Temperaturas em Manhattan

Julia Guazzelli Monteiro – 15465383
Vinícius de Sá Ferreira – 15491650

SME0205

Problema

$$\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T(x,t)}{\partial x^2}$$

$$\left.\begin{aligned}
T(x+1) &= T(x) + T'(x) + \frac{T''(x)}{2} + \frac{T^{(3)}(x)}{6} + \frac{T^{(4)}(x)}{24} + \cdots \\
T(x-1) &= T(x) - T'(x) + \frac{T''(x)}{2} - \frac{T^{(3)}(x)}{6} + \frac{T^{(4)}(x)}{24} - \cdots
\end{aligned}\right\} \text{Série de Taylor}$$

$$T(x+1) + T(x-1) = 2T(x) + T''(x) + \frac{T^{(4)}(x)}{12} + \cdots$$

$$T''(x) = T(x+1) - 2T(x) + T(x-1) - \mathcal{O}(1)$$

$$\frac{dT_i(t)}{dt} = \alpha(T_{i+1}(t) - 2T_i(t) + T_{i-1}(t))$$

Problema

$$LT = \left(\begin{bmatrix} G_1 & 0 & 0 & \cdots & 0 \\ 0 & G_2 & 0 & \cdots & 0 \\ 0 & 0 & G_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & G_n \end{bmatrix} - \begin{bmatrix} \\ \\ & A & \\ \\ \\ \end{bmatrix} \right)_{n\times n} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ \vdots \\ T_n \end{bmatrix}_{n\times 1} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_n \end{bmatrix}_{n\times 1}$$

$$C_i = G_i T_i - \sum_{j\to i} T_j$$

$$\frac{dT_i(t)}{dt} = \alpha(T_{i+1}(t) - 2T_i(t) + T_{i-1}(t)) = -\alpha(2T_i(t) - T_{i-1}(t) - T_{i+1}(t)) = -\alpha(G_i T_i(t) - \sum_{j\to i} T_j(t)) = -\alpha(LT)_i$$

