 1, 2, 3 \ldots, n.$$

Here '∇' is known as first backward difference.

Similarly

Second backward difference:

$\nabla y_2 - \nabla y_1 = \nabla^2 y_2$

$\vdots$

$\nabla y_{i+1} - \nabla y_i = \nabla^2 y_{i+1}$

Hence in general $\nabla^{n-1} y_{i+1} - \nabla^{n-1} y_i = \nabla^n y_{i+1}$.

So the backward difference can be represented in a tabular form as follows:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
x_0	y_0				
		∇y_1			
x_1	y_1		$\nabla^2 y_2$		
		∇y_2		$\nabla^3 y_3$	
x_2	y_2		$\nabla^2 y_3$		$\nabla^4 y_4$
		∇y_3		$\nabla^3 y_4$	
x_3	y_3		$\nabla^2 y_4$		
		∇y_4			
x_4	y_4				

This is also known as ascending difference table.

(3) Central difference: The central difference operator is denoted by 'δ'.

Date

NOTE: Alternative notations for the funⁿ $y = f(x)$.

IMP

Let us take $y = f(x)$ & suppose that we take the consecutive values of x differing by h, then

F

① $y_{x+h} - y_x = \Delta y_x$ i.e. $f(x+h) - f(x) = \Delta f(x)$

B

② $y_x - y_{x-h} = \nabla y_x$ i.e. $f(x) - f(x-h) = \nabla f(x)$

C

③ $y_{x+h/2} - y_{x-\frac{h}{2}} = \delta y_x$ ie. $f(x+\frac{h}{2}) - f(x-\frac{h}{2}) = \delta f(x)$.

27/03/19

The Shift Operator E :

MCQ IMP

$(\rightarrow)$ $E\, f(x) = f(x+h)$

$$
\begin{aligned}
E^2 f(x) &= E(E\{f(x)\}) \\
&= E(f(x+h)) \\
&= f((x+h)+h) \\
&= f(x+2h)
\end{aligned}
$$

In general,

$\therefore$ $\boxed{E^n f(x) = f(x+nh).}$; n is $(n>0)$

If h is the interval of differencing in the argument x, then the shift operator E is defined as, ①

Thus, when E operates on $f(x)$ then the result

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is the next value of the function. and hence it is known as shift operator or translation operator.

→ Inverse Shift Operator :

It is defined as $E^{-1} f(x) = f(x-h)$

$(n<0)$ Similarly $E^{-2} f(x) = f(x-2h)$

$\vdots$

In general $E^{-n} f(x) = f(x-nh)$

→ The Averaging Operator 'μ' :

The averaging operator μ is defined as

$$\mu f(x) = \frac{1}{2}\left[f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)\right]$$

Relation b/w all operators :

(1) $\Delta = E - 1$

Solⁿ : Here $\Delta f(x) = f(x+h) - f(x)$

$= E f(x) - f(x)$ ($\because$ by Shift operator)

$\Delta f(x) = f(x) [E-1]$

$\therefore \underline{\underline{\Delta = E - 1}}$

(2) $\nabla = I - E^{-1}$

$\nabla f(x) = f(x) - f(x-h)$ $\quad$ ($\because$ By inverse shift operator)

$= f(x) - E^{-1} f(x)$

$\nabla f(x) = f(x) [I - E^{-1}]$

$\therefore \boxed{\nabla = I - E^{-1}}$

(3) $\delta = E^{1/2} - E^{-1/2}$

Sol^n.

$\delta f(x) = f(x + h/2) - f(x - h/2)$

$= E^{1/2} f(x) - E^{-1/2} f(x)$ $\quad$ [$\because$ By Shift & Inverse Shift Optr]

$\delta f(x) = f(x) [E^{1/2} - E^{-1/2}]$

$\therefore \boxed{\delta = E^{1/2} - E^{-1/2}}$

(4) $\mu = \frac{1}{2} [E^{1/2} + E^{-1/2}]$

Sol^n:

$\mu f(x) = \frac{1}{2} [f(x + h/2) + f(x - h/2)]$

$= \frac{1}{2} [E^{1/2} f(x) + E^{-1/2} f(x)]$

$\mu f(x) = \frac{1}{2} f(x) [E^{1/2} + E^{-1/2}]$ $\quad$ [$\because$ By shift & Inv. shift Optr]

$\therefore \boxed{\mu = \frac{1}{2} [E^{1/2} + E^{-1/2}]}$

(5) $E\nabla = \nabla E = \Delta$.

