CS 2443: Quiz 4

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- Total marks: 10.
- Read the question carefully and answer to the question only.
- Maintain academic honesty.

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1. Consider the following algorithm, where input is a an undirected connected graph G and a weight function $w\colon E(G) \mapsto \mathbb{N}$ such that each edge gets distinct weight.

```
1 F := ∅;
2 Sort E(G) in the decreasing order of weight. Let this order be e_1, ..., e_m;
3 for i = 1 to m do
4 |   if G − F − e_i is connected then
5 |   |   F := F ∪ {e_i};
6 |   end
7 end
8 Output G − F;
```

   What is the output of the algorithm? Prove its correctness. [1+3]

2. Consider the following version of the Dijkstra's single source shortest path algorithm. Is the algorithm correct when the digraph has a weight function that is not necessarily non-negative? Justify your answer [3]

```
   Result: Given (G, w, s) output shortest paths from s to all other vertices
1  d(s) := 0;
2  d(v) := ∞ for all v ≠ s;
3  Insert(Q, (s, 0))                    ▷ Insert (s, 0) to a priority queue Q;
4  Insert(Q, (v, ∞)) for all v ≠ s      ▷ Insert (v, ∞) to the priority queue Q;
5  while the priority queue Q is not empty do
6  |   u :=ExtractMin(Q);
7  |   for all arcs (u, v) do
8  |   |   if d(v) > d(u) + w((u, v)) then
9  |   |   |   d(v) := d(u) + w((u, v));
10 |   |   |   DecreaseKey(Q, (v, d(v)))    ▷ Decrease the key value of v to d(v);
11 |   |   end
12 |   end
13 end
14 Output d(v) for all v ;
```

Algorithm 0.1: Dijkstra

3. Let G be a connected undirected graph and $w : E(G) \mapsto \mathbb{N}$ be a weight function that assigns *distinct* edge weights. There is a unique minimum weight spanning tree in G. Is this statement true? Justify your answer. [3]