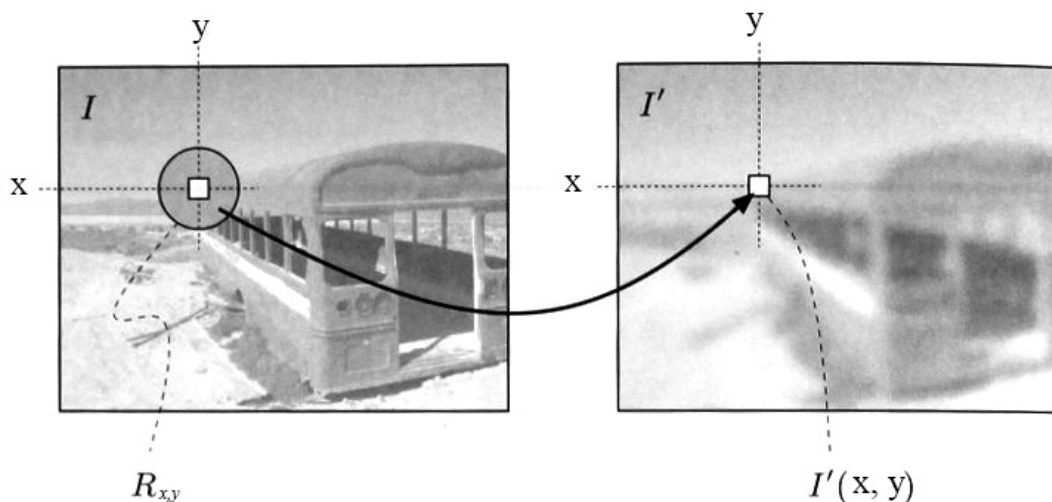# 影像空間濾波

Image Spatial Filtering

台科大色彩與照明科技研究所

色彩與影像實作技術 補充資料

孫沛立 副教授

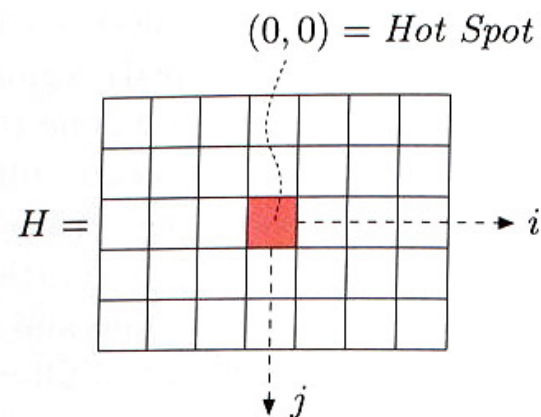# 空間濾波 Spatial filtering

FILTERS

Fig. 6.2
Principal filter operation. Each new pixel value $I'(x, y)$ is computed as a function of the pixels in a corresponding region of source pixels $R_{x,y}$ in the original image I.

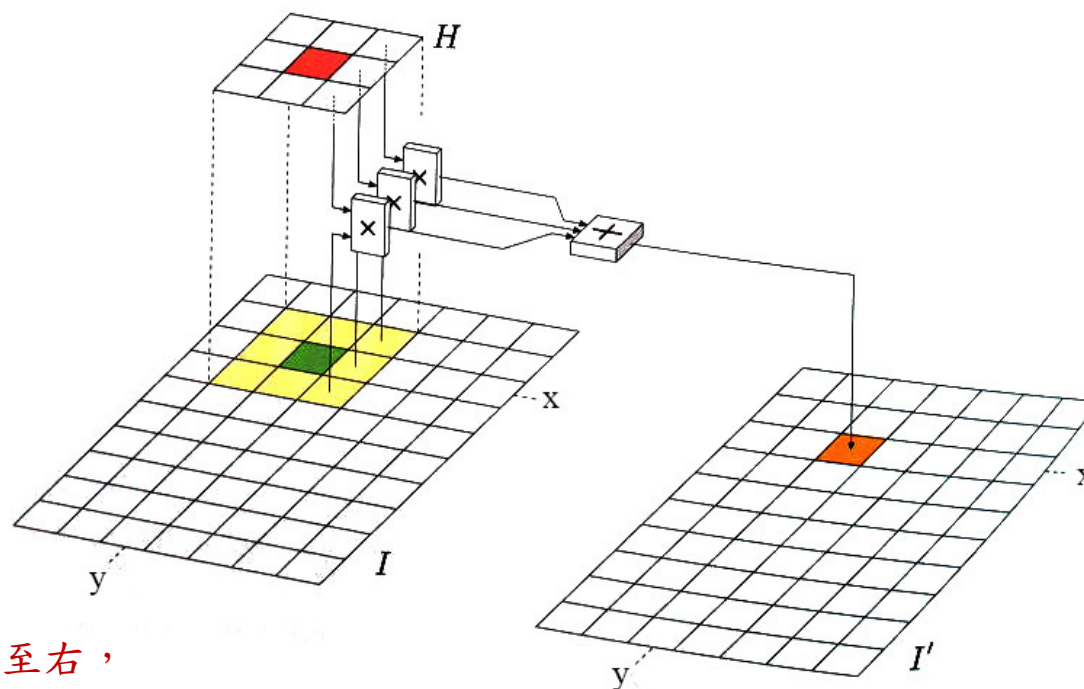

6 FILTERS

Fig. 6.3
Filter matrix and coordinate system.

1. 以Hot Spot為中心，由左至右，由上至下移動濾鏡矩陣。
2. 將濾鏡的數值乘上對應的影像灰階值。
3. 將數值總和填入輸出影像 Hot Spot 的位置。
4. 重覆步驟1~3至輸出影像被填滿。
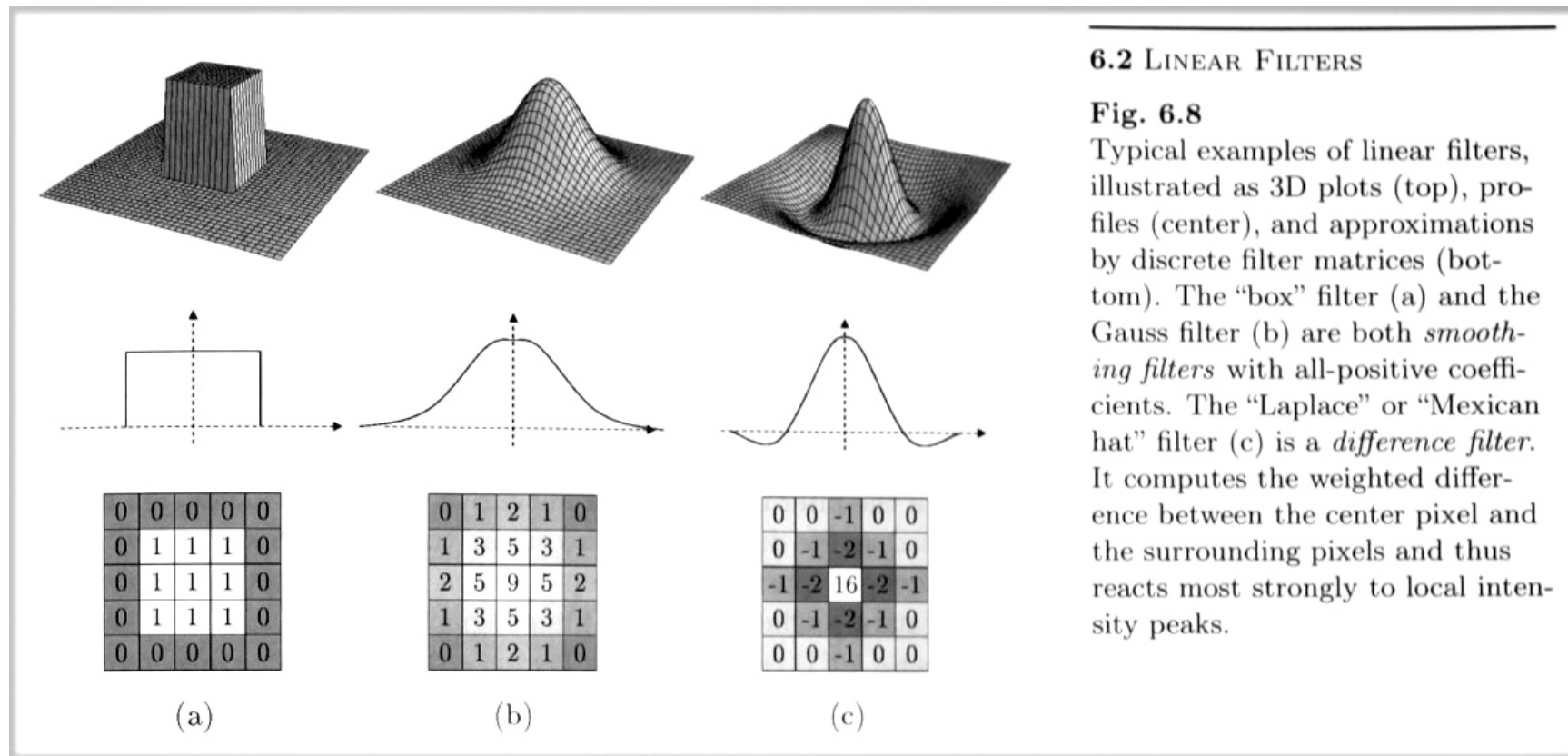
線性空間濾波 Linear Spatial filtering

