Université Gustave Eiffel

Reliability project

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M2 :SIA

Introduction:

The study of system resilience begins with the exploration of ecosystem self-recovery. Generally speaking, in the face of biological invasion or environmental changes, ecosystems can recover from these disturbances and have a certain degree of adaptability. The ability of such systems to recover from interference is called resilience.

In order to evaluate the indicators related to network resilience as comprehensively as possible, we add network service performance indicators, such as reliability, which includes O-D reliability , nodal reliability, the range between the maximum and minimum reliability and the system reliability.

Definitions:

Name	Expression	Definitions
OD reliability	$\mathrm{R^{od}(G.p)} = \sum_{k=1}^{n} p(D_k)$ $p(D_k) = \prod_{k=1}^{n} p(\overline{E_{k-1}}) \times p(E_k)$	Reliability between origin stop (O) and destination stop (D) on transit network.
Nodal reliability	$R_{Avr}^{node}(G.p) = \frac{1}{(N-1)} \times \sum_{d=1}^{n} R^{od}$	Average reliability : reliability from a specific origin (O) and all destinations.
Range reliability	$R_{range}^{node}(G.p) = \lvert R_{Max}^{od} - R_{Min}^{od} \rvert$	Range reliability : range between the maximal reliability and the minimal reliability at a node.
System Reliability	$R^{sys}(G.p) = \frac{1}{N(N-1)} \times \sum_{o=1}^{n} \sum_{d=1}^{n} R^{od}$	System's criticality with average reliability from all origin (O) to all other destinations (D)

Note : $p_{ij}(a)$ – number of shortest paths from node i to node j that pass through node a, N – number of nodes in network, d_{ij} – shortest path length between node i and node j, $p(E_k)$ - probability of an available path E_k of a pair of O-D, $p(\overline{E_{k-1}})$- complementary probability for $p(E_k)$, D_k - disjoint event path operation.

Methodology:

*For a pair of node origin-destination, P(ei) is the probability that it is the ei link that is "cut" among all those that make up the paths from origin to destination. Using the article “Network Reliability and Resilience of Rapid Transit Systems”, for the adjacency matrix provided “adjacency.mat” we calculated O-D reliability, nodal reliability, the range between the maximum and minimum reliability and the system reliability.

To do that we created 6 functions:

- Dijkstra function for shortest paths.

- YenAlgo function based on Yen's k-Shortest Path algorithm:

J. Y. Yen, "Finding the K shortest loopless paths in a network", Management Science 17:712–716, 1971.

-Rod function that compute the reliability for all pairs of origin-destination nodes.

- Rnode function that compute the nodal reliability.

- Rrange function that compute the range between the maximum and minimum reliability.

-Rsys function that compute the average reliability.

All the results are saved in the file “reliability.mat”

Steps taken in order to get to our results :

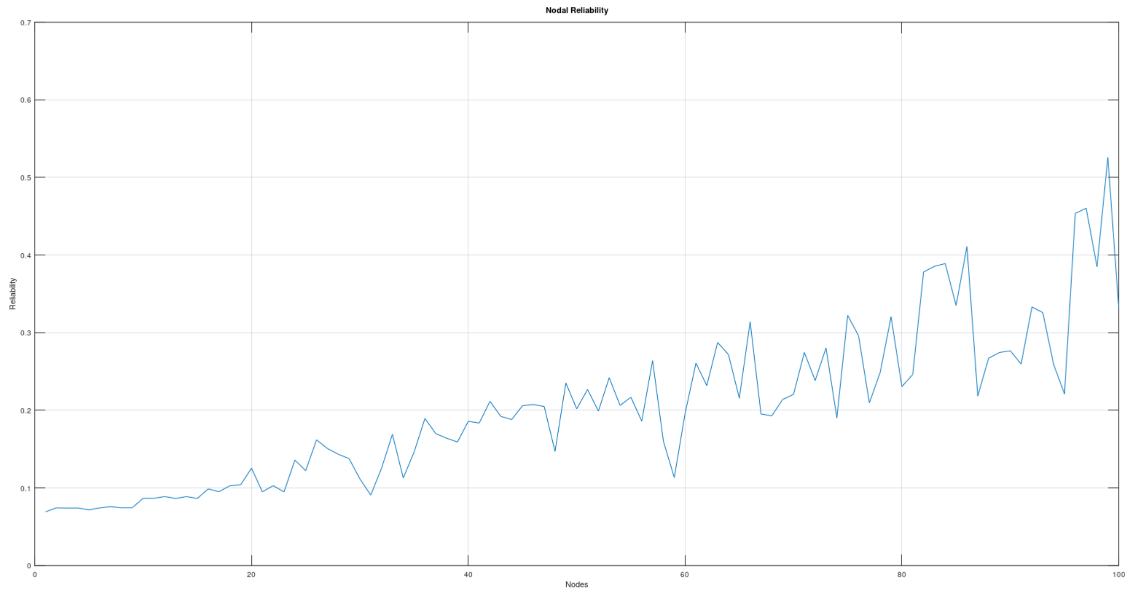
-In order to create RodMatrix.mat, we run the function “RodMatrix” which takes as parameters the adjacency matrix (RodMatrix(adjacency)), we used the K-shortest path algorithm (Yen’s algorithm with k=10), this step takes approximately 20~ minutes to run.

