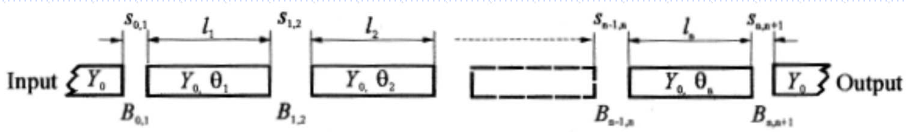
Microwave Filter Design
Chp. 5 Bandpass Filter

Prof. Tzong-Lin Wu

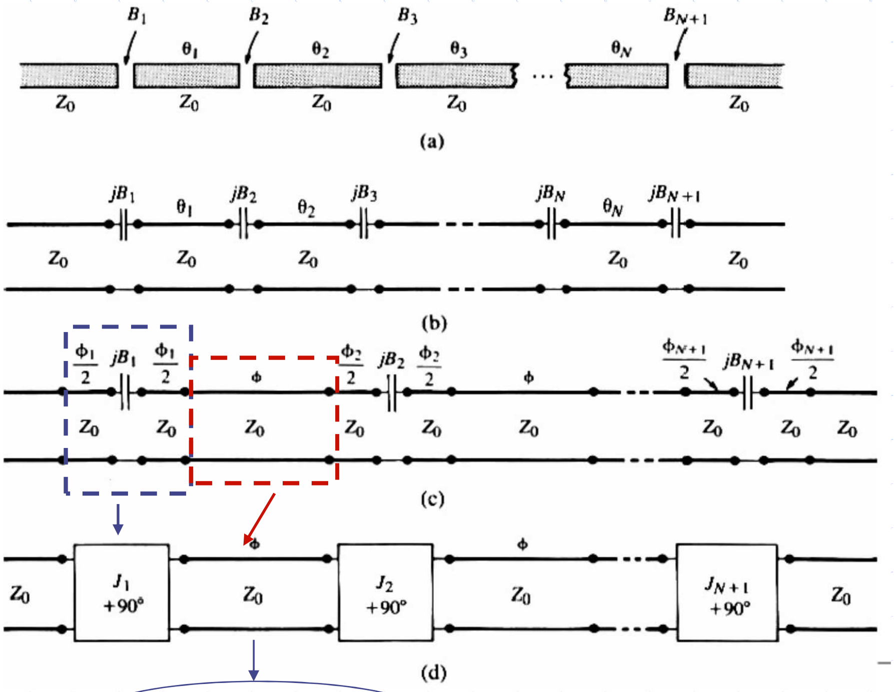
Department of Electrical Engineering
National Taiwan University

End-Coupled, Half-Wavelength Resonator Filters

Each open-end microstrip resonator is approximately a half guided wavelength long at the midband frequency f_0 of the bandpass filter.

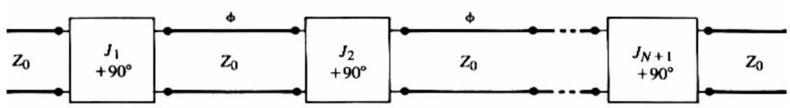


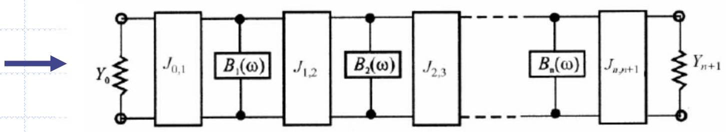
End-Coupled, Half-Wavelength Resonator Filters - Equivalent model



What kind of resonator ? Why?

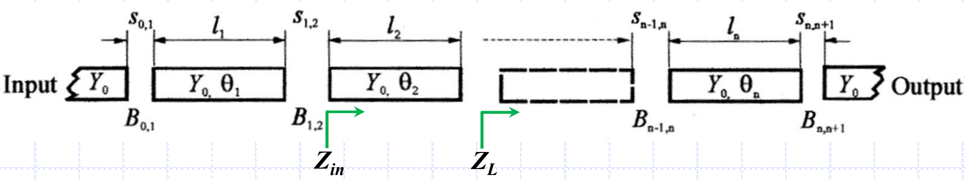
End-Coupled, Half-Wavelength Resonator Filters - Design theory



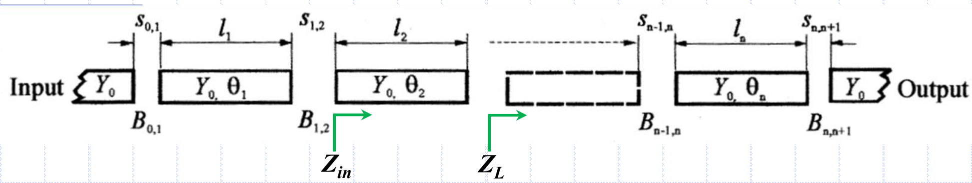


$J_{0,1}=\sqrt{\frac{Y_0 FBW b_1}{\Omega_c g_0 g_1}}$, $J_{i,i+1}=\frac{FBW}{\Omega_c}\sqrt{\frac{b_i b_{i+1}}{g_i g_{i+1}}}\Big|_{i=1 \text{ to } n-1}$, $J_{n,n+1}=\sqrt{\frac{FBW b_n Y_{n+1}}{\Omega_c g_n g_{n+1}}}$ $b_i=\frac{\omega_0}{2}\frac{dB_i(\omega)}{d\omega}\Big|_{\omega=\omega_0}$

1. Find J-inverter values (half-wavelength resonator)



End-Coupled, Half-Wavelength Resonator Filters - Design theory



$$Z_{in} = Z_0 \left. \frac{Z_L + jZ_0 \tan\left(\pi \frac{\omega}{\omega_0}\right)}{Z_0 + jZ_L \tan\left(\pi \frac{\omega}{\omega_0}\right)} \right|_{\omega \text{ near } \omega_0} = \left. \frac{1}{jY_0 \tan(\pi \frac{\omega}{\omega_0})} \right|_{|Z_L| = \infty} = \frac{1}{jB(\omega)}$$

- Susceptance slope parameters

$$b_i = \frac{\omega_0}{2} \left. \frac{dB(\omega)}{d\omega} \right|_{\omega=\omega_0} = \frac{\omega_0}{2} Y_0 \pi \frac{1}{\omega_0} = Y_0 \frac{\pi}{2}$$

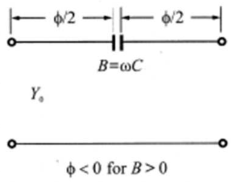
$$J_{01} = \sqrt{\frac{Y_0 b_1 FBW}{\Omega_c g_0 g_1}} = \left. \sqrt{\frac{Y_0 FBW}{\Omega_c g_0 g_1} Y_0 \frac{\pi}{2}} \right|_{\Omega_c = 1} = Y_0 \sqrt{\frac{\pi}{2} \frac{FBW}{g_0 g_1}}$$

$$J_{i,i+1} = \frac{FBW}{\Omega_c} \left. \sqrt{\frac{b_i b_{i+1}}{g_i g_{i+1}}} \right|_{i,i+1} = Y_0 \frac{\pi FBW}{2} \sqrt{\frac{1}{g_i g_{i+1}}}$$

$$J_{n,n+1} = \sqrt{\frac{Y_0 b_n FBW}{\Omega_c g_n g_{n+1}}} = \left. \sqrt{\frac{Y_0 FBW}{\Omega_c g_n g_{n+1}} Y_0 \frac{\pi}{2}} \right|_{\Omega_c = 1} = Y_0 \sqrt{\frac{\pi}{2} \frac{FBW}{g_n g_{n+1}}}$$

2. Find the susceptance of series gap capacitance

$$\frac{B_{j,j+1}}{Y_0} = \frac{\frac{J_{j,j+1}}{Y_0}}{1 - \left(\frac{J_{j,j+1}}{Y_0}\right)^2} \longrightarrow C_g^{j,j+1} = \frac{B_{j,j+1}}{\omega_0}$$



$$J = Y_0 \tan\left|\frac{\phi}{2}\right|$$

$$\phi = -\tan^{-1}\frac{2B}{Y_0}$$

$$\left|\frac{B}{Y_0}\right| = \frac{J/Y_0}{1-(J/Y_0)^2}$$

End-Coupled, Half-Wavelength Resonator Filters - Design theory

The physical lengths of resonators

$$\theta_j = \pi - \frac{1}{2}\left[\tan^{-1}\left(\frac{2B_{j-1,j}}{Y_0}\right) + \tan^{-1}\left(\frac{2B_{j,j+1}}{Y_0}\right)\right] \text{ radians}$$

The second term on the righthand side of the 2nd equation indicates the absorption of the negative electrical lengths of the *J*-inverters associated with the *j*th half-wavelength resonator.

$$\boxed{l_j = \frac{\lambda_{g0}}{2\pi}\theta_j - \Delta l_j^{e1} - \Delta l_j^{e2}}$$

where $\Delta l_j^{e1,e2}$ are the effective lengths of the shunt capacitances on the both ends of resonator j.

$$\Delta l_j^{e1} = \frac{\omega_0 C_p^{j-1,j}}{Y_0} \frac{\lambda_{g0}}{2\pi}$$

$$\Delta l_j^{e2} = \frac{\omega_0 C_p^{j,j+1}}{Y_0} \frac{\lambda_{g0}}{2\pi}$$

Example

a microstrip end-coupled bandpass filter is designed to have a fractional bandwidth FBW = 0.028 or 2.8% at the midband frequency f_0 = 6 GHz. A three-pole (n = 3) Chebyshev lowpass prototype with 0.1 dB passband ripple is chosen.

1. element values are $g_0 = g_4 = 1.0$, $g_1 = g_3 = 1.0316$, and $g_2 = 1.1474$.

$$\frac{J_{01}}{Y_0} = \frac{J_{3,4}}{Y_0} = \sqrt{\frac{\pi}{2} \times \frac{0.028}{1.0 \times 1.0316}} = 0.2065$$

$$\frac{J_{1,2}}{Y_0} = \frac{J_{2,3}}{Y_0} = \frac{\pi \times 0.028}{2} \frac{1}{\sqrt{1.0316 \times 1.1474}} = 0.0404$$

2.

$$\frac{B_{01}}{Y_0} = \frac{B_{3,4}}{Y_0} = \frac{0.2065}{1-(0.2065)^2} = 0.2157$$

$$\frac{B_{1,2}}{Y_0} = \frac{B_{2,3}}{Y_0} = \frac{0.0404}{1-(0.0404)^2} = 0.0405$$

$$C_g^{0,1} = C_g^{3,4} = 0.11443 \text{ pF}$$

$$C_g^{1,2} = C_g^{2,3} = 0.021483 \text{ pF}$$

$$\theta_1 = \theta_3 = \pi - \tfrac{1}{2}[\tan^{-1}(2 \times 0.2157) + \tan^{-1}(2 \times 0.0405)] = 2.8976 \text{ radians}$$

$$\theta_2 = \pi - \tfrac{1}{2}[\tan^{-1}(2 \times 0.0405) + \tan^{-1}(2 \times 0.0405)] = 3.0608 \text{ radians}$$

Example

3. Microstrip implementation

The line width for microstrip half-wave-

ε_r = 10.8 and a thickness h = 1.27 mm. The line width for microstrip half-wave-length resonators is also chosen as W = 1.1 mm, which gives characteristic impedance Z_0 = 50 ohm on the substrate.

How to determine the physical dimension of the coupling gap?

- Use the closed-form expressions for microstrip gap given in Chapter 4. However, the dimensions of the coupling gaps for the filter seem to be outside the parameter range available for these closed-form expressions. This will be the case very often when we design this type of microstrip filter.
- Utilize the EM simulation

Arrows indicate the reference planes for deembedding to obtain the two-port parameters of the microstrip gap.

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

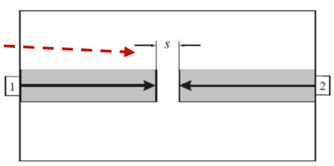
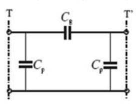


FIGURE 5.9 Layout of a microstrip gap for EM simulation.

$$C_g = -\frac{\text{Im}(Y_{21})}{\omega_0}$$

$$C_p = \frac{\text{Im}(Y_{11} + Y_{21})}{\omega_0}$$



Example

TABLE 5.4 Characterization of microstrip gaps with line width $W = 1.1$ mm on the substrate with $\varepsilon_r = 10.8$ and $h = 1.27$ mm

s (mm)	$Y_{11} = Y_{22}$ (mhos) at 6 GHz	$Y_{12} = Y_{21}$ (mhos) at 6 GHz	C_g (pF)	C_p (pF)
0.05	$j0.0045977$	$-j0.004434$	0.11762	0.00434
0.1	$j0.0039240$	$-j0.003604$	0.09560	0.00849
0.2	$j0.0032933$	$-j0.0026908$	0.07138	0.01598
0.5	$j0.0026874$	$-j0.0014229$	0.03774	0.03354
0.8	$j0.0025310$	$-j0.00081105$	0.02151	0.04562
1.0	$j0.0024953$	$-j0.00055585$	0.01474	0.05145
1.5	$j0.0024808$	$-j0.0001876$	0.00498	0.06083

by interpolation

$$s_{0,1} = s_{3,4} = 0.057 \text{ mm}$$
$$s_{1,2} = s_{2,3} = 0.801 \text{ mm}$$

$$C_p^{0,1} = C_p^{3,4} = 0.0049 \text{ pF}$$
$$C_p^{1,2} = C_p^{2,3} = 0.0457 \text{ pF}$$

At the midband frequency, $f_0 = 6$ GHz, the guided-wavelength of the microstrip line resonators is $\lambda_{g0} = 18.27$ mm. The effective lengths of the shunt capacitances are calculated using (5.17)

$$\Delta l_1^{e1} = \Delta l_3^{e2} = \frac{2\pi \times 6 \times 10^9 \times 0.0049 \times 10^{-12}}{(1/50)} \frac{18.27}{2\pi} = 0.0269 \text{ mm}$$

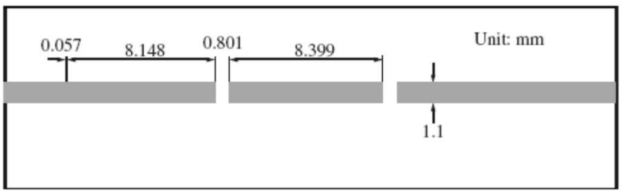
$$\Delta l_1^{e2} = \Delta l_3^{e1} = \frac{2\pi \times 6 \times 10^9 \times 0.0457 \times 10^{-12}}{(1/50)} \frac{18.27}{2\pi} = 0.2505 \text{ mm}$$

$$\Delta l_2^{e1} = \Delta l_2^{e2} = \Delta l_1^{e2}$$

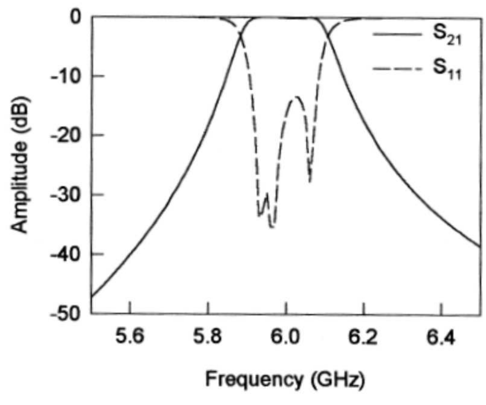
$$l_1 = l_3 = \frac{18.27}{2\pi} \times 2.8976 - 0.0269 - 0.2505 = 8.148 \text{ mm}$$

$$l_2 = \frac{18.27}{2\pi} \times 3.0608 - 0.2505 - 0.2505 = 8.399 \text{ mm}$$

Example



(a)

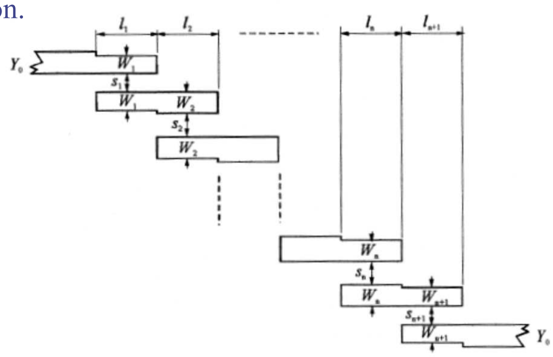


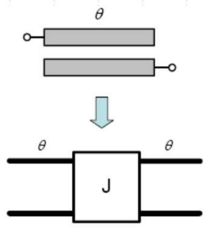
(b)

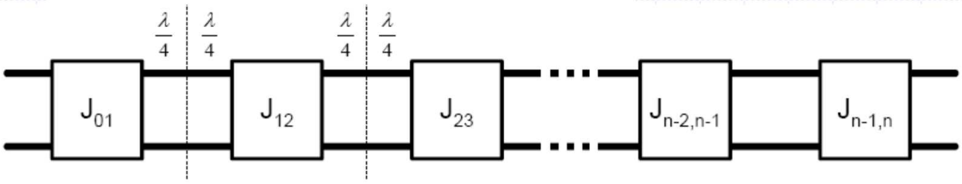
Parallel-Coupled, Half-Wavelength Resonator Filters

They are positioned so that adjacent resonators are parallel to each other along half of their length.