$$\frac{dT}{dt} = -\alpha LT$$

$$(L + P)T = Pb$$

Métodos Diretos

Cholesky: $M = LL^T \longrightarrow O(\frac{1}{3}N^3)$

$$Mx = k$$

$$(LL^T)x = k \begin{cases} y = L^T x \\ Ly = k \end{cases}$$

LU: $M = PLU \longrightarrow O(\frac{2}{3}N^3)$

$$MT = c$$

$$PLUT = c \begin{cases} UT = y \\ Ly = Pc \end{cases}$$

Métodos Diretos

QR: $M = QR \longrightarrow O(\,\frac{4}{3}N^3)$

$$MT = c$$

$$QRT = c$$

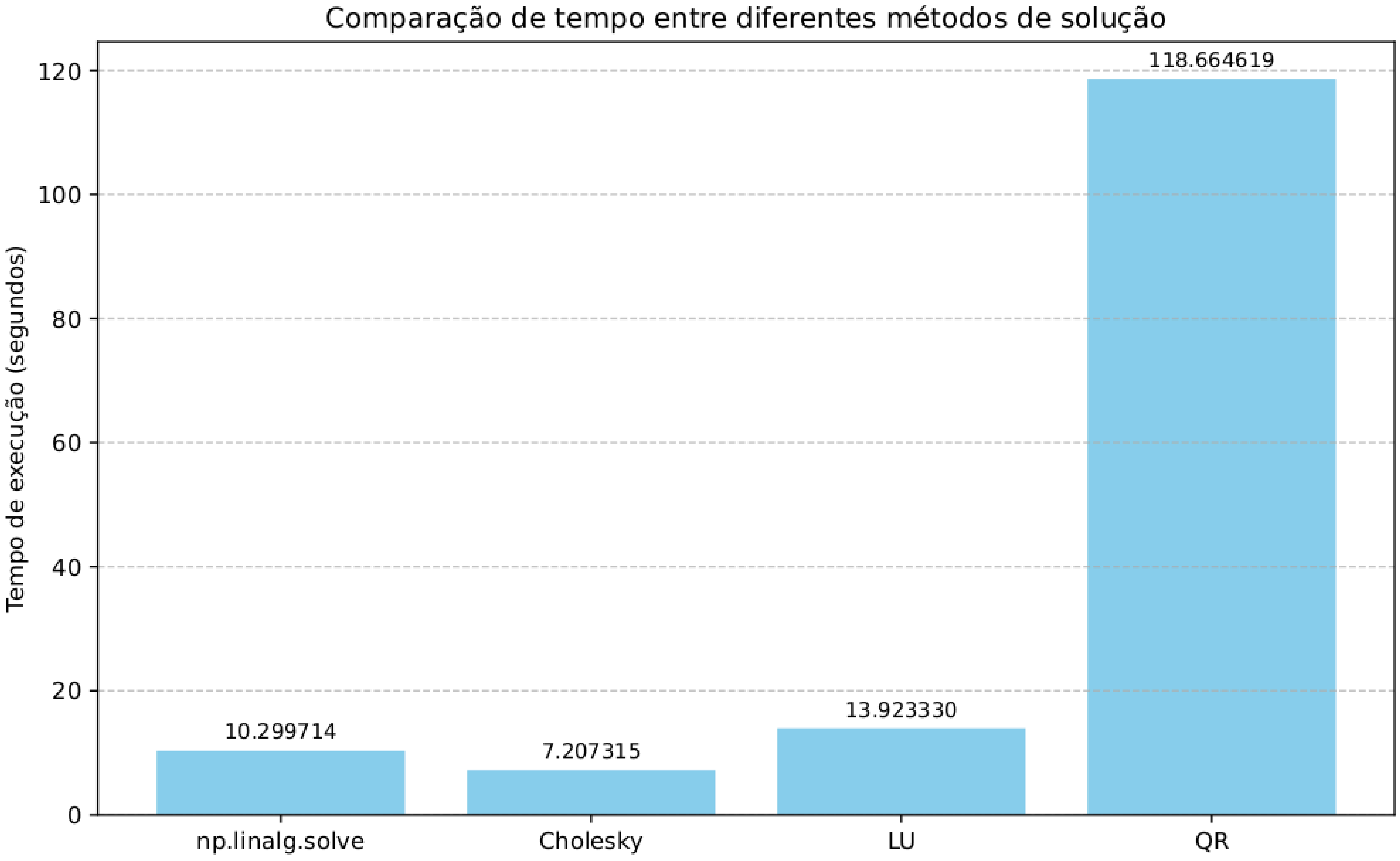
$$Q^T(QRT) = Q^T c$$

$$(Q^T Q)RT = Q^T c$$

$$IRT = Q^T c$$

$$RT = Q^T c$$

Métodos Diretos - Tempos



Métodos Iterativos

Gauss-Jacobi:

$$Mx = c$$

D é a diagonal de M

$$(M - D + D)x = c$$

$$(M - D)x + Dx = c$$

$$(M - D)x^{(k)} + Dx^{(k+1)} = c$$

$$Dx^{(k+1)} = (D - M)x^{(k)} + c$$

$$x^{(k+1)} = D^{-1}(D - M)x^{(k)} + D^{-1}c$$

$$x^{(k+1)} = (I - D^{-1}M)x^{(k)} + D^{-1}c$$

$$x^{(k+1)} = Cx^{(k)} + g$$

Métodos Iterativos

Gauss-Seidel:

$$Mx = c$$

$$(L + R)x = c$$

$$Lx + Rx = c$$

$$Lx^{(k+1)} + Rx^{(k)} = c$$

$$Lx^{(k+1)} = -Rx^{(k)} + c$$

$$x^{(k+1)} = (-L^{-1}R)x^{(k)} + L^{-1}c$$

$$x^{(k+1)} = Cx^{(k)} + g$$

L é a matriz triangular inferior de M e R, a triangular superior sem a diagonal

Métodos Iterativos

Gradientes Conjugados:

