Ans 3. Given: $T(n) = n^2 + 3 \cdot T(n-1)$

$T(0) = 1$

Proof:- $T(n) = n^2 + 3 \cdot T(n-1)$

$$T(n) = n^2 + 3 \cdot \left[(n-1)^2 + 3 \cdot T(n-2) \right]$$

$$T(n) = n^2 + 3^1 \cdot (n-1)^2 + 3^2 \cdot \left[(n-2)^2 + 3 \cdot T(n-3) \right]$$

$$\vdots$$

$$T(n) = \sum_{i=0}^{n} 3^i (n-i)^2 + 3^n \cdot \xrightarrow{1}{\cancel{T(0)}}$$

$$T(n) = 3^n + \sum_{i=0}^{n} 3^i (n-i)^2$$

$$= 3^n + \sum_{i=0}^{n} 3^i \cdot \left[n^2 + i^2 - 2ni \right]$$

$$3^n + n^2 \cdot \left[\frac{3^{n+1} - 1}{2} \right] - 2n \sum_{i=0}^{n} 3^i i + \sum_{i=0}^{n} i^2 3^i$$

~~Now obtaining the upper bound by considering~~

~~$S(n) = T(n)/3^n$ which satisfies recurrence~~

~~relation $S(n) = S(n-1) + \frac{n^2}{3^n}$~~

~~and it implies $T(n) = O(3^n)$ as~~

$$T(n) = 3^n + \sum_{i=0}^{n} 3^i (n-i)^2$$

$$\leq 3^n + \sum_{i=0}^{n} 3^i n^2$$

$$T(n) \leq 3^n + n^2 \sum_{i=0}^{n} 3^i$$

$$T(n) \leq 3^n + n^2 \cdot \frac{(3n-1)}{2}$$

$$\Rightarrow \boxed{T(n) = O(3^n)}$$ ar

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Ans 2. (a). Let $n = 2^k$, $\sqrt{n} = 2^{k/2}$

$k = \log n$.

Substituting in above eq^n:

$$T(n) = \sqrt{n} \cdot T(\sqrt{n}) + 5$$

$$\Rightarrow T(2^k) = 2^{k/2} \, T(2^{k/2}) + 5 \quad \text{— (1)}$$

(Master's Theorem: $T(n) = a \cdot T(\frac{n}{b}) + n^d$). — (3)

dividing eq^n (1) by 2^k.

$$\frac{T(2^k)}{2^k} = \frac{T(2^{k/2})}{2^{k/2}} + \frac{5}{2^{k/2}} \quad \text{— (2)}$$

Let $\frac{T(2^k)}{2^k} = y(k)$

$$\therefore \quad y(k) = y(k/2) + \frac{5}{2^{k/2}}$$

Ans 2. (b). $T(n) = 3T\left(\frac{n}{4}\right) + T\left(\frac{n}{6}\right) + n.$

$T(1) = T(2) = 5.$

Using recurrence rel^n we can see that the depth is $\log_6 n$.

$$\log_6 n = O(\log n).$$

We can also see that total work at each level is $11/12$ of upper level.

$\therefore$ The recursion method tree implies the upper bound

$$T(n) \leq n \sum_{i=0} \left(\frac{11}{12}\right)^i$$

$$T(n) \leq 12n$$

$$\therefore \boxed{T(n) = O(n)} \text{ as}$$

Ans 3. By Induction :-

Base case : $T(0) = O(1) \Rightarrow 1 \leq c \cdot 1$ (True)

Consider : $T(n) \leq c \cdot 3^n$. (Inductive step)

To prove : $T(n+1) \leq c \cdot 3^{n+1}$

Proof :- $T(n) \leq c \cdot 3^n$

$$3 \cdot T(n) \leq c \cdot 3^{n+1}$$

$$T(n+1) - (n+1)^2 \leq c \cdot 3^{n+1}$$

$$\Rightarrow T(n+1) \leq c \cdot 3^{n+1} + (n+1)^2$$

$\because (n+1)^2 > 0$, So it is clearly shown that

$$c' 3^{n+1} + (n+1)^2 \geq c' 3^{(n+1)} \quad \forall c, c'$$

$\therefore T(n+1) \leq c' 3^{n+1}$ $\therefore \exists c > 0$.

such that $T(n+1) \leq c' 3^{(n+1)}$

and hence proved that

$T(n)$ is of order 3^n

i.e $T(n) = O(3^n)$ as