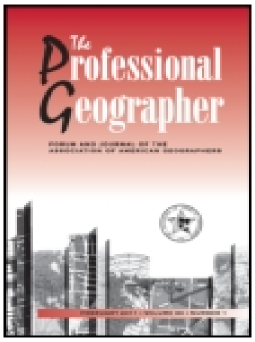

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Network Reliability and Resilience of Rapid Transit Systems

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The recent increase in demand and transportation security highlights the importance of the public transit system in the United States. This study explores how potential failures on nodal disruptions affect transit system flows and examines the change in the reliability of transit systems with a case study of the Greater Metropolitan Area of Washington, DC. For methodology, we employ network reliability and system flow loss and assess the criticality of stations under a variety of simulated nodal disruptions. We evaluate network resilience by identifying the best and worst geographical impact scenarios on networks. **Key Words: critical infrastructure, reliability, resilience.**

美国晚近对于需求及运输安全的增加，凸显出公共运输系统的重要性。本研究以华盛顿州特区的大都会地区作为案例研究，探讨节点分裂的潜在失败，如何影响运输系统的流动，并检视运输系统可靠性的改变。在研究方法方面，我们运用网络可靠性和系统流动损失，并评估各种模拟的节点分裂下的车站临界程度。我们指认对网络而言最佳与最糟的地理影响局面，以此评估网络的恢復力。 **关键词: 关键基础建设，可靠性，恢復力。**

El reciente incremento en la demanda y la seguridad del transporte destaca la importancia del sistema de tránsito público de los Estados Unidos. El presente estudio explora cómo fallas potenciales en las alteraciones nodales afectan los flujos del sistema de tránsito, y examina el cambio de confiabilidad en los sistemas de tránsito a través de un estudio de caso del Área Metropolitana Ampliada de Washington, DC. En cuanto a metodología, empleamos la confiabilidad de la red y la pérdida de flujo del sistema y evaluamos la criticidad de las estaciones bajo una variedad de perturbaciones nodales simuladas. Evaluamos la resiliencia de redes identificando los mejores y peores escenarios de impacto geográfico sobre las redes. **Palabras clave: infraestructura crítica, confiabilidad, resiliencia.**

The primary function of a rapid transit system is to facilitate the movement of a mass of people and goods among a set of origins and destinations across a network. Any disruption of network components in a network-based infrastructure has a negative impact on the entire system flow and network resilience (Murray and Grubesic 2007). If the system is a public transit system, component failure, regardless of failure type, entails a huge socioeconomic cost due to its criticality in serving a mass of people and goods (Angeloudis and Fisk 2006; Matisziw and Murray 2009; Matisziw, Murray, and Grubesic 2009). For example, abrupt flooding in subway systems in New York in 2007 initially only affected some stations' operations, but the flooding ultimately caused severe delays and suspensions for passengers across the system (Chan 2007). An intended attack targeting important stations could cause the total disruption of a massive public transit system, as exemplified by the 7 July 2005 bombing of the London underground system (Wrobel and Wrobel 2009; Maliszewski and Horner 2010). Even scheduled maintenance on a few stations becomes a significant factor in the delay and congestion of passenger movements on the network (Washington Metropolitan Area Transit Authority [WMATA] 2010b). In this context, researchers' interest in network vulnerability, any weakness in a network's design or operation exploited by an adversary, has grown (Bier and Azaiez 2009). Although many classical approaches to assessing the vulnerability of network-based systems have been proposed, gaps between theories and pragmatic analyses still exist in the literature (Derrible and Kennedy 2010b, 2010c; Taylor 2012).

The main goal of this article is to present how substantial analysis can capture the vulnerability of a public transit system, taking the Washington, DC, Greater Metropolitan Area as our case study. We identify the worst- and best-case scenarios and explore the capabilities of a system that faces possible interruptions at its stations. Employing a methodology that takes into account a network's typical characteristics is very important in assessing the impacts and resilience of a system (O'Kelly and Kim 2007). Accordingly, we provide a series of reliability measures for a transit system with a simulation-based approach.

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Scholars have labeled massive public transit systems *closed*, as most passengers tend to complete their journey without relying on other transport modes. Nontransfer and transfer stations, depending on their role in the network, are the two types of stations within the system. Nontransfer stations are channels for inbound and outbound passengers with low connectivity. The concern with nontransfer stations is the travel demand of passengers, as any inconvenience at these stations can result in the loss of flow within the network (Lewis 2006). In contrast, transfer stations are *hubs* because the passenger flows from other nontransfer stations or lines are collected and rerouted through these facilities, making possible the completion of passenger journeys. Researchers have highlighted the importance of hubs as a significant operational strategy because the efficiency of a network system is highly dependent on them (Moteff and Parfomak 2004; Kim and O'Kelly 2009). The advantages of operating hubs in public transit systems is to provide passengers with more alternative routes, which eventually improve the resilience of a network in the face of disruptive events. Note that the impact of failures at such stations results in their immediate inaccessibility followed by a serious degradation of the entire network's functionality, which might entail the termination of interactions in the network.

Every network-based system will inevitably experience a loss of traffic or delay owing to component disruptions. However, assessing the criticality of a system is a necessary preventive treatment, enabling governments or organizations to enhance the level of preparedness for unexpected situations at a site. We examine the impact of potential failures on a range of possible nodal disruption scenarios to characterize the resulting effects on system flows and resilience. Specifically, we focus on detecting the most critical stages of a set of key stations that result in serious degradation in network resilience. Our assessment of network criticality is twofold: the evaluation of system reliability in terms of structural functionality and the potential impact on system flows in various disruption scenarios. These assessments allow us to depict the resilience of the rapid system and geographical variations of network reliabilities for selected disruption scenarios and are ultimately used to determine a method for improving network resilience.