This parallel arrangement gives relatively large coupling for a given spacing between resonators, and thus, this filter structure is particularly convenient for constructing filters having a wider bandwidth as compared to the structure for the endcoupled microstrip filters described in the last section.







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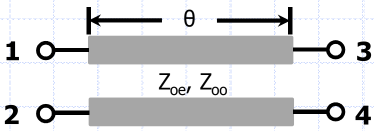
Parallel coupled line filters

$$Z_{11}=Z_{22}=Z_{33}=Z_{44}=\frac{-j}{2}\left(Z_{oe}+Z_{oo}\right)\cot\theta$$

$$Z_{12}=Z_{21}=Z_{34}=Z_{43}=\frac{-j}{2}\left(Z_{oe}-Z_{oo}\right)\cot\theta$$

$$Z_{13}=Z_{21}=Z_{24}=Z_{42}=\frac{-j}{2}\left(Z_{oe}+Z_{oo}\right)\csc\theta$$

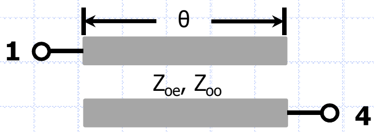
$$Z_{14}=Z_{41}=Z_{23}=Z_{32}=\frac{-j}{2}\left(Z_{oe}-Z_{oo}\right)\csc\theta$$



Apply boundary condition (open-end)

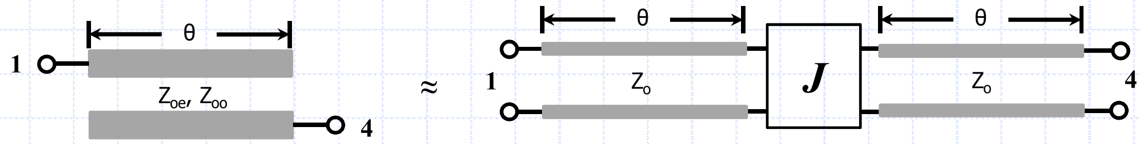
$$\begin{bmatrix}V_1\\V_4\end{bmatrix}=\begin{bmatrix}Z_{11}&Z_{14}\\Z_{41}&Z_{44}\end{bmatrix}\begin{bmatrix}I_1\\I_4\end{bmatrix}$$

$$\begin{bmatrix}Z_{11}&Z_{14}\\Z_{41}&Z_{44}\end{bmatrix}=\begin{bmatrix}\frac{-j}{2}\left(Z_{oe}+Z_{oo}\right)\cot\theta & \frac{-j}{2}\left(Z_{oe}-Z_{oo}\right)\csc\theta\\ \frac{-j}{2}\left(Z_{oe}-Z_{oo}\right)\csc\theta & \frac{-j}{2}\left(Z_{oe}+Z_{oo}\right)\cot\theta\end{bmatrix}$$



$$\begin{bmatrix}A&B\\C&D\end{bmatrix}=\begin{bmatrix}\frac{Z_{11}}{Z_{41}}&\frac{|Z|}{Z_{41}}\\ \frac{1}{Z_{41}}&\frac{Z_{44}}{Z_{41}}\end{bmatrix}=\begin{bmatrix}\frac{Z_{oe}+Z_{oo}}{Z_{oe}-Z_{oo}}\cos\theta & \frac{\left(Z_{oe}+Z_{oo}\right)^2\cot^2\beta\ell-\left(Z_{oe}-Z_{oo}\right)^2\csc^2\theta}{2j\left(Z_{oe}-Z_{oo}\right)\csc\theta}\\ 2\frac{j}{Z_{oe}-Z_{oo}}\sin\theta & \frac{Z_{oe}+Z_{oo}}{Z_{oe}-Z_{oo}}\cos\theta\end{bmatrix}$$

Parallel coupled line filters



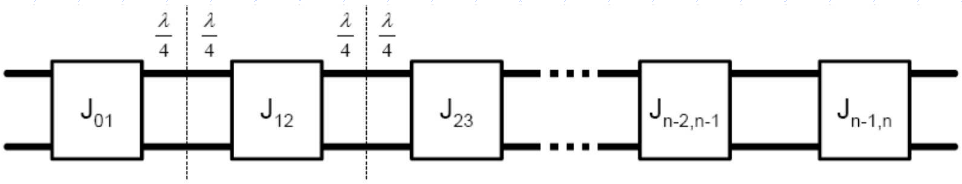
Left $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{Z_{oe}+Z_{oo}}{Z_{oe}-Z_{oo}}\cos\theta & j\dfrac{(Z_{oe}-Z_{oo})^2\csc^2\theta-(Z_{oe}+Z_{oo})^2\cot^2\theta}{2(Z_{oe}-Z_{oo})\csc\theta} \\ 2\dfrac{j}{Z_{oe}-Z_{oo}}\sin\theta & \dfrac{Z_{oe}+Z_{oo}}{Z_{oe}-Z_{oo}}\cos\theta \end{bmatrix}$

Right $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_o\sin\theta \\ j\dfrac{1}{Z_o}\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 & -j\dfrac{1}{J} \\ -jJ & 0 \end{bmatrix}\begin{bmatrix} \cos\theta & jZ_o\sin\theta \\ j\dfrac{1}{Z_o}\sin\theta & \cos\theta \end{bmatrix}$

$$= \begin{bmatrix} \left(JZ_o+\dfrac{1}{JZ_o}\right)\cos\theta\sin\theta & j\left(JZ_o^2\sin^2\theta-\dfrac{\cos^2\theta}{J}\right) \\ j\left(\dfrac{1}{JZ_o^2}\sin^2\theta-J\cos^2\theta\right) & \left(JZ_o+\dfrac{1}{JZ_o}\right)\cos\theta\sin\theta \end{bmatrix}$$

$\theta \approx \pi/2$ => $\begin{cases} \dfrac{Z_{oe}+Z_{oo}}{Z_{oe}-Z_{oo}} = JZ_o+\dfrac{1}{JZ_o} \\ \dfrac{Z_{oe}-Z_{oo}}{2} = JZ_o^2 \end{cases}$ => $\begin{cases} Z_{oe}+Z_{oo} = 2J^2Z_o^3+2Z_o = 2Z_o\left(1+J^2Z_o^2\right) \\ Z_{oe}-Z_{oo} = 2JZ_o^2 \end{cases}$ => $\begin{cases} Z_{oe} = Z_o\left(1+JZ_o+J^2Z_o^2\right) \\ Z_{oo} = Z_o\left(1-JZ_o+J^2Z_o^2\right) \end{cases}$

Parallel-Coupled, Half-Wavelength Resonator Filters -Theory



$$\frac{J_{01}}{Y_0} = \sqrt{\frac{\pi}{2}\frac{FBW}{g_0 g_1}}$$

$$\frac{J_{j,j+1}}{Y_0} = \frac{\pi FBW}{2}\frac{1}{\sqrt{g_j g_{j+1}}} \qquad j = 1 \text{ to } n-1$$

$$\frac{J_{n,n+1}}{Y_0} = \sqrt{\frac{\pi FBW}{2 g_n g_{n+1}}}$$

$$(Z_{0e})_{j,j+1} = \frac{1}{Y_0}\left[1+\frac{J_{j,j+1}}{Y_0}+\left(\frac{J_{j,j+1}}{Y_0}\right)^2\right] \qquad j = 0 \text{ to } n$$

$$(Z_{0o})_{j,j+1} = \frac{1}{Y_0}\left[1-\frac{J_{j,j+1}}{Y_0}+\left(\frac{J_{j,j+1}}{Y_0}\right)^2\right] \qquad j = 0 \text{ to } n$$

Parallel-Coupled, Half-Wavelength Resonator Filters -Example

Step 1:

Let us consider a design of five-pole (n = 5) microstrip bandpass filter that has a fractional bandwidth FBW = 0.15 at a midband frequency $f0$ = 10 GHz. Suppose a Chebyshev prototype with a 0.1-dB ripple is to be used in the design.

$$g_0 = g_6 = 1.0 \qquad g_1 = g_5 = 1.1468$$
$$g_2 = g_4 = 1.3712 \qquad g_3 = 1.9750$$

TABLE 5.5 Circuit design parameters of the five-pole, parallel-coupled half-wavelength resonator filter

j	$J_{j,j+1}/Y_0$	$(Z_{0e})_{j,j+1}$	$(Z_{0o})_{j,j+1}$
0	0.4533	82.9367	37.6092
1	0.1879	61.1600	42.3705
2	0.1432	58.1839	43.8661

Parallel-Coupled, Half-Wavelength Resonator Filters -Example

Step 2:

Find the dimensions of coupled microstrip lines that exhibit the desired even- and odd-mode impedances.

Assume that the microstrip filter is constructed on a substrate with a relative dielectric constant of 10.2 and thickness of 0.635 mm.

TABLE 5.6 Microstrip design parameters of the five-pole, parallel-coupled half-wavelength resonator filter

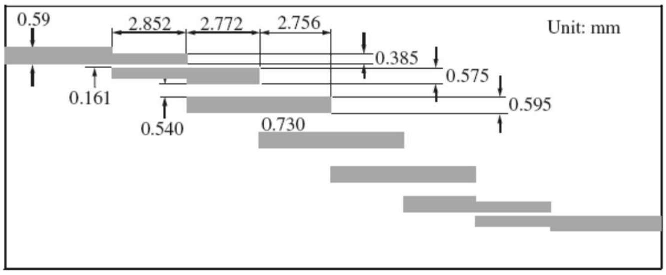
j	W_j (mm)	s_j (mm)	$(\varepsilon_{re})_j$	$(\varepsilon_{ro})_j$
1 and 6	0.385	0.161	6.5465	5.7422
2 and 5	0.575	0.540	6.7605	6.0273
3 and 4	0.595	0.730	6.7807	6.1260

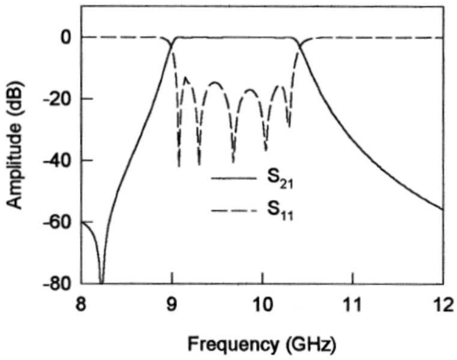
The actual lengths of each coupled line section are then determined by

$$l_j = \frac{\lambda_0}{4(\sqrt{(\varepsilon_{re})_j \times (\varepsilon_{ro})_j})^{1/2}} - \Delta l_j$$

where Δl_j is the equivalent length of microstrip open end,

Parallel-Coupled, Half-Wavelength Resonator Filters -Example





Hairpin-Line Bandpass Filters

They may conceptually be obtained by folding the resonators of parallel-coupled, half-wavelength resonator filters.

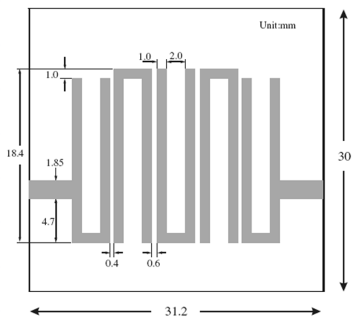
This type of "U" shape resonator is the so-called hairpin resonator.

The same design equations for the parallel-coupled, half-wavelength resonator filters may be used.

However, to fold the resonators, it is necessary to take into account the reduction of the coupled-line lengths, which reduces the coupling between resonators.

If the two arms of each hairpin resonator are closely spaced, they function as a pair of coupled line themselves, which can have an effect on the coupling as well.

To design this type of filter more accurately, a design approach employing full-wave EM simulation will be described.



Prof. T. L. Wu

Hairpin-Line Bandpass Filters - Example

A microstrip hairpin bandpass filter is designed to have a fractional bandwidth of 20% or *FBW* = 0.2 at a midband frequency *f*0 = 2 GHz. A five-pole (n = 5) Chebyshev lowpass prototype with a passband ripple of 0.1 dB is chosen.

The lowpass prototype parameters, given for a normalized lowpass cutoff frequency $\Omega_c = 1$, are $g_0 = g_6 = 1.0$, $g_1 = g_5 = 1.1468$, $g_2 = g_4 = 1.3712$, and $g_3 = 1.9750$.

$$Q_{e1} = \frac{g_0 g_1}{FBW}, \qquad Q_{en} = \frac{g_n g_{n+1}}{FBW}$$

$$M_{i,i+1} = \frac{FBW}{\sqrt{g_i g_{i+1}}} \quad \text{for } i = 1 \text{ to } n-1$$

$$Q_{e1} = Q_{e5} = 5.734$$

$$M_{1,2} = M_{4,5} = 0.160$$

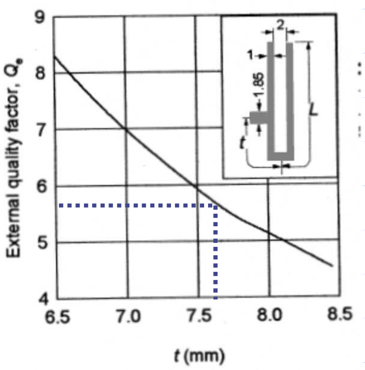
$$M_{2,3} = M_{3,4} = 0.122$$

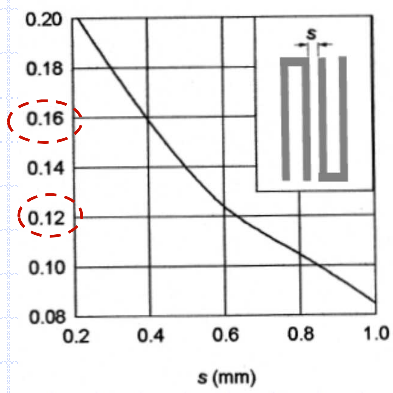
where Q_{e1} and Q_{en} are the external quality factors of the resonators at the input and output, and $M_{i,i+1}$ are the coupling coefficients between the adjacent resonators (see Chapter 8).

We use a commercial substrate (RT/D 6006) with a relative dielectric constant of 6.15 and a thickness of 1.27 mm for microstrip realization.

Hairpin-Line Bandpass Filters - Example

Using a parameter-extraction technique described in Chapter 8, we then carry out full-wave EM simulations to extract the external Q and coupling coefficient M against the physical dimensions.

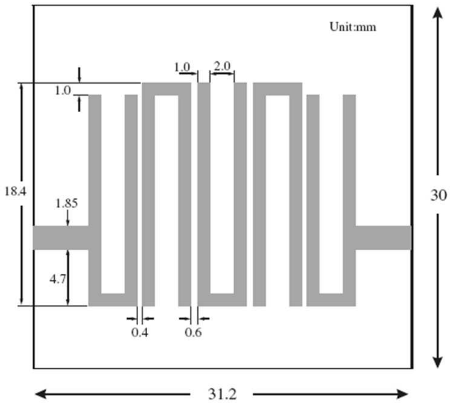


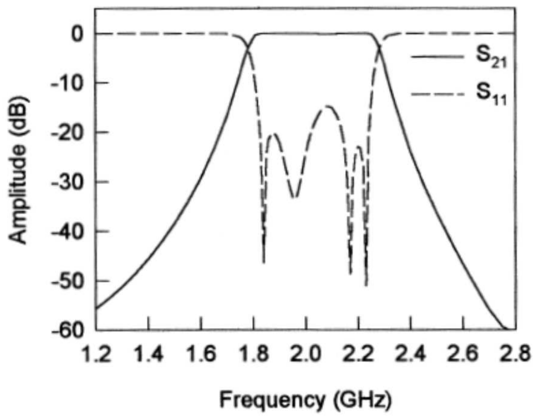


The hairpin resonators used have a line width of 1 mm and a separation of 2 mm between the two arms.

Dimension of the resonator as indicated by L is about $\lambda_{g0}/4$ long and in this case, L = 20.4 mm.

Hairpin-Line Bandpass Filters - Example





The filter is quite compact, with a substrate size of 31.2mm by 30 mm.

The input and output resonators are slightly shortened to compensate for the effect of the tapping line and the adjacent coupled resonator.

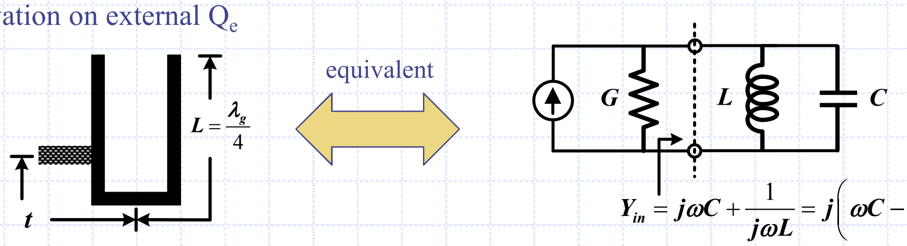
Hairpin-Line Bandpass Filters - Example

A design equation is proposed for estimating the tapping point t as

$$t = \frac{2L}{\pi}\sin^{-1}\left(\sqrt{\frac{\pi}{2}\frac{Z_0/Z_r}{Q_e}}\right)$$

in which Z_r is the characteristic impedance of the hairpin line, Z_0 is the terminating impedance, and L is about $\lambda_{g0}/4$ long, as mentioned above.

