Cubic

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$
$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$q(0) = a_0 = q_1(0)$$
$$\dot{q}(0) = a_1 = 0$$
$$\cancel{q(t) =}$$
$$q(T) = q_1(0) + a_2 T^2 + a_3 T^3 \quad - (1)$$
$$0 = \dot{q}(T) = 2a_2 T + 3a_3 T^2 \quad - (2)$$

$$\frac{2q(T)}{T} = \frac{2q_1(0)}{T} + \cancel{2a_2 T} + 2a_3 T^2$$
$$-\ 0 = -\ \cancel{2a_2 T} \pm 3a_3 T^2$$
$$\frac{2}{T}[q(T) - q(0)] = -a_3 T^2$$
$$\boxed{a_3 = \frac{-2}{T^3}(q(T) - q(0))}$$

$$\cancel{a_2(T)} \quad 2a_2 \cancel{T} = -3a_3 T^{\cancel{2}}$$
$$a_2 = \frac{-3}{\cancel{2}} \times \frac{-\cancel{2}}{T^{\cancel{3}2}}(q(T) - q(0))\cancel{T}$$
$$\boxed{a_2 = \frac{3(q(T) - q(0))}{T^2}}$$

Quintic Polynomial

$$\cancel{q(t) = q_0 + q_1 t + q_2 t^2 + q_3 t^3}$$
$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$
$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$
$$\ddot{q}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3$$

$$a_0 = q(0) \quad a_1 = 0 = \dot{q}(0) \quad a_2 = \ddot{q}(0) = 0$$
$$q(T) = q(0) + a_1 T + a_2 T^2 + a_3 T^3 + a_4 T^4 + a_5 T^5$$
$$q(T) = a_3 T^3 + a_4 T^4 + a_5 T^5 \quad - (1)$$
$$0 = \dot{q}(T) = \underset{0}{\cancel{2a_2 T}} + 3a_3 T^2 + 4a_4 T^3 + 5a_5 T^4 \quad (2)$$

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$$0 = 6a_3T + 12a_4T^2 + 20a_5T^3 \quad (3)$$

$$\frac{q(T)-q(0)}{T^2} = a_3T + a_4T^2 + a_5T^3 \quad (4)$$

$$0 = 3a_3T + 4a_4T^2 + 5a_5T^3 \quad (5)$$

Solving (3), (4), (5)

$$4a_4T^2 + 10a_5T^3 = 0$$

$$6a_4T^2 + 14a_5T^3 = -+\frac{6q(T)}{T^2}$$

$$\cancel{12a_4T^2} + 30a_5T^3 = 0$$

$$\cancel{12a_4T^2} + 28a_5T^3 = \frac{12(q(T)-q(0))}{T^2}$$

$$+2a_5T^3 = \frac{-12}{T^2}[q(T)-q(0)]$$

$$\boxed{a_5 = \frac{+6}{T^5}[q(T)-q(0)]} \quad a_4 = \frac{-\cancel{10}^{5}a_5T^{\cancel{3}}}{2\cancel{4}T^{\cancel{2}}}$$

$$\boxed{a_4 = \frac{-5}{2} \times \cancel{T} \times \frac{\cancel{6}^{3}}{T^{\cancel{5}4}} q(T) = \frac{-15(q(T)-q(0))}{T^4}}$$

$$3a_3T + \frac{60[q(T)-q(0)]}{T^2} + \frac{(-30)}{T^2}[q(T)-q(0)] = 0$$

$$\cancel{3}a_3T = \frac{-\cancel{30}^{10}[q(T)-q(0)]}{T^2}$$

$$\boxed{a_3 = \frac{+10}{T^3}[q(T)-q(0)]}$$

Linear Trajectory

- constant velocity
- but at the beginning the acceleration is ∞ because in actual the value will go from 0 to some constant value and similarly some constant ~~to~~ 0 again acceleration is ∞ again. sudden jerks

Smooth Trajectory

- velocity starts and ends at zero
- acceleration is finite always and ~~..~~ it has real values
- no sudden jerk but jerk has a constant value

Smoothness is important for real robots as this ensures that the real robots do not get damaged by sudden jerks and it works fine and it will not experience vibrations while moving. Maybe there is a possibility that frequency matches with the resonance frequency and vibrations get intense and system may get damaged.