$$I'(\mathrm{x}, \mathrm{y}) \leftarrow \sum_{(i,j) \in R_H} I(\mathrm{x}+i, \mathrm{y}+j) \cdot H(i,j), \qquad (6.5)$$

where R_H denotes the set of coordinates covered by the filter H. For a typical 3×3 filter with centered origin, this is

Fig. 6.4
Linear filter. The filter matrix H is placed with its origin at position (x, y) on the image I. Each filter coefficient $H(i,j)$ is multiplied with the corresponding image pixel $I(\mathrm{x}+i, \mathrm{y}+j)$, the results are added, and the final sum is inserted as the new pixel value $I'(\mathrm{x}, \mathrm{y})$.
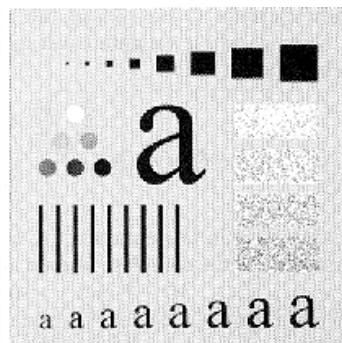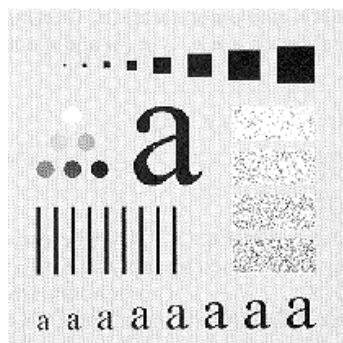
1. 以Hot Spot為中心，由左至右，
   由上至下移動濾鏡矩陣。
2. 將濾鏡的數值乘上對應的影像灰階值。
3. 將數值總和填入輸出影像 Hot Spot 的位置。
4. 重覆步驟1~3至輸出影像被填滿。

6.2 LINEAR FILTERS

Fig. 6.8
Typical examples of linear filters, illustrated as 3D plots (top), profiles (center), and approximations by discrete filter matrices (bottom). The "box" filter (a) and the Gauss filter (b) are both *smoothing filters* with all-positive coefficients. The "Laplace" or "Mexican hat" filter (c) is a *difference filter*. It computes the weighted difference between the center pixel and the surrounding pixels and thus reacts most strongly to local intensity peaks.

(a)   (b)   (c)

用平均值濾鏡(mean filter)作影像模糊化處理

原稿
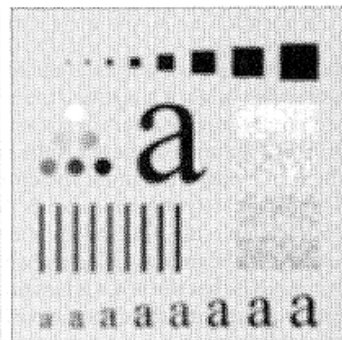
3x3
平均值濾鏡

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

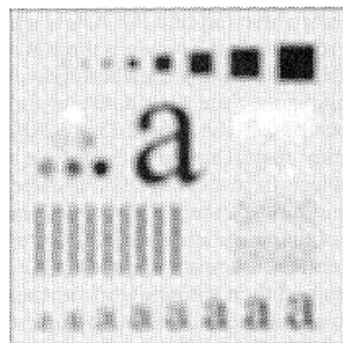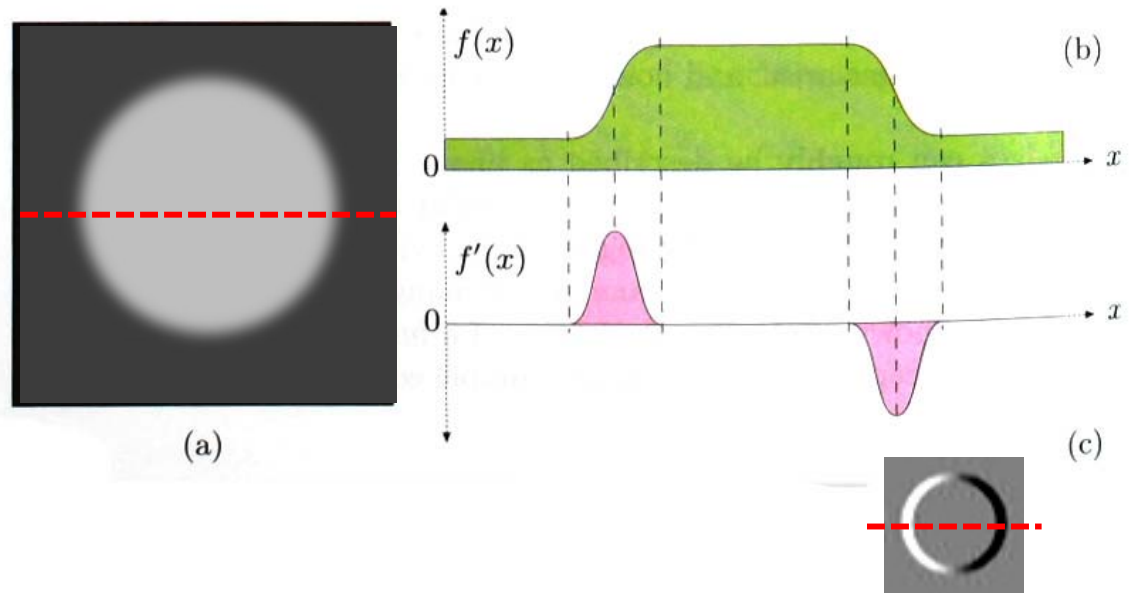$$\frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