Network Vulnerability: Measures and Approaches

Many vulnerability analyses have been applied to ground transportation systems such as highway and road networks (D'Este and Taylor 2003; Jenelius, Peterson, and Mattsson 2006). Most network vulnerability research in transportation focuses on measuring the degree to which a network component can operate or function correctly in the presence of stressful environmental conditions. Because scholars have constructed a myriad of definitions and strategies for evaluating the vulnerabilities of a network system, a vulnerability study must specify the measures that would work best for the network properties and environment (see Nicholson and Du 1994; Gribble 2001 in detail). Generally, two different measures are worth noting: deterministic and probabilistic. Deterministic measures represent the degree of vulnerabilities throughout *indexes* developed for a given network structure, which is represented with a set of nodes, links, and topology (Tahmasseby, van Nes, and van Oort 2009). Accessibility is a classical deterministic measure that presents the potential availability of nodes in a transportation network. Due to the conceptual flexibility of this measure, many variants have been proposed and used to assess network vulnerability. They view a rapid transit system with high accessibility as a less vulnerable system, as such a system offers more paths available in the case of network malfunction (Lee and Lee 1998; Kim 2009). Recently, Derrible and Kennedy (2010b) noted that metro networks with a higher number of transfer stations are more robust to failure than lower ones because transfer stations improve route availability in a network. As an extended deterministic measure, survivability focuses on evaluating the consequences in a network from possible disruptions by identifying crucial nodes or links causing severe connectivity loss in the systems (Kurauchi, Sumalee, and Seto 2009). As demonstrated by Li and Kim (2014) and Fortz and Labbé (2006), high network accessibility does not necessarily indicate high survivability because a network's survivability is more dependent on a few critical stations than high-performance components.

Researchers typically employ probabilistic measures to assess a network's criticality when they know the empirical or hypothetical operational or failure probability of a network component (Wakabayash and Iida 1992). For example, the importance of adopting *reliability*, a conventional probabilistic measure, in vulnerability research has been stressed. Despite its diverse definitions, in essence, reliability measures an entire system's soundness or ability to continue operation despite malfunctions in network components (Colbourn 1987; Yoo and Deo 1988; Bell and Iida 1997). Reliability measures assume that any network component operates within a certain level of operation probability rather than simply following all-or-nothing logic. Therefore, in theory, probabilistic measures are more realistic than deterministic measures (Lam, Zhang, and Lo 2007).

Three concerns are raised in previous probabilistic measures, however. First, many researchers have simply used a hypothetical probability of operation for network components when computing system reliability. Because obtaining or estimating the empirical probability of operation is often difficult, many studies construct reliability measures based on the assumption that the operational probabilities of all links or nodes are identical (Miller 2004). Using hypothetical probabilities can generate misleading results because the operational probability of links or in nodes varies with the length of the links, levels of connectivity, or traffic loads. Second, measures must reflect the spatial distribution of demands on network components when evaluating the criticality of network components. If it is possible to incorporate any socioeconomic attributes to a probabilistic measure, a disruption scenario can be modeled in a range of ways with knowledge of spatial demand and supply. For example, the stations covering densely populated regions of the network do not have the same criticality as those in lower demand areas (Transportation Research Board of the National Academies 2002). Unfortunately, the literature lacks analyses of reliability for rapid transit systems. Kondo, Shiomi, and Uno (2010) proposed a measure called road network reliability to evaluate network robustness against natural disasters to ensure emergency response in Kyoto, Japan. Kim (2009) assessed the network resilience of the subway system in Seoul using hypothetical network reliability. Both studies, however, focused more on reliability associated with network structure itself for selected hubs over networks. The third concern involves selecting an effective analytical strategy to find critical components that directly cause severe degradation of network performance. Two approaches are worthy of mention: the game theory and simulation approach. The game theory approach assumes that threats are not static and lead to successive responses between attackers and defenders (Bell et al. 2008). Combining game theory with a probabilistic measure could be beneficial for estimating the potential consequences of intelligent threats with time between the actions of two players. Although many models reflect reality, their application is limited to such situations in which the attacks or malfunctions only target the most important nodes or links to the defender's response due to the nature of game theory (Bier and Azaiez 2009). Thus, a game theory–based approach might only be effective for outlining plausible scenarios for limited disruption events. In contrast, the simulation approach focuses on diagnosing a network's potential consequences in a full range of disruption scenarios (Albert, Jeong, and Barabási 2000; Callaway et al. 2000; Pastor-Satorras and Vespignani 2004). Given enumerated probable instances, the network's performance is bounded by the best and worst cases; therefore, resilience is assessed to both random and targeted disruptions of network components. Note that researchers can apply a simulation approach alongside game theory or other approaches by embedding those methods within the simulation, but the main challenge of a simulation approach is the extremely intensive computation required to enumerate the probable scenarios to complete the analysis (Nicholson and Dalziell 2003; Delfino et al. 2005).

Given this context, we define a set of reliability measures in a simulation-based approach to evaluate the vulnerability and explore the resilience of the system for withstanding potential disruptive events using the case of the transit system in the Greater Metropolitan Area of Washington, DC (Figure 1), the second-largest rapid transit network in the United States, accounting for more than 700,000 daily passenger trips (WMATA 2009). An ideal transit system should avoid delays or possible intended threats. Therefore, identifying which stations contribute to the critical disruption scenarios is necessary to set up a feasible protection plan. Notice that the transit system in question was constructed using a star-shape topology, which is regarded as simple but efficient in flow management when compared to mesh-type structures (Gavish 1992). The eighty-six stations in the network serve spatial demands within the wide geographical areas of Washington, DC, but several suburbs are centralized to the downtown area served by five subway lines. Only eight transfer stations provide the opportunity for rerouting flow as hubs among lines. Our simulation results provide a clue to uncovering the relationship between the form of a network and its resilience, relating with other case studies (Turnquist and Bowman 1980; Derrible and Kennedy 2010a).

Analytical Framework

Reliability Measures

Although reliability can be defined differently based on the context of applications, it refers to the probability of the successful delivery of origin–destination flows without encountering delays in the network. Here, we define the term *reliability* as the probability that travelers will go from an origin to their destination(s) within a given period (Bell and Iida 2003). Given this definition, we introduce a set of reliability measures to examine network vulnerability: *o-d* reliability (R^{od}), nodal reliability (R^{node}), nodal flow (F^{node}), system reliability (R^{sys}), and loss of system flow (L^{sys}). First, R^{od} is defined as follows:

$$R^{od}(G,p) = \sum_{k=1}^{n} p(D_k) \tag{1}$$

$$p(D_k) = \prod_{k=1}^{n} p(\overline{E}_{k-1}) \cdot p(E_k), \tag{2}$$

where R^{od} (G, p) is the reliability between o (origin) and d (destination) on network G, where probability p is computed for a set of disjoint event paths D_k (k = 1 to n); $p(D_k)$ is the probability for a disjoint event path D_k, where D_k is enumerated based on the logic of Boolean algebra for the available paths E_k for a pair of o-d; and $p(E_k)$ is the probability of an available path E_k for o-d. A path E_k consists of a set of links e_i (i = 1 to m). The $p(E_k)$ is calculated using

