Temperaturas em Manhattan

Julia Guazzelli Monteiro – 15465383
Viníicius de Sá Ferreira – 15491650

SME0205

Problema

$$\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T(x,t)}{\partial x^2}$$

$$\left.\begin{aligned}
T(x+1) &= T(x) + T'(x) + \frac{T''(x)}{2} + \frac{T^{(3)}(x)}{6} + \frac{T^{(4)}(x)}{24} + \cdots \\
T(x-1) &= T(x) - T'(x) + \frac{T''(x)}{2} - \frac{T^{(3)}(x)}{6} + \frac{T^{(4)}(x)}{24} - \cdots
\end{aligned}\right\} \text{Série de Taylor}$$

$$T(x+1) + T(x-1) = 2T(x) + T''(x) + \frac{T^{(4)}(x)}{12} + \cdots$$

$$T''(x) = T(x+1) - 2T(x) + T(x-1) - \mathcal{O}(1)$$

$$\frac{dT_i(t)}{dt} = \alpha(T_{i+1}(t) - 2T_i(t) + T_{i-1}(t))$$

Problema

$$LT = \left(\begin{bmatrix} G_1 & 0 & 0 & \cdots & 0 \\ 0 & G_2 & 0 & \cdots & 0 \\ 0 & 0 & G_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & G_n \end{bmatrix} - \begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \right)_{n\times n} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ \vdots \\ T_n \end{bmatrix}_{n\times 1} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_n \end{bmatrix}_{n\times 1}$$

$$C_i = G_i T_i - \sum_{j\to i} T_j$$

$$\frac{dT_i(t)}{dt} = \alpha(T_{i+1}(t) - 2T_i(t) + T_{i-1}(t)) = -\alpha(2T_i(t) - T_{i-1}(t) - T_{i+1}(t)) = -\alpha(G_i T_i(t) - \sum_{j\to i} T_j(t)) = -\alpha(LT)_i$$