Sol^n: $\longrightarrow$ $E\nabla = \Delta$

$$E[\nabla f(x)] = E[f(x) - f(x-h)]$$
$$= Ef(x) - Ef(x-h)$$
$$= f(x+h) - f(x\cancel{+h}-\cancel{h})$$
$$= f(x+h) - f(x)$$
$$= \Delta f(x).$$

$\longrightarrow$ $\nabla E = \Delta$.

$$\nabla[E f(x)] = \nabla[f(x+h)]$$
$$= f(x+h) - f(x+h-h)$$
$$= f(x+h) - f(x)$$
$$= \Delta f(x).$$

(6) $\Delta = \delta E^{1/2}$

Sol^n:
$$\delta E^{1/2} = \delta[E^{1/2} f(x)]$$
$$= \delta[f(x+\tfrac{h}{2})]$$
$$= f(x+\tfrac{h}{2}+\tfrac{h}{2})] - f(x\cancel{-\tfrac{h}{2}}+\cancel{\tfrac{h}{2}})$$
$$= f(x+h) - f(x).$$

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03.19.

INTERPOLATION :

Suppose we are given a funⁿ $y = f(x)$ which takes the values $y_0, y_1, \ldots, y_n$ corresponding to $x_0, x_1, \ldots, x_n$. Then, the process of finding the value of a funⁿ y corresponding to any value of x between x_0 & x_n is known as interpolation.

IMP
imp

Thus interpolation is a technique of finding the value of a funⁿ for any intermediate value of the independent variable x.

There are 2 types of interpolation :

1. Newton's interpolation (common diffⁿ).
2. Lagrange's Interpolation

→ I Newton's interpolation : It is used when equally spaced arguments ~~that~~ means given in the problem that means there is common diffⁿ b/w each values of arguments only.

There are two types of Newton's interpolation :

1. Newton's forward Interpolation.
2. " backward "

→ Newton's forward interpolation formula :

If a funⁿ $y = f(x)$ takes the values $y_0, y_1, \ldots, y_n$ corresponding to $x_0, x_1, \ldots, x_n$ with common starting value. diff^n 'h' then $y = f(x) = y_0 + \frac{p}{1!}\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0$

$$+ \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \dots + \frac{p(p-1)(p-2)\dots(p-(n-1))}{n!}\Delta^n y_0.$$

where $p = \frac{x - x_0}{h}$.

→ Newton's backward interpolation formula :

ending value. If a fun^n $y = f(x)$ takes the values $y_0, y_1, y_2, \dots, y_n$ corresponding to $x_0, x_1, \dots, x_n$ with common diff^n 'h' then

$$y = f(x) = y_n + \frac{p}{1!}\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \dots +$$

$$\frac{p(p+1)\dots(p+(n-1))}{n!}\nabla^n y_n,$$

where $p = \frac{x - x_n}{h}$.

NOTE :

① N.F.I is used when the values of y near to the beginning of the tabular values, which we want to evaluate.

② N.B.I is used when the values for interpolating the values of y near to the end of the tabular values.

Q.1) The population of the town was given as below :
Then estimate the population for the year 1895

Year	1891	1901	1911	1921	1931
Population (in thousand)	46	66	81	93	101

Soln. Let us consider x as year and y as population
∴ Here the diffn table is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	46				
~~1901~~		20			
1901	66		-5		
~~1921~~		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

Here $x = 1895$ which is near to the beginning of the tabular values so we use N.F.I formula.

