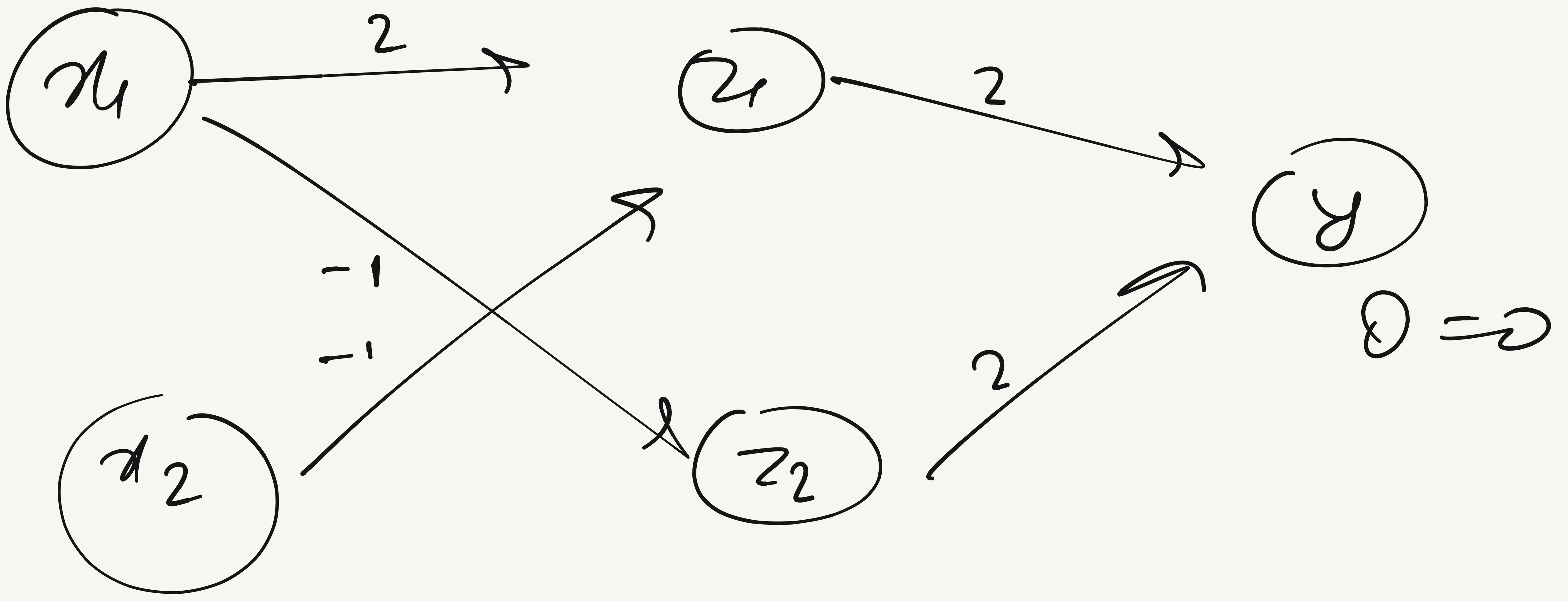
Ans 1. (a) Two layer perceptron can solve the XOR problem as follows $\rightarrow$



for $x_1 = 0$, $x_2 = 0$

we know that $\rightarrow$ $$z_1 = \begin{cases} 1, & \text{if } \sum_{i=1}^{n} x_i w_i > \theta \\ 0, & \text{if } \sum_{i=1}^{n} x_i w_i \leq \theta \end{cases}$$

here $\rightarrow$

$$\sum_{i=1}^{n} x_i w_i = 0(1) + (-1)(0)$$

$$= 0 < 1$$

So $z_1 = 0$, $z_2 = 0$

$$\& \ y = \begin{cases} 1, & \text{if } \sum_{i=1}^{n} z_i w_i > \theta \\ 0, & \text{if } \sum_{i=1}^{n} z_i w_i < \theta \end{cases}$$