$$\frac{dT}{dt} = -\alpha LT$$

$$(L+P)T = Pb$$

Métodos Diretos

Cholesky: $M = LL^T \longrightarrow O(\frac{1}{3}N^3)$

$$Mx = k$$

$$(LL^T)x = k \begin{cases} y = L^T x \\ Ly = k \end{cases}$$

LU: $PM = LU \longrightarrow O(\frac{2}{3}N^3)$

$$MT = c$$

$$LUT = Pc \begin{cases} UT = y \\ Ly = Pc \end{cases}$$

Métodos Diretos

QR: $M = QR \longrightarrow O(\frac{4}{3}N^3)$

$$MT = c$$

$$QRT = c$$

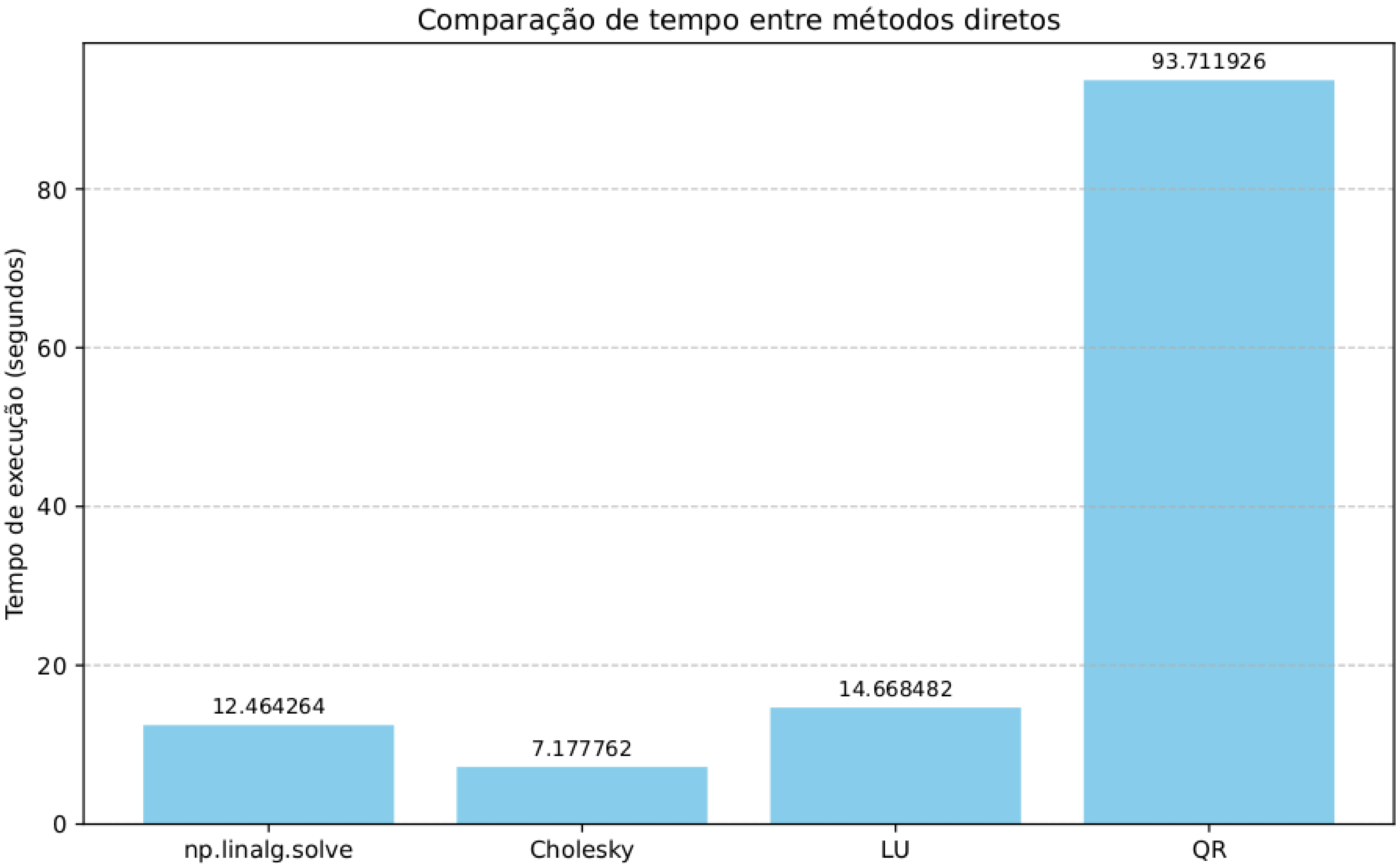
$$Q^T(QRT) = Q^Tc$$

$$(Q^TQ)RT = Q^Tc$$

$$IRT = Q^Tc$$

$$RT = Q^Tc$$

Métodos Diretos - Tempos



Métodos Iterativos

Gauss-Jacobi:

$$Mx = c$$

D é a diagonal de M

$$(M - D + D)x = c$$

$$(M - D)x + Dx = c$$

$$(M - D)x^{(k)} + Dx^{(k+1)} = c$$

$$Dx^{(k+1)} = (D - M)x^{(k)} + c$$

$$x^{(k+1)} = D^{-1}(D - M)x^{(k)} + D^{-1}c$$

$$x^{(k+1)} = (I - D^{-1}M)x^{(k)} + D^{-1}c$$