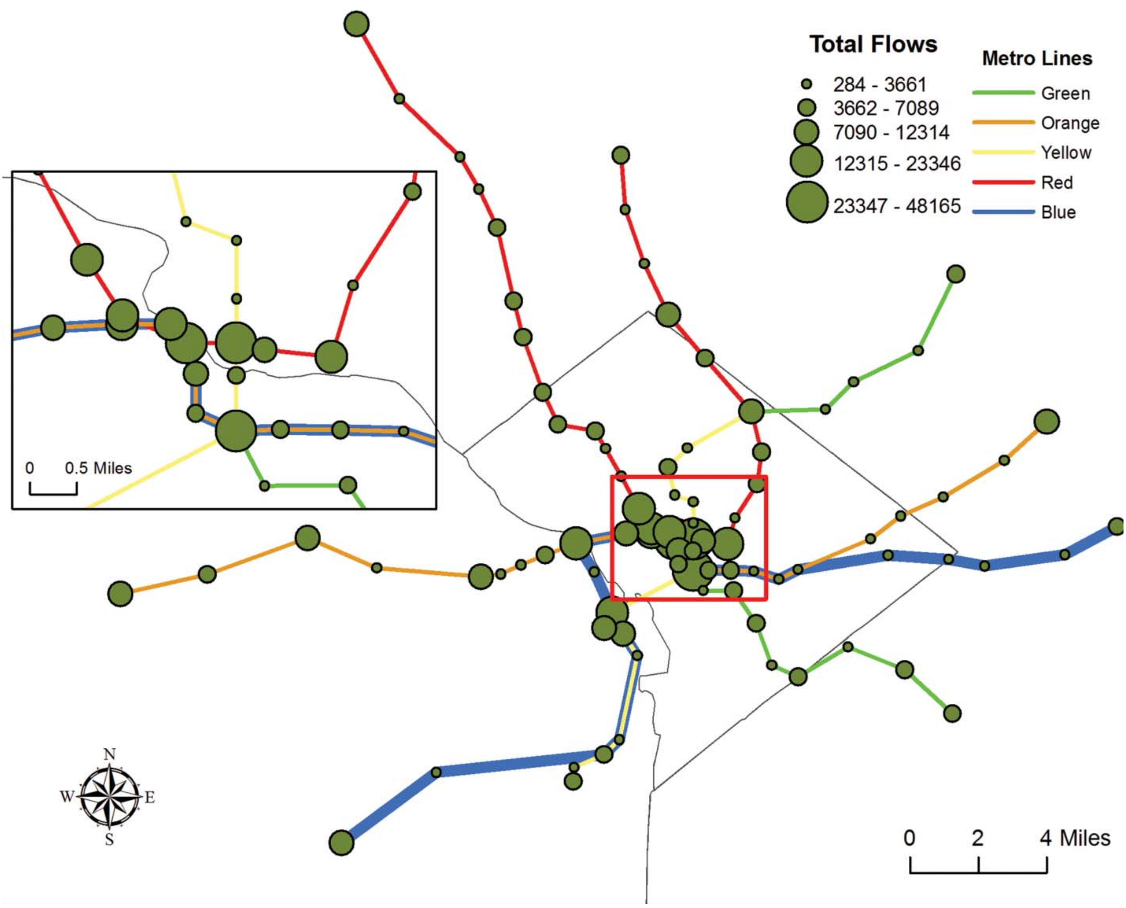

Figure 1 *Washington, DC's, rapid transit system and the geographical flow distribution at stations. (Color figure available online.)*

$E_k = \prod_{i=1}^{m} p(e_i)$, where e_i is the empirical reliability at link i and $p(\overline{E}_k)$ is the complementary probability for $p(E_k)$.

The R^{od} is computed by the summation of the probabilities of k disjoint event paths $p(D_k)$ for an *o-d* pair. Here, disjoint event paths are defined as mutually but statistically independent paths that are determined based on the paths E_k available between *o-d* (Colbourn 1987). In detail, the first step (Figure 2A) enumerates the set of state vectors with E_k that ensure routes between *o-d*. To enhance computational efficiency, we employed the k-shortest path algorithm to identify all available event paths for an *o-d* pair rather than exhaustive search. In the example, the three identified shortest paths ($k = 3$) are stored as E_1, E_2, and E_3, and the reliability of each path $p(E_k)$ is computed as follows based on the multiplication of $p(e_i)$ within each state vector: $p(E_1) = 0.81$, $p(E_2) = 0.81$, and $p(E_3) = 0.729$. The next steps are to make these paths disjoint events to remove the shared probability among E_k. To do this, we employ a most effective method developed by Yoo and Deo (1988), through which the set of disjoint event paths D_k is generated using Boolean algebra from the set of E_k. The main idea of this method is to work from any initial $p(E_k)$ to other unused paths E_k by finding its $p(\overline{E}_k)$. The method effectively removes the doubly counted probabilities of shared links among E_k, making all paths mutually and statistically disjoint. The $p(D_k)$ are calculated as $p(D_1) = p(E_1) = 0.81$ (Figure 2B), $p(D_2) = p(\overline{E}_1) \cdot p(E_2) = (1 - 0.81) \cdot 0.81 = 0.1539$ (Figure 2C), and $p(D_3) = p(\overline{E}_1) \cdot (\overline{E}_2) \cdot p(E_3) = 0.19 \cdot 0.19 \cdot 0.729 = 0.02632$ (Figure 2D). Notice that the method prevents the probability of e_4 shared in E_2 and E_3 from being overcounted. Finally, the summation of $p(D_k)$ represents the true R^{od}, meaning that 99.022 percent of passenger flows between *o-d* are expected to complete their journey without encountering any delay via the paths available in a network. All other things being equal, the more routes ensured by links, the higher the expected *o-d* reliability due to path redundancies. To evaluate nodal criticality among N nodes, two measures, R^{node}, the average reliability from a specific origin o to all destinations (Equation 3), and R^{range}, a range between the highest (max R^{od}) and the lowest reliability (min R^{od}) at node (Equation 4), are calculated, respectively. R^{sys} is defined as the system's