- Derivation on external Q_e



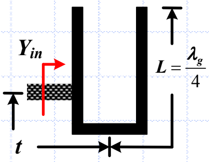
$$Y_{in} = j\omega C + \frac{1}{j\omega L} = j\left(\omega C - \frac{1}{\omega L}\right) = jB$$

External quality factor for parallel L/C resonator:

$$b = \frac{\omega_0}{2}\frac{\partial B}{\partial \omega}\bigg|_{\omega=\omega_0} = \frac{\omega_0}{2}\left(C + \frac{1}{\omega_0^2 L}\right) = \frac{\omega_0}{2}(C + C) = \omega_0 C$$

$$Q = \frac{\omega_0 C}{G} = \frac{b}{G}$$

Hairpin-Line Bandpass Filters - Example



External quality factor for hairpin line:

$$Y_{in} = jY_r \tan\left[\beta(L-t)\right] + jY_r \tan\left[\beta(L+t)\right] = jY_r\left\{\tan\left[\beta(L-t)\right] + \tan\left[\beta(L+t)\right]\right\} = jB(\omega)$$

$$b = \frac{\omega_0}{2}\frac{\partial B}{\partial \omega}\bigg|_{\omega=\omega_0} = \frac{\omega_0}{2}\frac{\partial B}{\partial \beta}\frac{\partial \beta}{\partial \omega}\bigg|_{\beta=\beta_0} = \frac{\omega_0}{2}\frac{Y_r}{u_p}\left\{(L-t)\sec^2\left[\beta_0(L-t)\right] + (L+t)\sec^2\left[\beta_0(L+t)\right]\right\}$$

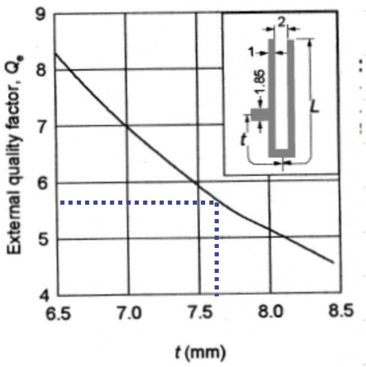
condition: $\beta_0 L = \frac{\pi}{2}$

$$= \frac{\omega_0}{2}\frac{Y_r}{u_p}\left[(L-t)\csc^2(\beta_0 t) + (L+t)\csc^2(\beta_0 t)\right] = Y_r\frac{\omega_0 L}{u_p}\frac{1}{\sin^2(\beta_0 t)} = \frac{\pi}{2}\frac{Y_r}{\sin^2\left(\frac{2\pi}{\lambda_g}t\right)} = \frac{\pi}{2}\frac{Y_r}{\sin^2\left(\frac{\pi t}{2L}\right)}$$

$$Q_e = \frac{b}{G} = \frac{\frac{\pi}{2}\frac{Y_r}{\sin^2\left(\frac{\pi t}{2L}\right)}}{\frac{1}{Z_0}} = \frac{\pi Z_0}{2Z_r\sin^2\left(\frac{\pi t}{2L}\right)} \Rightarrow t = \frac{2L}{\pi}\sin^{-1}\sqrt{\frac{\pi}{2}\frac{Z_0/Z_r}{Q_e}}$$

Example:

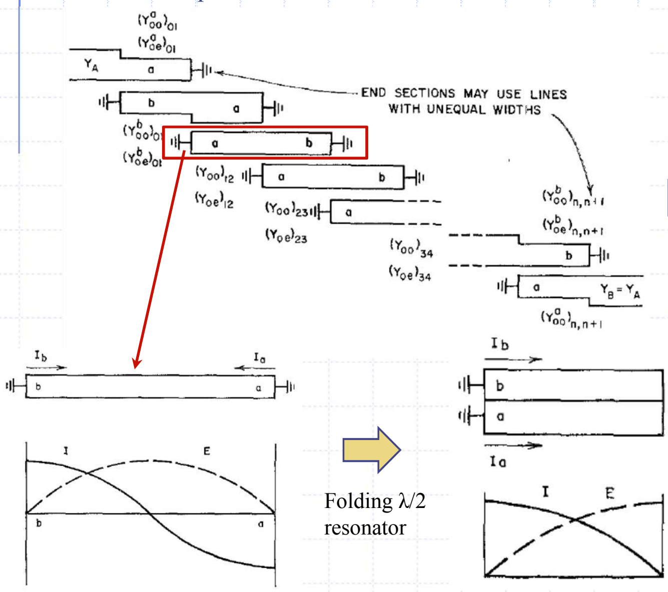
the hairpin line is 1.0 mm wide, which results in Z_r = 68.3 ohm on the substrate used. Recall that L = 20.4 mm, Z_0 = 50 ohm, and the required Q_e = 5.734. Substituting them into the eq. yields a t = 6.03 mm, which is close to the t of 7.625 mm found from the EM simulation above.



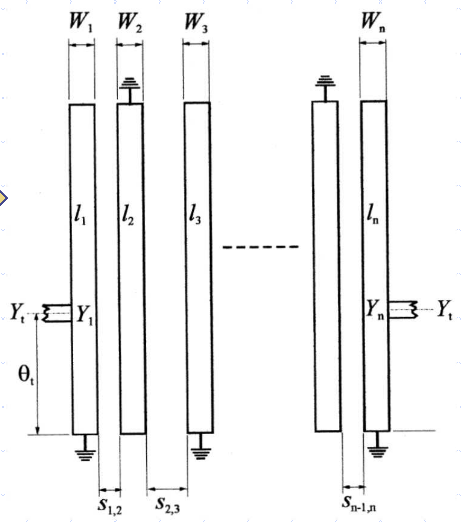
Interdigital Filter - Concept

The filter configuration, as shown, consists of an array of n TEM-mode or quasi-TEM-mode transmission line resonators, each of which has an electrical length of 90° at the midband frequency and is short-circuited at one end and open-circuited at the other end with alternative orientation.

- Parallel coupled-line BPF

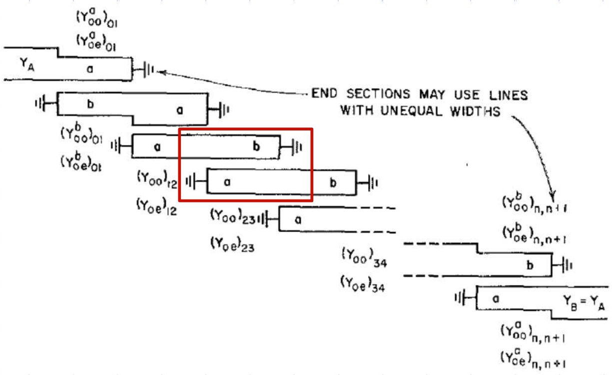


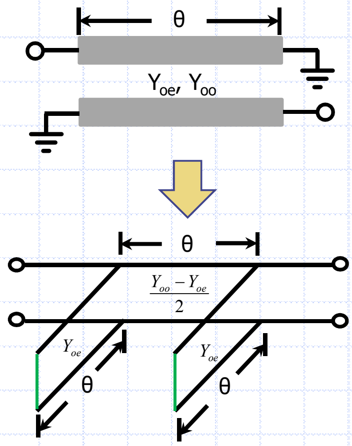
- Interdigital line filter with tapped line



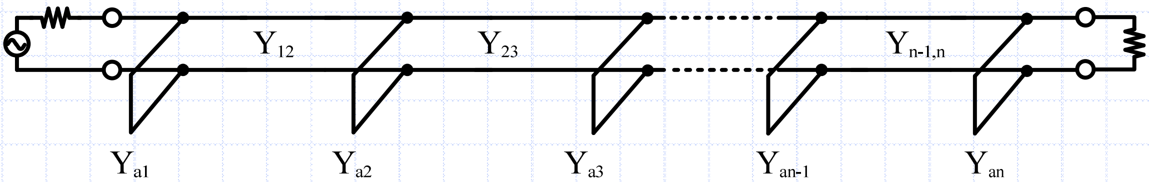
$Y_1 = Y_n$ denotes the single microstrip characteristic impedance of the input/output resonator.

Interdigital Filter
- Concept





- BPF with short-circuited stubs



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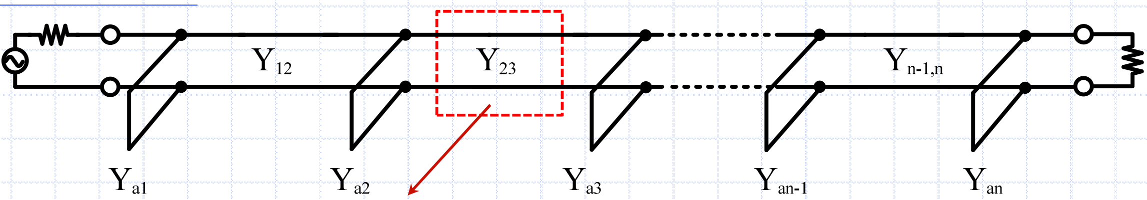
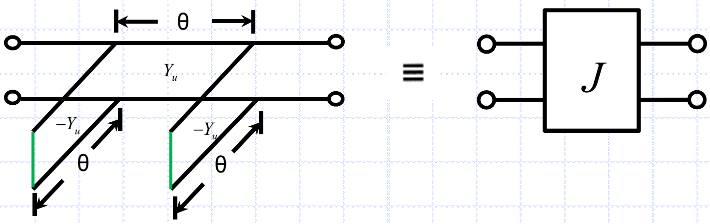
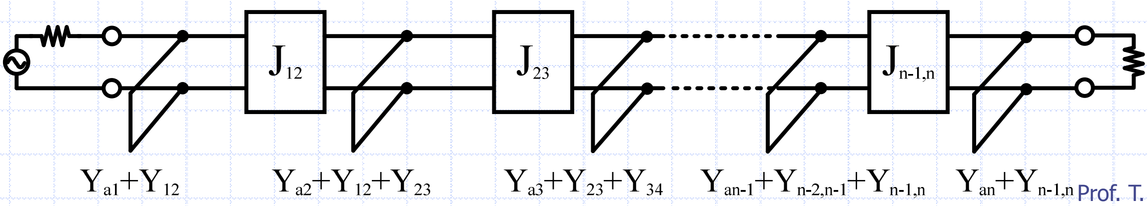
Interdigital Filter
- Design Equations





$$\begin{bmatrix} 1 & 0 \\ \frac{jY_u}{\tan\theta} & 1 \end{bmatrix}\begin{bmatrix} \cos\theta & \frac{j\sin\theta}{Y_u} \\ jY_u\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{jY_u}{\tan\theta} & 1 \end{bmatrix}=\begin{bmatrix} \cos\theta & \frac{j\sin\theta}{Y_u} \\ \frac{jY_u}{\sin\theta} & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{jY_u}{\tan\theta} & 1 \end{bmatrix}=\begin{bmatrix} 0 & \frac{j\sin\theta}{Y_u} \\ \frac{jY_u}{\sin\theta} & 0 \end{bmatrix}=\begin{bmatrix} 0 & \frac{j}{J} \\ jJ & 0 \end{bmatrix} \Rightarrow J=\frac{Y_u}{\sin\theta}$$



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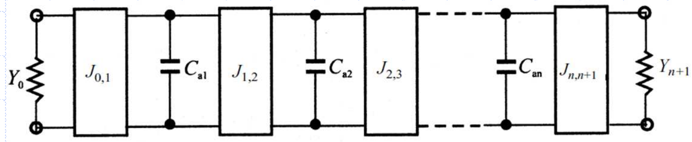
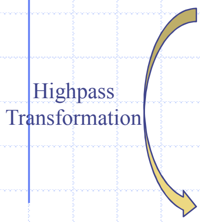
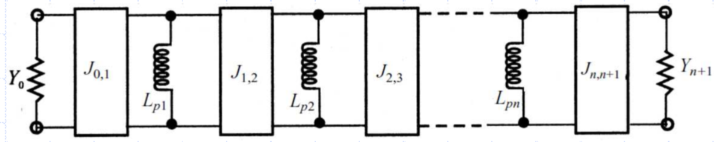
Interdigital Filter
- Design Equations

- Transform the lowpass prototype filter to the highpass prototype filter



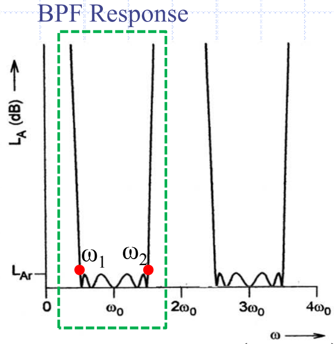


$$J_{0,1}=\sqrt{\frac{Y_0C_{a1}}{g_0g_1}}\ ,\quad J_{i,i+1}=\sqrt{\frac{C_{ai}C_{a(i+1)}}{g_ig_{i+1}}}\Bigg|_{i=1\text{ to }n-1}\ ,\quad J_{n,n+1}=\sqrt{\frac{C_{an}Y_{n+1}}{g_ng_{n+1}}}$$

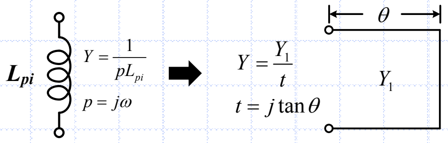


$$J_{0,1}=\sqrt{\frac{Y_0}{L_{p1}g_0g_1}}\qquad J_{i,i+1}=\frac{1}{\sqrt{L_{pi}L_{p(i+1)}g_ig_{i+1}}}\Bigg|_{i=1\text{ to }n-1}\qquad J_{n,n+1}=\sqrt{\frac{Y_{n+1}}{L_{pn}g_ng_{n+1}}}$$

BPF Response



- Richard's Transformation to decide the inductor value at edge frequency of passband

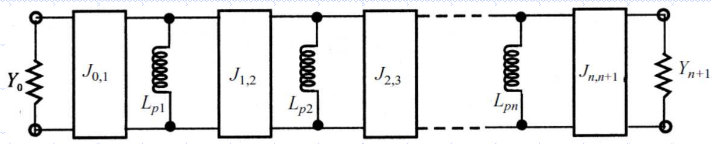


$$Y=\frac{1}{j\omega_1 L_{pi}}=\frac{Y_1}{j\tan\theta}\ \Rightarrow\ \frac{1}{L_{pi}}=\frac{Y_1}{\tan\theta}$$

$$\omega_1=\omega_0-\omega_0\frac{FBW}{2}\ \Rightarrow\ \theta=\frac{\omega_1}{u_p}\ell=\frac{\omega_0}{u_p}\ell\left(1-\frac{FBW}{2}\right)=\frac{\pi}{2}\left(1-\frac{FBW}{2}\right)$$

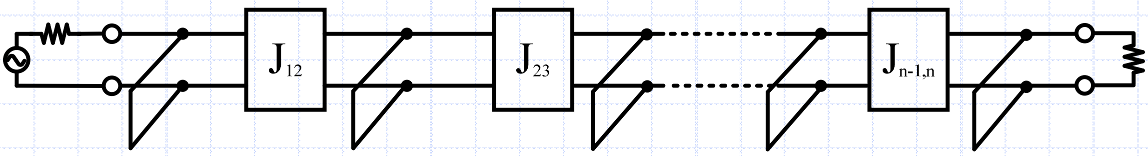
Interdigital Filter
- Design Equations

- J-inverter value for the BPF



$$J_{i,i+1}=\frac{1}{\sqrt{L_{pi}L_{p(i+1)}g_ig_{i+1}}}\Bigg|_{i=1\text{ to }n-1}=\frac{Y_1}{\tan\theta\sqrt{g_ig_{i+1}}}\Bigg|_{i=1\text{ to }n-1}=\frac{Y}{\sqrt{g_ig_{i+1}}}\Bigg|_{i=1\text{ to }n-1}$$

Y_1 denotes the characteristic impedance of the short-circuited stubs.



$Y_{a1}+Y_{12}$ $\quad Y_{a2}+Y_{12}+Y_{23}$ $\quad Y_{a3}+Y_{23}+Y_{34}$ $\quad Y_{an-1}+Y_{n-2,n-1}+Y_{n-1,n}$ $\quad Y_{an}+Y_{n-1,n}$

- From Richard's transformation and previous derivations on interdigital filter

Input part: $Y_1=Y_{a1}+Y_{12}$ ➡ $Y_{a1}=Y_1-Y_{12}$

Internal part: $Y_1=Y_{ai}+Y_{i-1,i}+Y_{i,i+1}$ ➡ $Y_{ai}=Y_1-Y_{i-1,i}-Y_{i,i+1}$

Output part: $Y_1=Y_{an}+Y_{n-1,n}$ ➡ $Y_{an}=Y_1-Y_{n-1,n}$

Y_{ai} denotes the characteristic impedance you have to find from the interdigital filter .

