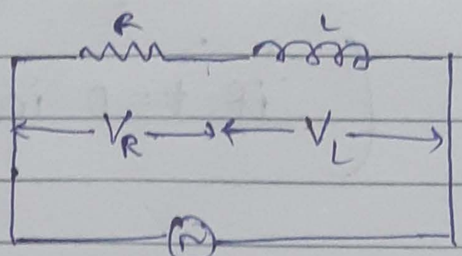
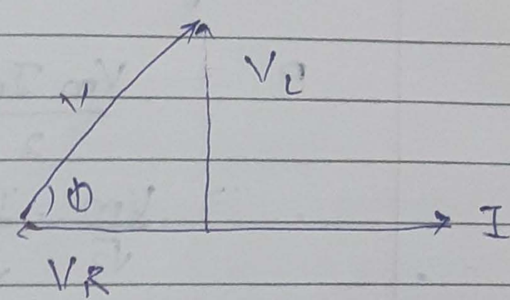
UNIT -

* AC Series Circuit : *

(1) R-L series circuit :



$V = V_m \sin \omega t$



$$V = \sqrt{V_R^2 + V_L^2}$$
$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

i) $\therefore Z = \sqrt{R^2 + X_L^2}$

$\therefore \frac{V}{I} = \sqrt{R^2 + X_L^2}$

ii) Phase angle $\tan \phi = \frac{V_L}{V_R} = \frac{IX_L}{IR}$

$$\boxed{\tan \phi = \frac{X_L}{R}}$$

iii) Equation of current i :

$$\boxed{i = I_m \sin(\omega t - \phi)}$$

iv) Instantaneous power :

$P = V \times i$

$P = V_m \sin \omega t \times I_m \sin(\omega t - \phi)$

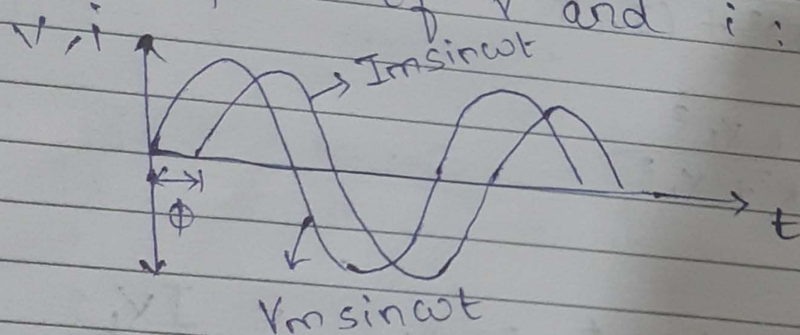
$= \frac{I_m \cdot V_m}{2} [\cos \phi - \cos(2\omega t - \phi)]$

$\therefore P = \frac{V_m I_m}{2} \cos\phi - \frac{V_m . I_m}{2} \cos(2\omega t - \phi)$

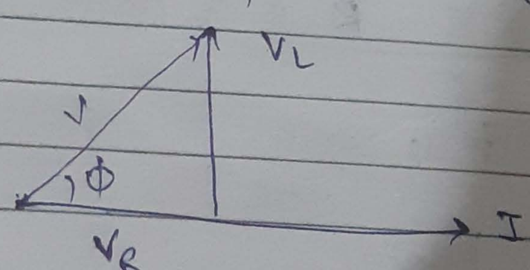
v) Average Power :

$$P = \frac{V_m I_m}{2} \cos\phi \quad \left(\because \text{if } t=0 \text{ in above eq}^n \right)$$
$$= \frac{V_m}{\sqrt{2}} . \frac{I_m}{\sqrt{2}} \cos\phi$$
$$= V_{rms} . I_{rms} \cos\phi$$

vi) Waveform of v and i :

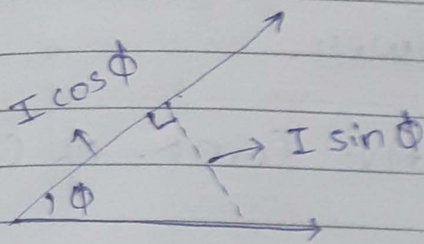


vii) Power factor : (lag always)



$$\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

* Active and Reactive Components of Current



— Active Component :
It is that component of current which is in phase with applied voltage V i.e I cos Φ. It is also known as <u>wattful</u> component.