$$f(x) = \frac{1}{2}x^T M x - c^T x$$

$$\nabla f(x) = Mx - c$$

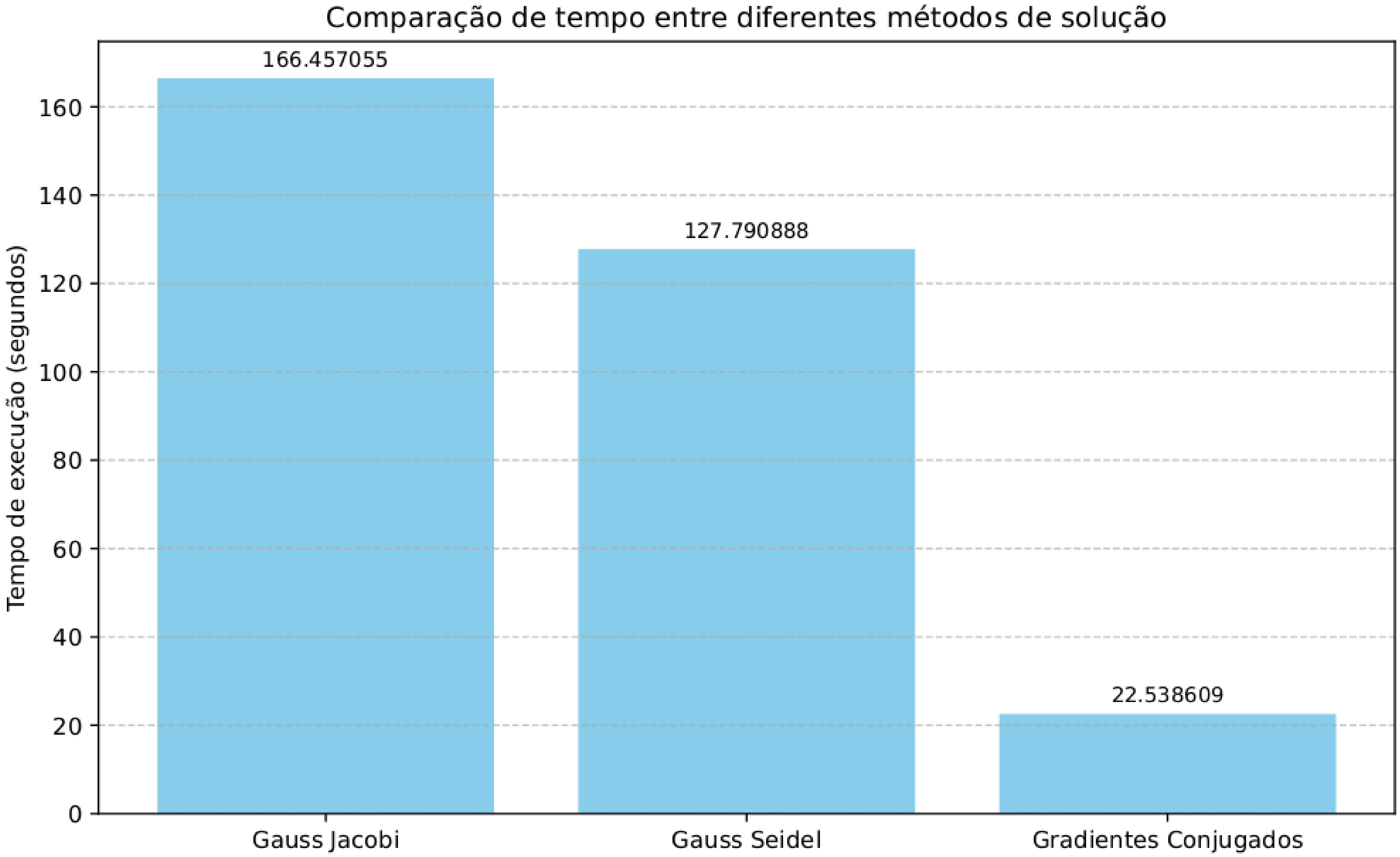
1. Chute inicial x_0;
2. $p_0 = r_0 = c - Mx_0$;
3. $\alpha = \dfrac{r_k^T \cdot r_k}{p_k^T \cdot M \cdot p_k}$;
4. $x_{k+1} = x_k + \alpha p_k$;
5. $r_{k+1} = r_k - \alpha(M \cdot p_k)$;
6. $\beta = \dfrac{r_{k+1}^T \cdot r_{k+1}}{r_k^T \cdot r_k}$;
7. $p_{k+1} = r_{k+1} + \beta p_k$;
8. Volta para o passo 3, enquanto $\|r_{k+1}\| > \text{tol}$.