$$x^{(k+1)} = Cx^{(k)} + g$$

Métodos Iterativos

Gauss-Seidel:

$$Mx = c$$

$$(L + R)x = c$$

$$Lx + Rx = c$$

$$Lx^{(k+1)} + Rx^{(k)} = c$$

$$Lx^{(k+1)} = -Rx^{(k)} + c$$

$$x^{(k+1)} = (-L^{-1}R)x^{(k)} + L^{-1}c$$

$$x^{(k+1)} = Cx^{(k)} + g$$

L é a matriz triangular inferior de M e R, a triangular superior sem a diagonal

Métodos Iterativos

Gradientes Conjugados:

$$f(x) = \frac{1}{2}x^T M x - c^T x$$

$$\nabla f(x) = Mx - c$$

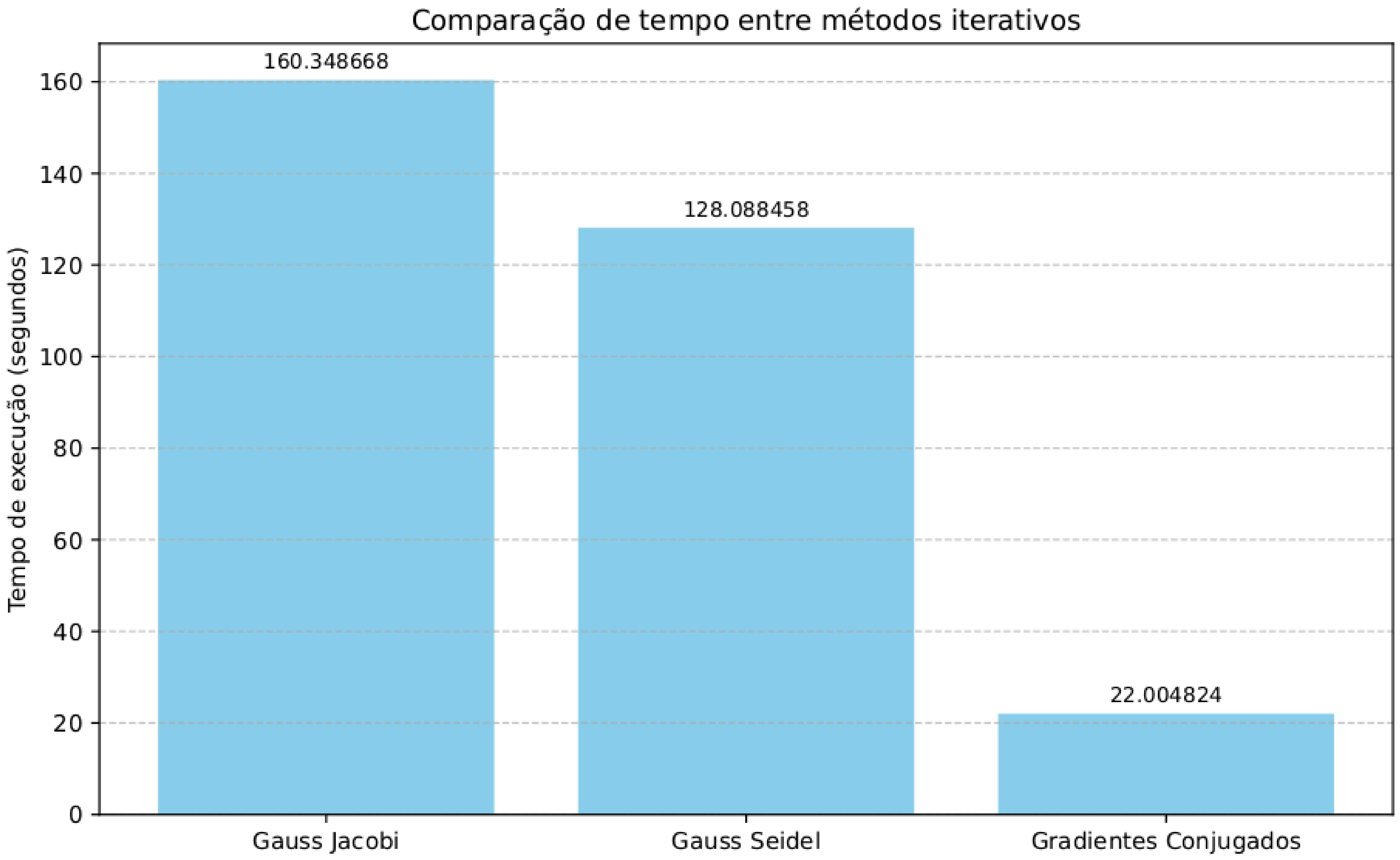
1. Chute inicial x_0;
2. $p_0 = r_0 = c - Mx_0$;
3. $\alpha = \dfrac{r_k^T \cdot r_k}{p_k^T \cdot M \cdot p_k}$;
4. $x_{k+1} = x_k + \alpha p_k$;
5. $r_{k+1} = r_k - \alpha(M \cdot p_k)$;
6. $\beta = \dfrac{r_{k+1}^T \cdot r_{k+1}}{r_k^T \cdot r_k}$;
7. $p_{k+1} = r_{k+1} + \beta p_k$;
8. Volta para o passo 3, enquanto $\|r_{k+1}\| > \text{tol}$.