5x5
平均值濾鏡
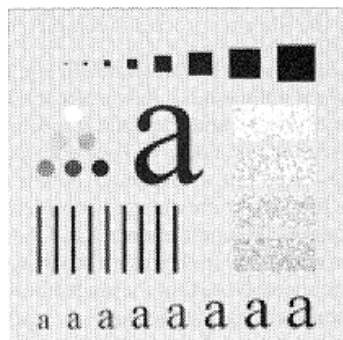
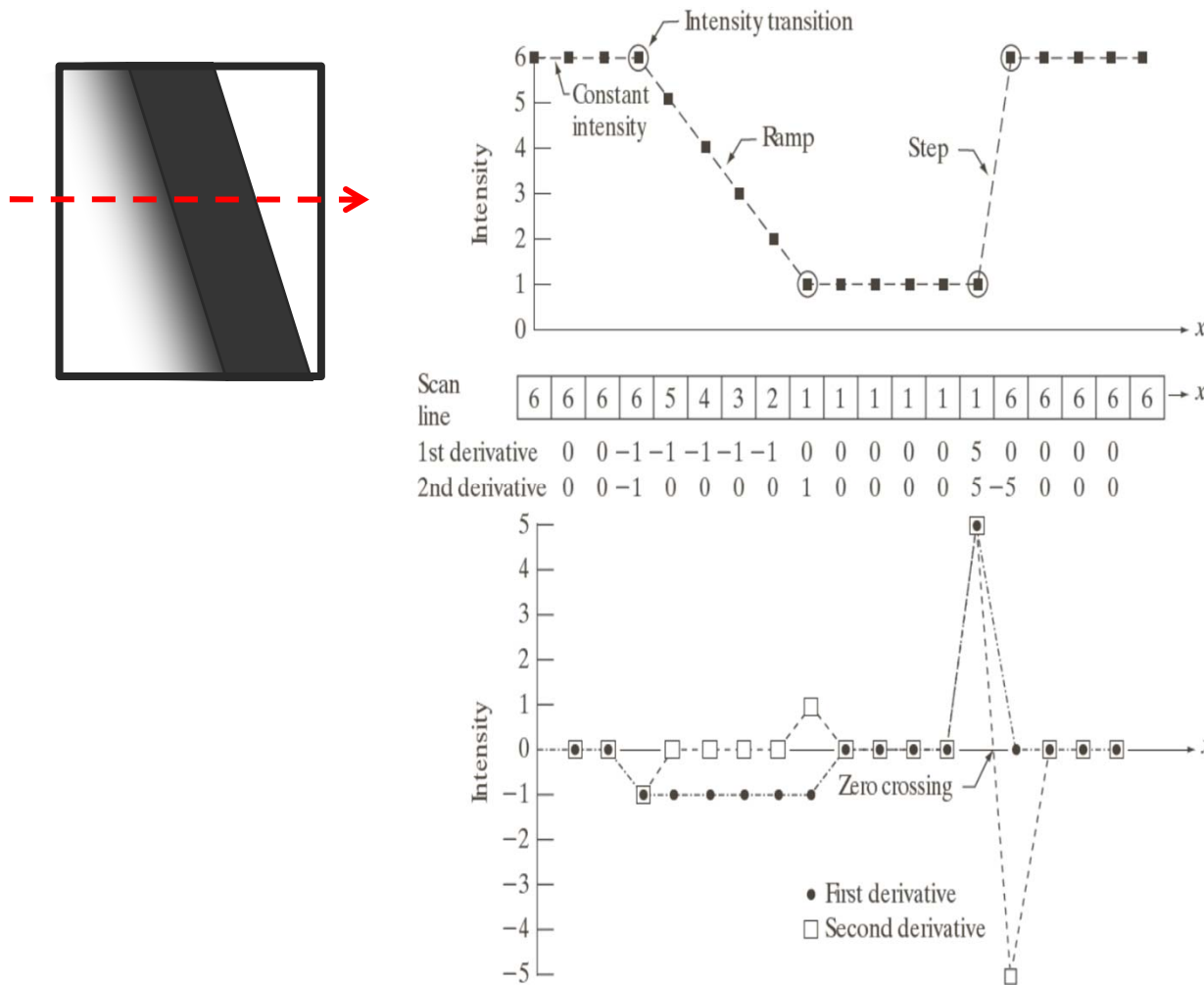7x7
平均值濾鏡

9x9
平均值濾鏡

11x11
平均值濾鏡

影像的梯度與一階導數

Fig. 7.2
Sample image and first derivative
in one dimension: original image
(a), horizontal intensity profile
$f(x)$ along the center image line
(b), and first derivative $f'(x)$ (c).

影像梯度(gradient)：像素單方向灰階值的落差
相當於影像的一階導數

影像梯度(Gradient)

$$f'(x) = f(x+1) - f(x)$$

-1	+1

-1	0	+1

Fig. 7.4
Partial derivatives of a two-dimensional function: synthetic image function I (a); approximate first derivatives in the horizontal direction $\partial I/\partial u$ (b) and the vertical direction $\partial I/\partial v$ (c); magnitude of the resulting gradient $|\nabla I|$ (d). In (b) and (c), the lowest (negative) values are shown black, the maximum (positive) values are white, and zero values are gray.

(a)

(b)

(c)

(d)
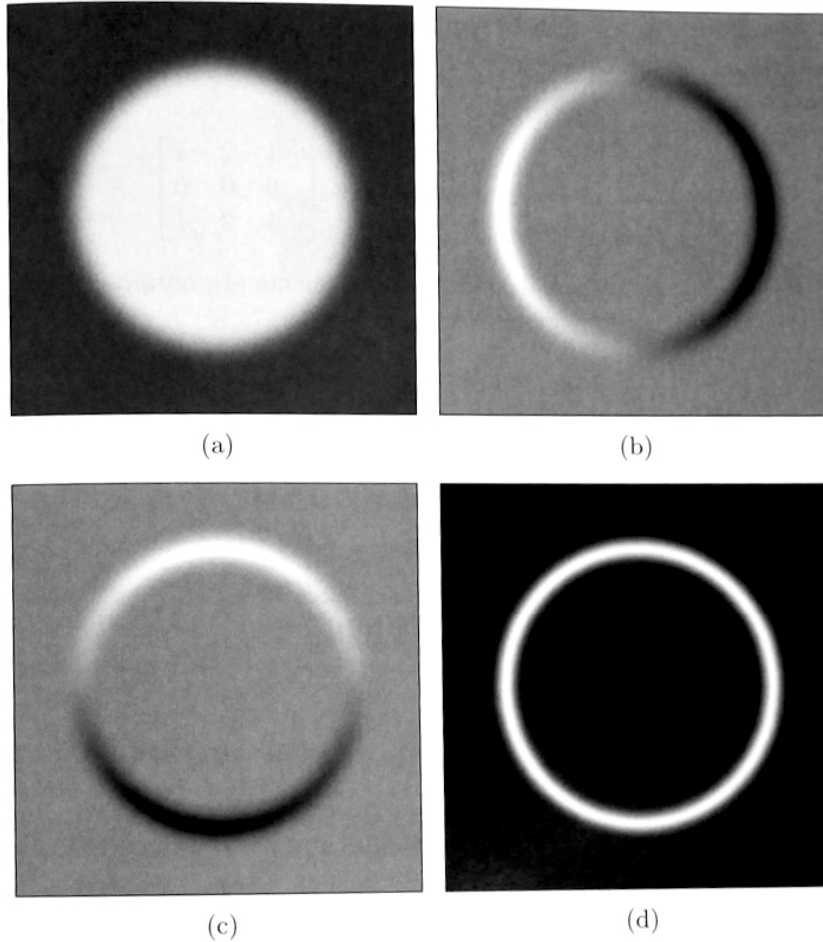
$$H_x^P = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad H_y^P = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad (7.8)$$
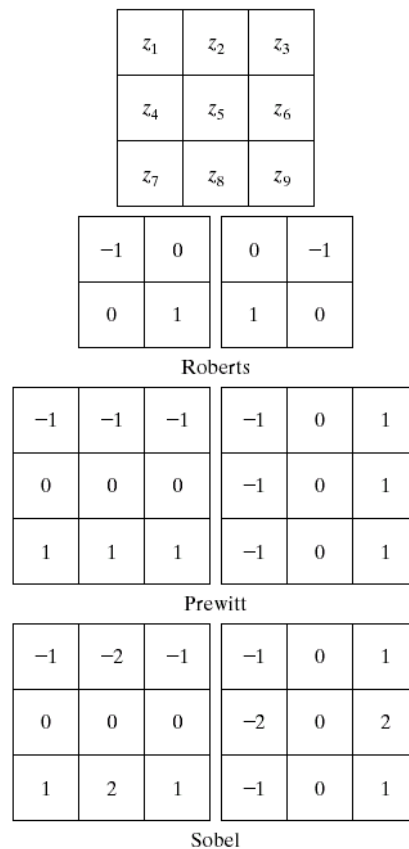
$$H_x^P = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad H_y^P = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad (7.9)$$
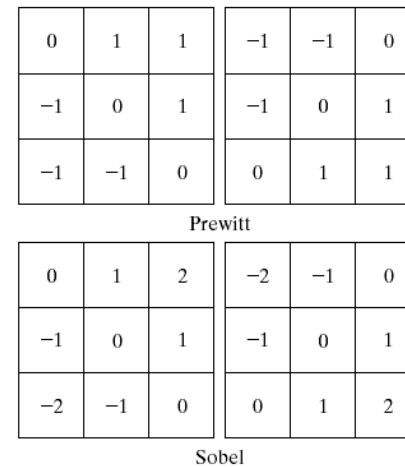
Edge detectors

a		
b	c	
d	e	
f	g	

FIGURE 10.8
A 3×3 region of an image (the z's are gray-level values) and various masks used to compute the gradient at point labeled z_5.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