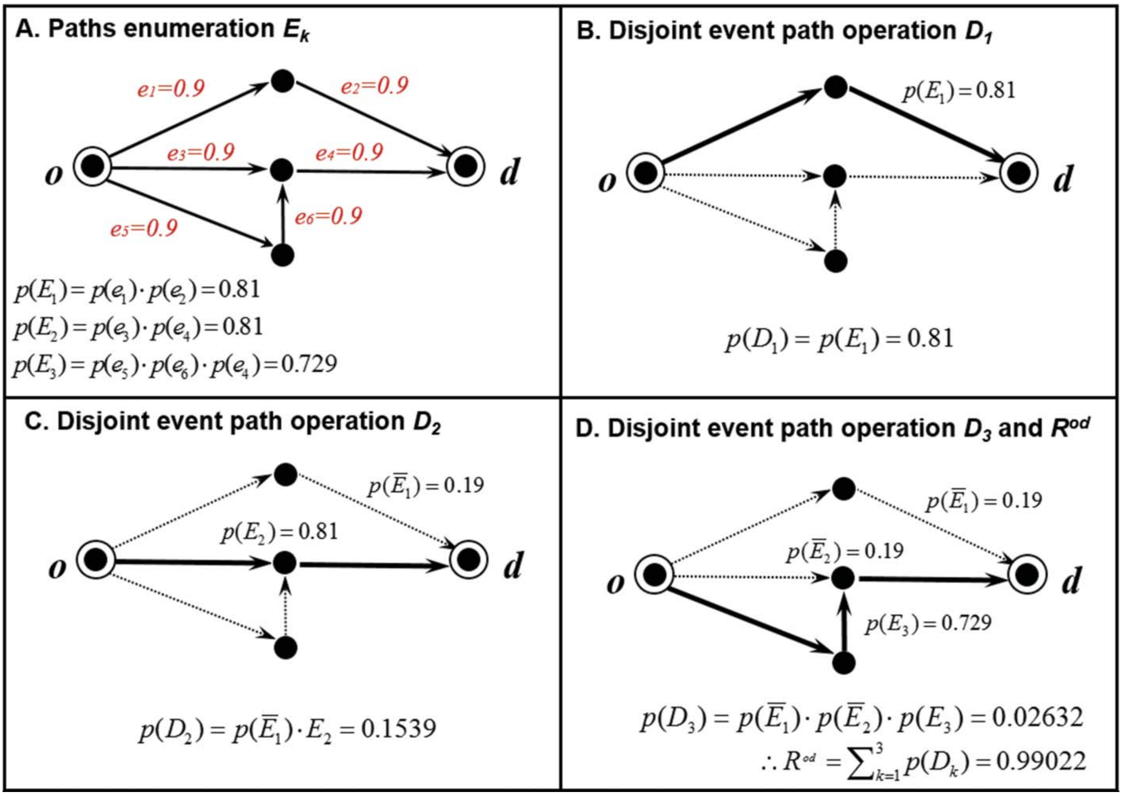

Figure 2 *Illustration of a reliability computation. (Color figure available online.)*

criticality with average reliability from all origins o to all other destinations d (Equation 5).

$$R^{\text{node}}(G,\ p)=\frac{1}{(N-1)}\sum_{d=1}^{n}R^{od} \tag{3}$$

$$R^{\text{range}}(G,\ p)=|\max R^{od}-\min R^{od}| \tag{4}$$

$$R^{\text{sys}}(G,\ p)=\frac{1}{N(N-1)}\sum_{o=1}^{n}\sum_{\substack{d=1\\(o\neq d)}}^{n}R^{od} \tag{5}$$

If we know W_{od}, the passenger flow among o and d, the estimated nodal flow (F^{node}) from a specific origin to all destinations is derived using Equation 6 and F^{range} informs the gap between the highest and the lowest traffic flows at a node (Equation 7). Using the complementary probability of R^{od}, the system's flow loss among all o-d pairs (L^{sys}) is calculated (Equation 8). Notice that R^{node} and F^{node} are the indicators of nodal criticality and R^{sys} and L^{sys} are the system criticality measures to examine the network resilience to disruption scenarios.

$$F^{\text{node}}(G,\ p)=\sum_{d=1}^{n}W_{od}R^{od} \tag{6}$$

$$F^{\text{range}}(G,\ p)=|\max W_{od}R^{od}-\min W_{od}R^{od}| \tag{7}$$

$$L^{\text{sys}}(G,\ p)=\sum_{o=1}^{n}\sum_{\substack{d=1\\(o\neq d)}}^{n}W_{od}(1-R^{od}) \tag{8}$$

Although our reliability measures do not directly consider the congestion effect in computation, they implicitly account for the potential degradation of system operation if a path needs more stops or the number of paths is reduced due to the disruptions.

We only consider nodal disruptions rather than link failures in this article because a link failure leads to the shutdown of the connected stations in reality, making it difficult to assess which station is more attributable to the network degradation. Second, the best- and worst-case scenarios are more clearly drawn during simulation with reduced computational complexity when we apply nodal disruptions.