Métodos Iterativos - Tempos

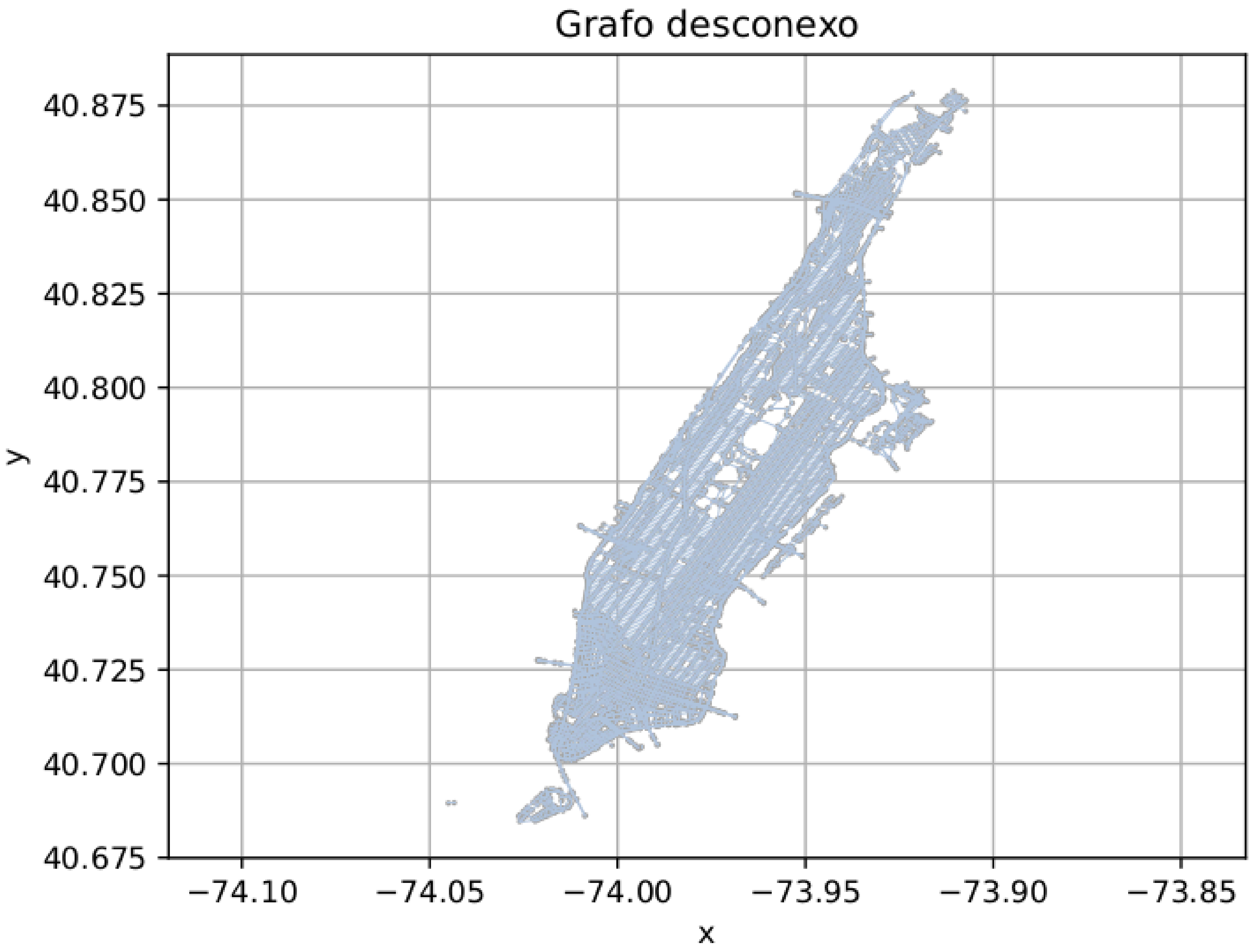
Tolerância: $\pm 10^{-3}$.