Roberts

−1	−1	−1
0	0	0
1	1	1

−1	0	1
−1	0	1
−1	0	1

Prewitt

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel

0	1	1
−1	0	1
−1	−1	0

−1	−1	0
−1	0	1
0	1	1

Prewitt

0	1	2
−1	0	1
−2	−1	0

−2	−1	0
−1	0	1
0	1	2

Sobel

a	b
c	d

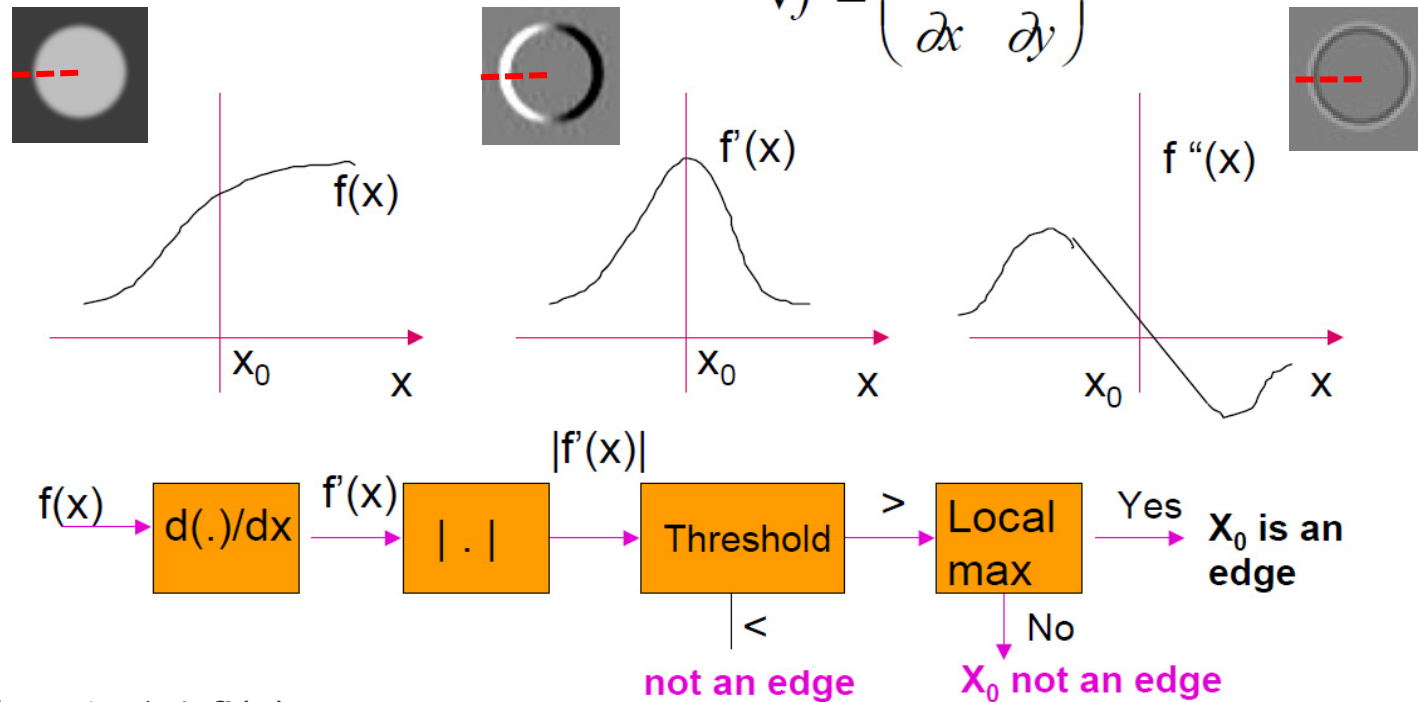**FIGURE 10.9** Prewitt and Sobel masks for detecting diagonal edges.

Gradient based methods

$$\nabla f = \left(\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y} \right)^{T}$$

1. 計算影像梯度f'(x)。
2. 取絕對值|f'(x)|。
3. 邊界線：|f'(x)|灰階影像二值化。高於門檻值的設為1 (edge)，其餘為0。
4. 邊界點：可取|f'(x)|的區域極大值，或其梯度(二階導數f''(x))的零穿越點，作為邊界點。

Edge detection: Compare 3 filters

Original   Robert   Prewitt

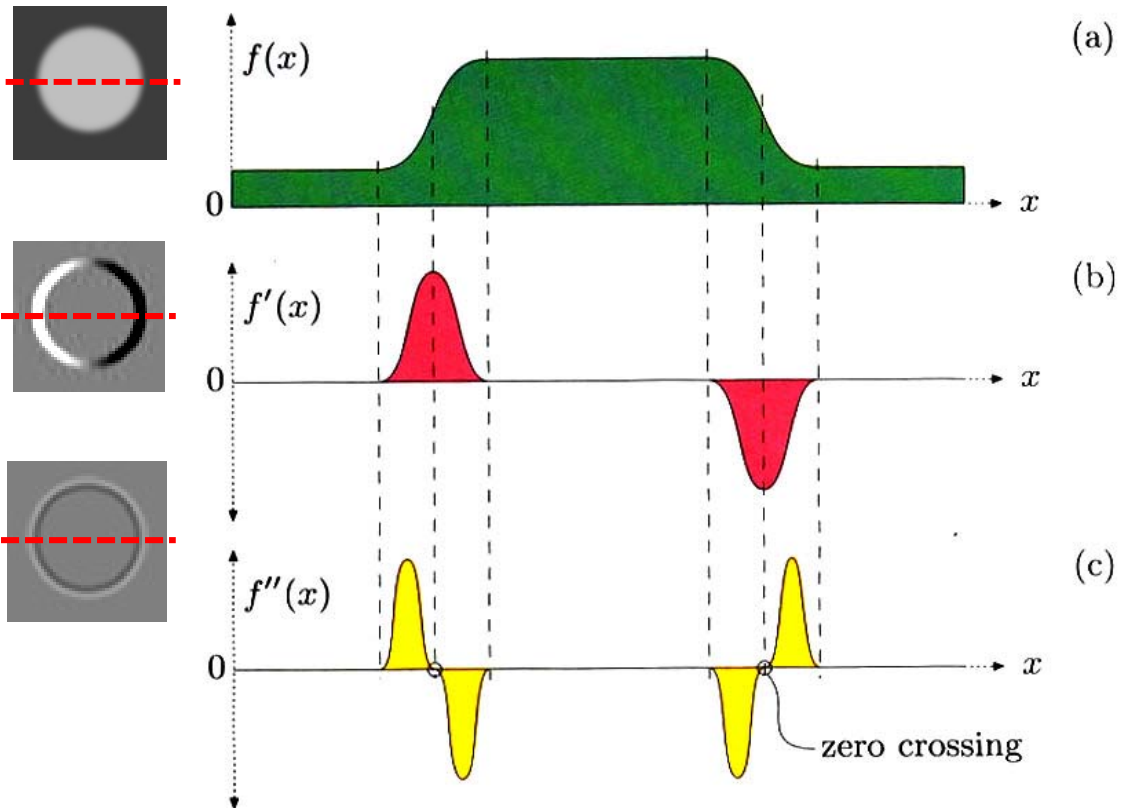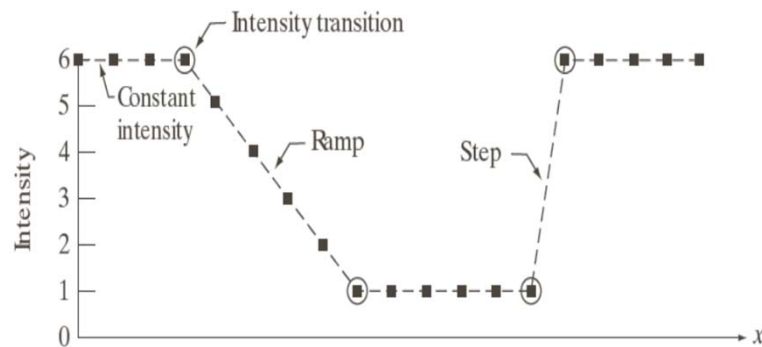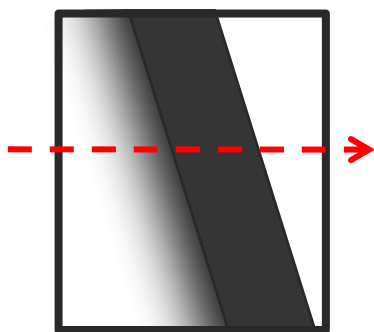Sobel (vertical)   Sobel (horizontal)   Sobel (horizontal) threshold=0.01