here $y = 0(2) + 0(+2)$
$= 0 = 0$

so $\boxed{y = 0}$

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for $x_1 = 0$, $x_2 = 1$

$$\sum_{i=1}^{n} x_i w_i = 0(2) + (-1)(1)$$
$$= -1 < 1$$

So $z_1 = 0$

& $\sum_{i=1}^{n} x_i w_i^2 = 0(-1) + 1(2) = 2 > 1$

$\Rightarrow$ $z_2 = 1$

Now for y

$$\sum_{i=1}^{n} x_i w_i = 2 > 0$$

$\Rightarrow$ $y = 1$

Also for $x_1 = 1$ & $x_2 = 0$ $\rightarrow$

Similarly we can say

$y = 1$

for $x_1 = 1$ & $x_2 = 1 \rightarrow$

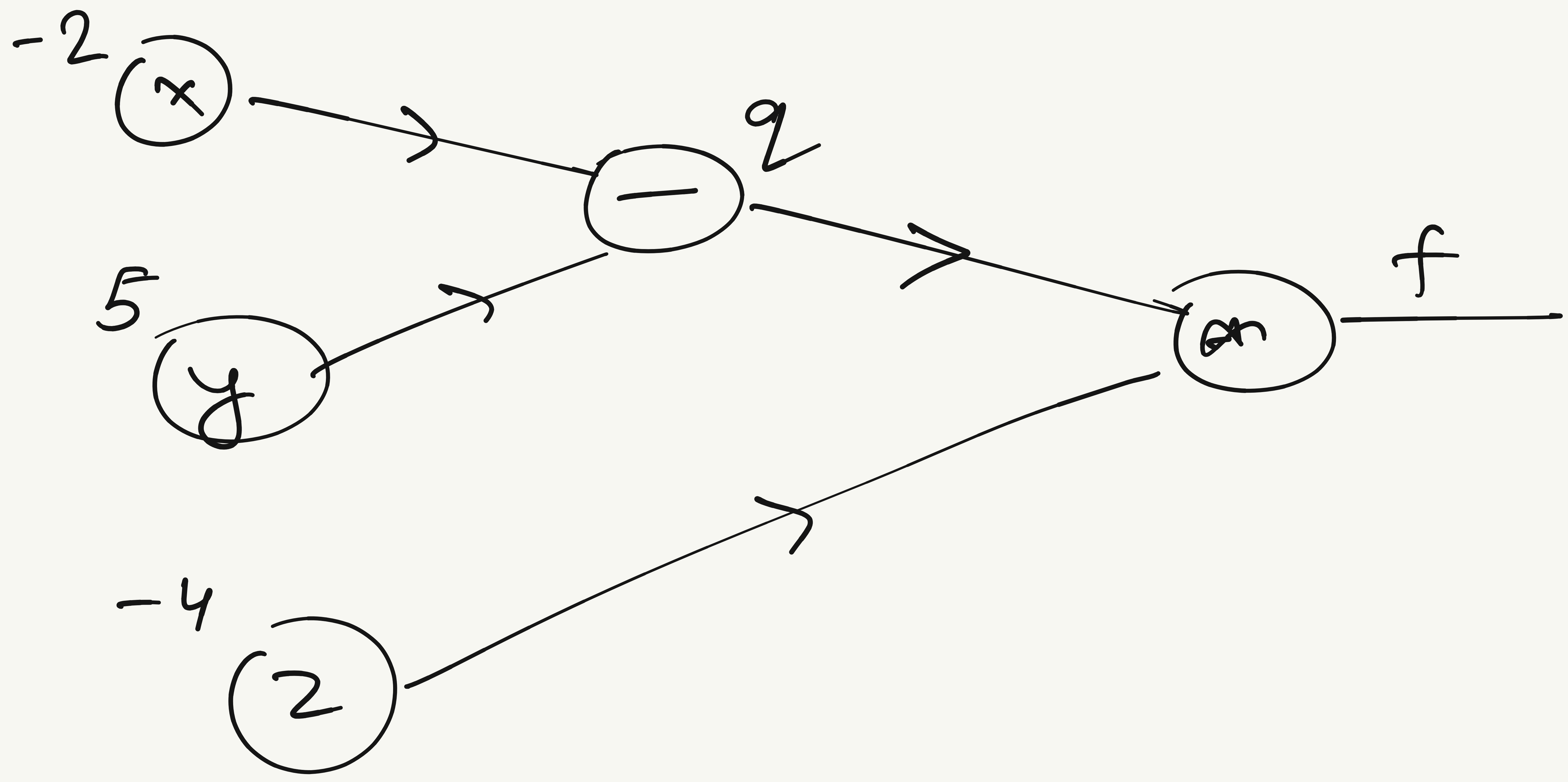
$$\sum_{i=1}^{n} x_i w_i = 1 = 1$$

$\Rightarrow z_1 = 0 \quad , z_2 = 0$

& $y = 0$

Ay

Ans 1.(b)



→ We can see that $q = x - y$

$\therefore \boxed{q = x - y = -2 - 5 = -7 = \frac{\partial f}{\partial z}}$

and similarly $f = q * z$

also → $\frac{\partial q}{\partial x} = 1$, $\frac{\partial q}{\partial y} = -1$, $\frac{\partial f}{\partial q} = z$

$$\therefore \frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial x} = z = -4 \quad \text{— (1)}$$

and $$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial y} = -z = 4 \quad \text{— (2)}$$

as

(1) & (2) by chain rule

Ans 2. $$\boxed{E(w) = -\sum_{n=1}^{N}\sum_{k=1}^{K} t_{kn} \ln(y_k(x_n, w))}$$

Given

For a given input

$$E = -\sum_{k=1}^{K} t_k \ln(y_k)$$

$$\Rightarrow \frac{\partial E}{\partial a_k} = \frac{-\partial}{\partial a_k}\left[\sum_{i=1}^{K} t_i \ln(y_i)\right]$$