$x = 1895$ $\quad x_0 = 1891$ $\quad p = \frac{x - x_0}{h} = \frac{1895 - 1891}{10}$

$h = 10$ $\quad y_0 = 46$

$= \frac{4}{10} = 0.4$

Now by N.F.I formula, we have

$$y = f(x) = 40 + \frac{p}{1!}\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \cdots + \frac{p(p-1)\cdots p(p-(n-1))}{n!}\Delta^n y_0$$

∴ we have

$$y = f(x) = 46 + \frac{0.4}{1}(20) + \frac{0.4(0.4-1)(-5)}{2} + \frac{0.4(-0.6)(-1.6)}{3}(2)$$

$$+ \frac{0.4(-0.6)(-1.6)(-2.6)(-3)}{24}$$

$= \cancel{24.1248}$

$= 46 + 8 + 0.6 + 0.128 + 0.1248$

$= \underline{54.85280}$

Hence in the year 1895 the population is 54.85280 thousand.

HW. Calculate above same e.g. for year 1925

20/04/19.

Q.2). The following table gives the distance in miles of the visible horizon for the given heights in feet above the earth surface. Find value of y when $x = 390$ ft.

Height	100	150	200	250	300	350	400
Dist^n	10.63	13.03	15.04	16.81	18.42	19.90	21.47

Sol^n: Let us consider height as x & dist^n as y.
Here x is 390 which is near to end of tabular values.
$\therefore$ we use N.B.I.

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
100	10.63					
		2.40				
150	13.03		−0.39			
		2.01		0.15		
200	15.04		−0.24		−0.07	
		1.77		0.08		0.02
250	16.81		−0.16		−0.05	
		1.61		0.03		
300	18.42		−0.13		0.19	
		1.48		0.22		
350	19.90		0.09			
		1.57				
400	21.47					

0.24

390−400

$x = 390$ $\quad x_n = 400$ $\quad p = \frac{x - x_n}{h} = \frac{\cancel{400-390}}{\cancel{50}\,50} = \frac{-1}{5}$

$h = 50$ $\quad y_n = 21.47$

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$p = -0.2$

Now by N.B.I formula, we have

$$f(x) = y_n + \frac{p}{1!}\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$$+ \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)}{5!}\nabla^5 y_n$$

$$+ \frac{p(p+1)(p+2)(p+3)(p+4)(p+5)}{6!}\nabla^6 y_n$$

$$= 21.47 + \frac{(-0.2)(1.57)}{1} + \frac{(-0.16)(0.09)}{2} - \frac{0.288\,(0.22)}{6}$$

$$+ \frac{(-0.8064)(0.19)}{24} + \frac{(-3.06432)(0.24)}{120} +$$

$$\frac{(-14.70873)(0.22)}{720}$$

$= 21.47 - 0.314 - 0.0072 - 0.01056 - 0.00638$
$- 0.00612 - 0.00449.$

$= 21.12125$

Q.3) From the foll. table estimate the no. of students who obtained marks between 40 & 45

IMP

Marks.	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
No. of students	31	42	51	35	31

Solⁿ: First we will generate the cumulative table from the given data.
let us consider x as marks less than & y as no. of students.

∴ Cumulative table is

Marks less than x	40	50	60	70	80
No. of students y	31	73	124	159	190.

Here we want to find the no. of students having marks less than 45.
i.e, Here we take $x = 45$, which is near to beginning of tabular values. so we use N.F.F.

∴ diffⁿ table is :

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
40	31	42	9	-25	37
50	73	51	-16	12	
60	124	35	-4		
70	159	31			
80	190				

Here $x = 45$ $y_0 = 31.$

$x_0 = 40$

$h = 10$ $p = \frac{x - x_0}{h} = \frac{45 - 40}{10} = \frac{5}{10}$

$= 0.5$

Now by N.F. formula,

$$y = f(x) = y_0 + \frac{p}{1!}\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0.$$

$$= 31 + \frac{(0.5)(42)}{1} + \frac{(-2.25)}{2} - \frac{84.375}{6} + \frac{333}{24}$$

$$= 31 + 21 - 1.125 - \cancel{14.0625} \; \cancel{+3.875}$$

-1.5625

-1.44531

$= \cancel{50.6875}$ $\cancel{35.36719}$ 47.86719 ≈ 48

Hence there are total 48 students having marks less than 45.

$<45 = 48$

$<40 = 31.$

Here no. of students with marks less than 40 are 31.

Hence the no. of students having marks b/w 40 & 45 % = ~~no.~~ 0

= no. of students having marks <45 − no. of students having marks <40

= 48 − 31

= 17.