First and second derivatives of an edge

Fig. 7.8
Principle of edge detection with the second derivative: original function (a), first derivative (b), and second derivative (c). Edge points are located where the second derivative crosses through zero and the first derivative has a high magnitude.
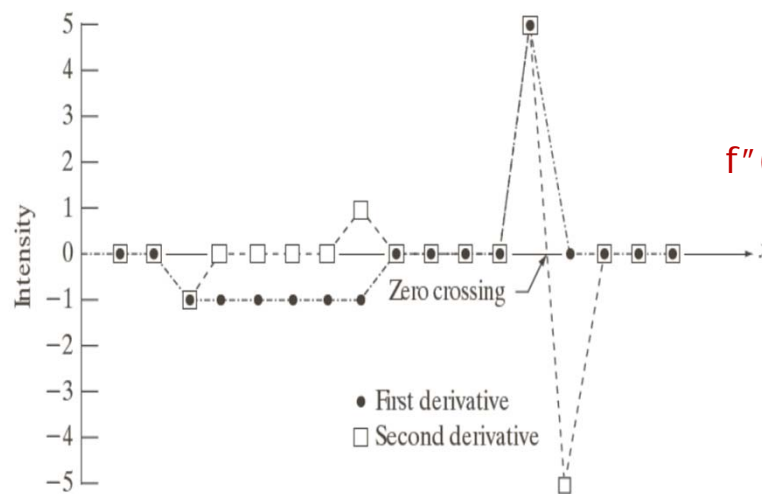
影像的二階導數：
梯度的梯度，用於檢測影像
細紋或小顆粒

影像梯度(Gradient)

$$f'(x) = f(x+1) - f(x)$$

$$f''(x) = f'(x+1) - f'(x)$$
$$= [f(x+1) - f(x)] - [f(x) - f(x-1)]$$
$$= f(x+1) - 2f(x) + f(x-1)$$

| +1 | -2 | +1 |

Laplacian filter（二階導數）

0	1	0	1	1	1
1	−4	1	1	−8	1
0	1	0	1	1	1

0	−1	0	−1	−1	−1
−1	4	−1	−1	8	−1
0	−1	0	−1	−1	−1

濾鏡加權值

位置
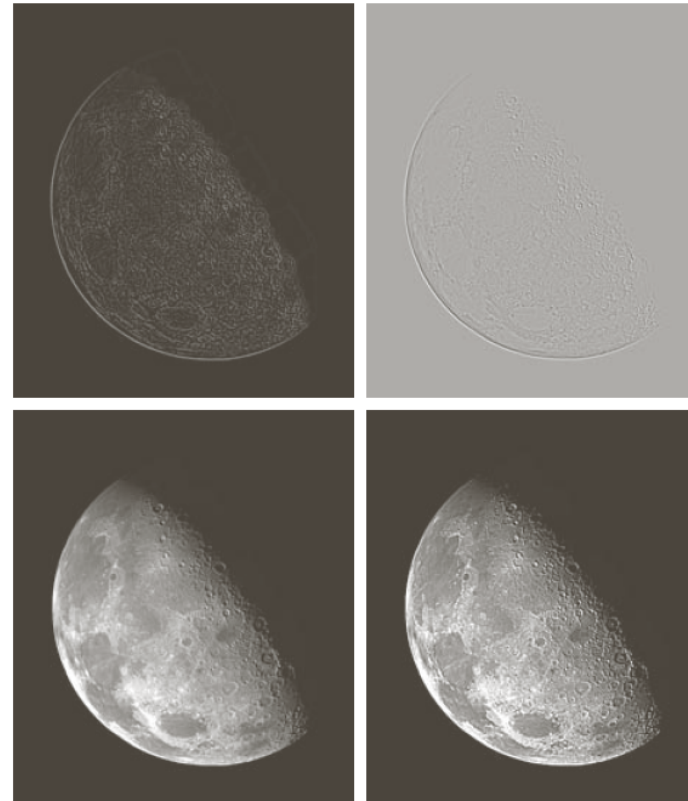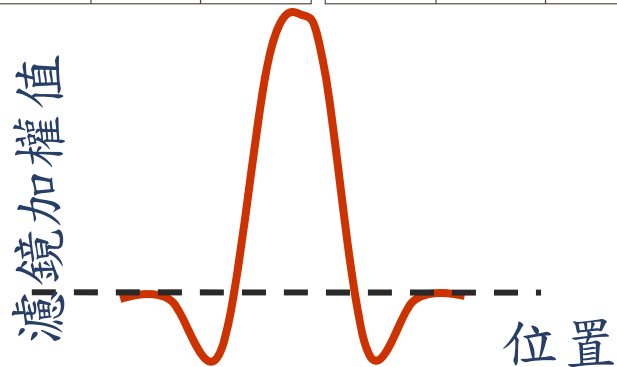
a
b c
d e