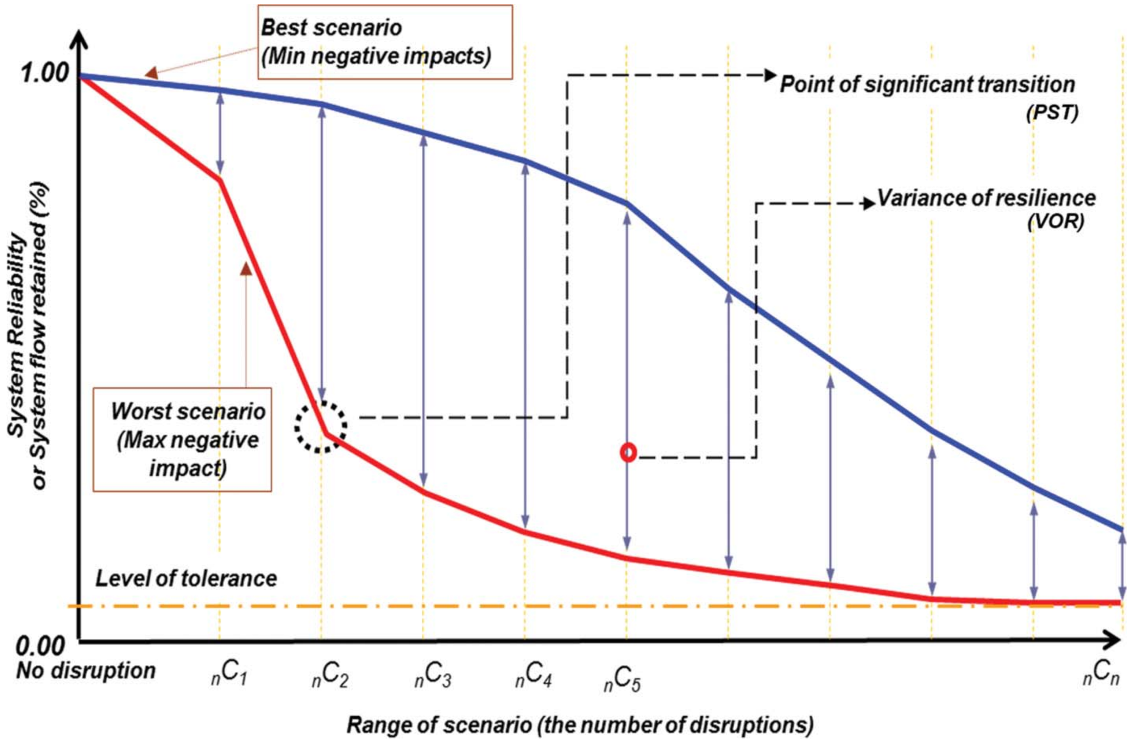

Figure 3 *Operational envelope. (Color figure available online.)*

Network Resilience

Prior attempts to draw the best and worst case scenarios for network disruptions have been made in other applications to identify the simulation results (Church and Scaparra 2007; Nagurney and Qiang 2009). We employ a graphic diagram called an operational envelope, as illustrated in Figure 3. The operational envelope has a range on the y axis that represents the network performance using R^{sys} or L^{sys}. The x axis shows the range of station disruptions (r) from the status quo ($r = 0$) to total disruption ($r = n$). For example, the first stage includes the scenario results for one nodal disruption among n given stations ($= {}_nC_1$), the second stage for those two nodal disruptions ($= {}_nC_2$), and so forth to define the best- and worst-case scenarios at each stage from the simulation results. We evaluate network resilience in terms of two points through the envelope. First, the gap between upper and lower bounds, named variance of resilience (VOR), shows the robustness of a network to disruptions. In theory, if we observe a large VOR in the envelope, the network is likely to significantly lose its robustness for the targeted disruption but resist random failures. In contrast, a small VOR means that the network maintains its functionality to both targeted and random failures. Second, a point of significant transition (PST) in the worst-case scenario, if observed, suggests that there is a particular set of nodes that causes a dramatic degradation in network performance before or after stages when those nodes are simultaneously disrupted. The PST also indicates that a fortification plan is necessary for the identified critical nodes to enhance resilience. A significant challenge in drawing operational envelopes is that the simulation requires large computations to enumerate all potential cases at each stage. To tackle the problem, we developed a rank-based resilience simulation algorithm to effectively complete simulations by finding the best and worst disruption scenario. The algorithm starts with the total enumeration of disruption events for the status quo to the fourth stage and the eighty-second to the eighty-sixth stage, which are manageable, to identify the exact best- and worst-case scenarios within our computing capabilities. For the rest of the stages, the algorithm approximates the best and worst scenarios by constructing the sets of the most and least contributing nodes to define them at each stage. The algorithm uses two selection criteria, the percentage of rankings from the best (U_{set}) and worst scenario (L_{set}) at each stage and the frequency of nodes identified in exact solution stages. Consequently, only several minutes are required for computation per stage.

Data

To measure network reliability, three matrices are used: network connectivity matrix, reliability matrix of links, and *o-d* flow matrix among the eighty-six stations. The connectivity matrix for the stations was constructed to reflect the complex characteristics of the transit system. For example, a transit line is not necessarily connected to all other lines at a transfer station, and a link can comprise multiple lines. Second, the reliability matrix of links was constructed using on-time performance reliability data for each transit line as of May 2009 (WMATA 2010a). Third, W_{od}, the *o-d* flows among the stations, are estimated based on the ridership survey data reported by WMATA (2009).

Results

Criticality of Stations at Status Quo

As a touchstone for comparing the results with disruption scenarios in terms of criticality, our first analysis focuses on evaluating nodal criticality using four measures (R^{node}, F^{node}, R^{range}, and F^{range}) at the individual station level assuming status quo. As presented in Table 1, most of the top ten R^{node} and F^{node} stations are located in the central area, whereas the bottom ten lie in the outskirts of the transit lines. This distinction suggests that areas of high- and low-ranked stations are geographically separated. Considering the fact that R^{node} is only the outcome of the topological structure of the network, not surprisingly, such downtown transfer stations as L'Enfant Plaza (first), Metro Center (second), Gallery Plaza (third), and the Pentagon (seventh) are highly ranked because they are hubs in a star-shaped structure. Notice that two nontransfer stations, Archives-Navy Memorial (fourth) and Smithsonian (fifth), are ranked within the top five because they have the benefit of nodal reliability by being directly connected to highly reliable transfer station(s). With regard to F^{node}, similar to the results of R^{node}, the criticality of the top three transfer stations is greatly stressed due to their role in handling a larger volume of transfer flows, but nontransfer stations in the downtown area (Union Station, Farragut West, and Farragut North) are newly ranked in the top ten. Notice that a dramatic change is found at such nontransfer stations as Mt. Vernon Square and Arlington Cemetery, whose rankings were in the top ten of R^{node} but in the bottom ten of F^{node} because of the significant volume of passengers using these stations. Second, R^{range} and F^{range} capture the different aspects of criticality. The highly ranked stations in R^{range}, for example, Capitol South, Waterfront, Pentagon City, and Farragut North, are characterized as bridge stations, which play the role of a gateway connecting the stations in branch lines with major hubs. The stations with low R^{range} are located at the edge of each transit line. The result of F^{range} is quite similar to that of F^{node}, but it also provides different criticality rankings among stations.

Based on the results, the stations in the center of the network are more reliable than stations at the edge of transit lines, implying that the passengers at outskirt lines might face a significant delay in reaching their destinations. In addition, transfer stations have higher reliability than nontransfer stations because of their better connectivity and redundancies, supporting previous findings by Gavish (1992) and Turnquist and Bowman (1980) that network structure affects the level of reliability as it does the routing of passenger transfers. Nodal criticality might not be as simple as reported in previous findings, however. As revealed by R^{range} and F^{range}, the bridge stations are also critical in maintaining system reliability and flows, indicating that multiple indicators from different aspects should be used in assessing nodal criticality.