— Reactive Component :
It is that component of current which is in quadrature with applied voltage V i.e I sin Φ. It is also known as <u>wattless</u> component.

(Power in circuit is consumed due to active component hence it is known as Wattful comp)

— <u>Apparent Power</u> :
The product of rms values of voltage & current in AC circuit is known as Apparent Power. i.e $\boxed{S = V_{rms} \times I_{rms}}$

— <u>Active Power</u> :
The power which is actually consumed in AC circuit is known as Active Power. It is denoted by P. ($P = V \cdot I \cos \phi$)
P = voltage × Active Comp. of Current

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Reactive Power : (Q) :

The power due to reactive component of current is known as reactive power.

$Q = VI \sin\phi$

(i.e Q = Voltage × Reactive Comp. of Current)

→ Examples :

(1) A coil takes 2.5 A when connected across 200 V 50 Hz supply. The power consumed is found to be 400 W. Calculate resistance & inductance of the coil.

$f = 50\ Hz \qquad V = 200\ V \qquad I = 2.5\ A$

$P = 400\ W$

~~$P = \frac{V^2}{R} \quad \therefore R = \frac{200 \times 200}{400} = 100$~~

~~$\omega = 2\pi \times 50 = 100 \times 3.14 = 314$~~

$P = I^2 R$

$400 = (2.5)^2 R$

$\therefore \boxed{R = 64\ \Omega}$

$Z = \frac{V}{I} = \frac{200}{2.5} = 80\ \Omega$

$Z = \sqrt{R^2 + X_L^2}$

$X_L^2 = Z^2 - R^2$

$= 6400 - 4096$

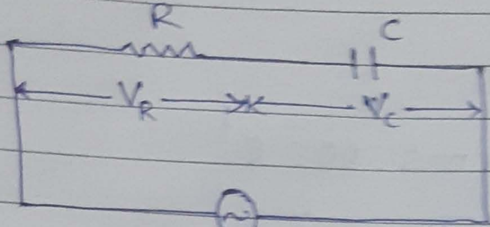
$X_L = 48\ \Omega$

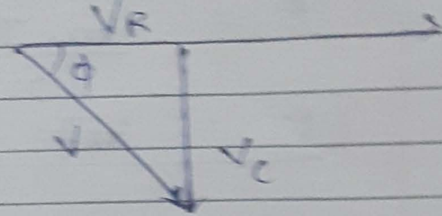
$X_L = \omega L = 314\ L$

$\therefore \boxed{L = \frac{48}{314} = 0.153\ H}$

(2)

R-C Series Circuit :





$V = V_m \sin\omega t$

(I) $V = \sqrt{V_R^2 + V_C^2}$

$V = \sqrt{(IR)^2 + (IX_C)^2}$

$\frac{V}{I} = \sqrt{R^2 + X_C^2}$

$$\boxed{Z = \sqrt{R^2 + X_C^2}}$$

(II) phase angle : $\boxed{\tan\phi = \frac{X_C}{R}}$

(III) Equation for current :

$$\boxed{i = I_m \sin(\omega t + \phi)}$$

(IV) Instantaneous Power :

$P = V \times i$

$= V_m \sin\omega t \times I_m \sin(\omega t + \phi)$

$= \frac{I_m . V_m}{2} . 2\sin(\omega t + \phi) . \sin\omega t$

$= \frac{I_m . V_m}{2} [\cos\phi - \cos(2\omega t + \phi)]$

$= \frac{V_m . I_m}{2} \cos\phi - \frac{I_m . V_m}{2} \cos(2\omega t + \phi)$

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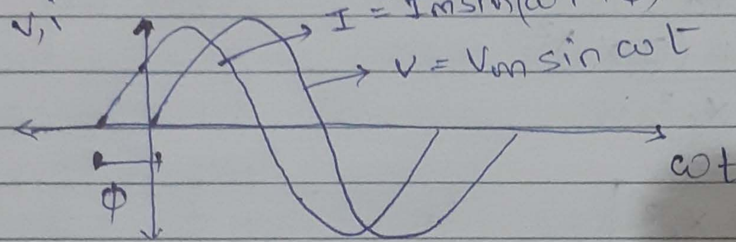
Average Power

$$P = \frac{V_m \cdot I_m}{2} \cos\phi$$

$$\boxed{P = \cancel{VI}\ V_{rms} \cdot I_{rms} \cos\phi}$$

(VI) Wave form of V and i :