FIGURE 3.38
(a) Blurred image of the North Pole of the moon.
(b) Laplacian without scaling.
(c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)
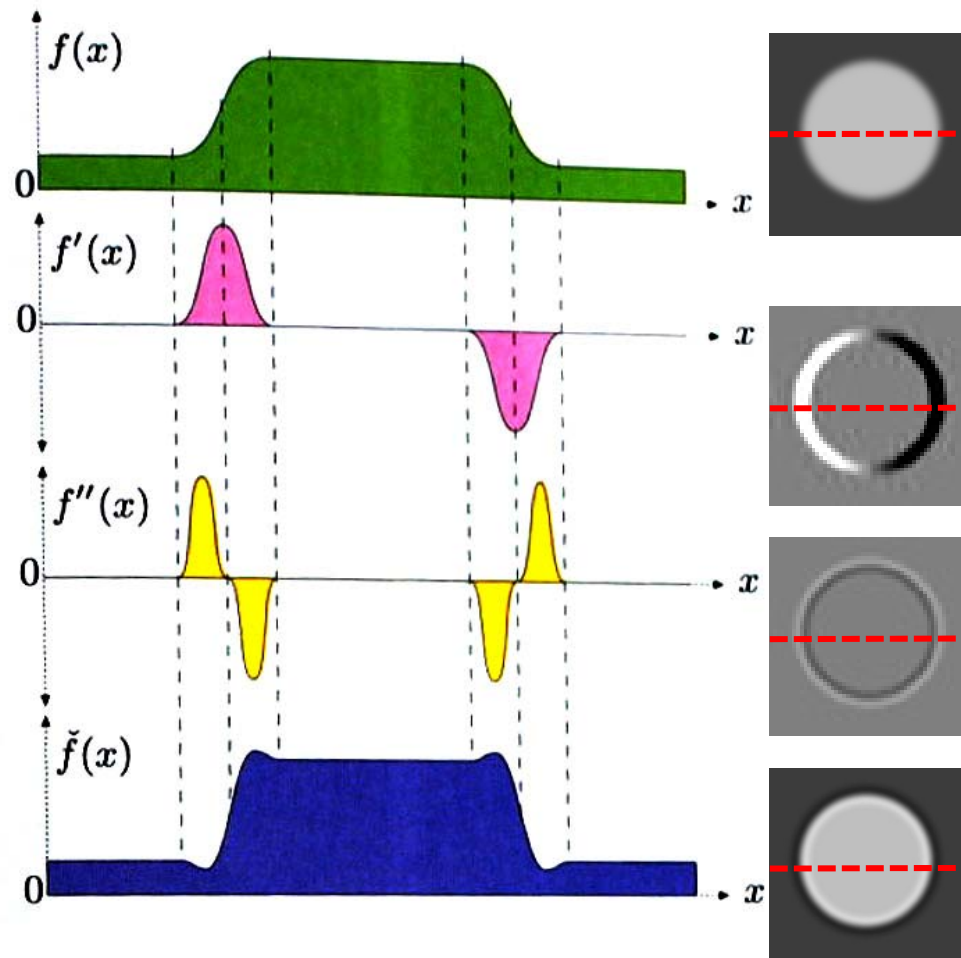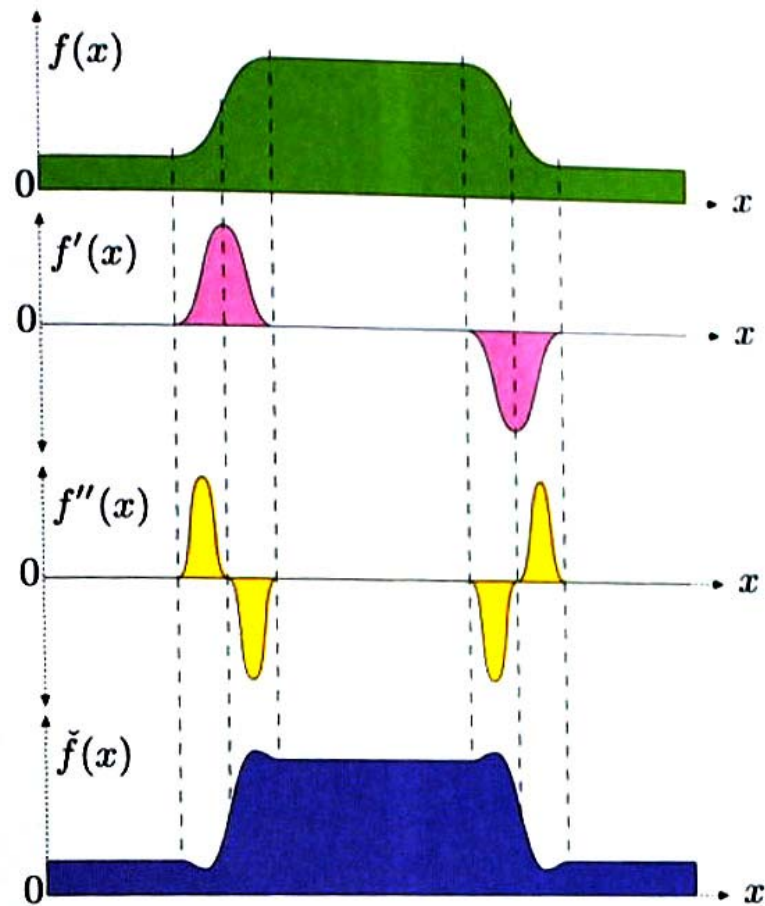
R.C. Gonzalez & R.E. Woods (2007)

7.6 EDGE SHARPENING

Fig. 7.11
Edge sharpening with the second derivative. The original intensity function $f(x)$, first derivative $f'(x)$, second derivative $f''(x)$, and sharpened intensity function $\check{f}(x) = f(x) - w \cdot f''(x)$ are shown.

影像銳化：
將影像減去二階導數紋理，
相當於影像疊加了輪廓細紋，
有銳化效果

7.6 EDGE SHARPENING

Fig. 7.11
Edge sharpening with the second derivative. The original intensity function $f(x)$, first derivative $f'(x)$, second derivative $f''(x)$, and sharpened intensity function $\check{f}(x) = f(x) - w \cdot f''(x)$ are shown.

灰階影像經由下列四種3x3濾波器處理所獲得結果

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

(a)

-1	-2	-1
0	0	0
1	2	1

(b)

0	-1	0
-1	4	-1
0	-1	0

(c)

0	-1	0
-1	5	-1
0	-1	0

(d)

影像模糊化

均值濾波器
Mean filter

輪廓邊界
(線偵測)

一階導數
Sobel
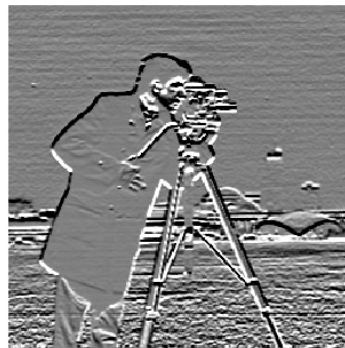
高頻資訊
(點偵測)

二階導數
Laplacian

影像銳利化

原影像
＋
二階導數

形態學影像處理
Morphological Image Processing
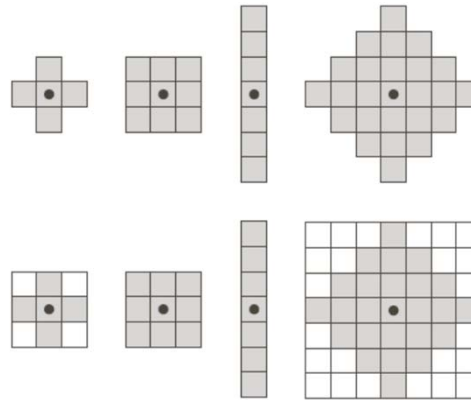
FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.
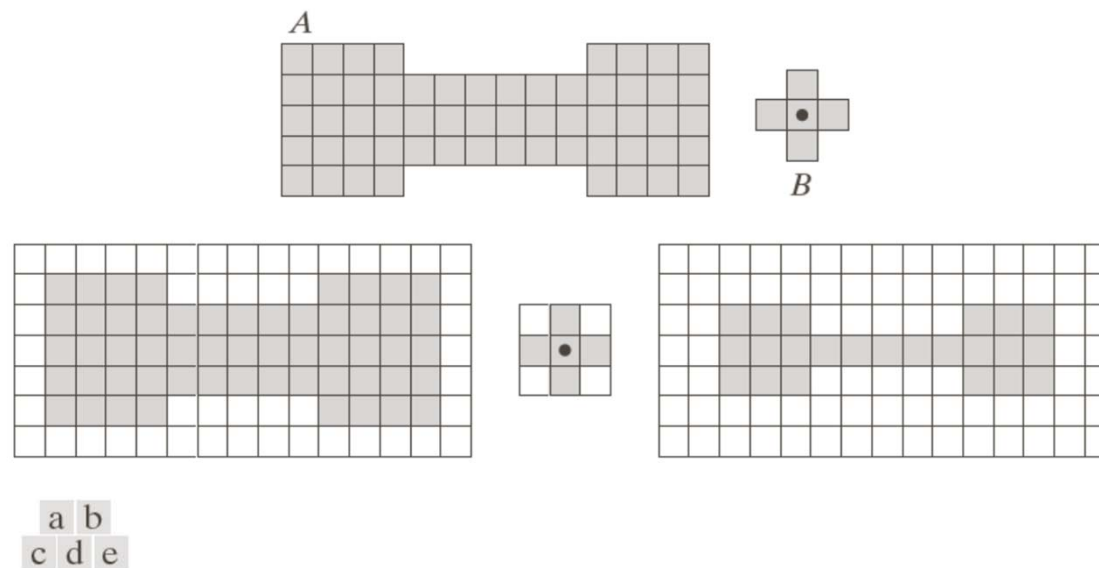
A

B

| a | b | |
| c | d | e |

FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

膨脹(dilation)

移動B, 所有A與B有交集的z位置

a b c
d   e

FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.