Interdigital Filter
- Design Equations

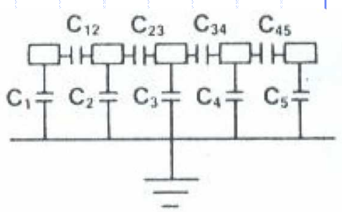
- Self- and mutual capacitances per unit length are used to find the corresponding physical dimensions

$$Y_{a1} = Y_1\text{-}Y_{12} \quad \Rightarrow \quad Y_{a1} = \sqrt{\frac{C_1}{L_1}} \times \sqrt{\frac{C_1}{C_1}} = u_p C_1 = Y_1\text{-}Y_{12} \Rightarrow C_1 = \frac{Y_1\text{-}Y_{12}}{u_p}$$

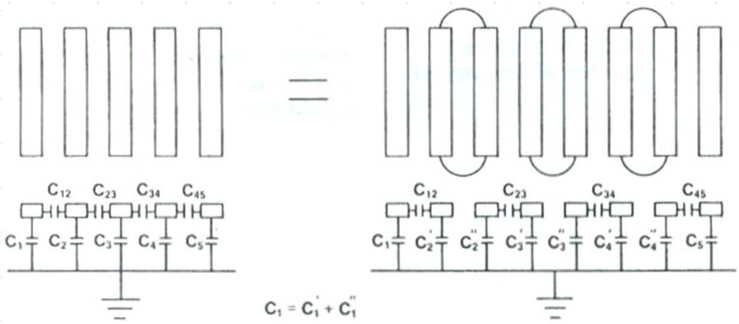
$$Y_{ai} = Y_1\text{-}Y_{i-1,i}\text{-}Y_{i,i+1} \quad \Rightarrow \quad Y_{ai} = Y_1\text{-}Y_{i-1,i}\text{-}Y_{i,i+1} \Rightarrow C_i = \frac{Y_1\text{-}Y_{i-1,i}\text{-}Y_{i,i+1}}{u_p}$$

$$Y_{an} = Y_1\text{-}Y_{n-1,n} \quad \Rightarrow \quad Y_{an} = Y_1\text{-}Y_{n-1,n} \Rightarrow C_n = \frac{Y_1\text{-}Y_{n-1,n}}{u_p}$$

Connecting line $Y_{i,i+1}$ ➡ $C_{i,i+1} = \dfrac{Y_{i,i+1}}{u_p}$ for i=1 to n-1



- Approximate design approach to find a pair of parallel-coupled lines



$$Z_{0e1,2} = \frac{1}{Y_1 - Y_{1,2}}, \qquad Z_{0o1,2} = \frac{1}{Y_1 + Y_{1,2}}$$

$$Z_{0ei,i+1} = \frac{1}{2Y_1 - 1/Z_{0ei-1,i} - Y_{i,i+1} - Y_{i-1,i}} \quad \text{for } i = 2 \text{ to } n-2$$

$$Z_{0oi,i+1} = \frac{1}{2Y_{i,i+1} + 1/Z_{0ei,i+1}} \quad \text{for } i = 2 \text{ to } n-2$$

$$Z_{0en-1,n} = \frac{1}{Y_1 - Y_{n-1,n}}, \qquad Z_{0on-1,n} = \frac{1}{Y_1 + Y_{n-1,n}}$$

Interdigital Filter
- Design theory

$$\theta = \frac{\pi}{2}\left(1 - \frac{FBW}{2}\right),\ Y = \frac{Y_1}{\tan\theta}$$

$$Y_t = Y_1 - \frac{Y_{1,2}^2}{Y_1}$$

$$J_{i,i+1} = \frac{Y}{\sqrt{g_i g_{i+1}}} \quad \text{for } i = 1 \text{ to } n-1$$

$$\theta_t = \frac{\sin^{-1}\left(\sqrt{\dfrac{Y\sin^2\theta}{Y_0 g_0 g_1}}\right)}{1 - \dfrac{FBW}{2}}$$

$$Y_{i,i+1} = J_{i,i+1}\sin\theta \quad \text{for } i = 1 \text{ to } n-1$$

$$C_1 = \frac{Y_1 - Y_{1,2}}{v},\ C_n = \frac{Y_1 - Y_{n-1,n}}{v}$$

$$C_t = \frac{\cos\theta_t \sin^3\theta_t}{\omega_0 Y_t\left(\dfrac{1}{Y_0^2} + \dfrac{\cos^2\theta_t \sin^2\theta_t}{Y_t^2}\right)}$$

$$C_i = \frac{Y_1 - Y_{i-1,i} - Y_{i,i+1}}{v} \quad \text{for } i = 2 \text{ to } n-1$$

$$C_{i,i+1} = \frac{Y_{i,i+1}}{v} \quad \text{for } i = 1 \text{ to } n-1$$

1. $C_i (i = 1 \text{ to } n)$ are the self-capacitances per unit length for the line elements, whereas $C_{i,i+1} (i = 1 \text{ to } n-1)$ are the mutual capacitances per unit length between adjacent line elements. Note that v denotes the wave phase velocity in the medium of propagation. The physical dimensions of the line elements may then be found from the required self- and mutual capacitances. C_t is the capacitance to be loaded to the input and output resonators in order to compensate for resonant frequency shift due to the effect of the tapped input and output.

Interdigital Filter - Design theory

Full-wave electromagnetic (EM) simulation technique can be employed to extract such even- and odd-mode impedances for the filter design,

$$S_{11} = S_{22} = |S_{11}|e^{j\phi_{11}}$$
$$S_{12} = S_{21} = |S_{21}|e^{j\phi_{21}}$$

$$\rightarrow \quad Z_c = \mathrm{Re}\left\{ \frac{Z_{in} - Z_0 + \sqrt{(Z_{in} - Z_0)^2 - 4Z_0 Z_{in} \tan^2 \phi_{21}}}{j2 \tan\phi_{21}} \right\}$$

$$\downarrow$$

$$\varepsilon_{re} = \left(\frac{\phi_{21}}{2\pi} \frac{\lambda_0}{L} \right)^2$$

$$Z_{in} = Z_0 \frac{1 + S_{11}}{1 - S_{11}}$$

Some commercial EM simulators such as *em* can automatically extract ε_{re} and Z_c.

For the even-mode excitation, the average even-mode impedance is then found by

$$Z_{0e} = 2Z_c$$

the average odd-mode impedance is determined by

$$Z_{0o} = Z_c/2$$

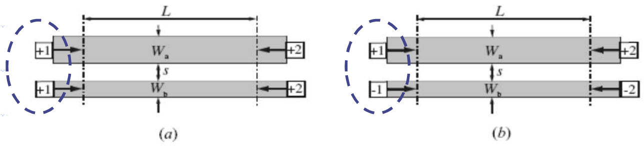


FIGURE 5.16 Microstrip layouts for full-wave EM simulations to extract even- and odd-mode impedances. (*a*) Even-mode excitation. (*b*) Odd-mode excitation.

Interdigital Filter - Design Example

the design is worked out using an n = 5 Chebyshev lowpass prototype with a passband ripple 0.1 dB. The bandpass filter is designed for a fractional bandwidth *FBW* = 0.5 centered at the midband frequency $f0$ = 2.0 GHz.
The prototype parameters are

$$g_0 = g_6 = 1.0 \qquad g_1 = g_5 = 1.1468$$
$$g_2 = g_4 = 1.3712 \qquad g_3 = 1.9750$$

using the design equations above given the table 5.7

TABLE 5.7 Circuit design parameters of the five-pole, interdigital bandpass filter with asymmetric coupled lines

i	$Z_{0ei,i+1}$	$Z_{0oi,i+1}$
1	65.34	34.78
2	59.16	36.83
3	59.16	36.83
4	65.34	34.78

$Y_1 = 1/45.5$ mhos.
$Y_t = 1/50$ mhos.
$\theta_t = 0.82929$ radians.
$C_t = 3.45731 \times 10^{-13}$ F.

TABLE 5.8 Microstrip design parameters of the five-pole, interdigital bandpass filter with asymmetric coupled lines

W_a (mm)	W_b (mm)	s (mm)	Z_{0e} (ohm)	Z_{0o} (ohm)	ε_{re}^e	ε_{re}^o
2.2	1.2	0.2	63.6	34.15	4.68	3.76
2.2	1.1	0.2	64.92	34.83	4.66	3.75
2.0	1.1	0.5	74.7	41.82	4.70	3.80
2.4	1.1	0.5	59.8	40.44	4.73	3.82
2.6	1.1	0.5	57.6	39.86	4.75	3.83
2.8	1.1	0.5	55.6	39.33	4.76	3.84
2.8	1.1	0.4	56.4	37.85	4.76	3.82
2.8	1.1	0.3	57.2	36.01	4.75	3.80
2.7	1.1	0.3	58.26	36.22	4.75	3.79
2.6	1.1	0.3	59.36	36.43	4.74	3.79

The characteristic admittance Y_1 is so chosen that the characteristic impedance of the tapped lines $Z_t = 1/Y_t$ is equal to 50 ohms.

A commercial dielectric substrate (RT/D 6006) with a relative dielectric constant of 6.15 and a thickness of 1.27 mm is chosen for the filter design, and table 5.8 is obtained.

Design Example with asymmetric coupled lines

Next, we need to decide the lengths of microstrip interdigital resonators.

$$l_i = \lambda_{g0i}/4 - \Delta l_i$$

where λ_{g0i} is the guided wavelength and Δl_i is the equivalent line length of microstrip open end associated with resonator i.

unequal effective dielectric constants for the both modes

$$\lambda_{g0i} = \lambda_0(\sqrt{\varepsilon^e_{rei}\varepsilon^o_{rei}})^{-1/2}$$

there is a capacitance C_t that needs to be loaded to the input and output resonators

capacitive loading may be achieved by an open-circuit stub, namely, an extension in length of the resonators.

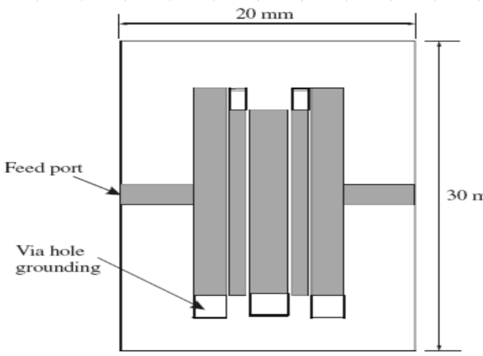
$$l_1 = l_n = \lambda_{g01}/4 - \Delta l_1 + \Delta l_C$$

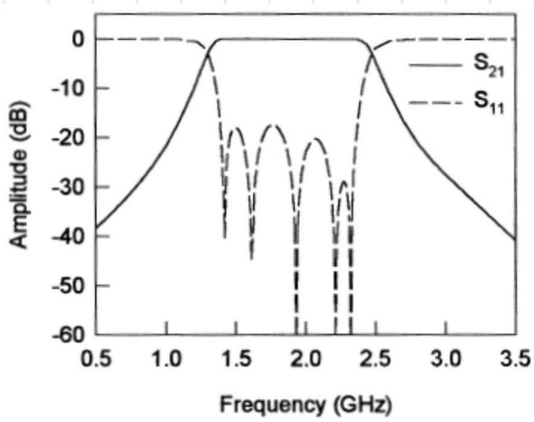
$$\Delta l_C = \frac{\lambda_{g01}}{2\pi}\tan^{-1}\left(\frac{2\pi f_0 C_t}{Y_1}\right)$$

Design Example with asymmetric coupled lines

TABLE 5.9 Filter dimensions (mm) on substrate with $\varepsilon_r = 6.15$ and $h = 1.27$ mm

$W_1 = W_5 = 2.2$	$W_2 = W_4 = 1.1$	$W_3 = 2.6$	$s_{1,2} = s_{4,5} = 0.2$	$W_t = 1.85$
$l_1 = l_5 = 20.14$	$l_2 = l_4 = 17.85$	$l_3 = 17.72$	$s_{2,3} = s_{3,4} = 0.3$	$l_t = 9.68$





Combline Filters

The combline bandpass filter is comprised of an array of coupled resonators.

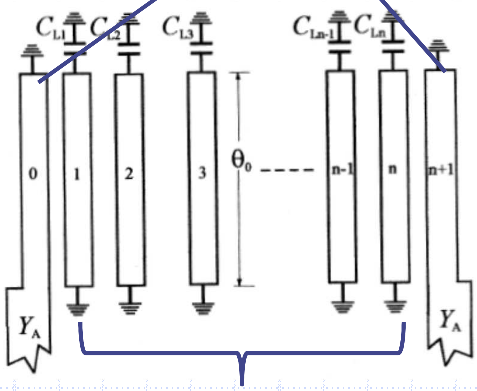
The resonators consist of line elements 1 to n, which are short-circuited at one end, with a lumped capacitance C_{Li} loaded between the other end of each resonator line element and ground.

The input and output of the filter are through coupled-line elements 0 and $n+1$, which are not resonators.

Not resonator, but input and output of the filter through the coupled line elements.

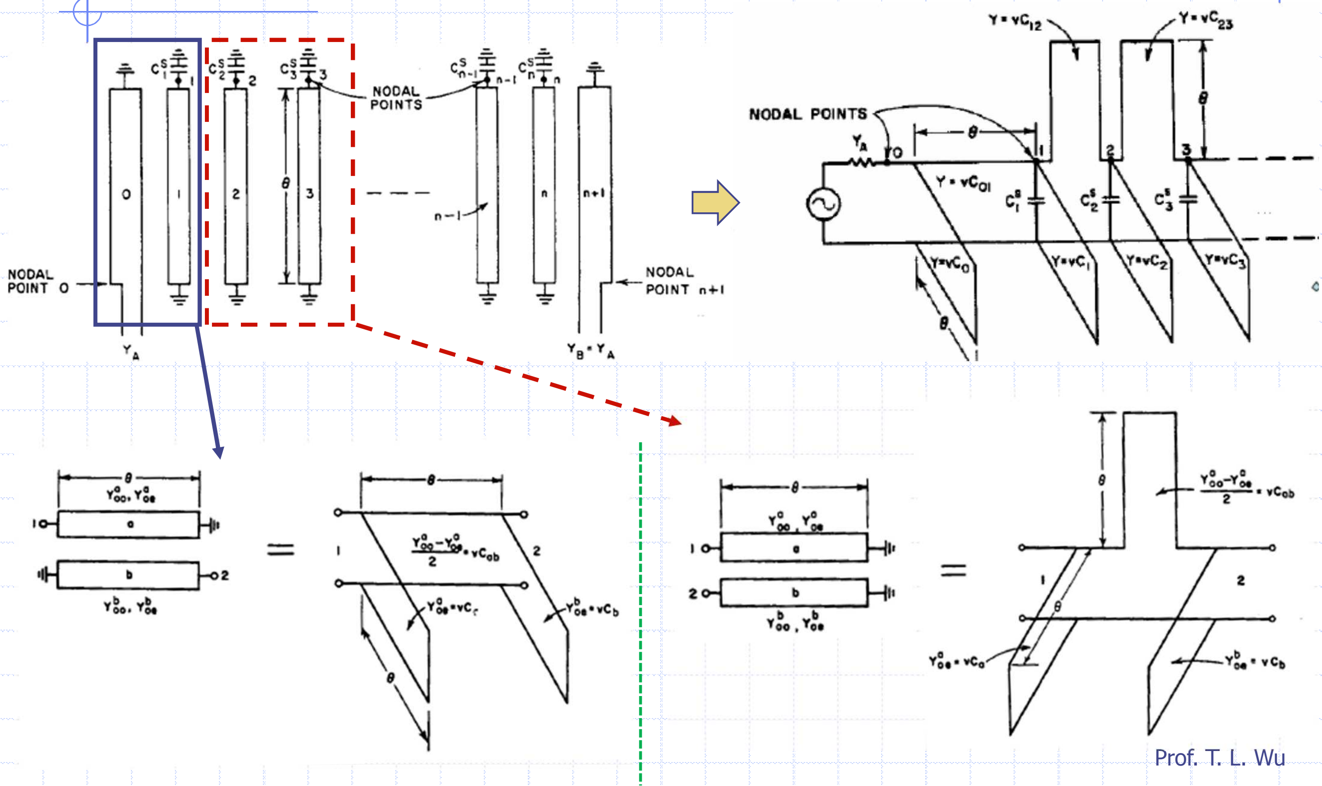
The larger the loading capacitances C_{Li}, the shorter the resonator lines, which results in a more compact filter structure with a wider stopband between the first passband (desired) and the second passband (unwanted).

The minimum resonator line length could be limited by the decrease of the unloaded quality factor of resonator and a requirement for heavy capacitive loading.

