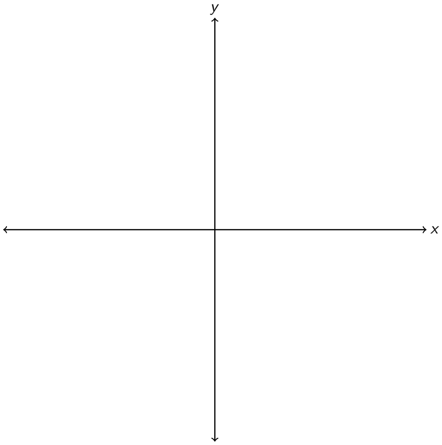
Single Joint Arm Dynamics

July 31, 2016

Introduction



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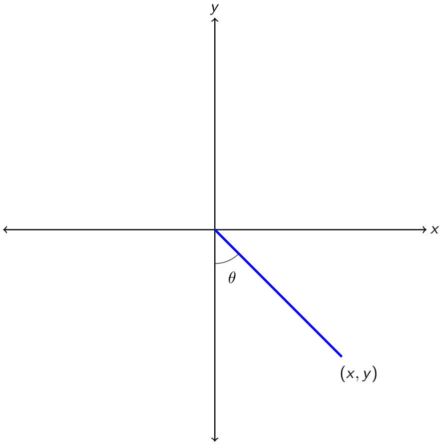

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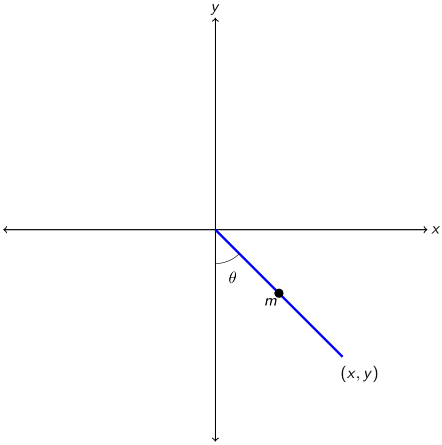

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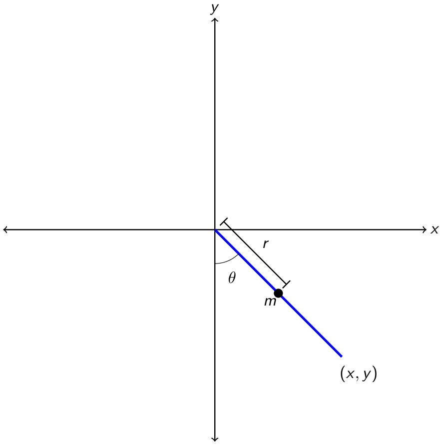

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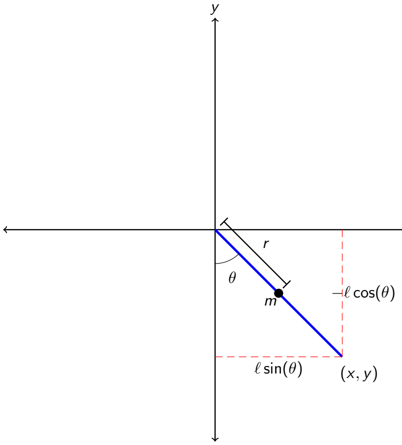

Single Joint Arm

$$x = \ell \sin(\theta_t) \qquad y = -\ell \cos(\theta_t)$$

Single Joint Arm

$$x = \ell \sin(\theta_t) \qquad y = -\ell \cos(\theta_t)$$

$$v_x = \ell \cos(\theta_t)\dot{\theta}_t \qquad v_y = \ell \sin(\theta_t)\dot{\theta}_t$$

Kinetic Energy

$$T_{lin} = 0.5m\left(r^2\sin^2(\theta_t)\dot{\theta_t}^2 + r^2\cos^2(\theta_t)\dot{\theta_t}\right)^2$$

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Kinetic Energy

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$$T_{lin} = 0.5mr^2\dot{\theta_t}^2$$

$$T_{rot} = 0.5i\dot{\theta_t}^2$$

Kinetic Energy

$$
\begin{aligned}
T_{lin} &= 0.5m\left(r^2\sin^2(\theta_t)\dot{\theta_t}^2 + r^2\cos^2(\theta_t)\dot{\theta_t}\right)^2 \\
T_{lin} &= 0.5mr^2\dot{\theta_t}^2 \\
T_{rot} &= 0.5i\dot{\theta_t}^2 \\
T &= T_{lin} + T_{rot}
\end{aligned}
$$

Kinetic Energy

$$
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T_{lin} &= 0.5m\left(r^2\sin^2(\theta_t)\dot{\theta_t}^2 + r^2\cos^2(\theta_t)\dot{\theta_t}\right)^2 \\
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T_{rot} &= 0.5i\dot{\theta_t}^2 \\
T &= T_{lin} + T_{rot} \\
T &= 0.5i\ \dot{\theta_t}^2 + 0.5mr^2\dot{\theta_t}^2
\end{aligned}
$$

Kinetic Energy

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T_{lin} &= 0.5m\left(r^2\sin^2(\theta_t)\dot{\theta_t}^2 + r^2\cos^2(\theta_t)\dot{\theta_t}\right)^2 \\
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T &= T_{lin} + T_{rot} \\
T &= 0.5i\;\dot{\theta_t}^2 + 0.5mr^2\dot{\theta_t}^2 \\
T &= 0.5\left(i + mr^2\right)\dot{\theta_t}^2
\end{aligned}
$$

Euler-Lagrange equation

- Potential Energy

$$U = m\ g\ r\ \left(1 - \cos\left(\theta_t\right)\right)$$

Euler-Lagrange equation

- Potential Energy

$$U = m\ g\ r\ \ (1 - \cos(\theta_t))$$

- Lagrangian

$$L = T - U = 0.5\left(i + mr^2\right)\dot{\theta_t}^2 - gmr\,(1 - \cos(\theta_t))$$

Euler-Lagrange equation

- Potential Energy

$$U = m\,g\,r\,\left(1 - \cos\left(\theta_t\right)\right)$$

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$$L = T - U = 0.5\left(i + mr^2\right)\dot{\theta_t}^2 - gmr\left(1 - \cos\left(\theta_t\right)\right)$$

- Euler-Lagrange equation

$$Q = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right)$$

$$Q = g\,m\,r\,\sin\left(\theta_t\right) + \left(i + mr^2\right)\ddot{\theta}_t$$

Euler-Lagrange equation

- Potential Energy

$$U = m\ g\ r\ \ (1 - \cos(\theta_t))$$

- Lagrangian

$$L = T - U = 0.5\left(i + mr^2\right)\dot{\theta_t}^2 - gmr\,(1 - \cos(\theta_t))$$

- Euler-Lagrange equation

$$Q = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right)$$

$$Q = g\ m\ r\ \sin(\theta_t) + \left(i + mr^2\right)\ddot{\theta}_t$$

- Forward Dynamics

$$\ddot{\theta}_t = \frac{Q - g\ m\ r\ \sin(\theta_t)}{(i + mr^2)}$$