Table 1 *The criticality rankings of stations at status quo*

Rank	Station	R^{node}	Station	F^{node}	Station	R^{range}	Station	F^{range}
1	**L'Enfant Plaza**	0.6208	**Metro Center**	44,173	Capitol South[a]	0.7559	**L'Enfant Plaza**	3,413
2	**Metro Center**	0.6167	**L'Enfant Plaza**	38,458	Waterfront[a]	0.7455	**Metro Center**	3,409
3	**Gallery Place**	0.6166	**Gallery Place**	30,697	Pentagon City[a]	0.7452	**Gallery Place**	2,680
4	Archives-Navy Memorial	0.6142	Union Station	22,405	Tenleytown-AU	0.7428	Union Station	1,924
5	Smithsonian	0.6081	Farragut West	17,291	Federal Center SW	0.7397	Farragut West	1,470
6	Federal Triangle	0.6067	Farragut North[a]	17,209	Van Ness-UDC	0.7244	Farragut North[a]	1,463
7	**Pentagon**	0.6053	**Rosslyn**	15,343	Cleveland Park	0.7036	**Rosslyn**	1,300
8	McPherson Square[a]	0.5935	**Pentagon**	13,213	Farragut North[a]	0.6807	**Pentagon**	1,114
9	Mt. Vernon Square	0.5878	McPherson Square[a]	13,134	Woodley Park-Zoo	0.6801	McPherson Square[a]	1,107
10	Arlington Cemetery	0.5859	Dupont Circle	12,876	Dupont Circle	0.6542	Dupont Circle	1,086
77	Vienna	0.3058	Reagan Nat'l Airport	2,151	New Carrollton	0.8475	Benning Road	165
78	New Carrollton	0.2902	Capitol Heights	1,541	Landover	0.8414	Forest Glen	158
79	Morgan Blvd.	0.2832	Morgan Blvd.	1,265	Vienna	0.8352	Eisenhower Avenue	147
80	Medical Center	0.2827	Congress Heights	1,723	Cheverly	0.8347	Congress Heights	142
81	Grosvenor	0.2590	Deanwood	1,330	Shady Grove	0.8312	Waterfront[a]	140
82	Largo Town Center	0.2505	Cheverly	1,222	Huntington	0.8294	Capitol Heights	127
83	White Flint	0.2365	Shaw-Howard	2,151	Dunn Loring	0.8279	Deanwood	109
84	Twinbrook	0.2148	Mt. Vernon Square	2,262	Deanwood	0.8272	Morgan Blvd.	104
85	Rockville	0.1936	Waterfront[a]	1,700	Largo Town Center	0.8271	Cheverly	100
86	Shady Grove	0.1726	Arlington Cemetery	281	Branch Avenue	0.8257	Arlington Cemetery	23

Note: The stations highlighted in bold are transfer stations.
[a]Bridge stations.

Table 2 *Impact on system reliability and system flow loss based on a one-nodal disruption scenario*

	Impact on the system reliability		Impact on the system flow loss	
Rank	**Station**	**R^{sys} impact (%)**	**Station**	**L^{sys} impact (%)**
1	**L'Enfant Plaza**	0.23388 (–47.6)	**Metro Center**	–68.8
2	**Gallery Place**	0.27103 (–39.3)	**L'Enfant Plaza**	–68.3
3	**Metro Center**	0.30840 (–31.0)	**Gallery Place**	–66.6
4	**Pentagon**	0.33468 (–25.1)	**Pentagon**	–56.2
5	Federal Center SW[a]	0.34739 (–22.8)	**Rosslyn**	–54.1
6	**Fort Totten**	0.34921 (–21.8)	Farragut North[a]	–53.2
7	**Rosslyn**	0.35596 (–20.4)	**Fort Totten**	–50.7
8	Capitol South[a]	0.35644 (–20.2)	Union Station	–50.2
9	Eastern Market	0.36459 (–18.4)	McPherson Square[a]	–50.1
10	Farragut North[a]	0.36489 (–18.3)	Farragut West	–50.0
11	Potomac Avenue	0.37195 (–16.7)	Pentagon City[a]	–49.7
12	Dupont Circle	0.37457 (–16.1)	Judiciary Square	–49.5
13	Stadium-Armory	0.37505 (–16.0)	Foggy Bottom	–48.8
14	Pentagon City[a]	0.37510 (–15.9)	Court House[a]	–48.5
15	Waterfront[a]	0.37770 (–15.4)	Federal Center SW[a]	–48.4

Note: The stations highlighted in bold are transfer stations.
[a]Bridge stations.

Impact of Nodal Disruptions

A disruption of highly reliable nodes might not necessarily have a considerable impact on the network. A node with low reliability can bring about a significant degradation of an entire system's reliability or traffic loss to the disruption. Table 2 clearly confirms that this scenario is probable in the case of one-node disruptions. Six major transfer stations are identified in the top fifteen of both criteria R^{sys} and L^{sys}, highlighting that these are the most important stations during disruption as well as at status quo. Among them, any single disruption at L'Enfant Plaza, Gallery Place, and Metro Center is anticipated to bring more than 30 percent loss of system reliability with 60 percent of total system flow loss simultaneously. Particular attention is paid to the impact of losing bridge stations on network performance because their loss directly disconnects a set of peripheral stations in the branch line from the main body of the system. Notice that the main reason for this huge impact in the "just one disruption" scenario is attributed to the inherent weakness in the star-shaped topology structured in the current transit system because passengers from suburban stations are not likely to have an opportunity to switch routes until they reach a transfer station downtown. Clearly, a certain combination of nodal disruptions can have a more severe impact on network performance. For example, the simultaneous malfunction of such stations as Farragut North (Red Line), Court House (Orange Line), and Waterfront (Green Line) means passengers cannot complete their journey because no alternative paths are available among the lines.

Network Resilience to Disruption Scenarios

We examined successive disruption scenarios with operational envelopes and present network resilience in Figure 4 in terms of system reliability (Figure 4A) and the percentage of system flows retained on the network (Figure 4B) with increases in disruptions. The retained system flow rate is calculated based on L^{sys} to the nodal disruptions over the total amount of *o-d* flows at status quo. Note that our developed algorithm's performance, how confidently it determines the range of the impact of disruption from the fifth to eighth stages, is validated by comparing it with the VOR, which is bounded by more than 100,000 randomly generated disruption events per stage. As clearly displayed in both envelopes, the VORs with our algorithm are much wider than those of random simulations, confirming that the developed algorithm very effectively and correctly approximates the best and worst scenarios among potential scenarios. Given the results, two observations are noteworthy in terms of VOR and PST. First, the VOR significantly increases in the first stage, and grows larger with each stage. As more disruption scenarios continue, the eighth stage can bring the system reliability below 0.05 and 90 percent of system flow loss where only local passenger interactions are possible so that access to downtown is not allowed. As indicated by the two PSTs in the worst-case scenarios, the first critical moment appears just in the second stage in both envelopes, where the R^{sys} falls below 0.15 with only 20 percent of the flow retention. This fact indicates that the current rapid transit system could be vulnerable to disruption of a few targeted stations. Interestingly, the second PSTs are observed in the fifth stage of R^{sys} but the fourth stage in system flow retention, highlighting protecting system flows is much critical concern in network resilience. A positive side should be noted, however: The network is very resistant to most disruption events according to the best case scenarios, as the system might lose only a small degree of functionality (0.05 percent of R^{sys} and 6 percent of L^{sys}) by the eighth stage. In our investigation, only 0.2 percent of particular node sets associate with the severe degradation of network resilience observed at the worst-case