-We then call the “Reliability” function in order to store the values of Rnode,Rrange and Rsys in “Reliabity.mat” .

-Finally we plotted the results as show below in order to better analyse them.

Discuss the relevance of this indicator:

1) Rnode :



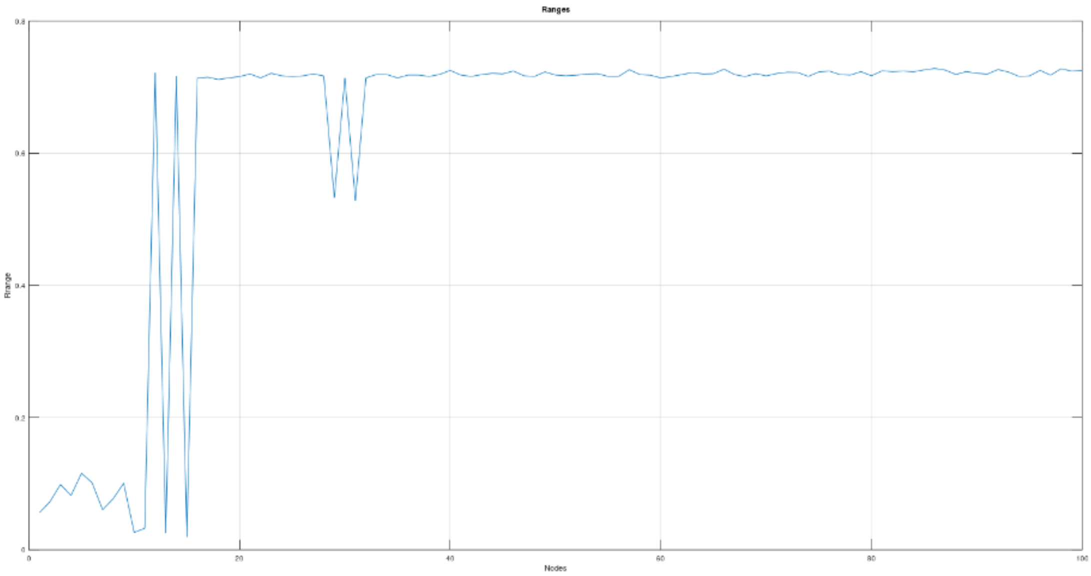
We can observe a great variance in our data, reliability varies greatly from node to node, we can see a pattern emerge though, the reliabilty increases with the node index, almost in a linear fashion, we can distinguish 3 dintinct zones in this graph:

-node 0 to 20: low nodal reliability, these nodes have little to no reliability, if we take the analogy of transit systems, these nodes would represent stations that lie in the outskirts of the transit line and have a little to no transfer rate.

-node 20 to 80: reliability varies from 0.1 to 0.3 , these nodes would represent stations of low transfer rate.

-node 80 to 100: reliability varies 0.2 to 0.5 , meaning that these nodes have relatively high reliability, these nodes would correspond stations that lie in central areas and have high transfer rate.

2) Rrange :



In this graph, we can see that value of Rrange suddenly peaks at around the 18th node in order to converge to a value of ~0.7, the most reliable nodes also have a high Rrange value in contrast to the least reliable nodes which have a low Rrange value, the correlation between node reliability and Rrange values isn't a strong one though, as we have nodes with relatively low reliability but a high rRange value, meaning that nodal reliability is an inconsistant indicator of the Rrange values.

3) Rsys :

Rsys= 0.2022, this value is low indicating that our adjacency matrix not very dense, as a matter of fact , we can calculate its density using the density formula for directed graphs: D= E/n(n-1) = 761/100*99 =0.076 .

A low value of Rsys is generally an indicator of low resilience to disruptions within our system, this means that our system could totally collapse in the face of one or few disruptions depending on the criticality of the node affected. Since the Rsys is the average of Rnode for all the different nodes, it is not a very good indicator in our case at all since we have a great variance between our different nodal reliability values and we are working in the simplest case which is "Status quo" . This indicator would have been valuable to us in case of disruptions happening in our system, as we could compare the Rsys value at status quo to the one calculated after a disruption has occurred in a particular node, this would then allow us to determine if this node is critical or not to the reliability of our entire system, as a matter of fact, "A disruption of highly reliable nodes might not necessarily have a considerable impact on the network. A node with low reliability can bring about a significant degradation of an entire system's reliability or traffic loss to the disruption" . This means that the value we have for Rsys at status quo isn't enough to determine which nodes are the most critical to our system and therefore we have no knowledge as to which nodes should be protected the most.