(VII) Power Factor (leading)

$$\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

Q: Resistor & capacitor are connected in series across 230 V ~~AC~~ AC supply. The current taken by the circuit is 6 A for 50 Hz frequency. The current is reduced to 5 A when the frequency of supply is decreased to 40 Hz. Determine the value of resistor & capacitor.

$V = 230$ V $\quad I_1 = 6$ A $\quad f_1 = 50$ Hz $\therefore$ ~~ω = 314~~

$R = ?\quad C = ?\quad I_2 = 5$ A $\quad f_2 = 40$ Hz

~~L = Iω~~

$$Z = \sqrt{R^2 + X_C^2}$$

$$\therefore Z_1^2 = R^2 + X_{C_1}^2 \quad \text{--- (1)}$$

$$Z_2^2 = R^2 + X_{C_2}^2 \quad \text{--- (2)}$$

$$Z_1 = \frac{230}{6} = 38.3\ \Omega$$

$$Z_2 = \frac{230}{5} = 46\ \Omega$$

From 1 & 2,

$$Z_2^2 - Z_1^2 = X_{c_2}^2 - X_{c_1}^2$$

$$(46)^2 - (38.3)^2 = \left(\frac{1}{2\pi f_2 C}\right)^2 - \left(\frac{1}{2\pi f_1 C}\right)^2$$

$$2116 - 1466.89 = \frac{1}{(6.28\,C)^2}\left(\frac{900}{1600 \times 2500}\right)$$

$$\cancel{\frac{649.11 \times 6.28 \times 90}{10}} = \frac{1}{C^2}$$

$$\therefore \cancel{\frac{1}{C} = 36687.69}$$

$$\therefore \cancel{C =}$$

$$\therefore C = 93.9\ \mu F$$

$$(46)^2 = R^2 + \frac{1}{(2\pi f \times 93.9\,\mu)^2}$$

$$\therefore \boxed{R = 17.9\ \Omega}$$

Q: Capacitor & resistor are connected in series to an AC supply of 50 V, 50 Hz. The current is 2A and power dissipated in the circuit is 80 W. Calculate values of R & C.

$$Z = \frac{50}{2} = 25\ \Omega \qquad R = 20\ \Omega$$

$$Z^2 = R^2 + X_c^2$$

$$\therefore (25)^2 = (20)^2 + X_c^2$$

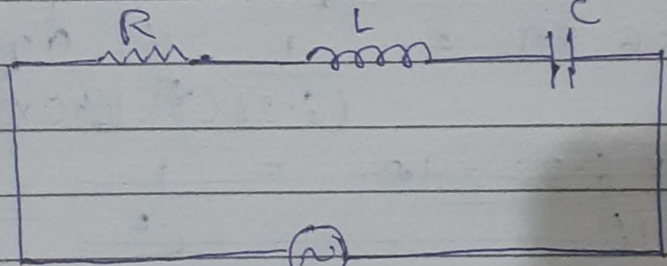
$$225 = X_c^2 \Rightarrow X_c = 15$$

$$\frac{1}{2\pi \times 50\, C} = 15$$

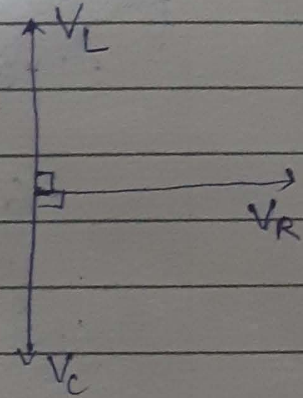
$$\therefore \quad C = \frac{1}{30 \times 50 \times \pi}$$

$$C = 212\ \mu F$$

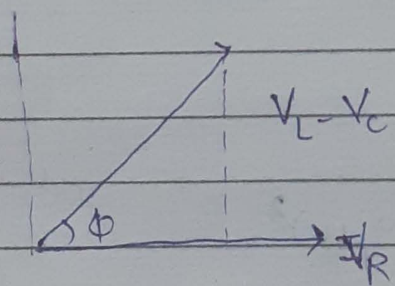
(3) R-L-C Series Circuit :