$$A \oplus B = \left\{ z \mid \left(\hat{B} \right)_z \cap A \neq \phi \right\}$$
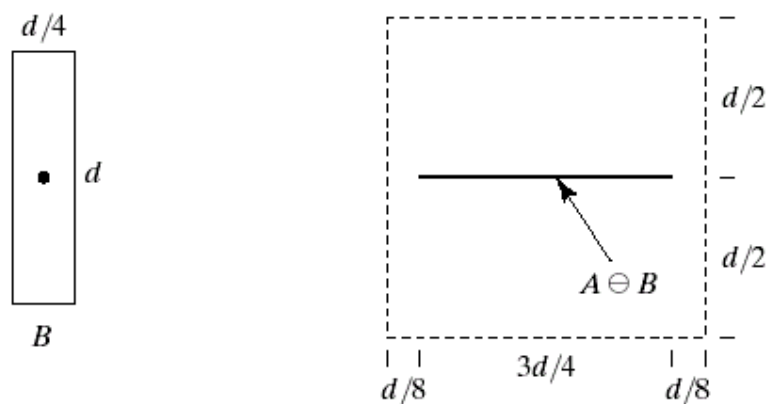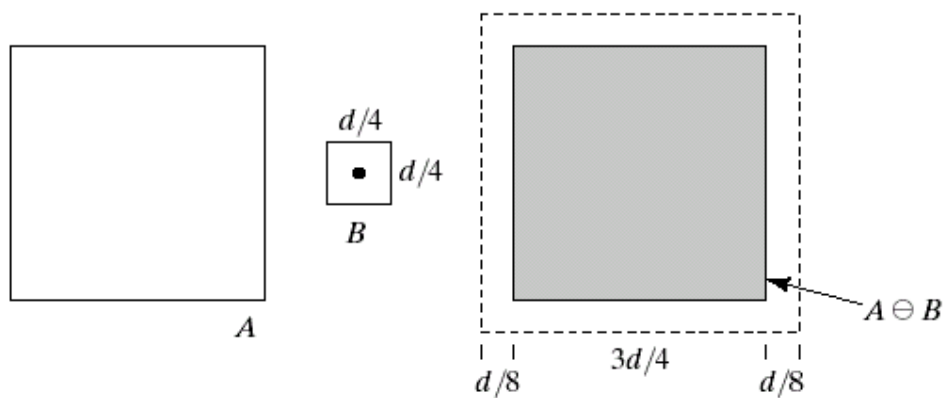$$= \left\{ z \mid \left[\left(\hat{B} \right)_z \cap A \right] \subseteq A \right\}$$

Matlab: imdilate()
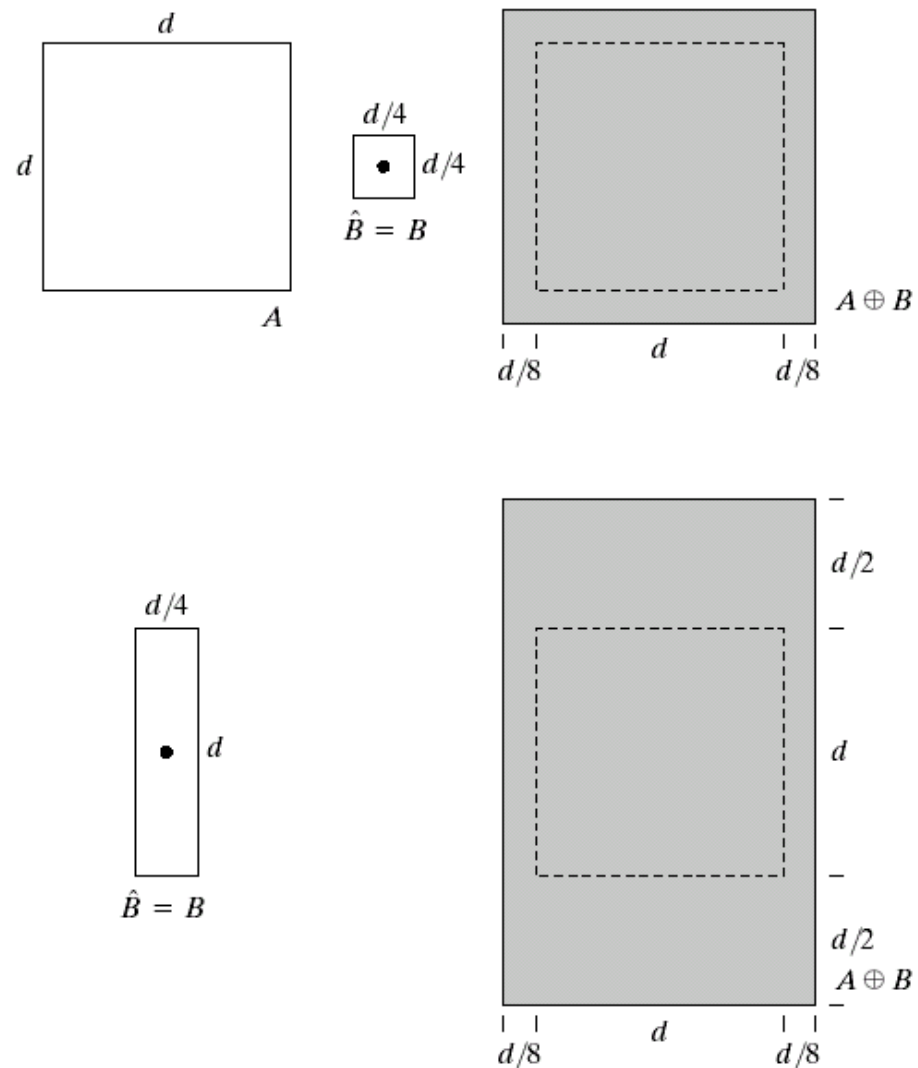
侵蝕(erosion)　　移動B，使B完全沒入A中的所有z位置

$$A \Theta B = \left\{ z \mid (B)_z \subseteq A \right\}$$
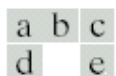$$= \left\{ z \mid (B)_z \cap A^c = \phi \right\}$$

Matlab: imerode()

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.
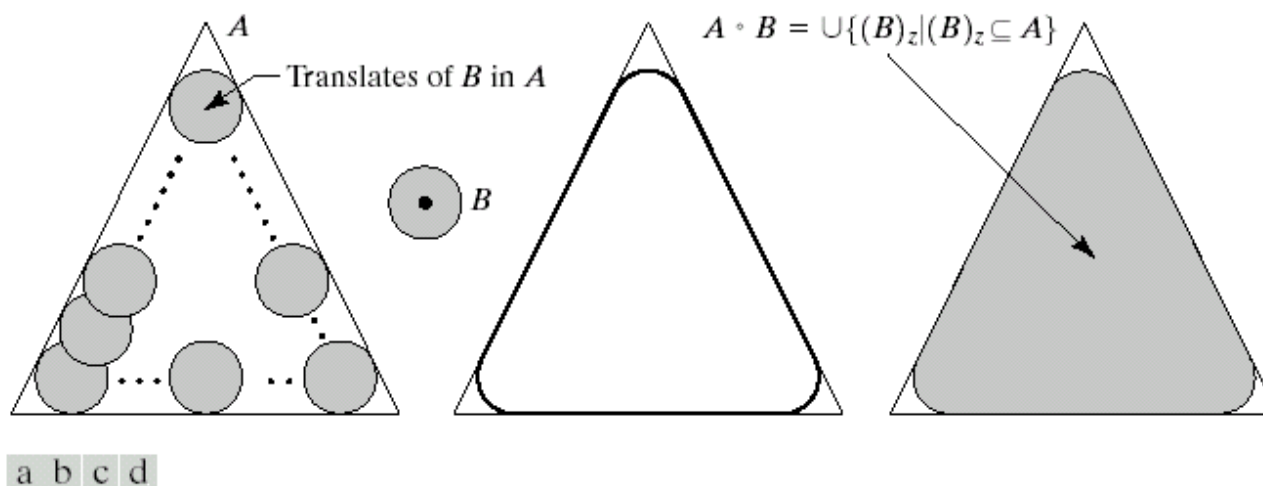
斷開(Opening): 使物體輪廓平滑，吃掉細小連線