Ans.

Q4) Estimate the values f(42) and f(22) from the foll. data.

x	20	25	30	35	40	45
y	354	332	291	260	231	204

Sol^n: Here we want to find f(42) & f(22) which are near to end & starting of the tabular values respectively. So we use N.B.I & N.F.I.

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
20	354					
		-22				
25	332		-19			
		-41		29		
30	291		10		-37	45
		-31		-8		
35	260		2		+8	
		-29		0		
40	231		2			
		-27				
45	204					

$x = 42$ $\quad x_n = 45$ $\quad p = \frac{x - x_n}{h} = \frac{42-45}{5}$

$h = 5$ $\quad y_n = 204$

$= \frac{-3}{5} = \underline{-0.6}$

$f(x) = y_n + \frac{p}{1!}\nabla y_n + \frac{p(p+1)\nabla^2 y_n}{2!} + \frac{p(p+1)(p+2)\nabla^3 y_n}{3!}$

$+ \frac{p(p+1)(p+2)(p+3)\nabla^4 y_n}{4!} + \frac{p(p+1)(p+2)(p+3)(p+4)}{5!}\nabla^5 y_n$

$= 204 + \frac{(-0.6)(-27)}{1} + \frac{(-0.6)(0.4)(2)}{2} + \frac{(-0.6)(0.4)(1.4)(0)}{3!}$

$+ \frac{(-0.6)(0.4)(1.4)(2.4)(8)}{24} + \frac{(-0.6)(0.4)(1.4)(2.4)(3.4)(45)}{120}$

$= 204 + 16.2 - 0.24 + 0 - 0.2688 - 1.02816$

$= \underline{218.66304}$

$$y = f(x) = y_0 + \frac{p}{1!}\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \frac{p(p-1)(p-2)(p-3)(p-4)}{5!}\Delta^5 y_0$$

$$\cancel{= 354 + \frac{(-0.6)(-22)}{1} + \frac{(-0.6)(-1.6)(-19)}{2}}$$

$$\cancel{+ \frac{(-0.6)(-1.6)(-2.6)(29)}{6} + \frac{(-0.6)(-1.6)(-2.6)(-3.6)(-37)}{24}}$$

$$\cancel{+ \frac{(-0.6)(-1.6)(-2.6)(-3.6)(-4.6)(45)}{120}}$$

$\underline{352.22304}$

$\cancel{= 354 + 13.2 - 9.12 - 12.064 - 13.8528 - 15.50016}$

$\cancel{= \underline{316.66304}}$

$$p = \frac{x - x_0}{h} = \frac{22-20}{5} = \frac{2}{5} = 0.4.$$

$$= 354 + \frac{(0.4)(-22)}{1} + \frac{(0.4)(-0.6)(-19)}{2} + \frac{(0.4)(-0.6)(-1.6)(29)}{6}$$

$$+ \frac{(0.4)(-0.6)(-1.6)(-2.6)(-37)}{24} + \frac{(0.4)(-0.6)(-1.6)(-2.6)(-3.6)(45)}{120}$$

$= 354 - 8.8 + 2.28 + 1.856 + 1.5392 + 1.34784.$

$= 352.22304$

04/04/19

When the arguments are not equally spaced in the given problem then we use Lagrange's Interpolation.
That means the values of independent variables x have no any common difference.

Lagrange's Interpolation formula:

Statement: If a function $y=f(x)$ takes the values $y_0, y_1, \ldots, y_n$ corresponding to $x_0, x_1, \ldots, x_n$, which are not equally spaced, then

$$y=f(x)=\frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}y_0+\frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)\cdots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\cdots(x_2-x_n)}y_2+\cdots+$$

$$\frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}y_n$$

NOTE: Lagrange's Interpolation formula is used for both equally and unequally spaced argument.

IMP MCQ.

Q.1) Use Lagrange's Interpolation formula to find the value of y when $x = 10$ from the following values of x and y are given below.

x	5	6	9	11
y	12	13	14	16

Soln: Here unequally spaced argument given so we use Lagrange's Interpolation formula.

