

Ans 3.

Graph: G.

u , and v are edge disjoint paths.

$$\therefore E(P_1) \cap E(P_2) = \phi$$

Relation $R := \sim$ ~~is also an eqv. rel.~~

\sim is Reflexive. $\therefore u \sim u$ ✓ True.
 \sim is also symmetric.

and

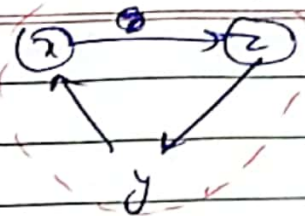
$$u \sim x, x \sim v \Rightarrow u \sim v$$

\therefore it is also transitive.

Hence \sim is an equivalence relⁿ.

Ans 4.

SCC's :



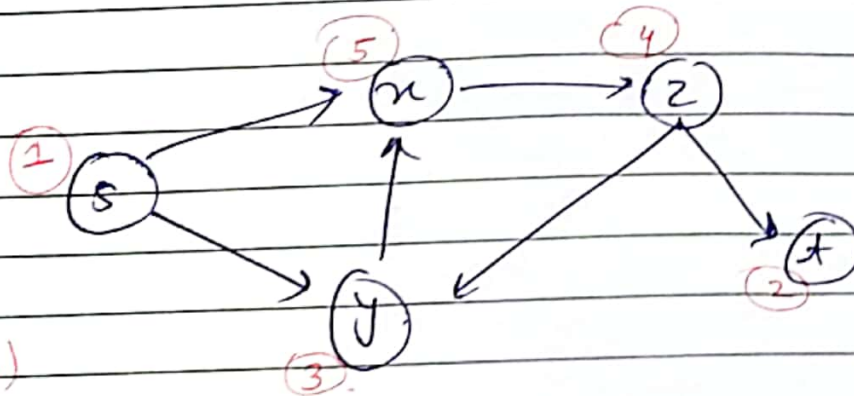
, {t}, {s}.

these 3 are the SCC's in ~~above~~ given graph.

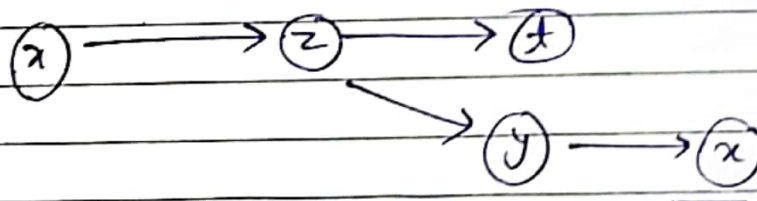
Graph :

Let's mark
the vertex
no.

(random marking)



∴ Starting from highest number vertex i.e. x.



$$f(3) = f(y) = 1$$

$$f(2) = f(t) = 2$$

$$f(4) = f(z) = 3$$

$$f(5) = f(x) = 4$$

$$f(7) = f(s) = 5$$

Finishing time of each vertex.

Ans 1. $\text{Subset sum}(i, T') = \begin{cases} 0 & \text{if } i = 0 \\ \text{Subset sum}(i-1, T') & \text{if } x(i) > T' \\ \max \left(\text{Subset sum}(i-1, T'), \text{Subset sum}(i-1, T' - x[i] + w(i)) \right) & \text{otherwise} \end{cases}$

Pseudo code: - as

The following recurrence relationship takes $x[i]$ if it is greater than T .

The algo considers both cases, when $x(i)$ was considered and not considered. The corresponding weight is added.

Then func. ensures that max. values is outputted.

If there's no o/p, result is $-\infty$, the individual value since it is simple brute force which considers every subset and considers the max. value. Thus the algo is correct.

Ans 2. To prove: For all i $L_i = \{x \mid d(s, x) = i\}$

Proof: - Base case ($i=0$)

$$\therefore L_0 = \underline{s.}$$

s : source vertex.

Inductive Cond'n: \downarrow

~~Cond'n~~ if y was added to L_{i+1} , then $d(s, y) = \underline{i+1}$
~~Cond'n~~ ~~$d(s, y) = i+1$~~ ~~y is added to L_{i+1}~~

above

If we prove ^{above} "cond" ~~Cond'n~~ then we can conclude that $L_{i+1} = \{y \mid d(s, y) = i+1\}$.

Proof ~~Cond'n~~ Cond'n:- if y is added to L_{i+1} ,

it was added by traversing an edge (x, y) where $x \in L_i$ so that there's path from s to y .
 So taking shortest path from s to x followed by (x, y) .

$$d(s, y) \leq d(s, x) + \underline{1.}$$

Since $x \in L_i$, by induction hypothesis.
 $d(s, x) = \underline{i.}$

$$\therefore \underline{d(s, y) \leq i+1.}$$

However since $y \notin L_j$ for any $j \leq i$, by induction hypothesis $d(s, y) > i$

$$\therefore d(s, y) = i+1$$