Métodos Iterativos - Tempos

Tolerância: $\pm 10^{-3}$.

	Gauss-Jacobi	Gauss-Seidel	Gradientes Conjugados
k	1280	657	255
tempo	$3min$	$2min$	$30s$

Métodos Iterativos - Tempos



Resultados

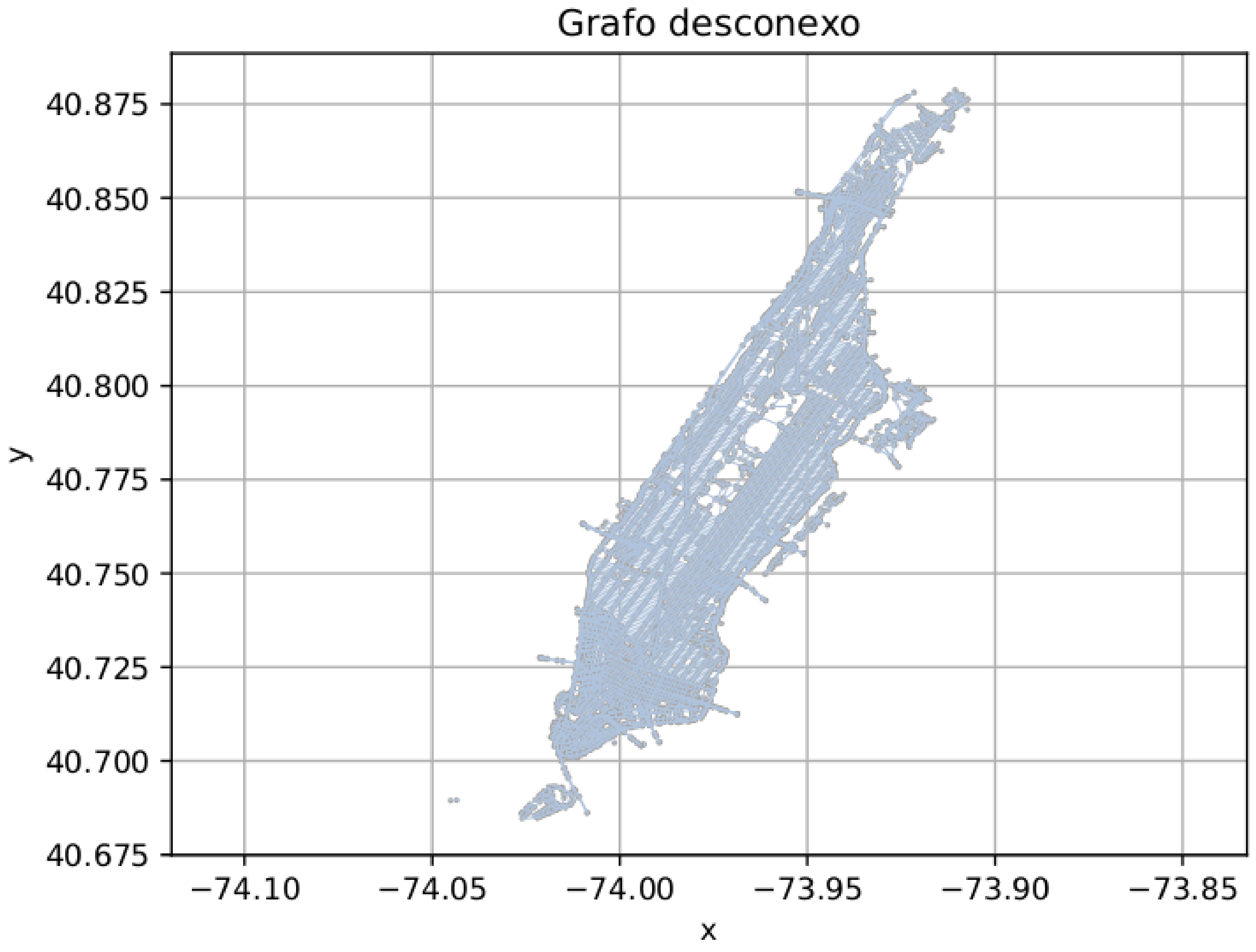
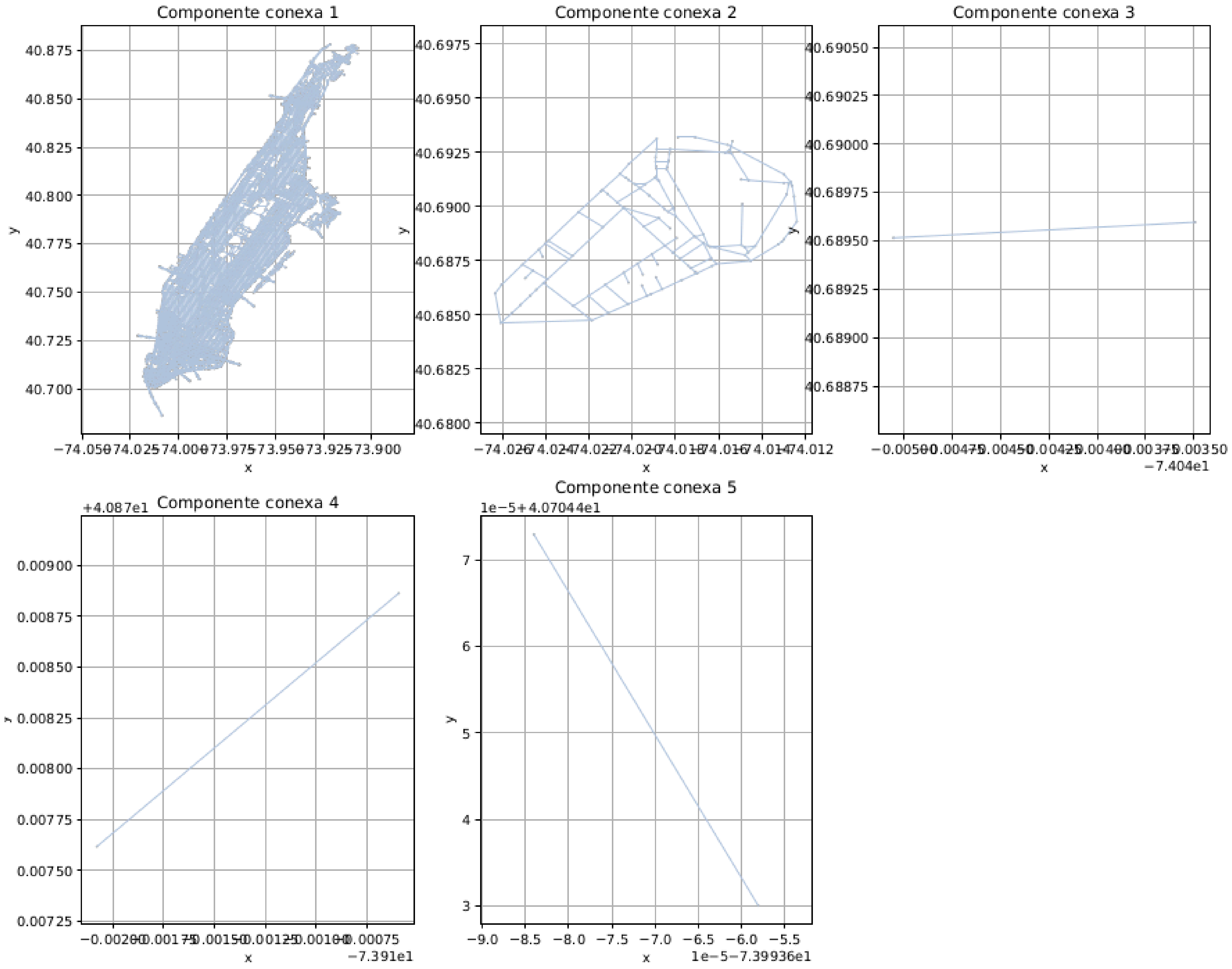
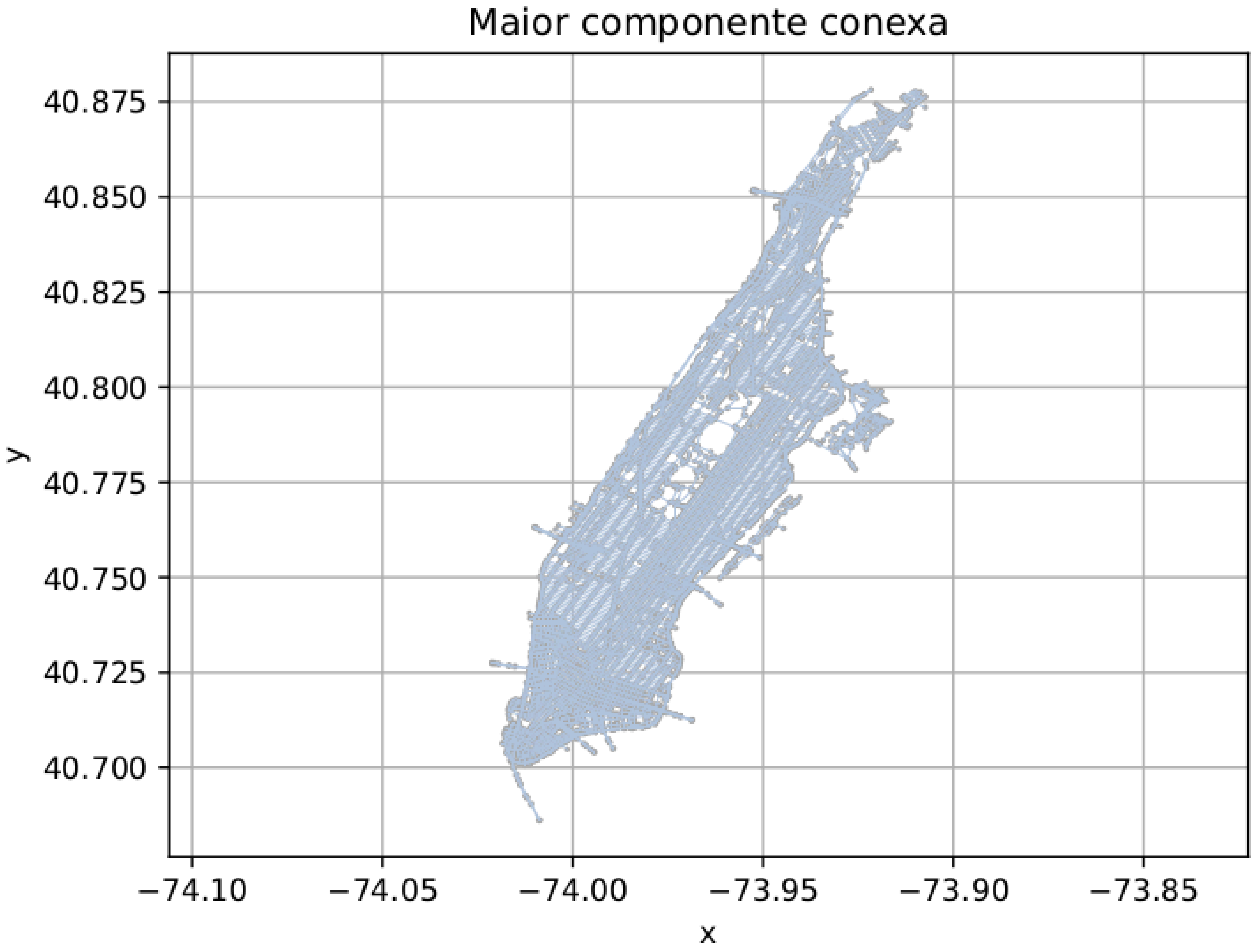