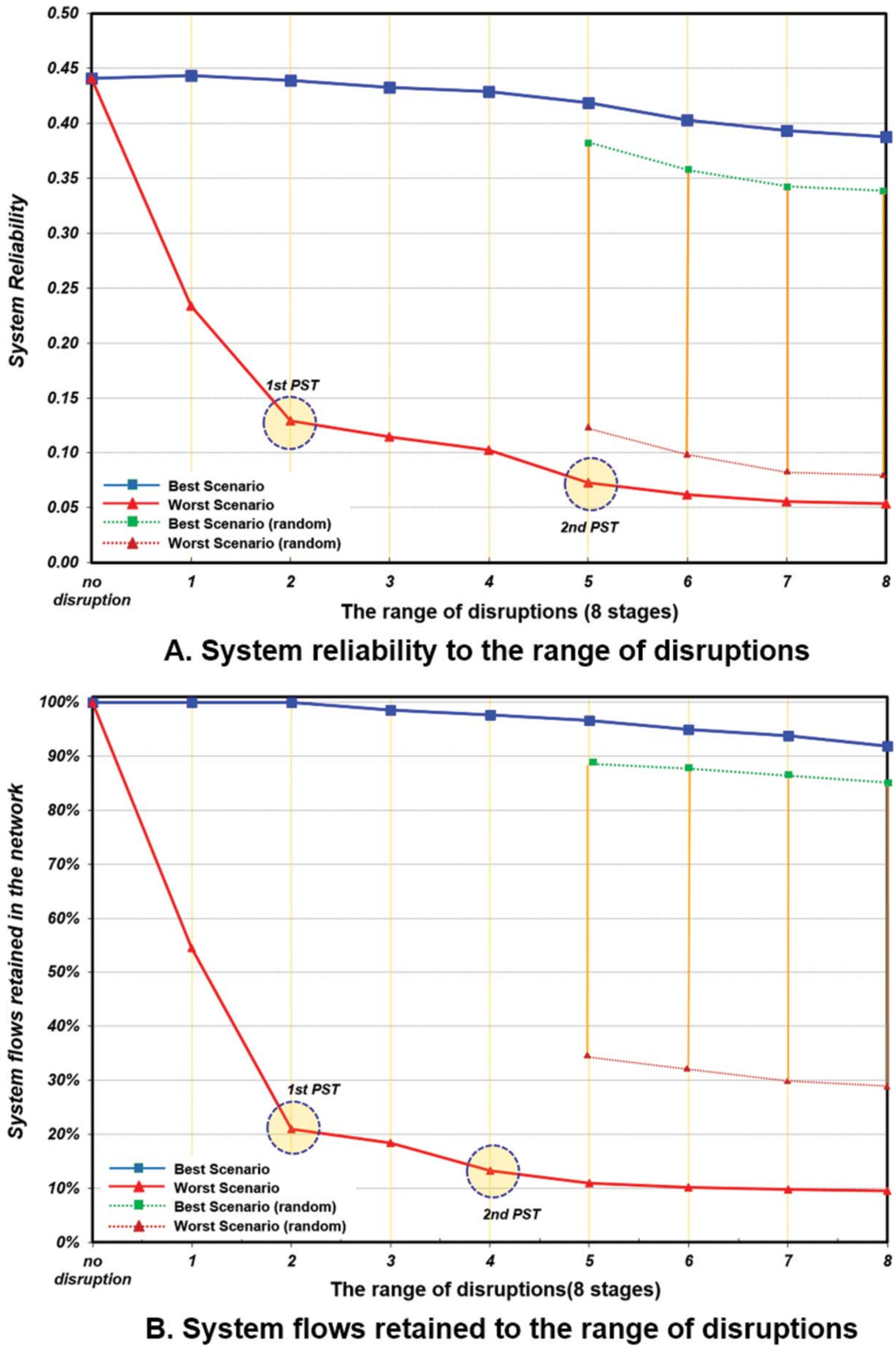

Figure 4 *Network resilience based on operational envelopes. (Color figure available online.)*

scenarios. Then, the question is this: What nodes are critical for the targeted disruption? Not surprisingly, the stations among the top five in Table 2 continuously comprise the critical node sets causing the worst-case scenarios. For example, for the worst-case scenarios of both PSTs in both envelopes, three transfer stations (Metro Center, L'Enfant Plaza, and Gallery Place) are common members of the sets; thus, such nodes should be strategically protected to maintain network resilience to potential disruption events.