	Gauss-Jacobi	Gauss-Seidel	Gradientes Conjugados
k	1280	657	255
tempo	$3min$	$2min$	$30s$

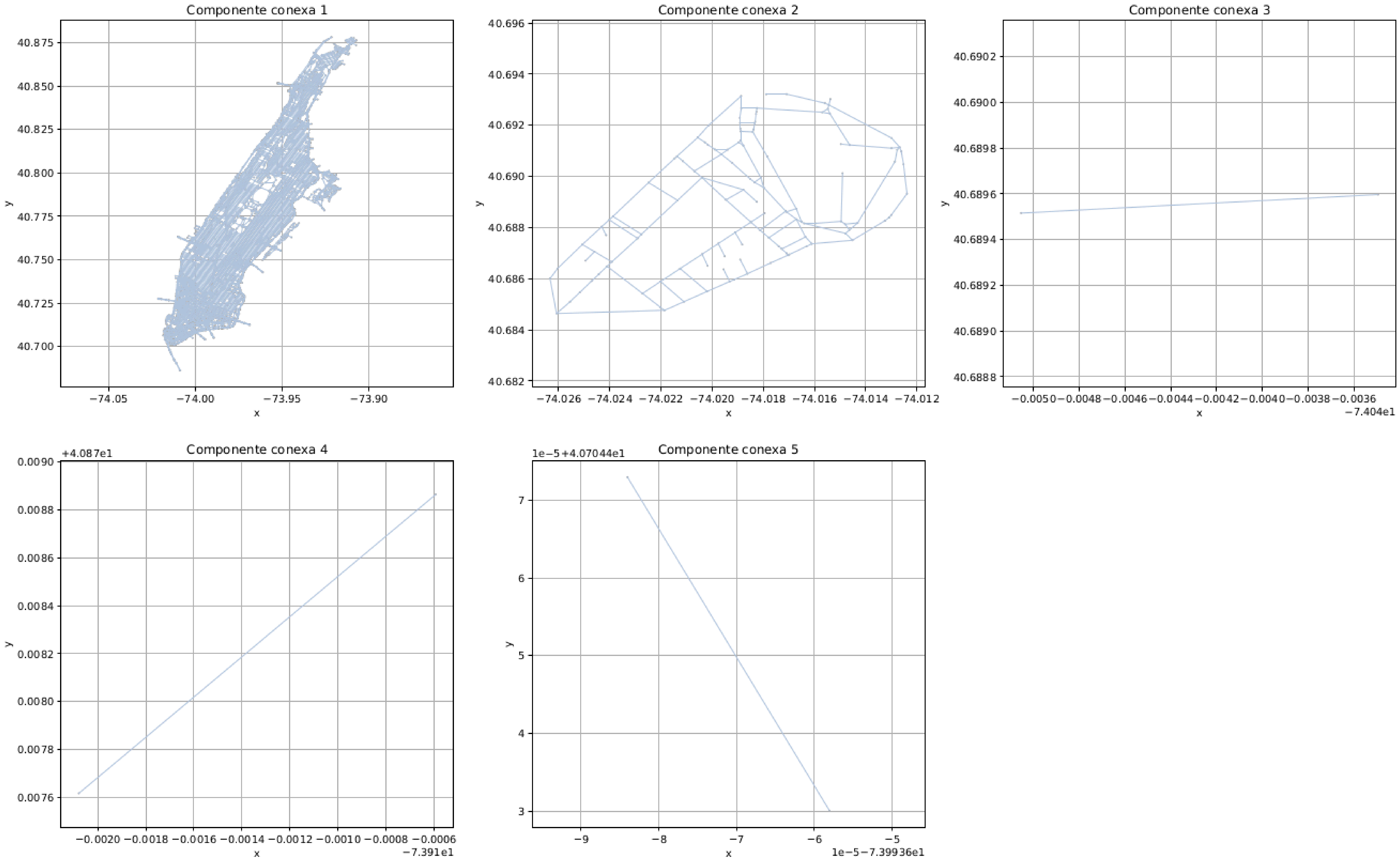
Métodos Iterativos - Tempos



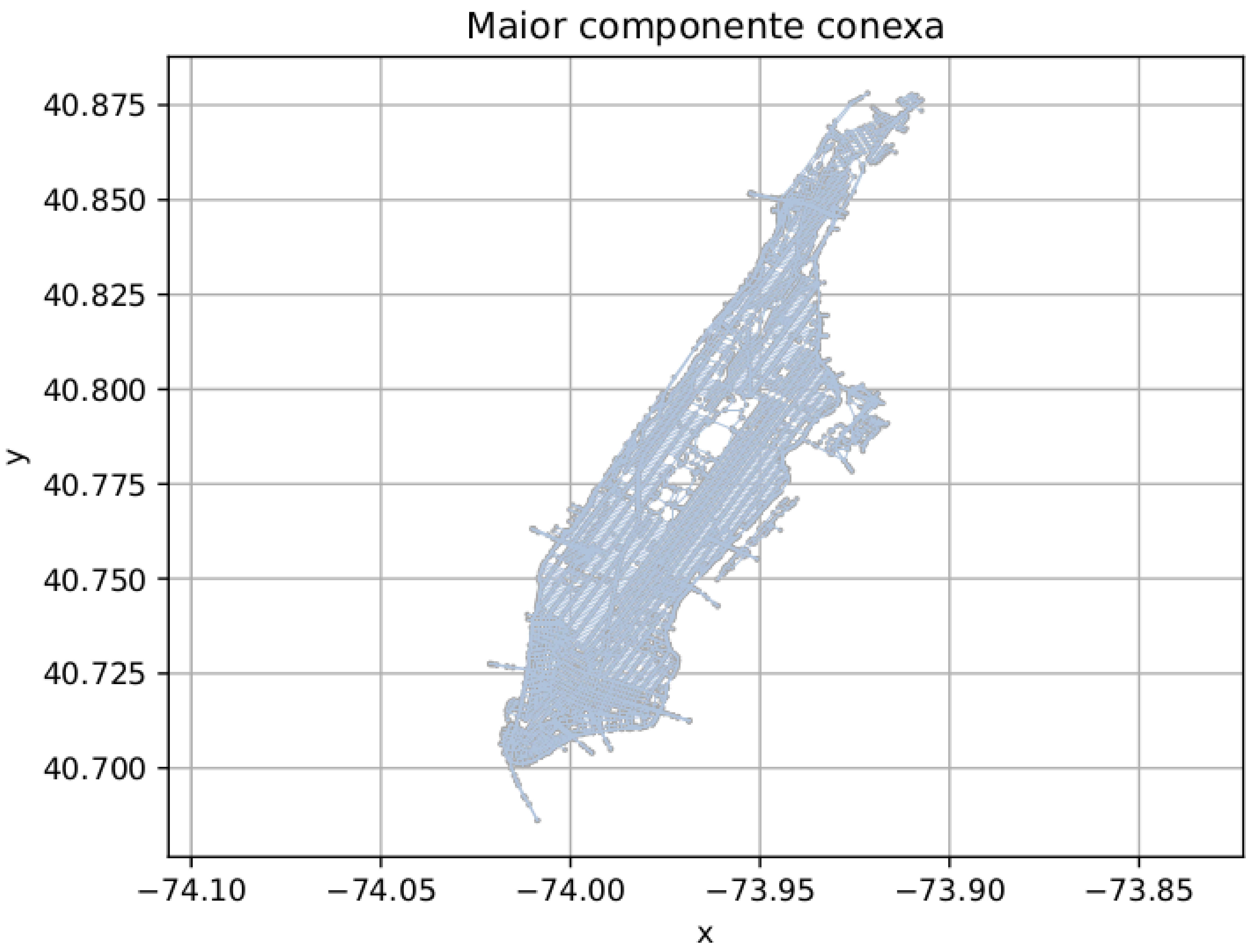
Resultados


Grafo desconexo
40.875
40.850
40.825
40.800
40.775
40.750
40.725
40.700
40.675
y
−74.10
−74.05
−74.00
−73.95
−73.90
−73.85
x

