

Indian Institute of Technology Hyderabad

Department of Physics

Complex Analysis, EP2197/PH5197

Final exam, Sep – Dec 2020

Answer all the questions

Time:  $1\frac{1}{4}$  hours

Maximum marks: 100

1. Let  $f(z)$  be a analytic function. Show that the derivative of  $f(z)$  with respect to  $z^*$  exist only if  $f(z)$  is constant.

(25 marks)

2. Let  $C$  be a closed contour defined by  $|z| = R$ , where  $R$  is a constant. For the cases of  $R < 1$  and  $R > 1$ , evaluate the following integral using the Cauchy's integral formula and Cauchy's integral theorem.

$$\oint_C \frac{dz}{z^2 + z}$$

**Hint:** You can use partial fractions.

(25 marks)

3. (i) Show that the derivative of the function  $\ln(1 + z)$  exist, except for  $z = -1$ . Find the derivative of this function.

(ii) Develop the Taylor series expansion for  $\ln(1 + z)$  around the point  $z = 0$ .

**Hint:** For an integer  $m$ , you can use

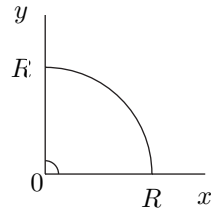
$$\frac{d}{dz}(1 + z)^m = m(1 + z)^{m-1}$$

(25 marks)

4. Let  $\alpha, t, s$  are real variables. For  $-1 < \alpha < 0$ , Show that

$$\int_0^\infty e^{it} t^\alpha dt = i^{\alpha+1} \int_0^\infty e^{-s} s^\alpha ds$$

**Hint:** You can use the below contour



(25 marks)