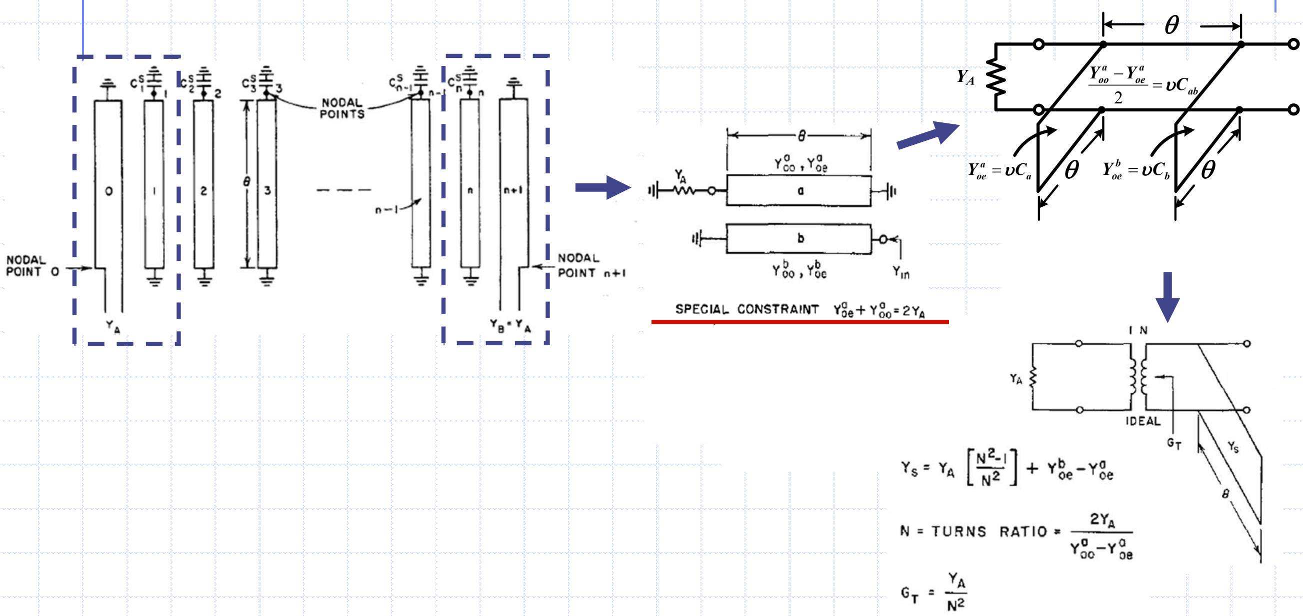


resonators

Combline Filters - Design Concept



Combline Filters
- Design Concept



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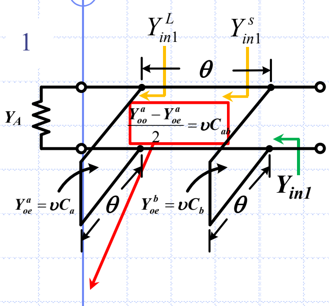
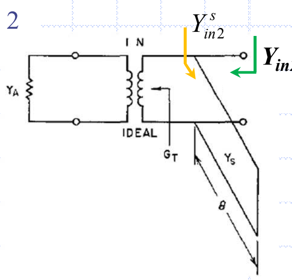


$$Y = \frac{Y_{oo}^a - Y_{oe}^a}{2}$$

Constraint: $\frac{Y_{oo}^a + Y_{oe}^a}{2} = Y_A$

2



$$Y_{in1}^L = Y_A + \frac{Y_{oe}^a}{j\tan\theta}$$

$$Y_{in1}^s = Y\frac{Y_{in1}^L + jY\tan\theta}{Y + jY_{in1}^L\tan\theta} = Y\frac{\left(Y_A+\frac{Y_{oe}^a}{j\tan\theta}\right)+jY\tan\theta}{Y+j\left(Y_A+\frac{Y_{oe}^a}{j\tan\theta}\right)\tan\theta} = Y\frac{\left(Y+Y_{oe}^a+\frac{Y_{oe}^a}{j\tan\theta}\right)+jY\tan\theta}{Y+Y_{oe}^a+jY_A\tan\theta}$$

$$= Y\frac{Y(1+j\tan\theta)+Y_{oe}^a\left(1+\frac{1}{j\tan\theta}\right)}{Y_A(1+j\tan\theta)} = \frac{Y^2}{Y_A}+\frac{YY_{oe}^a}{Y_A j\tan\theta}$$

$$Y_{in1} = \frac{Y^2}{Y_A}+\frac{YY_{oe}^a}{Y_A j\tan\theta}+\frac{Y_{oe}^b}{j\tan\theta} = \frac{Y^2}{Y_A}+\frac{1}{j\tan\theta}\left(\frac{YY_{oe}^a}{Y_A}+Y_{oe}^b\right)$$

$$Y_{in2} = G_T // Y_{in2}^s = \frac{Y_A}{N^2}+\frac{Y_s}{j\tan\theta}$$

$$Y_{in1} = Y_{in2} \Rightarrow N = \frac{Y_A}{Y} = \frac{2Y_A}{Y_{oo}^a - Y_{oe}^a}$$

$$Y_s = \frac{YY_{oe}^a}{Y_A}+Y_{oe}^b = \frac{YY_{oe}^a}{Y_A}+Y_{oe}^a - Y_{oe}^a + Y_{oe}^b = Y_{oe}^a\left(1+\frac{Y}{Y_A}\right) - Y_{oe}^a+Y_{oe}^b$$

$$= (Y_A - Y)\left(\frac{Y_A+Y}{Y_A}\right) - Y_{oe}^a+Y_{oe}^b = \left(\frac{Y_A^2-Y^2}{Y_A}\right)-Y_{oe}^a+Y_{oe}^b = Y_A\left(\frac{N^2-1}{N^2}\right)-Y_{oe}^a+Y_{oe}^b$$

Combline Filters
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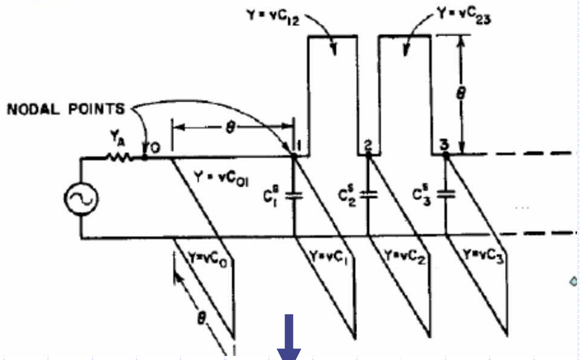
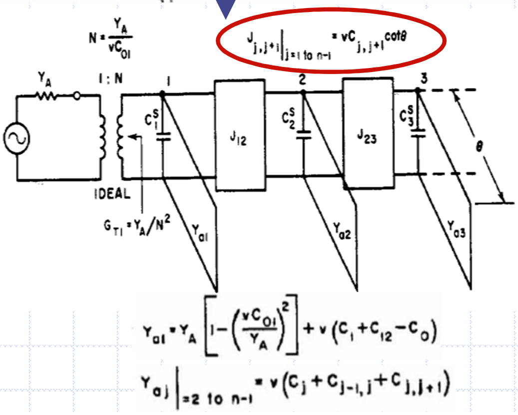
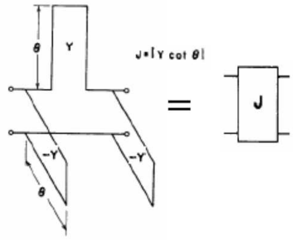
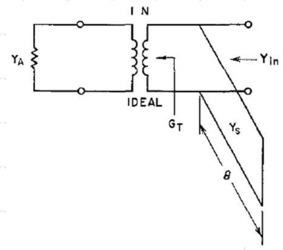
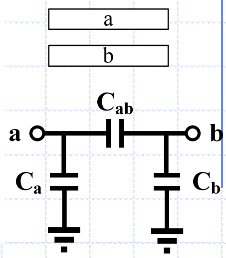










$$Y_{oe}^{a}=\upsilon C_a$$

$$Y_{oe}^{b}=\upsilon C_b$$

$$Y_{oo}^{a}=\upsilon\left(C_a+2C_{ab}\right)$$

$$N=\frac{2Y_A}{\upsilon\left(C_a+2C_{ab}-C_a\right)}=\frac{Y_A}{\upsilon C_{ab}}$$

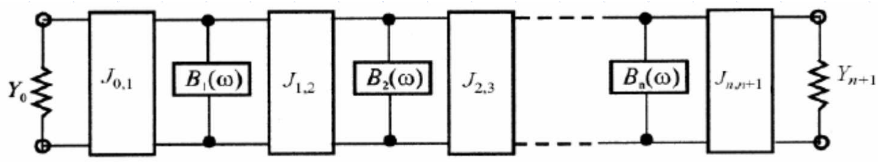
Design formula for coupled-line:

$$Y_{a1}=Y_A\left(\frac{N^2-1}{N^2}\right)-Y_{oe}^{a}+Y_{oe}^{b}+Y_{12}$$

$$=Y_A\left[1-\left(\frac{\upsilon C_{01}}{Y_A}\right)^2\right]+\upsilon\left(-C_0+C_1+C_{12}\right)$$

$$Y_{aj}\Big|_{j=2\ to\ n-1}=Y_j+Y_{j-1,j}+Y_{j,j+1}=\upsilon\left(C_j+C_{j-1,j}+C_{j,j+1}\right)$$

Combline Filters
- Formulation



$$J_{0,1}=\sqrt{\frac{Y_0 FBW b_1}{\Omega_c g_0 g_1}}\ ,\quad J_{i,i+1}=\frac{FBW}{\Omega_c}\sqrt{\frac{b_i b_{i+1}}{g_i g_{i+1}}}\Bigg|_{i=1\ to\ n-1}\ ,\quad J_{n,n+1}=\sqrt{\frac{FBW b_n Y_{n+1}}{\Omega_c g_n g_{n+1}}}$$

$$b_i=\frac{\omega_0}{2}\frac{dB_i(\omega)}{d\omega}\Bigg|_{\omega=\omega_0}$$

A design procedure starts with choosing the resonator susceptance slope parameters ***bi***

(Parallel LC)

$$\frac{b_i}{Y_A}=\frac{Y_{ai}}{Y_A}\left(\frac{\cot\theta_0+\theta_0\csc^2\theta_0}{2}\right)\qquad \text{for } i=1 \text{ to } n$$

$$\boxed{B_i(\omega)=\omega C_{Li}-Y_{ai}\cot\theta}$$

$$\because B(\omega_0)=\omega_0 C_{Li}-Y_{ai}\cot\theta_0\equiv 0$$

$$\boxed{\therefore C_{Li}=Y_{ai}\frac{\cot\theta_0}{\omega_0}}$$

where Y_A is the terminating line admittance, θ_0 is the midband electrical length of the resonators, and Y_{ai} is interpreted physically as the admittance of line with the adjacent lines $i-1$ and $i+1$ grounded. The choice of Y_{ai} fixes the admittance level within the filter and can influence the unloaded quality factors of the resonators.

Combline Filters - Formulation

$$\frac{J_{0,1}}{Y_A}=\sqrt{\frac{FBW\frac{b_1}{Y_A}}{g_0g_1}},\qquad \frac{J_{n,n+1}}{Y_A}=\sqrt{\frac{FBW\frac{b_n}{Y_A}}{g_ng_{n+1}}}$$

$$\frac{J_{i,i+1}}{Y_A}=FBW\sqrt{\frac{(b_i/Y_A)(b_{i+1}/Y_A)}{g_ig_{i+1}}}\qquad \text{for } i=1 \text{ to } n-1$$

The lumped capacitances C_{Li} are:

$$C_{Li}=Y_A\left(\frac{Y_{ai}}{Y_A}\right)\frac{\cot\theta_0}{\omega_0}\qquad \text{for } i=1 \text{ to } n$$

where ω_0 is the angular frequency at the midband.

The self-capacitances C_i are given by

$$\frac{C_0}{\varepsilon_0\sqrt{\varepsilon_{re}}}=\eta_0Y_A\left(1-\frac{J_{0,1}}{Y_A}\right),\qquad \frac{C_{n+1}}{\varepsilon_0\sqrt{\varepsilon_{re}}}=\eta_0Y_A\left(1-\frac{J_{n,n+1}}{Y_A}\right)$$

$$\frac{C_1}{\varepsilon_0\sqrt{\varepsilon_{re}}}=\eta_0Y_A\left(\frac{Y_{a1}}{Y_A}-1+\left(\frac{J_{0,1}}{Y_A}\right)^2-\frac{J_{1,2}}{Y_A}\tan\theta_0\right)+\frac{C_0}{\varepsilon_0\sqrt{\varepsilon_{re}}}$$

$$\frac{C_n}{\varepsilon_0\sqrt{\varepsilon_{re}}}=\eta_0Y_A\left(\frac{Y_{an}}{Y_A}-1+\left(\frac{J_{n,n+1}}{Y_A}\right)^2-\frac{J_{n-1,n}}{Y_A}\tan\theta_0\right)+\frac{C_{n+1}}{\varepsilon_0\sqrt{\varepsilon_{re}}}$$

$$\frac{C_i}{\varepsilon_0\sqrt{\varepsilon_{re}}}=\eta_0Y_A\left(\frac{Y_{ai}}{Y_A}-\frac{J_{i-1,i}}{Y_A}\tan\theta_0-\frac{J_{i,i+1}}{Y_A}\tan\theta_0\right)\qquad \text{for } i=2 \text{ to } n-2$$

and the mutual capacitances $C_{i,i+1}$ are

$$\frac{C_{0,1}}{\varepsilon_0\sqrt{\varepsilon_{re}}}=\eta_0Y_A-\frac{C_0}{\varepsilon_0\sqrt{\varepsilon_{re}}},\qquad \frac{C_{n,n+1}}{\varepsilon_0\sqrt{\varepsilon_{re}}}=\eta_0Y_A-\frac{C_{n+1}}{\varepsilon_0\sqrt{\varepsilon_{re}}}$$

$$\frac{C_{i,i+1}}{\varepsilon_0\sqrt{\varepsilon_{re}}}=\eta_0Y_A\frac{J_{i,i+1}}{Y_A}\tan\theta_0$$

Design Procedure (1/3)

- Find the g_i ($i = 1$ to n) coefficients for the chosen prototype filter.
- Choose the admittance of each resonator, Y_{ai}.
- Choose the midband electrical length of each resonator, θ_0.
- Obtain the susceptance slope parameter of each resonator using

$$\frac{b_i}{Y_A}=\frac{Y_{ai}}{Y_A}\left(\frac{\cot\theta_0+\theta_0\csc^2\theta_0}{2}\right)\qquad \text{for } i=1 \text{ to } n$$

- Obtain the required lumped capacitance using

The lumped capacitances C_{Li} are:

$$C_{Li}=Y_A\left(\frac{Y_{ai}}{Y_A}\right)\frac{\cot\theta_0}{\omega_0}\qquad \text{for } i=1 \text{ to } n$$

where ω_0 is the angular frequency at the midband.

Design Procedure (2/3)

- Obtain the J-inverter values using the formula

$$\frac{J_{0,1}}{Y_A} = \sqrt{\frac{FBW\,\frac{b_1}{Y_A}}{g_0 g_1}}, \qquad \frac{J_{n,n+1}}{Y_A} = \sqrt{\frac{FBW\,\frac{b_n}{Y_A}}{g_n g_{n+1}}}$$

$$\frac{J_{i,i+1}}{Y_A} = FBW\sqrt{\frac{(b_i/Y_A)(b_{i+1}/Y_A)}{g_i g_{i+1}}} \qquad \text{for } i = 1 \text{ to } n-1$$

- Obtain the required self and mutual capacitance for n + 2 lines using

The self-capacitances C_i are given by

$$\frac{C_0}{\varepsilon_0\sqrt{\varepsilon_{re}}} = \eta_0 Y_A\left(1 - \frac{J_{0,1}}{Y_A}\right), \qquad \frac{C_{n+1}}{\varepsilon_0\sqrt{\varepsilon_{re}}} = \eta_0 Y_A\left(1 - \frac{J_{n,n+1}}{Y_A}\right)$$

$$\frac{C_1}{\varepsilon_0\sqrt{\varepsilon_{re}}} = \eta_0 Y_A\left(\frac{Y_{a1}}{Y_A} - 1 + \left(\frac{J_{0,1}}{Y_A}\right)^2 - \frac{J_{1,2}}{Y_A}\tan\theta_0\right) + \frac{C_0}{\varepsilon_0\sqrt{\varepsilon_{re}}}$$

$$\frac{C_n}{\varepsilon_0\sqrt{\varepsilon_{re}}} = \eta_0 Y_A\left(\frac{Y_{an}}{Y_A} - 1 + \left(\frac{J_{n,n+1}}{Y_A}\right)^2 - \frac{J_{n-1,n}}{Y_A}\tan\theta_0\right) + \frac{C_{n+1}}{\varepsilon_0\sqrt{\varepsilon_{re}}}$$

$$\frac{C_i}{\varepsilon_0\sqrt{\varepsilon_{re}}} = \eta_0 Y_A\left(\frac{Y_{ai}}{Y_A} - \frac{J_{i-1,i}}{Y_A}\tan\theta_0 - \frac{J_{i,i+1}}{Y_A}\tan\theta_0\right) \qquad \text{for } i = 2 \text{ to } n-2$$

and the mutual capacitances $C_{i,i+1}$ are

$$\frac{C_{0,1}}{\varepsilon_0\sqrt{\varepsilon_{re}}} = \eta_0 Y_A - \frac{C_0}{\varepsilon_0\sqrt{\varepsilon_{re}}}, \qquad \frac{C_{n,n+1}}{\varepsilon_0\sqrt{\varepsilon_{re}}} = \eta_0 Y_A - \frac{C_{n+1}}{\varepsilon_0\sqrt{\varepsilon_{re}}}$$

$$\frac{C_{i,i+1}}{\varepsilon_0\sqrt{\varepsilon_{re}}} = \eta_0 Y_A \frac{J_{i,i+1}}{Y_A}\tan\theta_0$$

Design Procedure (3/3)

- Find the dimensions of the coupled lines, namely the width and spacing that gives the required self and mutual capacitance. (similar to the interdigital BPF)

Combline filters
- Another design approach

Instead of working on the self- and mutual capacitances, an alternative design approach is to determine the dimensions in terms of another set of design parameters consisting of external quality factors and coupling coefficients.

$$Q_{e1} = \frac{b_1}{J_{0,1}^2/Y_A} = \frac{g_0 g_1}{FBW}, \qquad Q_{en} = \frac{b_n}{J_{n,n+1}^2/Y_A} = \frac{g_n g_{n+1}}{FBW}$$

$$M_{i,i+1} = \frac{J_{i,i+1}}{\sqrt{b_i b_{i+1}}} = \frac{FBW}{\sqrt{g_i g_{i+1}}} \qquad \text{for } i = 1 \text{ to } n-1$$

where Q_{e1} and Q_{en} are the external quality factors of the resonators at the input and output, and $M_{i,i+1}$ are the coupling coefficients between the adjacent resonators. The

Combline filters
- Another design approach

Assume that a five-pole Chebyshev lowpass prototype filter with 0.1 dB passband ripple has been chosen for the bandpass filter design. The lowpass prototype parameters are $g_0 = g_6 = 1.0$, $g_1 = g_5 = 1.1468$, $g_2 = g_4 = 1.3712$, and $g_3 = 1.9750$. The bandpass filter is designed to have a fractional bandwidth $FBW = 0.1$ at a midband frequency $f_0 = 2$ GHz. From (5.45) we obtain

$$Q_{e1} = Q_{e5} = 11.468$$
$$M_{1,2} = M_{4,5} = 0.07975$$
$$M_{2,3} = M_{3,4} = 0.06077$$

As mentioned above, the combline resonators cannot be $\lambda_{g0}/4$ long if they are realized with TEM transmission lines. However, this is not necessarily the case for a microstrip combline filter because the microstrip is not a pure TEM transmission line.