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

$$A \circ B = \left(A \Theta B \right) \oplus B$$
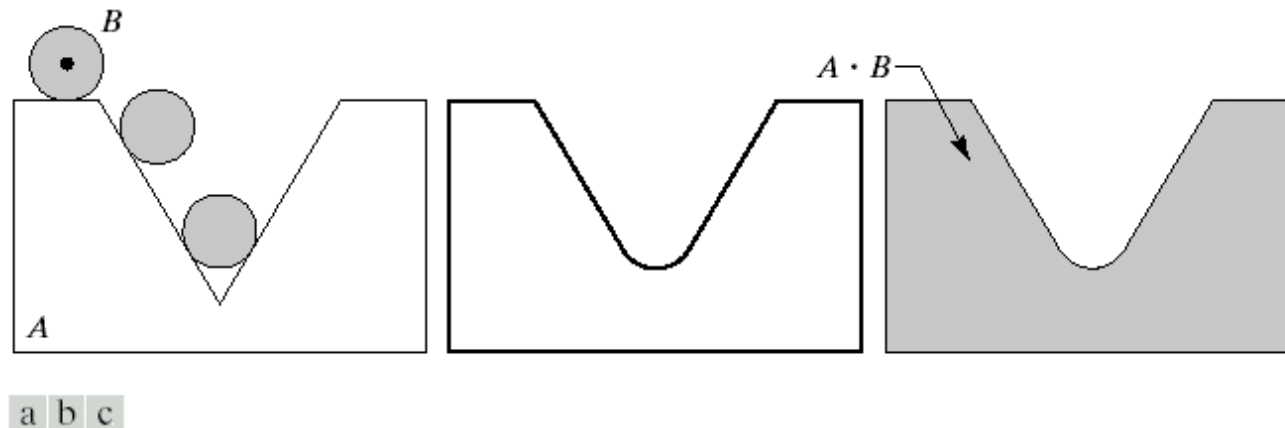
Matlab: imopen()

閉合(Closing): 消除小洞，連結細小連線

a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).
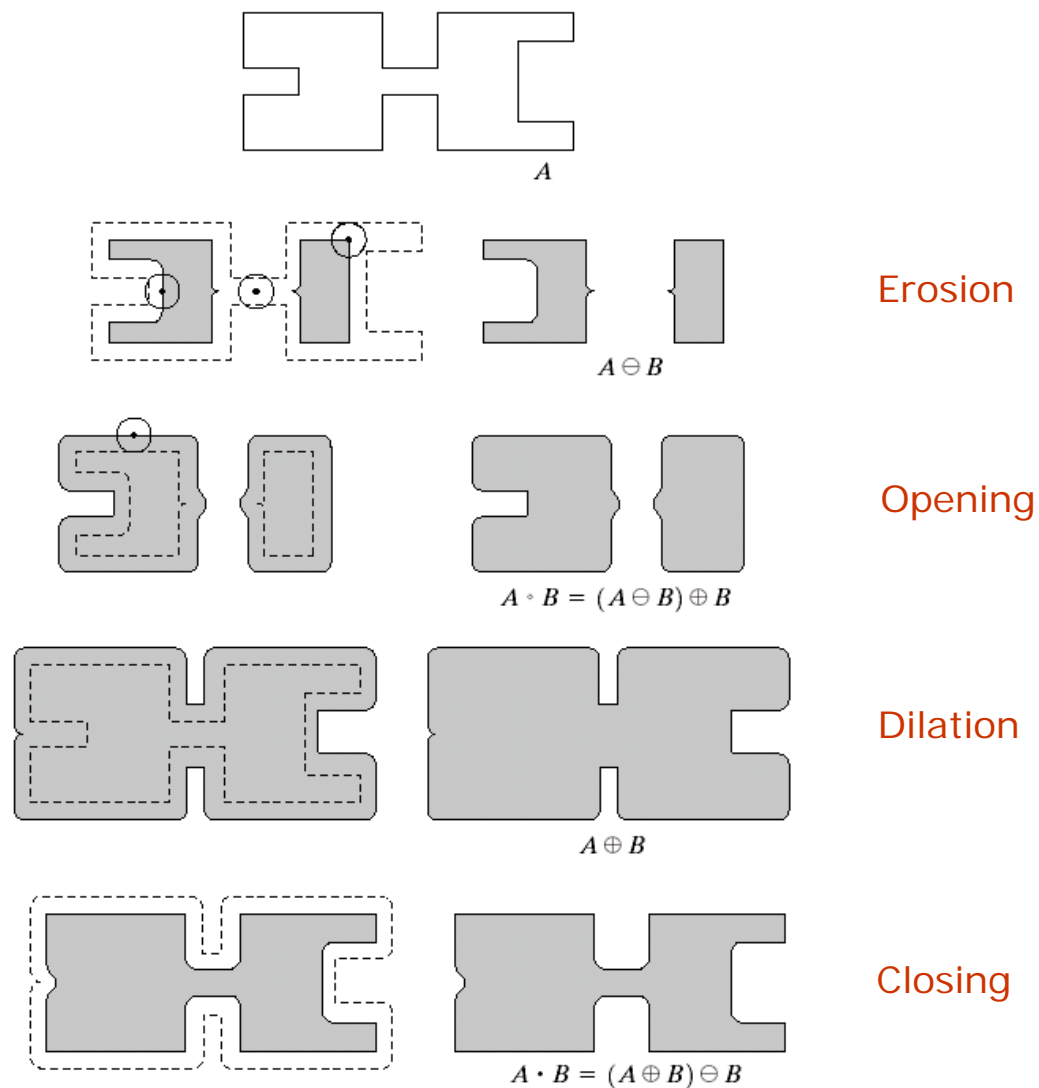
$$A \bullet B = (A \oplus B) \Theta B$$

Matlab: imclose()

比較(b)侵蝕 (c)斷開 (d)膨脹 (e)閉合

a
b c
d e
f g
h i

FIGURE 9.10
Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

Erosion

$A \ominus B$

Opening

$A \circ B = (A \ominus B) \oplus B$

Dilation

$A \oplus B$

Closing

$A \cdot B = (A \oplus B) \ominus B$

實例 1(白色部份為處理對象)

Original

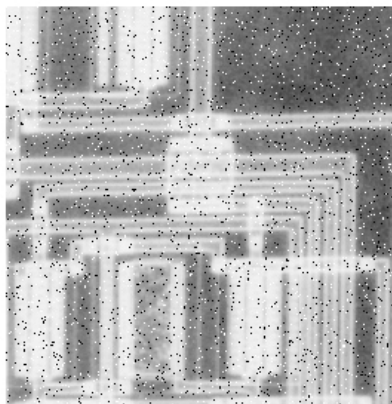Dilation

Erosion

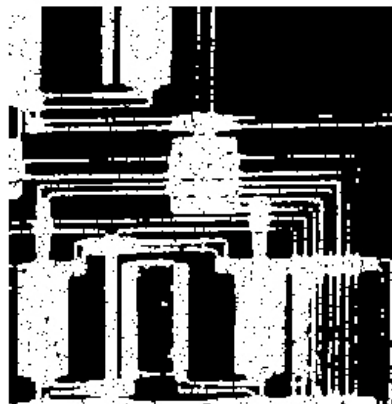Opening

Closing

實例 2 (白色部份為處理對象)
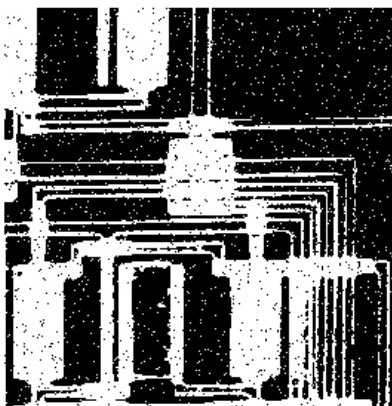
Original (BW)　　Dilation　　Erosion

Original (Grayscale)　　Opening　　Closing