$V = V_m \sin \omega t$



(1) $V_L > V_C$

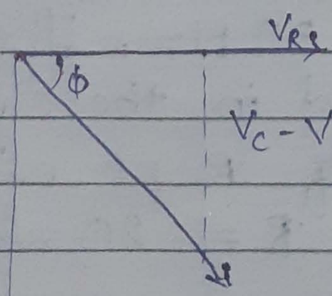


$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\therefore \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore \boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

(2) $V_C > V_L$



$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$\frac{V}{I} = \sqrt{R^2 + (X_C - X_L)^2}$$

$$\therefore \boxed{Z = \sqrt{R^2 + (X_C - X_L)^2}}$$

* Resonance in R-L-C Series Circuit:

- R-L-C are connected in series across variable frequency AC source.

$X_L = 2\pi f L \qquad X_C = \dfrac{1}{2\pi f C}$

- With increase in frequency X_L increases while X_C decreases. At one frequency, the value of inductive reactance X_L becomes equal to capacitive reactance X_C.
- This frequency is known as resonance frequency

i.e $X_L = X_C$

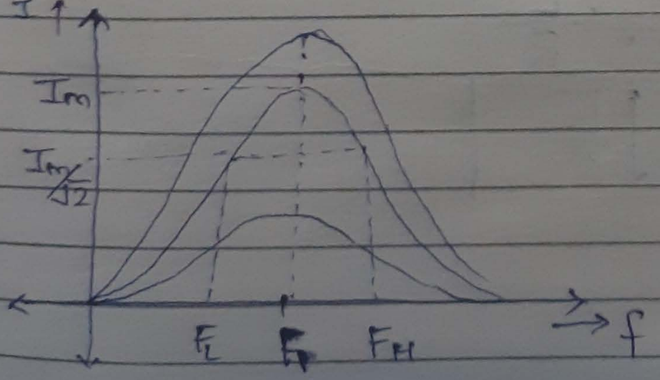
$$\therefore 2\pi f L = \frac{1}{2\pi f C}$$

$$\therefore F^2 = \frac{1}{(2\pi)^2 LC}$$

$$\therefore \boxed{F = \frac{1}{2\pi\sqrt{LC}}}$$

At resonance condition, $\boxed{Z = R}$ $(\because X_L - X_C = 0)$

* Resonance Curve:



The curve b/w frequency and current is known as resonance curve.

Bandwidth $(B_w) = F_H - F_L$

For smaller values of R, the resonance curve has sharp peak, but for larger values of R, the curve is flat.

* Q - Factor :

Q-factor is also known as voltage magnificat^n

$$Q\text{-factor} = \frac{\text{Voltage across L or C}}{\text{Applied Voltage}}$$

$$= \frac{V_L}{V} \text{ or } \frac{V_C}{V}$$

$$= \frac{IX_L}{IR} \text{ or } \frac{IX_C}{IR}$$

$$= \frac{X_L}{R} \text{ or } \frac{X_C}{R}$$

$$\boxed{\therefore Q \text{ factor} = \frac{\omega L}{R} \text{ or } \frac{1}{\omega C R}}$$

But we have considered resonance cond^n

$$\Rightarrow \omega_r = \frac{1}{\sqrt{LC}}$$

$$\therefore Q \text{ factor} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R}$$

$$\therefore \boxed{Q \text{ factor} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

* Bandwidth of Resonance Circuit :

Bandwidth of a circuit is given by band of frequencies which lies between two points on either side of resonant frequency, where current falls to $1/\sqrt{2}$ times of its maximum value i.e ($I = I_m/\sqrt{2}$).

$$\text{Bandwidth (Bw)} = \frac{\text{Resonant frequency}}{\text{Q-factor}}$$

eg: Coil of $R = 10\,\Omega$ and $L = 1\,H$ and $C = 15.83\,\mu F$ are connected in series across 100 V supply. The current is found to be 10 A. Determine frequency of supply.