Figura 1

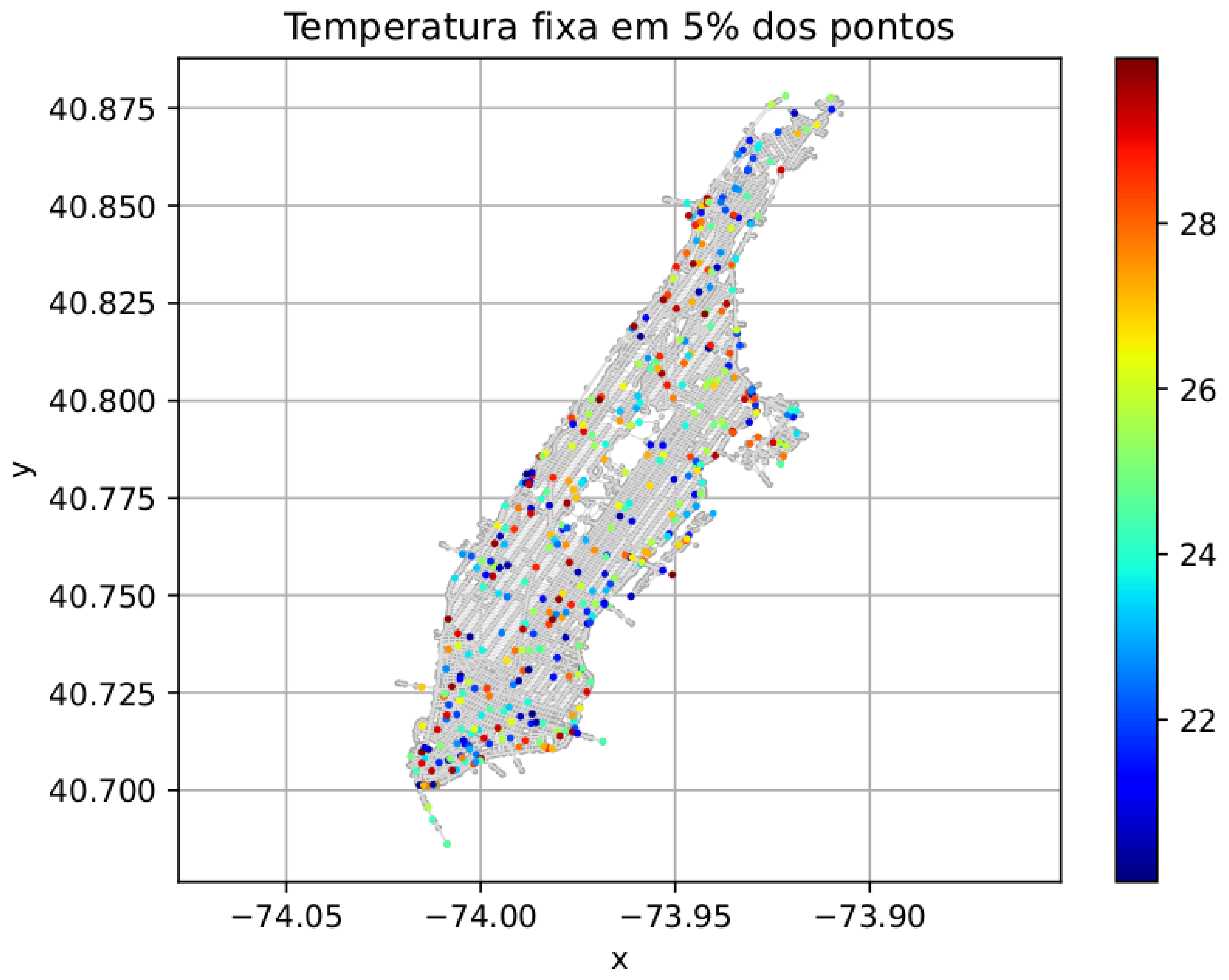


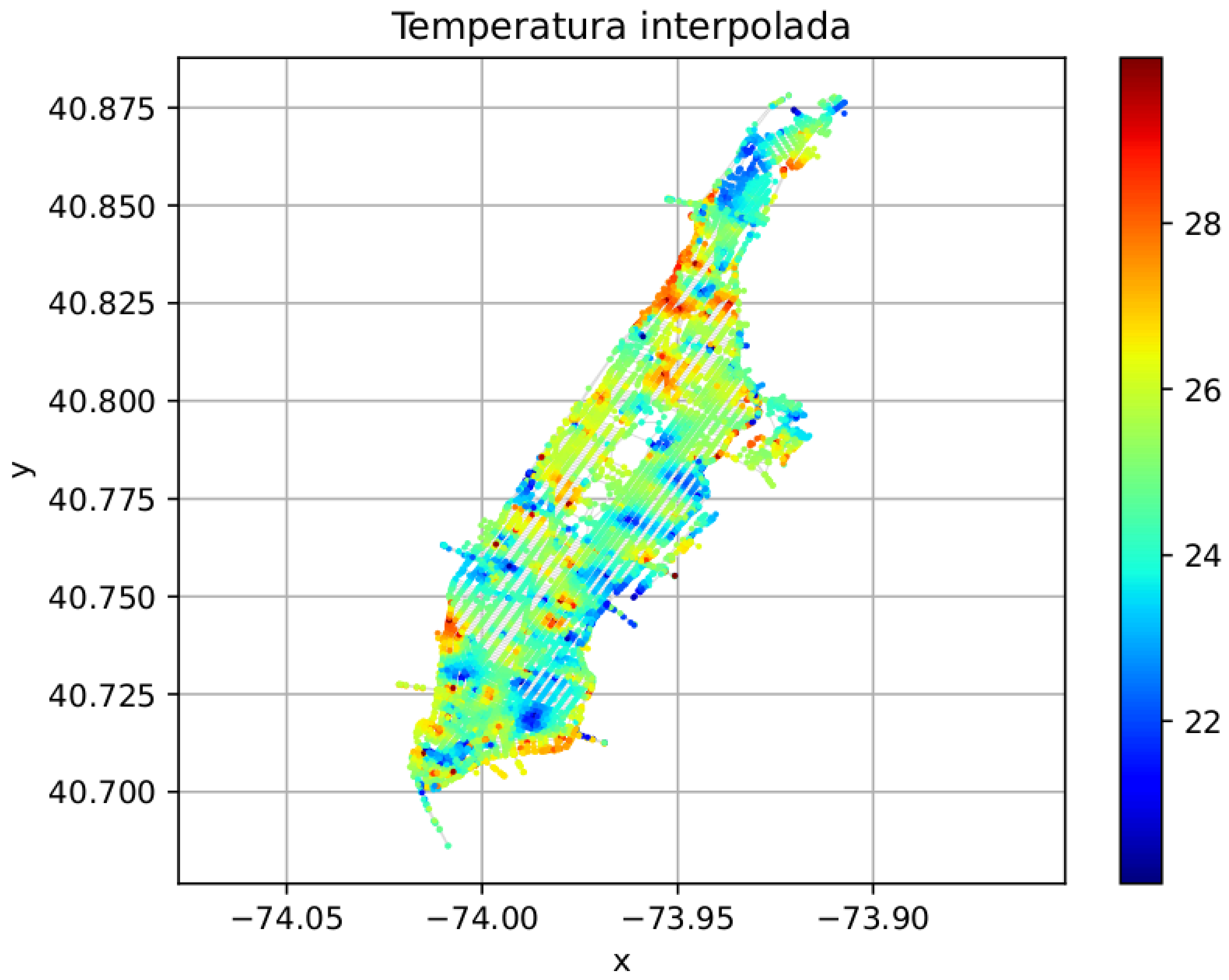
Resultados



Maior componente conexa
40.875
40.850
40.825
40.800
40.775
40.750
40.725
40.700
y
−74.10
−74.05
−74.00
−73.95
−73.90
−73.85
x

Resultados


Temperatura fixa em 5% dos pontos
40.875
40.850
40.825
40.800
40.775
40.750
40.725
40.700
y
−74.05
−74.00
−73.95
−73.90
x
28
26
24
22


Temperatura interpolada
40.875
40.850
40.825
40.800
40.775
40.750
40.725
40.700
y
−74.05
−74.00
−73.95
−73.90
x
28
26
24
22