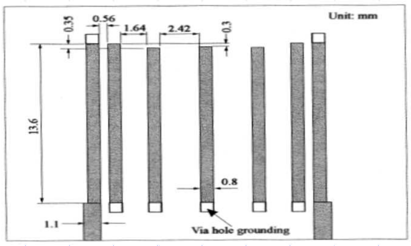
For demonstration, we also let the microstrip resonators be $\lambda_{g0}/4$ long and require no capacitive loading for this filter design.

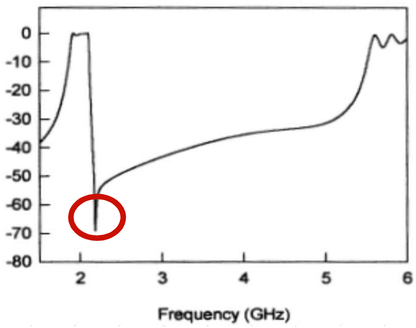
The microstrip filter is designed on a commercial substrate (RT/D 6010) with a relative dielectric constant of 10.8 and a thickness of 1.27 mm.

With the EM simulation we are able to work directly on the dimensions of the filters. Therefore, we first fix a line width $W = 0.8$ mm for the all line elements except for the terminating lines.

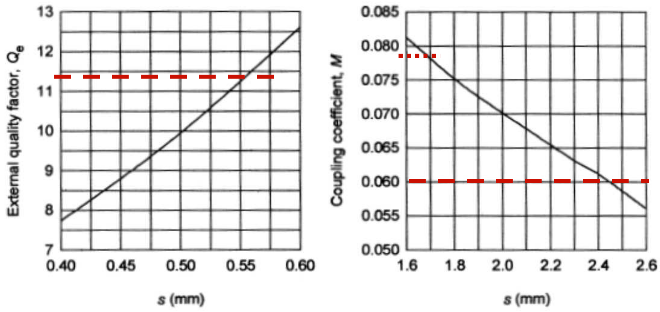
The terminating lines are 1.1mm wide, which matches to 50 ohm terminating impedance.

Combline filters





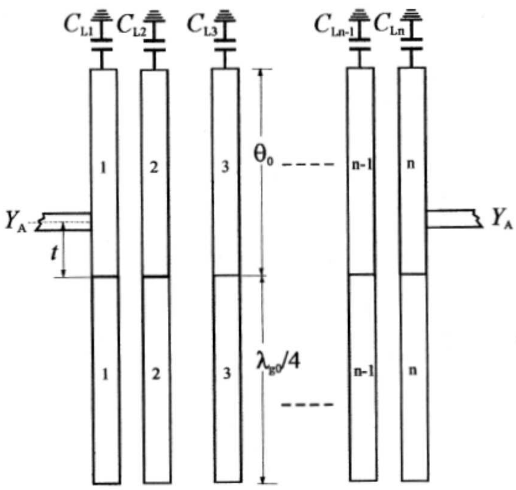
$$Q_{e1} = Q_{e5} = 11.468$$
$$M_{1,2} = M_{4,5} = 0.07975$$
$$M_{2,3} = M_{3,4} = 0.06077$$

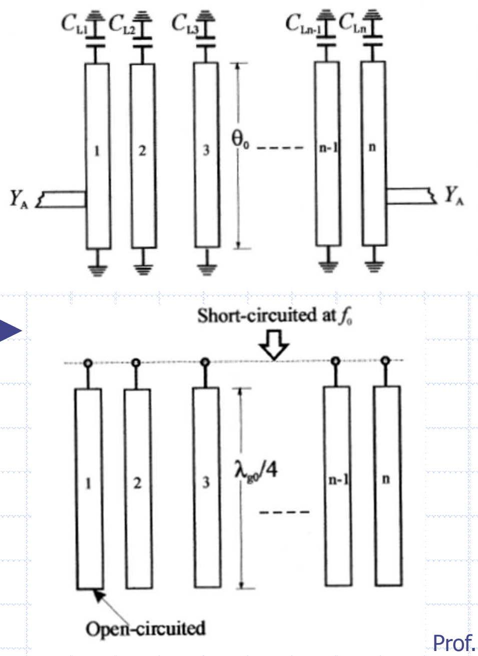


It is interesting to note that there is an attenuation pole near the high edge of the passband, resulting in a higher selectivity on that side. This attenuation pole is likely due to cross couplings between the nonadjacent resonators.

Pseudocombline Filters

Pseudocombline bandpass filter that is comprised of an array of coupled resonators. The resonators consist of line elements 1 to n, which are open-circuited at one end, with a lumped capacitance C_{Li} loaded between the other end of each resonator line element and ground.





Pseudocombline Filters

Similar to the combline filter, if the capacitors were not present, the resonator lines would be a full $\lambda_{g0/2}$ long at resonance.

The filter structure would have no passband at all when it is realized in stripline, due to a total cancellation of electric and magnetic couplings. For microstrip realization, this would not be the case

The type of filter in Figure 5.22 can be designed with a set of bandpass design parameters consisting of external quality factors and coupling coefficients.

$$Q_{e1} = \frac{g_0 g_1}{FBW}, \qquad Q_{en} = \frac{g_n g_{n+1}}{FBW}$$

$$M_{i,i+1} = \frac{FBW}{\sqrt{g_i g_{i+1}}} \qquad \text{for } i = 1 \text{ to } n-1$$

where Q_{e1} and Q_{en} are the external quality factors of the resonators at the input and output, and $M_{i,i+1}$ are the coupling coefficients between the adjacent resonators. The required dimensions can be found by using full-wave EM simulation (see Chapter 8

Pseudocombline Filters - Example

For this design, we work with a five-pole Chebyshev lowpass prototype filter with 0.1 dB passband ripple. The lowpass prototype parameters are $g_0 = g_6 = 1.0$, $g_1 = g_5 = 1.1468$, $g_2 = g_4 = 1.3712$, and $g_3 = 1.9750$. The bandpass filter is designed to have a 15% fractional bandwidth or $FBW = 0.15$ at a midband frequency $f_0 = 2$ GHz.

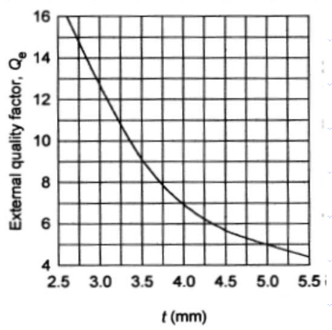
$$Q_{e1} = Q_{e5} = 7.645$$

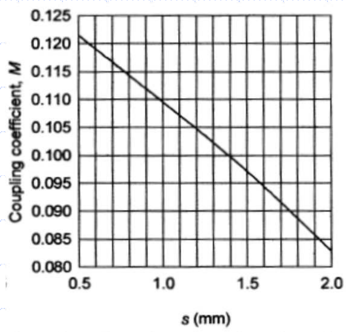
$$M_{1,2} = M_{4,5} = 0.11962$$

$$M_{2,3} = M_{3,4} = 0.09115$$

Design on commercial substrate (RT/D 6010) with a relative dielectric constant of 10.8 and a thickness of 1.27 mm. Fix a line width $W = 0.8$ mm. The tapped lines are 1.1mm wide, which matches to 50 ohm terminating impedance.

Using the parameter extraction technique described in Chapter 8, the design curves for external quality factor and coupling coefficient against spacing s can be obtained



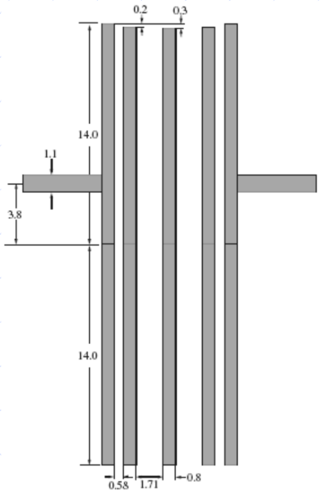


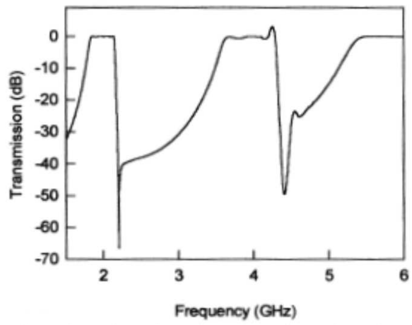
Pseudocombline Filters - Example

Similar to the combline filter discussed previously, there is an attenuation pole near the high edge of the passband, resulting in a higher selective on that side.

This attenuation pole is likely to be due to cross couplings between the nonadjacent resonators.

However, the second passband of the pseudocombline filter is centered at about 4 GHz, which is only twice the midband frequency. This is expected because the $\lambda_{g0/2}$ resonators are used without any lumped capacitor loading.





Bandpass Filters Using Quarter-Wave Coupled Quarter-Wave Resonators

Since quarter-wave short-circuited transmission line stubs look like parallel resonant circuits, they can be used as the shunt parallel LC resonators for bandpass filters.

Quarter wavelength connecting lines between the stubs will act as admittance inverters, effectively converting alternate shunt stubs to series resonators.

For a narrow passband bandwidth (small Δ), the response of such a filter using N stubs is essentially the same as that of a lumped element bandpass filter of order N. The circuit topology of this filter is convenient in that only shunt stubs are used, but a disadvantage in practice is that the required characteristic impedances of the stub lines are often unrealistically low. A similar design employing open-circuited stubs can be used for bandstop filters

How to design Z_{0n} ?

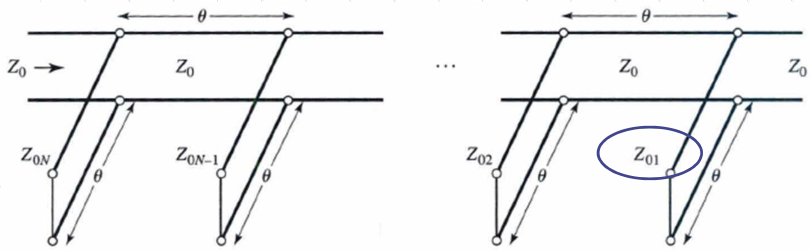
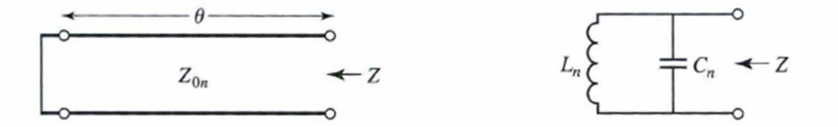


FIGURE 5.21 A bandpass filter using shunt short-circuited quarter-wave resonators with quarter-wave connecting sections.

Bandpass Filters Using Quarter-Wave Coupled Quarter-Wave Resonators

Note that a given LC resonator has <u>two degrees of freedom</u>: L and C, or equivalently, ω_0, and the slope of the admittance at resonance.

For a stub resonator the corresponding degrees of freedom are the resonant length and <u>characteristic impedance of the transmission line</u> **Z_{0n}** .



(a)

$$Y = \frac{-j}{Z_{0n}} \cot\theta,$$

where $\theta = \pi/2$ for $\omega = \omega_0$. If we let $\omega = \omega_0 + \Delta\omega$, where $\Delta\omega \ll \omega_0$, then $\theta = \frac{\pi}{2}(1 + \frac{\Delta\omega}{\omega_0})$, which allows the admittance of (5.43) to be approximated as

$$Y = \frac{-j}{Z_{0n}} \cot\left(\frac{\pi}{2} + \frac{\pi\Delta\omega}{2\omega_0}\right) = \frac{j}{Z_{0n}} \tan\frac{\pi\Delta\omega}{2\omega_0} \cong \frac{j\pi\Delta\omega}{2Z_{0n}\omega_0}, \qquad (5.44)$$

Bandpass Filters Using Quarter-Wave Coupled Quarter-Wave Resonators

(a)

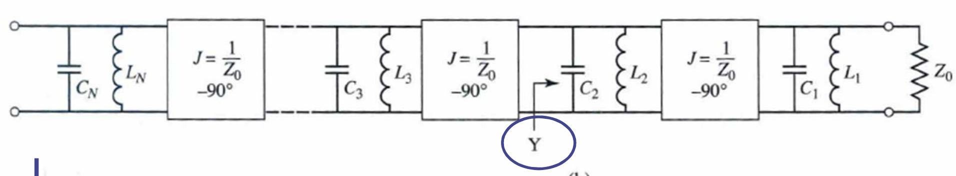
$$Y = \frac{-j}{Z_{0n}} \cot\left(\frac{\pi}{2} + \frac{\pi\Delta\omega}{2\omega_0}\right) = \frac{j}{Z_{0n}} \tan\frac{\pi\Delta\omega}{2\omega_0} \cong \frac{j\pi\Delta\omega}{2Z_{0n}\omega_0},$$

$$\boxed{Z_{0n} = \frac{\pi\omega_0 L_n}{4} = \frac{\pi}{4\omega_0 C_n}.}$$

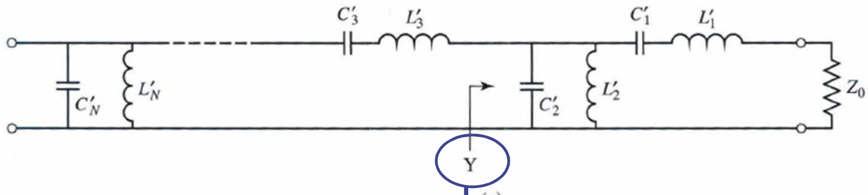
$$Y = j\omega C_n + \frac{1}{j\omega L_n} = j\sqrt{\frac{C_n}{L_n}}\left(\omega\sqrt{C_n L_n} - \frac{1}{\omega\sqrt{C_n L_n}}\right)$$

$$= j\sqrt{\frac{C_n}{L_n}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \cong 2jC_n\Delta\omega,$$

where $C_n L_n = 1/\omega_0^2$.