Here $x_0 = 5$ $y_0 = 12$

$x_1 = 6$ $y_1 = 13$

$x_2 = 9$ $y_2 = 14$

$x_3 = 11$ $y_3 = 16$

$$y = f(x) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} \cdot 12 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} \cdot 13$$

$$+ \frac{(10-5)(10-\cancel{9}6)(10-11)}{(9-5)(9-6)(9-11)} \cdot 14 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} \cdot 16$$

$$= \frac{(4)(1)(-1)(12)}{(-1)(-4)(-6)} + \frac{(5)(1)(-1)(13)}{(1)(-3)(-5)} + \frac{(5)(-1)(\cancel{8}4)(14)}{(4)(3)(-2)} + \frac{(5)(4)(1)(16)}{(6)(5)(2)}$$

$$= \frac{-48}{-24} + \frac{(-65)}{15} + \frac{\cancel{80}\,280}{-24} + \frac{320}{60}$$

$$= 2 - 4.33333 \;\cancel{+}\; \cancel{2.91666}\; + 11.66666 + 5.33333$$

$$= \cancel{5.91666}. \quad \cancel{18}\,14.66666.$$

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Q.2) Using Lagrange's interpolation formula find a polynomial of degree 3 from the foll. data.

x	-1	0	1	3
$y=f(x)$	2	1	0	-1

Sol^n: Here $x_0=-1$ $\quad y_0=2$

$x_1=0$ $\quad y_1=1$

$x_2=1$ $\quad y_2=0$

$x_3=3$ $\quad y_3=-1$

$$y=f(x)=\frac{(x-0)(x-1)(x-3)\cdot(2)}{(-1-0)(-1-1)(-1-3)}+\frac{(x+1)(x-1)(x-3)\cdot(1)}{(0+1)(0-1)(0-3)}$$

$$+\frac{(x+1)(x-0)(x-3)\cdot(0)}{(1+1)(1-0)(1-3)}+\frac{(x+1)(x-0)(x-1)\,(-1)}{(3+1)(3-0)(3-1)}$$

$$\Rightarrow \frac{(x)(x-1)(x-3)\,2}{(-1)\cdot(-2)(-4)}+\frac{(x+1)(x-1)(x-3)}{(1)(-1)(-3)}$$

$$+\cancel{x(x+1)(x-3)}\;0+\frac{(x+1)(x)(x-1)\,(-1)}{(4)(3)(2)}$$

$$\Rightarrow x-1\left[\frac{2x(x-3)}{-8}+\frac{(x+1)(x-3)}{3}-\frac{x(x+1)}{24}\right]$$

$$\Rightarrow x-1\left[\frac{2x^2-6x}{-8}+\frac{x^2-2x-3}{3}-\frac{x^2-x}{24}\right]$$

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$$= \frac{(x-0)(x-1)(x-3)(2)}{(-1-0)(-1-1)(-1-3)} + \frac{(x+1)(x-1)(x-3)(1)}{(0+1)(0-1)(0-3)}$$

$$= \frac{(x+1)(x-0)(x-3)(0)}{(1+1)(1-0)(1-3)} + \frac{(x+1)(x-0)(x-1)(-1)}{(3+1)(3-0)(3-1)}$$

$$\Rightarrow \frac{(x)(x-1)(x-3)(\cancel{2})}{(-1)(-\cancel{2})(-4)} + \frac{(x+1)(x-1)(x-3)}{(1)(-1)(-3)}$$

$$+ \frac{(x+1)(x-0)(x-3)(0)}{(2)(1)(-2)} + \frac{(x+1)(x)(x-1)(-1)}{(4)(3)(2)}$$

$$\Rightarrow x-1\left[\frac{x(x-3)}{-4} + \frac{(x+1)(x-3)}{3} - \frac{x(x+1)}{24}\right]$$

$$\Rightarrow \frac{x-1}{24}\left[-6x(x-3) + 8\left[x^2-2x-3\right] - x^2-x\right]$$

$$= \frac{x-1}{24}\left[-6x^2+18x+8x^2-16x-24-x^2-x\right]$$

$$\Rightarrow \frac{x-1}{24}\left[x^2+x-24\right]$$

$$\Rightarrow \frac{x^3+\cancel{x^2}-24x-\cancel{x^2}-x+24}{24}$$

$$\Rightarrow \frac{x^3-25x+24}{24}.$$

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Curve Fitting.