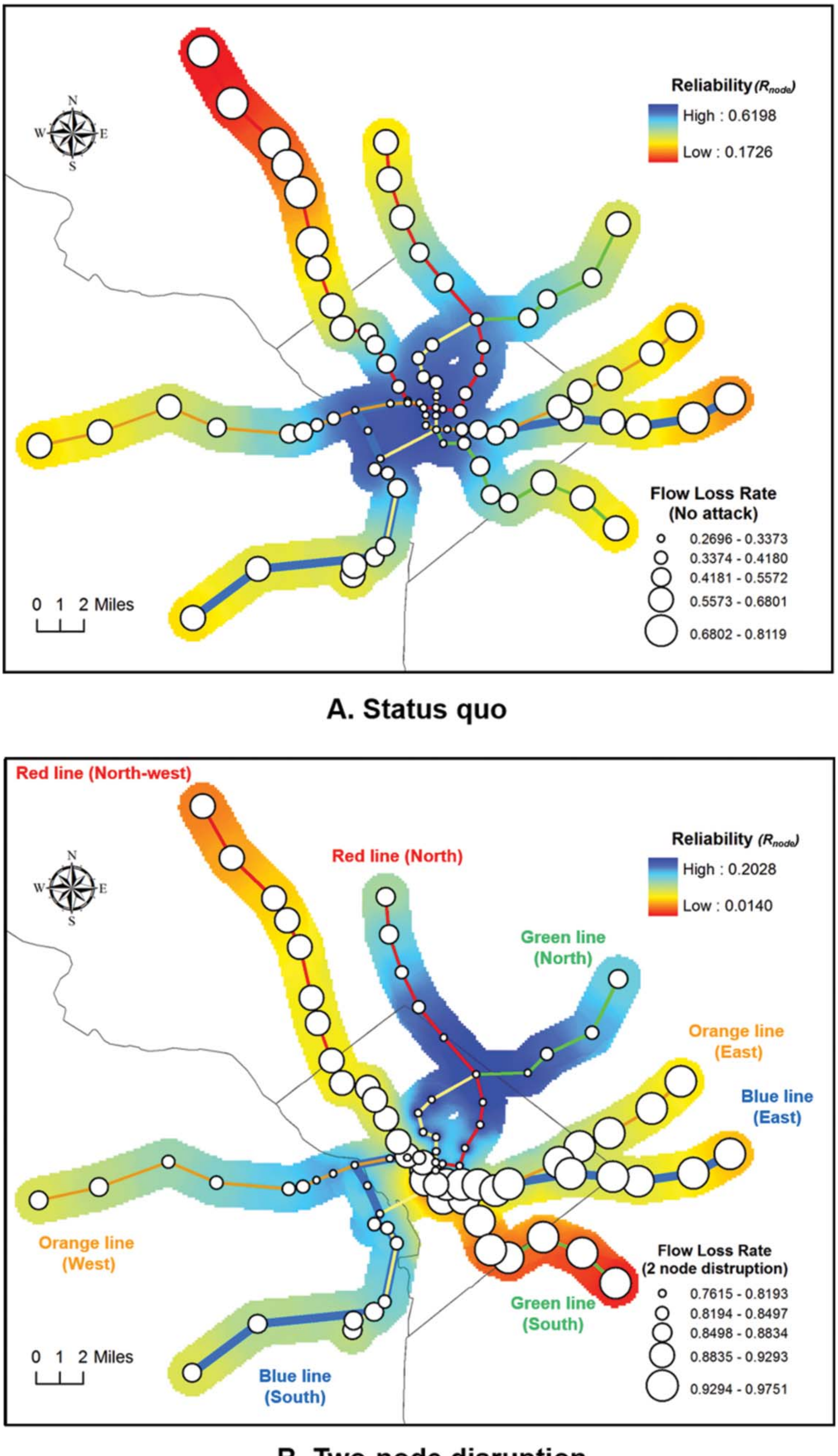

Figure 5 *Comparison of reliability potentials (no disruption vs. the worst-case scenario of a two-node disruption). (Color figure available online.)*

Geographical Variation of Reliability Potentials

Recall that the indicators only show the outcomes based on the topological characteristics of the system, ignoring the detailed geographic dynamics of reliability potentials that help identify the most and least affected areas within the disruption scenarios. Figure 5 visualizes two reliability potentials for the status quo and the first PST, to compare the change of pattern using two indicators, nodal reliability (R^{node}) and nodal flow loss rate (L^{node}). The regions in blue represent the more reliable areas, whereas red represents the less reliable areas in terms of R^{node}. The L^{node} is symbolized proportionally with circles to represent the degree of negative impact at stations. For the status quo (Figure 5A), the stations in central areas are highly reliable but become less reliable in outskirt areas as expected in a star-shaped network. In the worst-case scenario of a two-node disruption (Figure 5B), the negative impact from the central area spreads out along all transit lines in general; however, geographical discrepancies in impact are clearly identified between the lines. For example, in terms of R^{node}, the Red Line (northwest) is observed as the most vulnerable to disruption, and its impact stretches to the Green Line of the south, whereas the Red and Green Lines of the north maintain their reliability well comparatively. The stations at the Orange and Blue Lines in the east experience a moderate impact, but those in the Orange (west) and Blue (south) are less affected in the disruption scenario. Notice that traffic loss is worst in the downtown area but the impact is significantly developed from the Red Line (northwest) and directly stretched to the Green Line in the south as well as Orange and Blue Lines (east), indicating that stations at both lines will lose functionality more quickly than those at other lines.

Conclusions

We produced several important findings that contribute to network vulnerability research. First, although existing literature stresses that transfer stations are a critical asset to be protected from disruption in a star-shape network where the structural weakness mainly accounts for the current rapid system's vulnerability, our results indicate that all hub stations are not equally critical and that selective nontransfer stations with larger passenger flows or bridge stations adjacent to hubs could be more critical for targeted disruption. Such nodes should be fortified to avoid the worst-case scenarios because of the implications of such scenarios. Second, the network's resilience should be evaluated considering not only structural characteristics but system flow as well, to draw the scenarios correctly. As demonstrated, two indicators, the PST and VORs, reveal that flow loss is more sensitive to the disruptions in capturing critical moments. Finally, from a geographical perspective, the impact of failures is not evenly manifested over the network, as demonstrated in Figure 5. The proposed reliability measures can be used to benchmark any expansion plans from the current rapid system. Obviously, adding more transfer stations or lines enhances network reliability and resilience; however, the best plan in practice can be prepared if the evaluation is made using multiple indicators. Future research should also include a link-based simulation with advanced algorithms to identify specific links associated with network reliability to facilitate strategic network fortification. ■

Notes

[1] Computing reliability measures is manageable for the fourth or eighty-second stages with their complete enumeration of events (${}_{86}C_4 = {}_{86}C_{82} = 2{,}123{,}555$) based on the total twenty-four Intel Core 3.47 GHz processors on three Windows Servers with 24 GB of RAM. The solution times ranged from only 3.5 minutes for the status quo to up to 39 hours for the fourth stage to confirm the exact solution.

[2] On-time performance reliability is defined as the percentage of station stops made within the scheduled headway plus two minutes during peak periods or off-peak periods within 50 percent of the scheduled headway of a weekday (WMATA 2009).

[3] Because the data only report the amount of passenger flow at the station of origin (o_i), destination (d_j), and transfer stations T_i or T_j, we used a flow relaxation technique developed by Taaffe, Gauthier, and O'Kelly (1996) to specify the estimated flow W_{ij} using this equation: $W_{ij} = \frac{(O_i + T_i).(d_j + T_j)}{\sum_i d_i} = \frac{(O_i + T_i).(d_j + T_j)}{\sum_j d_j}$

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