(b)



(c)

$$Y = j\omega C_2 + \frac{1}{j\omega L_2} + \frac{1}{Z_0^2}\left[j\omega C_1 + \frac{1}{j\omega L_1} + \frac{1}{Z_0}\right]^{-1}$$
$$= j\sqrt{\frac{C_2}{L_2}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) + \frac{1}{Z_0^2}\left[j\sqrt{\frac{C_1}{L_1}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) + \frac{1}{Z_0}\right]^{-1}$$

$$Y = j\omega C'_2 + \frac{1}{j\omega L'_2}\left[j\omega L'_1 + \frac{1}{j\omega C'_1} + Z_0\right]^{-1}$$
$$= j\sqrt{\frac{C'_2}{L'_2}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) + \left[j\sqrt{\frac{L'_1}{C'_1}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) + Z_0\right]^{-1}$$

These two results are exactly equivalent for all frequencies if the following conditions are satisfied:

$$\sqrt{\frac{C_2}{L_2}} = \sqrt{\frac{C'_2}{L'_2}}, \qquad Z_0^2\sqrt{\frac{C_1}{L_1}} = \sqrt{\frac{L'_1}{C'_1}}.$$

Using the fact that $L'_1C'_1 = L'_2C'_2 = 1/\omega_0^2$ allows these two equations to be solved for L_1 and L_2:

$$L_1 = \frac{Z_0^2}{\omega_0^2 L'_1}, \tag{5.50a}$$
$$L_2 = L'_2. \tag{5.50b}$$

Then using (5.46) and the impedance-scaled bandpass filter elements from Table 5.4 gives the required characteristic impedances for the first two stubs as

$$Z_{01} = \frac{\pi\omega_0 L_1}{4} = \frac{\pi Z_0^2}{4\omega_0 L'_1} = \frac{\pi Z_0 \Delta}{4 g_1}, \tag{5.51a}$$
$$Z_{02} = \frac{\pi\omega_0 L_2}{4} = \frac{\pi\omega_0 L'_2}{4} = \frac{\pi Z_0 \Delta}{4 g_2}. \tag{5.51b}$$

By extension, it can be shown that the general result for the characteristic impedance of the nth stub in a filter of order N is given by

$$Z_{0n} = \frac{\pi Z_0 \Delta}{4 g_n}. \tag{5.52}$$

These results apply only to filters having input and output impedances of Z_0, and so cannot be used for equal-ripple designs with N even.

EXAMPLE 5.6 BANDPASS FILTER DESIGN USING QUARTER-WAVE COUPLED RESONATORS

Design a third-order bandpass filter with a 0.5 dB equal-ripple response using quarter-wave coupled quarter-wave short-circuited stub resonators. The center frequency is 2.5 GHz, and the bandwidth is 15%. The impedance is 50 Ω. What is the resulting attenuation at 2.0 GHz?

Solution

We first calculate the attenuation at 2.0 GHz. Using (5.22) to convert 2.0 GHz to normalized low-pass form gives

$$\omega \leftarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{0.15}\left(\frac{2.0}{2.5} - \frac{2.5}{2.0}\right) = -3.00.$$

Then, to use Figure 5.6a, the value on the horizontal axis is

$$\left|\frac{\omega}{\omega_c}\right| - 1 = |-3.00| - 1 = 2.00,$$

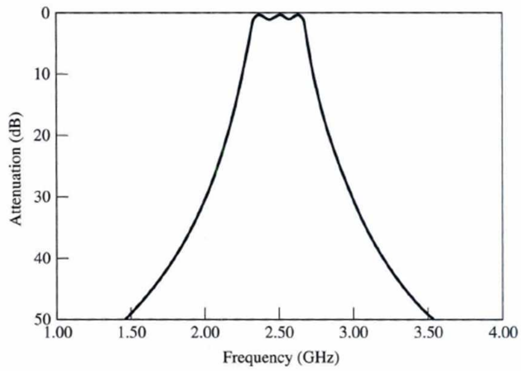
from which we find the attenuation as 30 dB.

From Table 5.2 we find the required g_n's for 0.5 dB ripple and $N = 3$. Then (5.52) gives the necessary characteristic impedances:

From Table 5.2 we find the required g_n's for 0.5 dB ripple and $N = 3$. Then (5.52) gives the necessary characteristic impedances:

n	g_n	Z_{0n} (Ω)
1	1.5963	3.69
2	1.0967	5.37
3	1.5963	3.69

All stubs and connecting lines are $\lambda/4$ long at 2.5 GHz. The calculated response of the filter is shown in Figure 5.23. Note that 30 dB attenuation is achieved at 2 GHz, as expected. Also note that the characteristic impedances for the stubs are very low, making practical implementation of this type of filter very difficult. This difficulty is avoided with the capacitively coupled resonator filter discussed in the following section. ○



Stub Bandpass Filters

$$\theta = \frac{\pi}{2}\left(1 - \frac{FBW}{2}\right)$$

$$h = 2$$

$$\frac{J_{1,2}}{Y_0} = g_0\sqrt{\frac{hg_1}{g_2}}, \qquad \frac{J_{n-1,n}}{Y_0} = g_0\sqrt{\frac{hg_1 g_{n+1}}{g_0 g_{n-1}}}$$

$$\frac{J_{i,i+1}}{Y_0} = \frac{hg_0 g_1}{\sqrt{g_i g_{i+1}}} \qquad \text{for } i = 2 \text{ to } n-2$$

$$N_{i,i+1} = \sqrt{\left(\frac{J_{i,i+1}}{Y_0}\right)^2 + \left(\frac{hg_0 g_1 \tan\theta}{2}\right)^2} \qquad \text{for } i = 1 \text{ to } n-1$$

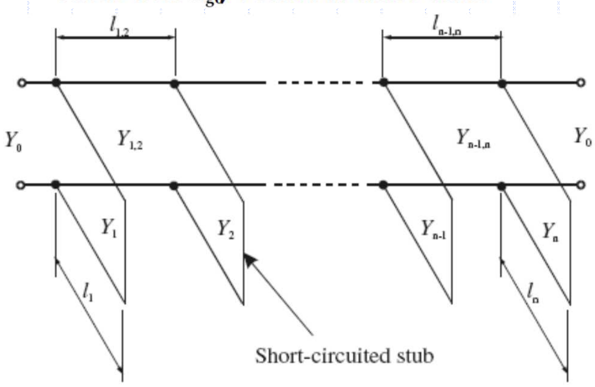
$$Y_1 = g_0 Y_0\left(1 - \frac{h}{2}\right)g_1 \tan\theta + Y_0\left(N_{1,2} - \frac{J_{1,2}}{Y_0}\right)$$

$$Y_n = Y_0\left(g_n g_{n+1} - g_0 g_1 \frac{h}{2}\right)\tan\theta + Y_0\left(N_{n-1,n} - \frac{J_{n-1,n}}{Y_0}\right)$$

$$Y_i = Y_0\left(N_{i-1,i} + N_{i,i+1} - \frac{J_{i-1,i}}{Y_0} - \frac{J_{i,i+1}}{Y_0}\right) \qquad \text{for } i = 2 \text{ to } n-1$$

$$Y_{i,i+1} = Y_0\left(\frac{J_{i,i+1}}{Y_0}\right) \qquad \text{for } i = 1 \text{ to } n-1$$

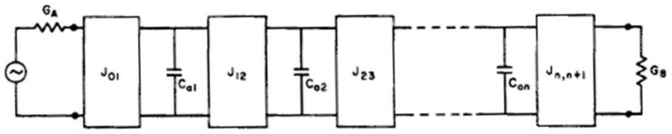
Filters with $\lambda_{g0}/4$ Short-circuited Stubs



h is a dimensionless constant which may be assigned to another value so as to give a convenient admittance level in the interior of the filter.

Derivation concept (1/6)

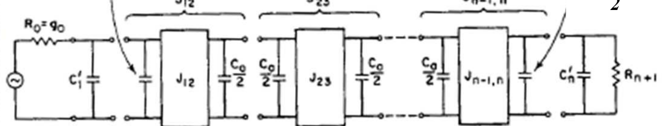
- Lowpass filter with only one kind of reactive element



$$J_{01}=\sqrt{\frac{G_A C_{a1}}{g_0 g_1}}\ ,\quad J_{k,k+1}\Big|_{k=1\ \text{to}\ n-1}=\sqrt{\frac{C_{ak}\ C_{a(k+1)}}{g_k\ g_{k+1}}}\ ,\quad J_{n,n+1}=\sqrt{\frac{C_{an}\ G_B}{g_n \cdot g_{n+1}}}$$

C_{ai} can be arbitrary chosen and the inverters J_{01} and $J_{n,\ n+1}$ are eliminated!

$$C_1''=\frac{C_a}{2}=\frac{h}{2}g_1 \qquad C_1''=\frac{C_a}{2}$$



$$C_1'=\left(1-\frac{h}{2}\right)g_1 \qquad C_a=hg_1 \qquad C_n'=\frac{g_n g_{n+1}-(h/2)\,g_1 R_{n+1}}{R_{n+1}}$$

$$C_1=g_1=C_1'+C_1'' \qquad C_n=g_n\frac{g_{n+1}}{R_{n+1}}=C_n'+C_n''$$

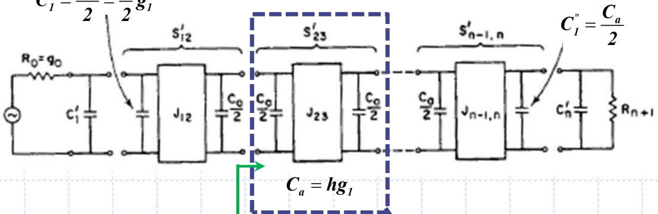
$$J_{12}=\sqrt{\frac{C_1 C_a}{g_1 g_2}}=Y_A\sqrt{\frac{hg_1}{g_2}} \qquad J_{k,k+1}\Big|_{2\ to\ n-1}=\frac{C_a}{\sqrt{g_k g_{k+1}}}=Y_A\frac{hg_1}{\sqrt{g_k g_{k+1}}} \qquad J_{n-1,n}=\sqrt{\frac{C_a C_n}{g_{n-1} g_n}}=Y_A\sqrt{\frac{hg_1 g_{n+1}}{g_{n-1}}}$$

C_{ai} is scaled by terminal impedance Y_A!

Derivation concept (2/6)

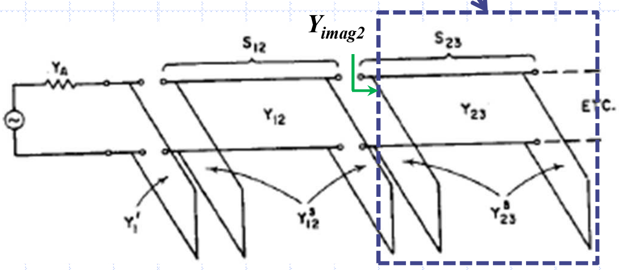
- For interior sections

$$C_1''=\frac{C_a}{2}=\frac{h}{2}g_1 \qquad C_1''=\frac{C_a}{2} \qquad C_a=hg_1$$



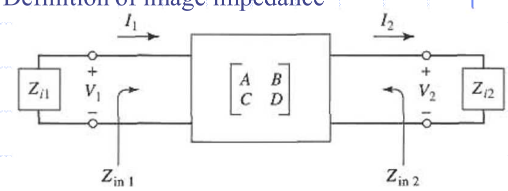
Y_{imag1}

Y_{imag2}



- Using image impedance

Definition of image impedance



Z_{i1} = input impedance at port 1 when port 2 is terminated with Z_{i2}

Z_{i2} = input impedance at port 2 when port 1 is terminated with Z_{i1}

$$Z_{i1}=\sqrt{\frac{AB}{CD}} \qquad Z_{i2}=\sqrt{\frac{BD}{AC}}$$

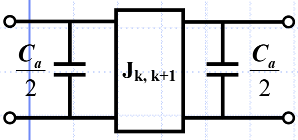
If the network is symmetric, then $A=D$ and $Z_{i1}=Z_{i2}$ as expected.

$$Y_{imag1}=\frac{1}{Z_{imag1}}=\sqrt{\frac{C_1 D_1}{A_1 B_1}}$$

$$Y_{imag2}=\frac{1}{Z_{imag2}}=\sqrt{\frac{C_2 D_2}{A_2 B_2}}$$

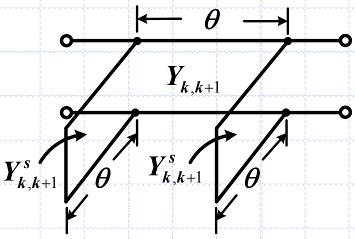
Derivation concept (3/6)

- Derivations on the image parameters



$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j\frac{\omega C_a}{2} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\frac{1}{J_{k,k+1}} \\ jJ_{k,k+1} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j\frac{\omega C_a}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{-\omega C_a}{2J_{k,k+1}} & j\frac{1}{J_{k,k+1}} \\ jJ_{k,k+1} - j\frac{(\omega C_a)^2}{J_{k,k+1}} & \frac{-\omega C_a}{2J_{k,k+1}} \end{bmatrix}$$

$$\Rightarrow Y_{imag1} = \sqrt{\frac{C_1 D_1}{A_1 B_1}} = J_{k,k+1}\sqrt{1-\left(\frac{\omega C_a}{2J_{k,k+1}}\right)^2}$$



$$\theta = \frac{\pi}{2}\frac{\omega}{\omega_0}$$

$$\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -jY^s_{k,k+1}\cot\theta & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \frac{j}{Y_{k,k+1}}\sin\theta \\ jY_{k,k+1}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -jY^s_{k,k+1}\cot\theta & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta + \frac{Y^s_{k,k+1}}{Y_{k,k+1}}\cos\theta & \frac{j}{Y_{k,k+1}}\sin\theta \\ -2jY^s_{k,k+1}\frac{\cos^2\theta}{\sin\theta} + jY_{k,k+1}\sin\theta - j\frac{\left(Y^s_{k,k+1}\right)^2}{Y_{k,k+1}}\frac{\cos^2\theta}{\sin\theta} & \cos\theta + \frac{Y^s_{k,k+1}}{Y_{k,k+1}}\cos\theta \end{bmatrix}$$

$$\Rightarrow Y_{imag2} = \sqrt{\frac{C_2 D_2}{A_2 B_2}} = \frac{\sqrt{Y^2_{k,k+1} - \left(Y_{k,k+1} + Y^s_{k,k+1}\right)^2\cos^2\theta}}{\sin\theta}$$

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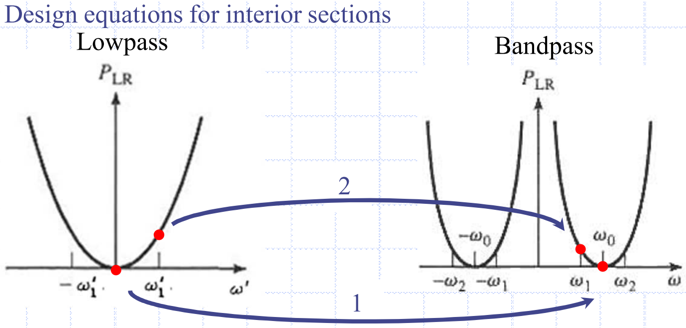
Derivation concept (4/6)

- Design equations for interior sections



1. $Y_{imag1}\left(\omega' = 0\right) = Y_{imag2}\left(\omega = \omega_0\right)$
2. $Y_{imag1}\left(\omega' = \omega_1'\right) = Y_{imag2}\left(\omega = \omega_1\right)$

1. $Y_{imag1}\left(\omega' = 0\right) = Y_{imag2}\left(\omega = \omega_0\right) \Rightarrow J_{k,k+1}\sqrt{1-\left(\frac{\omega C_a}{2J_{k,k+1}}\right)^2} = \frac{\sqrt{Y^2_{k,k+1} - \left(Y_{k,k+1} + Y^s_{k,k+1}\right)^2\cos^2\theta}}{\sin\theta} \Rightarrow J_{k,k+1} = Y_{k,k+1}$

2. $Y_{imag1}\left(\omega' = \omega_1'\right) = Y_{imag2}\left(\omega = \omega_1\right) \Rightarrow J_{k,k+1}\sqrt{1-\left(\frac{\omega_1' C_a}{2J_{k,k+1}}\right)^2} = \frac{\sqrt{Y^2_{k,k+1} - \left(Y_{k,k+1} + Y^s_{k,k+1}\right)^2\cos^2\theta_1}}{\sin\theta_1}$

$$\Rightarrow \left(J_{k,k+1}\sin\theta_1\right)^2\left[1-\left(\frac{\omega_1' C_a}{2J_{k,k+1}}\right)^2\right] = Y^2_{k,k+1} - \left(Y_{k,k+1} + Y^s_{k,k+1}\right)^2\cos^2\theta_1$$

$$\Rightarrow \left(J_{k,k+1} + Y^s_{k,k+1}\right)^2\cos^2\theta_1 = J^2_{k,k+1}\left(1-\sin^2\theta_1\right) + \left(\frac{\omega_1' C_a \sin\theta_1}{2}\right)^2$$

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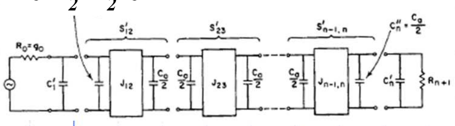
Derivation concept (5/6)

2. $Y_{imag1}\left(\omega'=\omega_1'\right)=Y_{imag2}\left(\omega=\omega_1\right) \Rightarrow \left(J_{k,k+1}+Y_{k,k+1}^{s}\right)^2\cos^2\theta_1=J_{k,k+1}^2\left(1-\sin^2\theta_1\right)+\left(\frac{\omega_1' C_a\sin\theta_1}{2}\right)^2$

$C_1''=\frac{C_a}{2}=\frac{h}{2}g_1$



$C_a=hg_1$

$$\Rightarrow \left(J_{k,k+1}+Y_{k,k+1}^{s}\right)^2=J_{k,k+1}^2+\left(\frac{\omega_1' C_a\tan\theta_1}{2}\right)^2$$