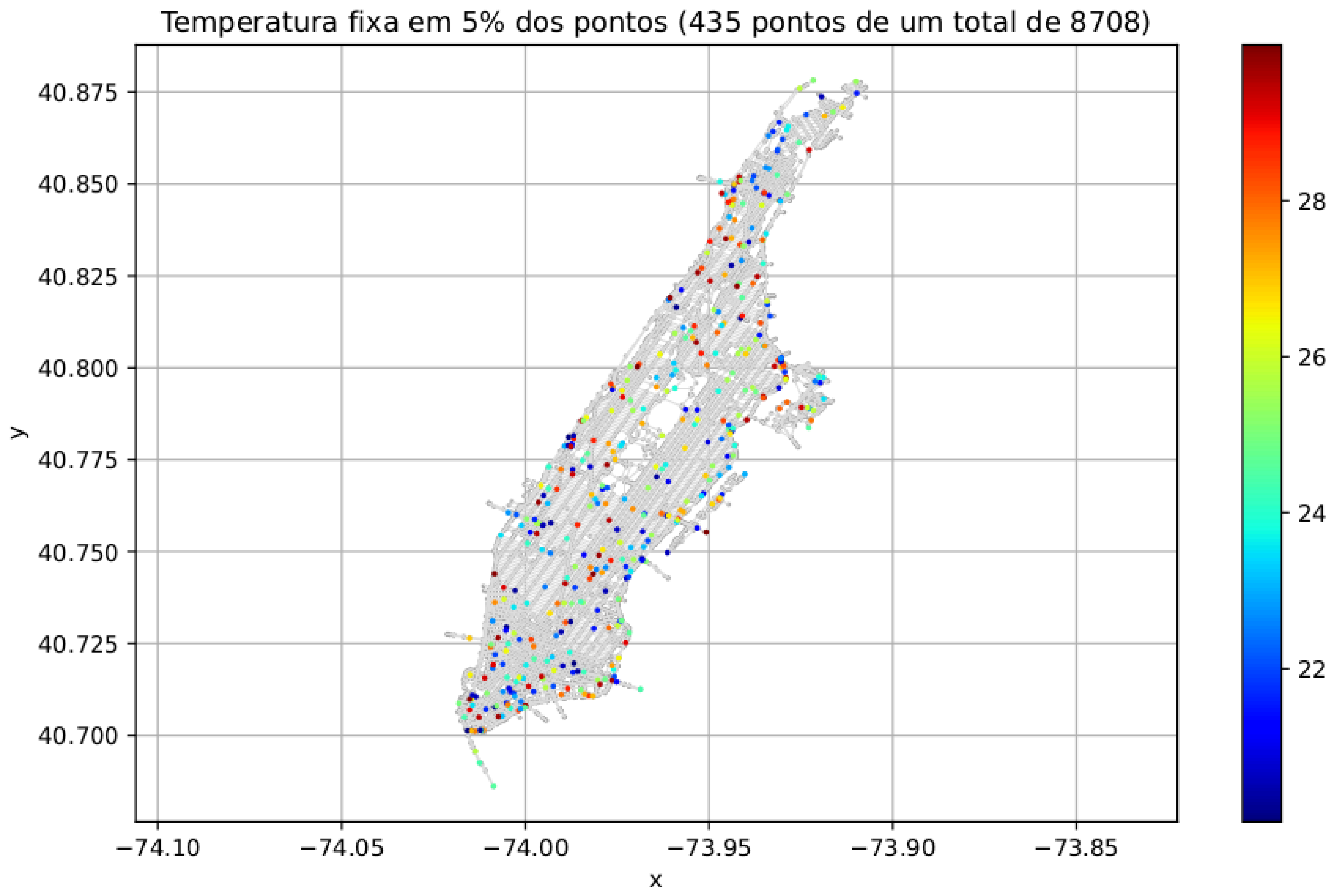
Resultados


Componente conexa 1
40.875
40.850
40.825
40.800
40.775
40.750
40.725
40.700
y
−74.05 −74.00 −73.95 −73.90
x
Componente conexa 2
40.696
40.694
40.692
40.690
40.688
40.686
40.684
40.682
y
−74.026 −74.024 −74.022 −74.020 −74.018 −74.016 −74.014 −74.012
x
Componente conexa 3
40.6902
40.6900
40.6898
40.6896
40.6894
40.6892
40.6890
40.6888
y
−0.0050 −0.0048 −0.0046 −0.0044 −0.0042 −0.0040 −0.0038 −0.0036
x
−7.404e1
Componente conexa 4
+4.087e1
0.0090
0.0088
0.0086
0.0084
0.0082
0.0080
0.0078
0.0076
y
−0.0020 −0.0018 −0.0016 −0.0014 −0.0012 −0.0010 −0.0008 −0.0006
x
−7.391e1
Componente conexa 5
1e−5+4.07044e1
7
6
5
4
3
y
−9 −8 −7 −6 −5
x
1e−5−7.39936e1