$$= \frac{-\partial}{\partial a_k}\left[t_k \ln(y_k) + \sum_{i \neq k} t_i \ln(y_i)\right]$$

$$\Rightarrow \frac{\partial E}{\partial a_k} = -\frac{t_k}{y_k} \times \frac{\partial y_k}{\partial a_k} - \sum_{i \neq k} \frac{t_i}{y_i}\frac{\partial y_i}{\partial a_k}$$

$$= -\frac{t_k}{\cancel{y_k}} \times \cancel{y_k} \times (1 - y_k) - \sum_{i \neq k} \frac{t_i}{\cancel{y_i}}(-\cancel{y_i} \cdot y_k)$$

$$= -t_k + t_k y_k + \sum_{i \neq k} t_i y_k$$

$$= -t_k + y_k\left(t_k + \sum_{i \neq k} t_i\right)$$

②

$$\Rightarrow \frac{\partial E}{\partial a_k} = -t_k + y_k\left(\sum t_i\right)$$

$$\because \sum t_i = \underline{1}$$

$\Rightarrow$ Hence, $\boxed{\dfrac{\partial E}{\partial a_k} = -t_k + y_k = y_k - t_k}$

Ans 3. By substituting the values of E_{AV} and E_{ENS} and by expanding the squares we get $\rightarrow$

$$E_x\left[\frac{1}{M^2}\left(\sum_{m=1}^{M} y_m(x)\right)^2 + f^2(x) - \frac{2}{M} f(x) \sum_{m=1}^{M} y_m(x)\right]$$

$$\leq \frac{1}{M}\sum_{m=1}^{M} E_x\left[y_m^2(x) + f^2(x) - 2y_m(x)f(x)\right] \quad - (1)$$

Now using linearity of expansion :-

LHS $\leq$ RHS (from eqn (1))

$$\text{LHS} :\rightarrow \frac{1}{M^2} E_x\left[\left(\sum_{m=1}^{M} y_m(x)\right)^2\right] + E_x\left[f^2(x)\right] - \frac{2}{M} E_x\left[f(x)\sum_{m=1}^{M} y_m(x)\right]$$

and

$$\text{RHS} :\rightarrow \frac{1}{M}\sum_{m=1}^{M} E_x\left[y_m^2(x)\right] + \frac{1}{M}\sum_{m=1}^{M} E_x\left[f^2(x)\right] - \frac{2}{M}$$

$\because$ $E_x[f^2(x)]$ is independent of m

$\therefore$ $\frac{1}{M}\sum_{m=1}^{M} E_x[f^2(x)]$ can be written as $\rightarrow$

$$\frac{1}{M}\sum_{m=1}^{M} E_x[f^2(x)] = \frac{1}{M} E_x[f^2(x)] \sum_{m=1}^{M} 1$$

$$= E_x[f^2(x)]$$

So the above then cancels the term on the LHS of inequality and we get $\rightarrow$

$$\frac{1}{M^2} E_x\left[\left(\sum_{m=1}^{M} y_m(x)\right)^2\right] - \frac{2}{M} E_x\left[f(x)\sum_{m=1}^{M} E_x(y_m^2(x))\right]$$

$$\leq$$

$$\frac{1}{M}\sum_{m=1}^{M} E_x[y_m^2(x)] - \frac{2}{M}\sum_{m=1}^{M} E_x[y_m(x) f(x)] \quad \text{— } (2)$$

Again using the linearity of expectation in eq^n (2) we get $\rightarrow$

$$\frac{2}{M} E_x\left[f(x)\sum_{m=1}^{M} y_m(x)\right] = \frac{2}{M}\sum_{m=1}^{M} E_x[f(x)\, y_m(x)]$$

So cancelling above terms both side $\rightarrow$

We finally get →

$$\frac{1}{M^2} E_x\left[\left(\sum_{m=1}^{M} y_m(x)\right)^2\right] \leq \frac{1}{M} \sum_{m=1}^{M} E_x\left[y_m^2(x)\right]$$

$$\Rightarrow \left(\sum_{m=1}^{M} y_m(x)\right)^2 \leq M \sum_{m=1}^{M} y_m^2(x) \quad \text{--- } (3)$$

Now we can use Cauchy-Schwarz inequality to ~~probe~~ prove eq^n (3)'s inequality.

i.e. $(x_1 y_1 + x_2 y_2 + \cdots x_m y_m)^2 \leq (x_1^2 + x_2^2 + \cdots x_m^2) \times (y_1^2 + y_2^2 + \cdots y_m^2)$

$\therefore$ Substituting $x_1 = x_2 = x_3 \cdots = x_m = 1$

$$\Rightarrow (y_1 + y_2 + y_3 \cdots y_m)^2 \leq M(y_1^2 + y_2^2 + \cdots y_m^2)$$

$$\boxed{\Rightarrow \left(\sum_{m=1}^{M} y_m(x)\right)^2 \leq M \sum_{m=1}^{M} \left(y_m^2(x)\right)}$$

Hence Proved.