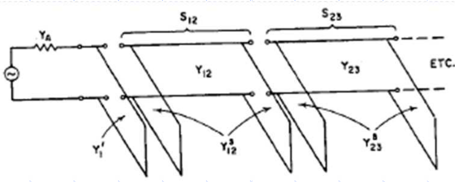
Assume $\omega_1'=1$

$$\Rightarrow Y_{k,k+1}^{s}=\sqrt{J_{k,k+1}^2+\left(\frac{\omega_1' C_a\tan\theta_1}{2}\right)^2}-J_{k,k+1}=\sqrt{J_{k,k+1}^2+\left(\frac{Y_A hg_1\tan\theta_1}{2}\right)^2}-J_{k,k+1}$$

$$=Y_A\sqrt{\left(\frac{J_{k,k+1}}{Y_A}\right)^2+\left(\frac{hg_1\tan\theta_1}{2}\right)^2}-J_{k,k+1}=Y_A N_{k,k+1}-J_{k,k+1}$$

Note: $\theta_1=\frac{\pi}{2}\frac{\omega_1}{\omega_0}=\frac{\pi}{2}\frac{(\omega_0-\omega_0 FBW/2)}{\omega_0}=\frac{\pi}{2}\left(1-\frac{FBW}{2}\right)$

Required stub admittance

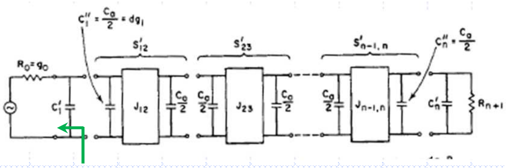


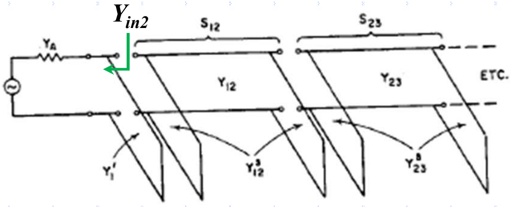
$$Y_k=Y_{k-1,k}^{s}+Y_{k,k+1}^{s}=\left(Y_A N_{k-1,k}-J_{k-1,k}\right)+\left(Y_A N_{k,k+1}-J_{k,k+1}\right)$$

$$=Y_A\left(N_{k-1,k}+N_{k,k+1}-\frac{J_{k-1,k}}{Y_A}-\frac{J_{k,k+1}}{Y_A}\right)\qquad for\ k=2\ to\ n-1$$

Derivation concept (6/6)

- Design equations for end sections





Take input end for example

$$\frac{\mathrm{Im}\{Y_{in1}\}}{\mathrm{Re}\{Y_{in1}\}}=\frac{\mathrm{Im}\{Y_{in2}\}}{\mathrm{Re}\{Y_{in2}\}} \Rightarrow \frac{\omega_1' C_1'}{1/R_0}=\frac{Y_1'\cot\theta_1}{Y_A} \Rightarrow Y_1'=Y_A g_0\omega_1' C_1'\tan\theta_1=Y_A g_0 g_1(1-\frac{h}{2})\tan\theta$$

Required stub admittance for Y_1

$$Y_1=Y_1'+Y_{12}^{s}=Y_A g_0 g_1(1-\frac{h}{2})\tan\theta+Y_A N_{12}-J_{12}=Y_A g_0 g_1(1-\frac{h}{2})\tan\theta+Y_A\left(N_{12}-\frac{J_{12}}{Y_A}\right)$$

- Since h can be assigned to be arbitrary number, the best way to simplify the calculation process is h = 2.

Stub Bandpass Filters - Example

- Design a stub bandpass filter using five-order chebyshev prototype with a passband ripple L_{Ar} = 0.1 dB at f_0 = 2 GHz with a fractional bandwidth of 0.5. A 50 ohm terminal impedance is chosen.

For passband ripple L_{Ar} = 0.1 dB

n	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
1	0.3052	1.0								
2	0.8431	0.6220	1.3554							
3	1.0316	1.1474	1.0316	1.0						
4	1.1088	1.3062	1.7704	0.8181	1.3554					
5	1.1468	1.3712	1.9750	1.3712	1.1468	1.0				
6	1.1681	1.4040	2.0562	1.5171	1.9029	0.8618	1.3554			

- Using previous equations to find the required Y_i and $Y_{i,\,i+1}$

TABLE 5.12 Circuit design parameters of a five-pole, stub bandpass filter with $\lambda_{g0}/4$ short-circuited stubs

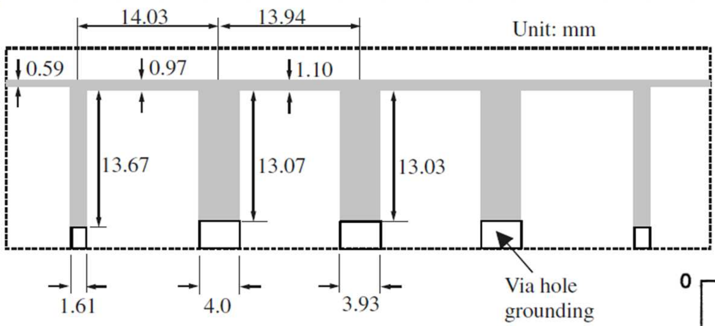
i	Y_i (mhos)	$Y_{i,i+1}$ (mhos)
1	0.03525	0.02587
2	0.06937	0.02787
3	0.06824	0.02787
4	0.06937	0.02587
5	0.03525	

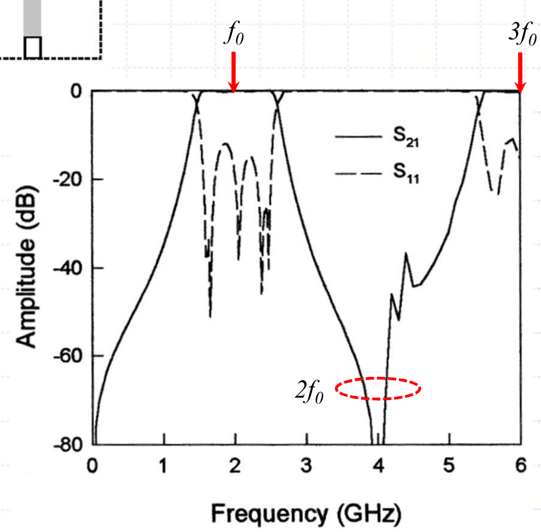
TABLE 5.13 Microstrip design parameters of a five-pole, stub bandpass filter with $\lambda_{g0}/4$ short-circuited stubs

i	W_i (mm)	$\lambda_{g0i}/4$ (mm)	$W_{i,i+1}$ (mm)	$\lambda_{g0i,i+1}/4$ (mm)
1	1.61	13.67	0.97	14.03
2	4.00	13.07	1.10	13.97
3	3.93	13.03	1.10	13.97
4	4.00	13.07	0.97	14.03
5	1.61	13.67		

fabricated on a substrate with a relative dielectric constant of 10.2 and a thickness of 0.635 mm.

Stub Bandpass Filters - Example





Stub Bandpass Filters without shorting via

- Additional passbands in the vincity of $f = 0$ and $f = 2f_0$
- Better passband response with transmission zeros
  - If $Y_{ia} = Y_{ib}$
    ➔ at $f_0/2$ and $3f_0/2$
  - If $Y_{ia} \neq Y_{ib}$
    ➔ transmission zeros are controlled
- Design equation:

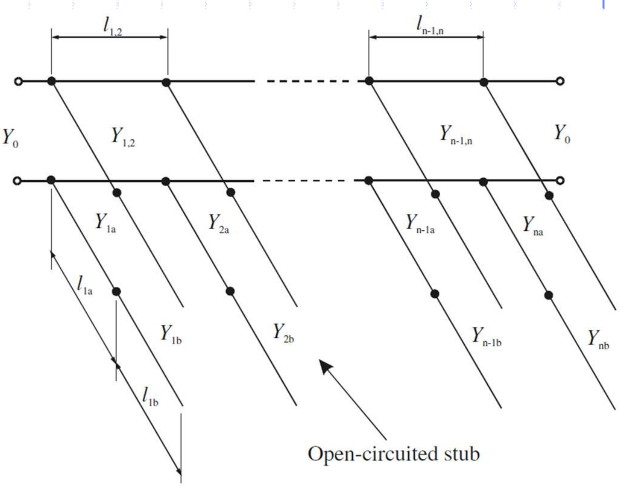
$$Y_{ia} = \frac{Y_i(\alpha_i \tan^2\theta - 1)}{(\alpha_i + 1)\tan^2\theta}$$

$$Y_{ib} = \alpha_i Y_{ia}$$

$$\alpha_i = \cot^2\left(\frac{\pi f_{zi}}{2f_0}\right)$$

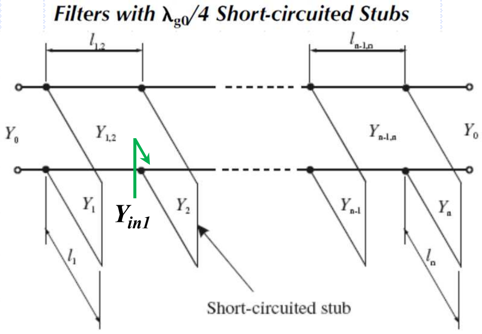
where f_{zi} is the assigned transmission zero at the lower edge of passband

Filters with $\lambda_{g0}/2$ Open-circuited Stubs

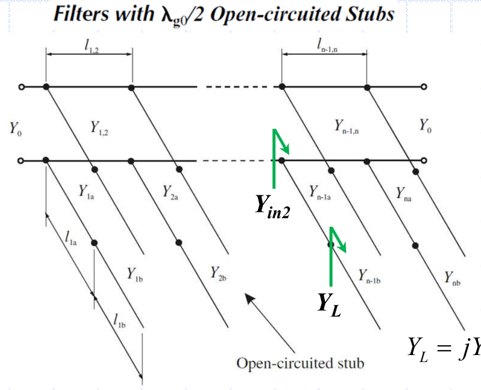


Design Equation

Filters with $\lambda_{g0}/4$ Short-circuited Stubs



Filters with $\lambda_{g0}/2$ Open-circuited Stubs



$$Y_L = jY_{ib}\tan\theta$$

$$Y_{in1} = \frac{Y_i}{j\tan\theta}$$

$$Y_{in2} = Y_{ia}\frac{Y_L + jY_{ia}\tan\theta}{Y_{ia} + jY_L\tan\theta} = Y_{ia}\frac{jY_{ib}\tan\theta + jY_{ia}\tan\theta}{Y_{ia} - Y_{ib}\tan^2\theta}$$

◆ Equivalence of the two input admittance using assumption of $Y_{ib} = \alpha_i Y_{ia}$

$$Y_{in1} = Y_{in2} \Rightarrow \frac{Y_i}{j\tan\theta} = Y_{ia}\frac{jY_{ib}\tan\theta + jY_{ia}\tan\theta}{Y_{ia} - Y_{ib}\tan^2\theta} \Rightarrow \frac{Y_i}{j\tan\theta} = Y_{ia}\frac{j\alpha_i Y_{ia}\tan\theta + jY_{ia}\tan\theta}{Y_{ia} - \alpha_i Y_{ia}\tan^2\theta}$$

$$\Rightarrow \frac{Y_i}{j\tan\theta} = Y_{ia}\frac{j(\alpha_i + 1)\tan\theta}{1 - \alpha_i\tan^2\theta} \Rightarrow Y_{ia} = \frac{Y_i(\alpha_i\tan^2\theta - 1)}{(1 + \alpha_i)\tan^2\theta}$$

◆ Transmission zero in the $\lambda_g/2$ case

$$Y_{in2} = \infty \Rightarrow Y_{ia} - Y_{ib}\tan^2\theta = 0 \Rightarrow Y_{ia} - \alpha_i Y_{ia}\tan^2\theta = 0 \Rightarrow \alpha_i = \cot^2\theta = \cot^2\left(\frac{2\pi}{\lambda_g}\frac{\lambda_0}{4}\right) = \cot^2\left(\frac{\pi}{2}\frac{f_z}{f_0}\right)$$

Stub Bandpass Filters without shorting via - Example

- Specifications for the $\lambda_g/2$ stubs are the same as the ones using $\lambda_g/4$ stubs . Choose the f_{zi} = 1 GHz.

$$\alpha_i = \cot^2\left(\frac{\pi f_{zi}}{2f_0}\right) \rightarrow \alpha_i = 1$$

  - Using previous equations to find the required Y_{ia}, Y_{ib} and $Y_{i,\,i+1}$

$$Y_{ia} = \frac{Y_i(\alpha_i \tan^2\theta - 1)}{(\alpha_i + 1)\tan^2\theta}$$

$$Y_{ib} = \alpha_i Y_{ia}$$

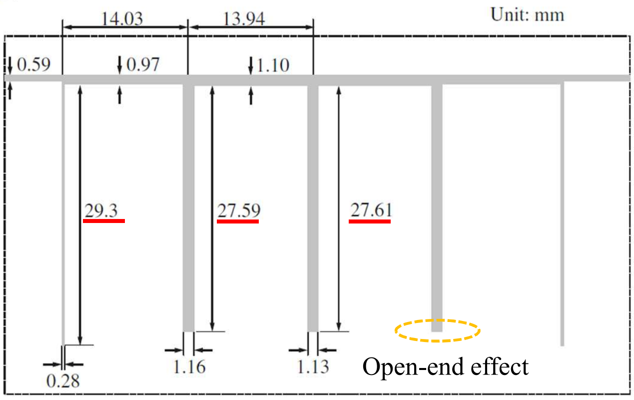
$$Y_{i,i+1} = Y_0\left(\frac{J_{i,i+1}}{Y_0}\right)$$

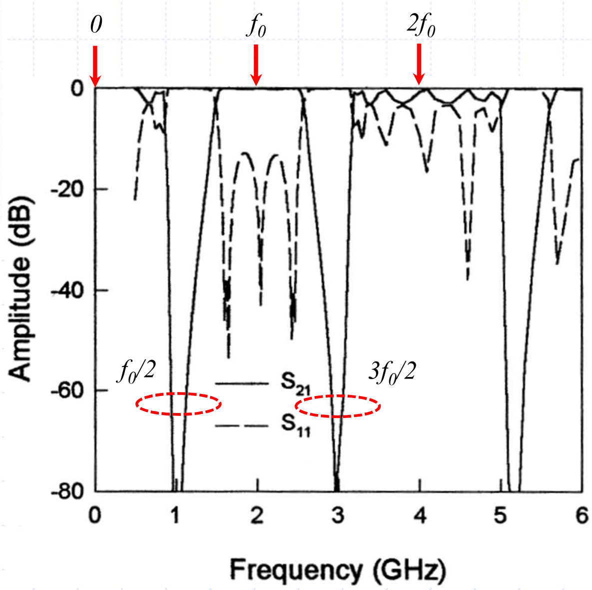
TABLE 5.14 Microstrip design parameters of a five-pole, stub bandpass filter with $\lambda_{g0}/2$ open-circuited stubs

i	$Y_{ai} = Y_{bi}$ (mhos)	W_i (mm)	$\lambda_{g0i}/4$ (mm)	$Y_{i,i+1}$ (mhos)	$W_{i,i+1}$ (mm)	$\lambda_{g0i,i+1}/4$ (mm)
1	0.01460	0.28	14.73	0.02587	0.97	14.03
2	0.02873	1.16	13.91	0.02787	1.10	13.97
3	0.02826	1.13	13.92	0.02787	1.10	13.97
4	0.02873	1.16	13.91	0.02587	0.97	14.03
5	0.01460	0.28	14.73			

fabricated on a substrate with a relative dielectric constant of 10.2 and a thickness of 0.635 mm.

Stub Bandpass Filters without shorting via - Example





HW III

Please design a BPF based on Chebyshev prototype with following specifications using end-coupled and parallel-coupled half-wavelength resonators:

1. Passband ripple < 0.1 dB
2. Center frequency : 5 GHz
3. FBW : 0.1
4. Insertion loss > 20 dB at 4 GHz and 6 GHz.

The properties of the substrate is $\varepsilon_r = 4.4$ and loss tangent of 0. The substrate thickness is 1.6 mm.

a. Please decide the order of the BPF and calculate the J-inverter values.
b. Find out the required series capacitances for end-coupled resonator case and the even- and odd-mode characteristic impedances of the coupled-line for parallel-coupled resonator case.
c. Plot the return loss and insertion loss for the designed BPF using HFSS and ADS.
d. Considering the discontinuities in the two cases, and using HFSS and ADS again to compare the results with those in (c). Please also discuss the reasons for the discrepancy between them.
e. Discuss the sizes and frequency responses using the two methods.

HW IV

1. According to the specification on HW III, design a interdigital filter. Let $Y_1 = 1/49.74$ mho.
   a. Calculate the per unit length self- and mutual-capacitances.
   b. Obtain the required even- and odd-mode impedance.
   c. Using EM solver to find corresponding dimensions based on problem (b).
   d. Considering the discontinuities and using EM solver to plot the return loss and insertion loss.
   e. Discuss the sizes and frequency responses for the three methods. (interdigital filter, end-coupled, and parallel-coupled resonator)

2. Please derive the Y_s and turns ratio N for the equivalence of the short-circuted parallel coupled-line and a transformer with short-circuited stub in combline filter.