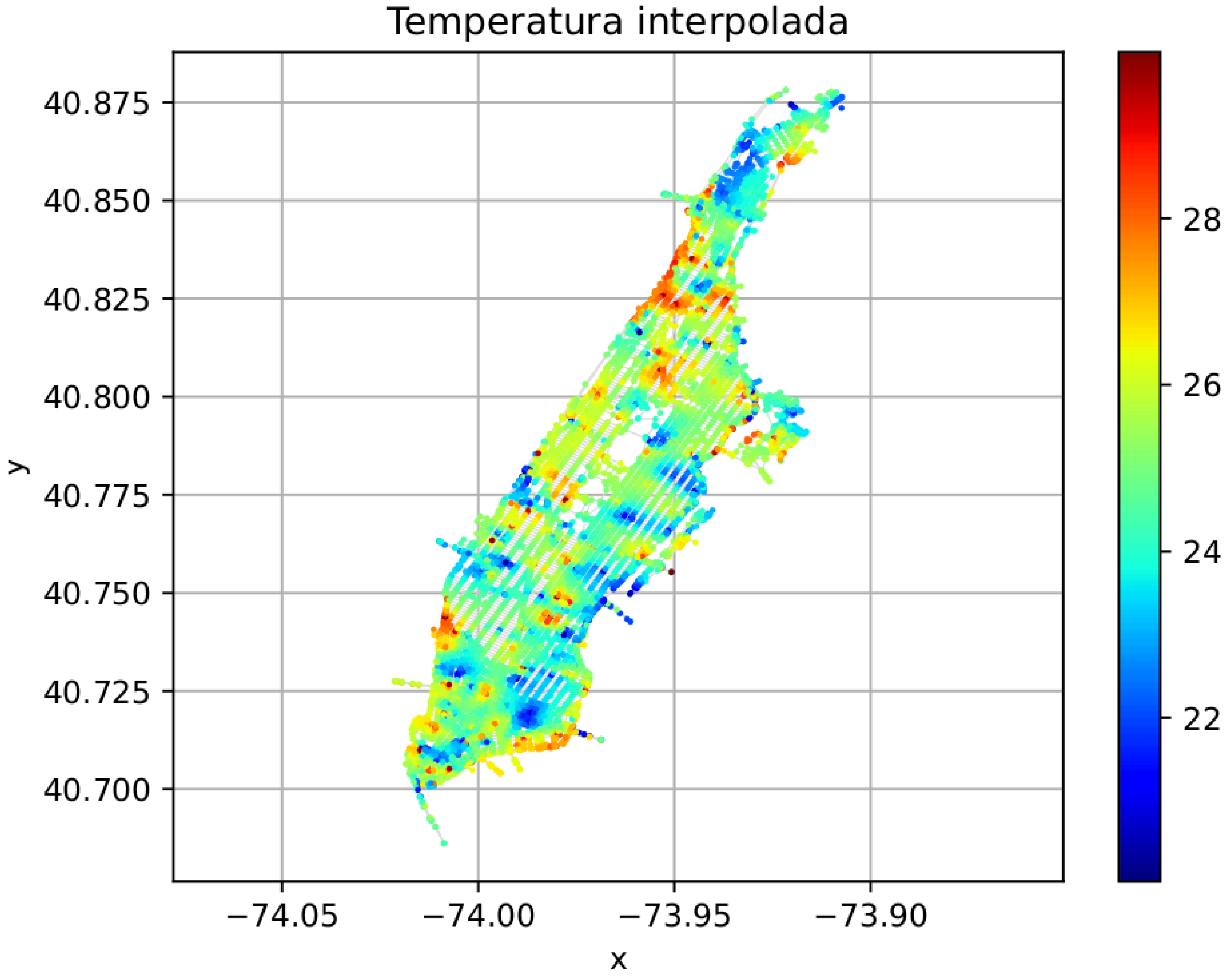
Resultados


Maior componente conexa
40.875
40.850
40.825
40.800
40.775
40.750
40.725
40.700
y
−74.10
−74.05
−74.00
−73.95
−73.90
−73.85
x