$Z = V/I = 100/10 = 10$

$\therefore R = 10\,\Omega$ ($\because$ it is in resonance)

$$\therefore f = \frac{1}{2\pi\sqrt{LC}}$$

$$f = \frac{1}{6.28\sqrt{1\times 15.83\times 10^{-6}}}$$

$$\boxed{f = 40\ Hz}$$

(2) Calculate power, power factor & reactive power in a series circuit consisting of $R = 5\,\Omega$, $L = 0.1\,H$ and $C = 200\,\mu F$ when connected to 250 V, 50 Hz supply.

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$X_L = \omega L = 2\pi \times 50 \times 0.1 = 31.4$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times 200 \times 10^{-6}} = 15.9$

~~$P = VI \cos\phi$~~

~~$= 250 I \cos\phi$~~

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$Z = \sqrt{25 + (15.5)^2}$

$Z = 16.28$

$\therefore \quad I = \frac{V}{Z} = \frac{250}{16.28} = 15.35 \text{ A}$

1) $\cos\phi = \frac{R}{Z} = \frac{5}{16.28} = 0.3071$

2) $P = VI \cos\phi$

$= 250 \times 15.35 \times 0.3071$

$= 1178.49 \text{ W}$

3) $P_R = VI \sin\phi$

$= 250 \times 15.35 \times 0.951$

$= 3649.4 \text{ W}$

3 A series circuit has $R = 10\,\Omega$, $L = 200/\pi$ mH and $C = 1000/\pi$ μF. Calculate :

1) Current flowing in circuit (250 V, 50 Hz)
2) Power factor
3) Power drawn from supply
4) Draw phasor diagram.

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$$X_L = \omega L = 2\pi f L = 2\cancel{\pi} \times 50 \times \frac{200 \times 10^{-3}}{\cancel{\pi}}$$

$$= 20\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{\cancel{1}\ \cancel{\pi} \times 10^{6}}{2\cancel{\pi} \times 50 \times 1000}$$

$$= 10\ \Omega$$

$$\therefore Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{100 + (20-10)^2}$$

$$Z = 10\sqrt{2}\ \Omega$$

1) $I = \frac{V}{Z} = \frac{250}{10\sqrt{2}} = \underline{\underline{17.6\ A}}$

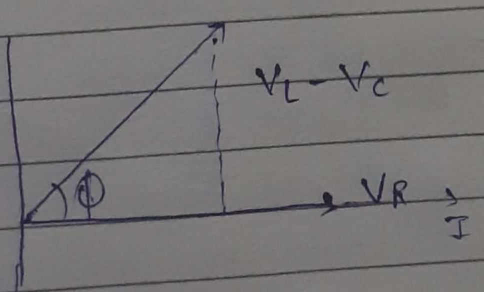
2) $\cos\phi = \frac{R}{Z} = \frac{10}{10\sqrt{2}} = \underline{\underline{0.7\ \Omega}}$

3) $P = VI\cos\phi$

$= 250 \times 10\sqrt{2} \times 0.7$

$= \underline{\underline{3.125\ kW}}$

4)



eg 4) Capacitor & resistor are connected in series across an AC supply of 50 V, 50 Hz. Current is 2 A & power dissipated in circuit is 80 W. Calculate R and C.

$P = I^2 R$

$\therefore R = \dfrac{80}{2 \times 2} = 20\ \Omega$

$Z = \dfrac{V}{I} = \dfrac{50}{2} = 25\ \Omega$

$Z^2 = R^2 + X_c^2$

$625 = 400 + X_c^2$

$\therefore X_c^2 = 225 \Rightarrow X_c = 15$

$X_c = \dfrac{1}{2\pi f C}$

$\therefore C = \dfrac{1}{6.28 \times 15 \times 50}$

$\therefore C = 212.2\ \mu F$