↳ Suppose we want to find a possible relation b/w the values of x & y for the provided data then we need the concept of curve fitting.

↳ With the help of concept of curve fitting we will find a specific relation $y = f(x)$ for the given data which satisfy it as accurate as possible and such eqⁿ or relation is known as 'best fitting curve' or 'curve of best fit'

↳ For that we will use the method of least square approximation
We will use foll. two methods of least square approximation for curve fitting

1) Linear curve fitting → $y = ax + b$
2) Non-linear curve fitting. → $y = ax^2 + bx + c$.

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Linear Curve Fitting :

In this curve fitting, we will fit a straight line of the form $y = ax + b$ such that y fits to a given values (x, y) of the provided data as near as possible.

→ For this fitting we will use the foll. normal equations.

$$ax + b = y \longrightarrow (1)$$

$$b \frac{\Sigma 1}{n}$$

$$a\Sigma x + bn = \Sigma y \longrightarrow (i)$$

$$a\Sigma x^2 + b\Sigma x = \Sigma xy \longrightarrow (ii)$$

IMP//.

$$\left[\because \Sigma = \sum_{K=1}^{n} 1 = 1+1+\dots+1 \text{ (n times)} = n \right]$$

The value of a & b can be obtained by solving normal eq^n (1) & (2)

flency $y = ax + b$ becomes the best fitting straight line to the given data.

For evaluation of a & b, we will use foll. two methods :

i) Direct method : In this method for given values (x, y) of data we will directly use the normal eq^n. & evaluate the values of a & b.

ii) Step-deviation method : This method can be preferred when the value of either x or y or both x & y are large.

In this method, we will change the values of x & y to fit a straight line

→ Here we consider n = no. of entries of x in the given problem.

→ If n is odd numbers, then we define new x & y as $X = x - A$, $Y = y - A'$, where A is middle term of x. & A' is the middle term of y.

→ If n is even number, then x & y as defined as

$$X = x - \left(\frac{A+B}{2}\right) \qquad Y = y - \left(\frac{A'+B'}{2}\right)$$

where A & B are two middle values of x & A' & B' are two middle values of y.

→ Write the normal eqn for X & Y.

Q.1) Find a straight line $y = ax + b$ that fits to the foll. data.

x	1	2	3	4	5
y	14	27	40	55	68

Solⁿ: Let $y = ax + b$ be a straight line
Then the normal eqⁿs are :

$$a\sum x + nb = \sum y \rightarrow \text{i}$$
$$a\sum x^2 + b\sum x = \sum xy \rightarrow \text{ii}$$

$\therefore$

x	y	xy	x^2
1	14	14	1
2	27	54	4
3	40	120	9
4	55	220	16
5	68	340	25
15	204	748	55

Here $n = 5$

Mode
Eqn
unknown 2

$\therefore$ From eqn ① & ②

$a_1 = 15$ then :
$b_1 = 5$ then :
$c_1 = 204$ then :

$$a(15) + b(5) = 204$$
$$a(55) + b(15) = 748.$$

$a_2 = 55$
$b_2 = 15$
$c_2 = 748$

$\therefore$ $a = 13.6$
$b = 0.$

On solving above eqn we will get $a = 13.6$ & $b = 0.$

$a = 13.6$
$b = 0.$

Hence the required straight line is
$$y = 13.6x + 0$$
$$\therefore \boxed{y = 13.6x.}$$

if $x = 2$
$y = 27.2$

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Step deviation method :

Here $n = 5$ which is odd no. so let us define $X = x - A$ & $Y = y - A'$

A & A' are middle values of x & y.

$\therefore$ Here

$$X = x - 3 \qquad Y = y - 40.$$

let

$Y = aX + b$ be a straight line.

then the normal eqns are.

$$a \sum X + nb = \sum Y$$

$$a \sum X^2 + b \sum X = \sum XY$$

$\therefore$ We have

x	y	$X = x - 3$	$Y = y - 40$	XY	X^2
1	14	-2	-26	52	4
2	27	-1	-13	13	1
3	40	0	0	0	0
4	55	$+1$	$+15$	15	1
5	68	$+2$	$+28$	56	4
		$\sum X = 0$	$\sum Y = 4$	136	10.