Resultados

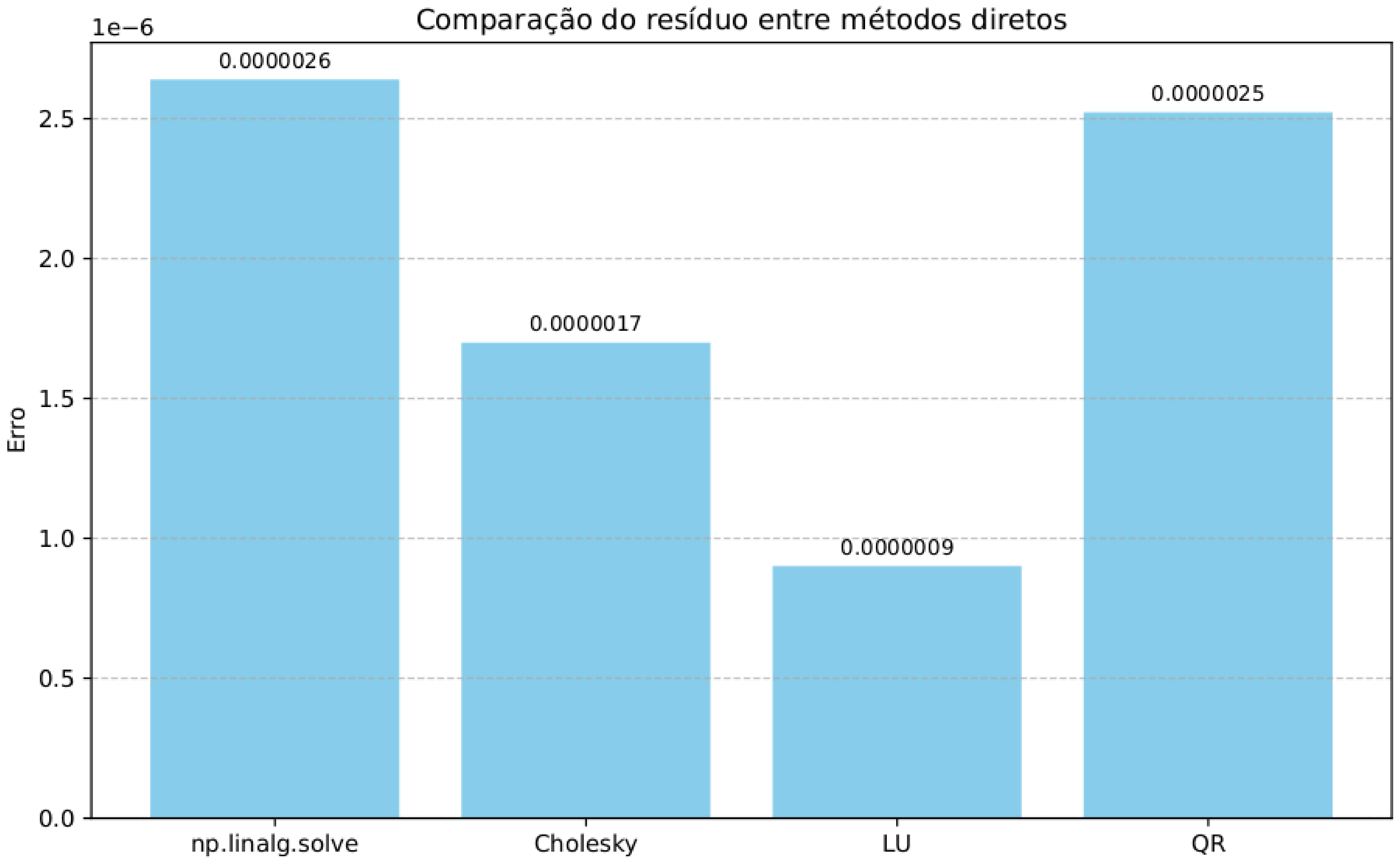


Temperatura fixa em 5% dos pontos (435 pontos de um total de 8708)
y
x
40.875
40.850
40.825
40.800
40.775
40.750
40.725
40.700
−74.10
−74.05
−74.00
−73.95
−73.90
−73.85
28
26
24
22

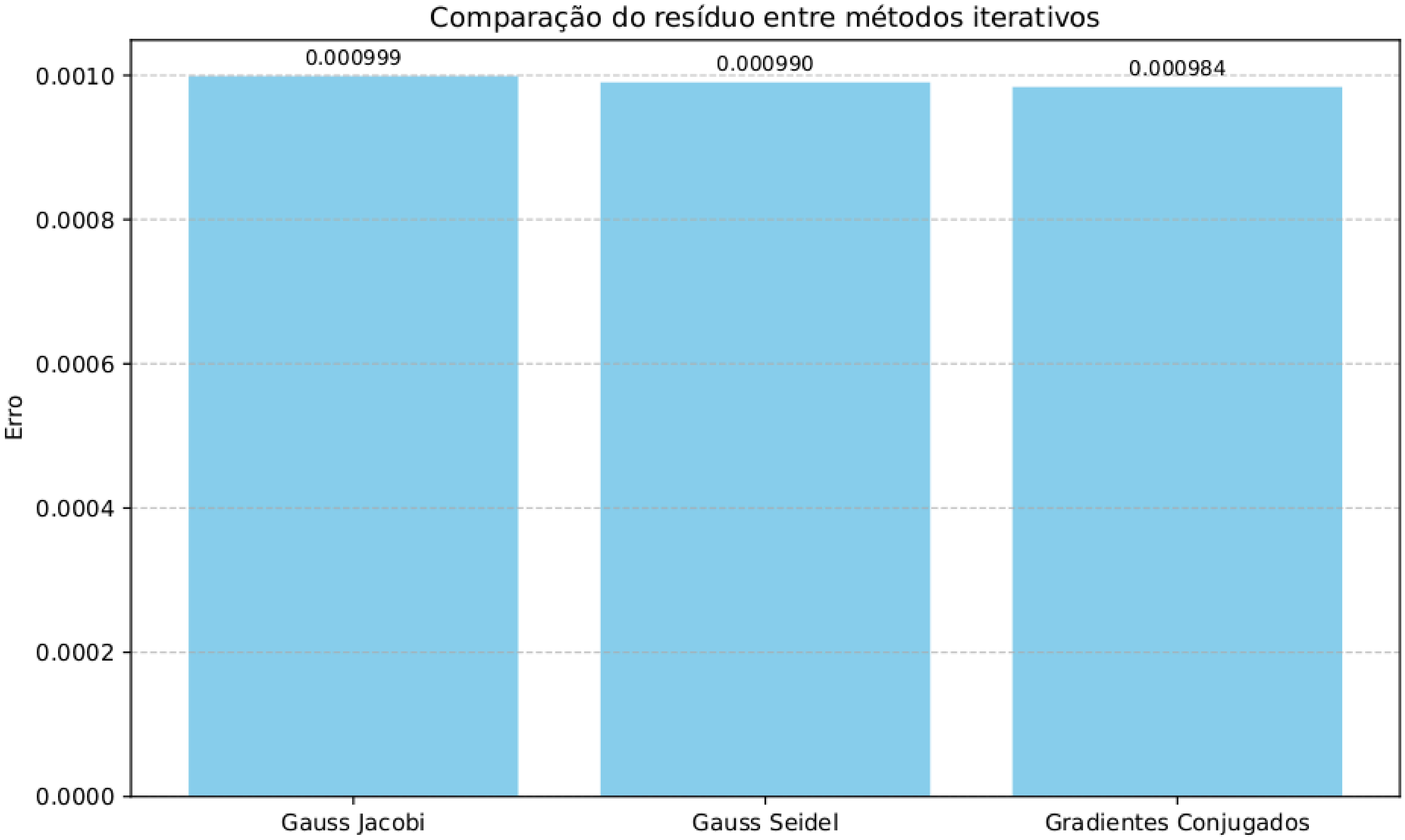


Temperatura interpolada
y
x
40.875
40.850
40.825
40.800
40.775
40.750
40.725
40.700
−74.05
−74.00
−73.95
−73.90
28
26
24
22

Resultados


Comparação do resíduo entre métodos diretos
1e−6
0.0000026
0.0000017
0.0000009
0.0000025
2.5
2.0
1.5
1.0
0.5
0.0
Erro
np.linalg.solve
Cholesky
LU
QR

Resultados



Comparação do resíduo entre métodos iterativos
0.000999
0.000990
0.000984
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
Erro
Gauss Jacobi
Gauss Seidel
Gradientes Conjugados

FIM