From eqn 1 & 2

$$a(0) + (5)b = 4 \qquad a(0) + b(0) = 136$$

$$b = 4/5 \qquad a = \frac{136}{10}.$$

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$\therefore$ Required straight line is

$$Y = 13.6X + 0.8$$

~~= 13.6~~

$$(y-40) = 13.6(x-3) + 0.8.$$

$$y = 40 + 13.6x - 40.8 + 0.8$$

$$\boxed{y = 13.6x}$$

Q.2) Fit a straight line $y = ax + b$ using least square method for foll. data.

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9.

Solⁿ:

$$a\Sigma x + nb = \Sigma y$$
$$a\Sigma x^2 + b\Sigma x = \Sigma xy$$

x	y	xy	x^2
1	1	1	1
3	2	6	9
4	4	16	16
6	4	24	36
8	5	40	64
9	7	63	81
11	8	88	121
14	9	126	196
56	40	364	524

So

$$56a + 8b = 40$$
$$524a + 56b = 364.$$

$a = 0.6$
$b = 0.54545.$

$y = \cancel{0.6x} + 0.54545.$
$0.63636x$

Q.3). Find a straight line $y = ax + b$ which fits to the foll. data.

x	100	120	140	160	180	200
Y	0.45	0.55	0.60	0.70	0.80	0.85

Sol^n.

x	y	xy	x^2
100	0.45	45	10000
120	0.55	66	14400
140	0.60	84	19600 ~~25600~~
160	0.70	112	25600
180	0.80	144	32400
200	0.85	170	40000
900	3.95	621	142000

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Here the values of x are large so we preferred step deviation method

$n = 6$ which is even no. so we define

X & Y as: $X = x - \left(\frac{A+B}{2}\right)$ $Y = y - \left(\frac{A'+B'}{2}\right)$

; A & B are two middle values of x.

A' & B' " " y.

$$X = x - \left(\frac{140+160}{2}\right) \qquad Y = y - \left(\frac{0.60+0.70}{2}\right)$$

$$= x - 150 \qquad = y - 0.65$$

let $Y = aX + b$ be a straight line

then the normal eq^n are: $a\Sigma X + nb = \Sigma Y$

$a\Sigma X^2 + b\Sigma X = \Sigma XY$

x	y	$X = x - 150$	$Y = y - 0.65$	XY	X^2
100	0.45	-50	-0.20	10	2500
120	0.55	-30	-0.10	3	900
140	0.60	-10	-0.05	0.5	100
160	0.70	10	0.05	0.5	100
180	0.80	30	0.15	4.5	900
200	0.85	50	0.20	10	2500
		0	0.05	28.5	7000

$a(0) + 6b = 0.05$

$b = 0.00833$

$7000(a) + b(0) = 28.5$ $\qquad a = 0.00407$

Hence required straight line is

$Y = (0.00407)X + (0.00833)$

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$$(y-0.65) = (0.00407)(x-150) + 0.00833$$

$$y = 0.65 + 0.00407x - 0.61050 + 0.00833$$
$$= \cancel{0.04783} \quad 0.00407x + 0.04783$$

Non-linear Curve fitting :

In this curve fitting we will fit a parabola of the form $y = ax^2 + bx + c$ such that y fits to the given values (x, y) of given problem as accurate as possible.

For that, the normal eq^n's are:

$$ax^2 + bx + c = y \longrightarrow x \quad x^2$$

$$a\Sigma x^2 + b\Sigma x + nc = \Sigma y \quad \text{i}$$
$$a\Sigma x^3 + b\Sigma x^2 + c\Sigma x = \Sigma xy \quad \text{ii}$$
$$a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 = \Sigma x^2 y \quad \text{iii}$$

On solving above normal eq^n's we will get the values of a, b & c so that $y = ax^2 + bx + c$ becomes the best fitting parabola of second degree to the given data.

Here we use earlier methods for the sol^n.