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Approximation of Exponential function

By

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Maclaurin Series

$$H\left(Z\right) = \exp\left(F(z)\right) = \exp\left(\sum_{m=0}^{M} C(m) Z^{-m}\right)$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}(x)^n$$

$$= f(0) + \frac{f'(0)}{1!}(x) + \frac{f''(0)}{2!}(x)^2 + \frac{f'''(0)}{3!}(x)^3 + \cdots$$

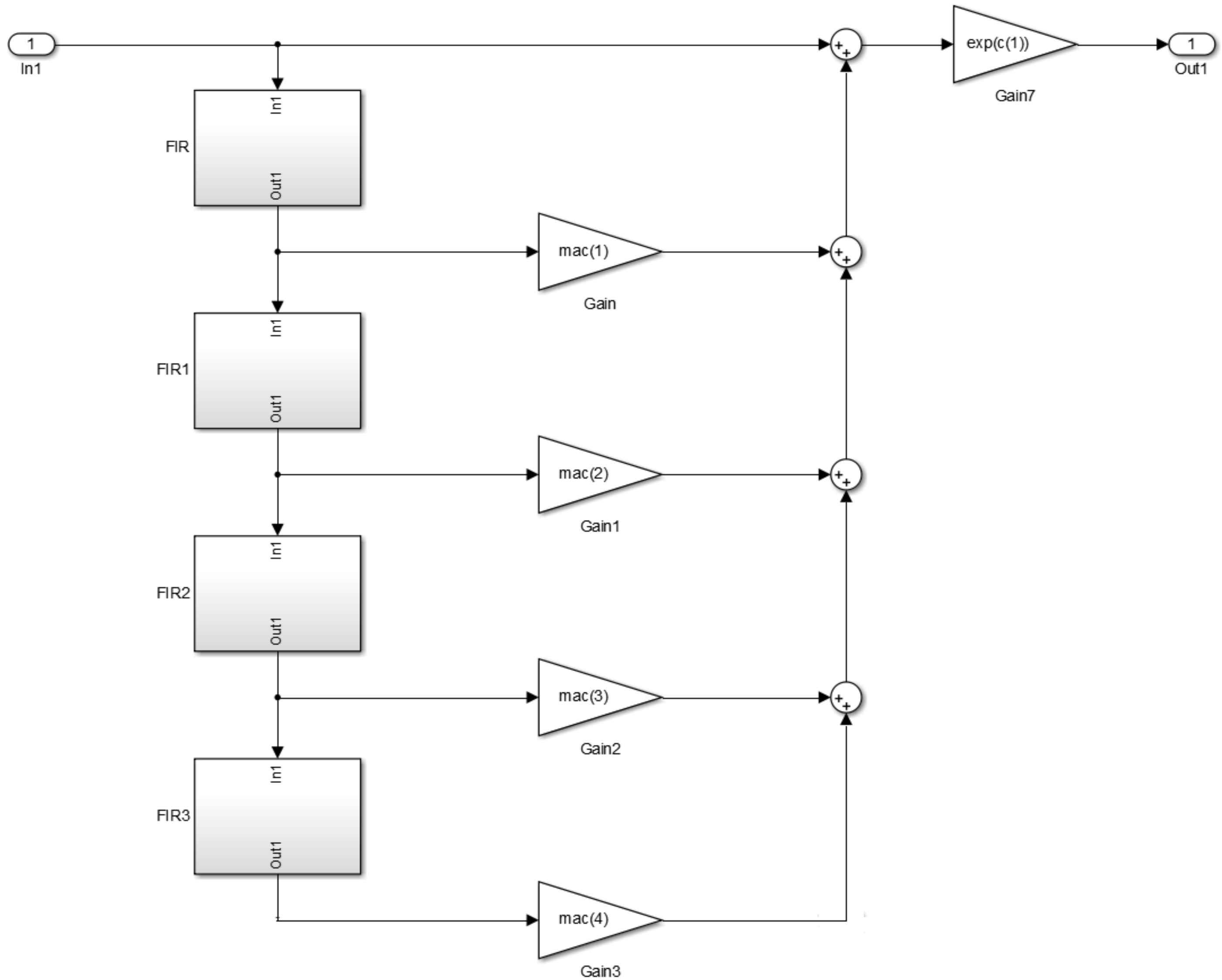
$$f(x) = \exp(x) = \sum_{n=0}^{\infty} \frac{(x)^n}{n!}$$

$$= 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$H(z) = \exp\left(F(z)\right) = \sum_{n=0}^{\infty} \frac{\left(F(z)\right)^n}{n!}$$

$$= 1 + \frac{F(z)}{1!} + \frac{F(z)^2}{2!} + \frac{F(z)^3}{3!} + \cdots$$

$$H(z)=1+F(z)+\frac{1}{2!}F(z)^2+\frac{1}{3!}F(z)^3+\frac{1}{4!}F(z)^4$$



Pade Approximation

$$R(x) = \frac{P_m(x)}{Q_n(x)} = \frac{p_0 + p_1 x + p_2 x^2 + p_3 x^3 + \ldots + p_m x^m}{q_0 + q_1 x + q_2 x^2 + q_3 x^3 + \ldots + q_n x^n}$$

- The unknown coefficients are determined from the condition that the first m+n+1 terms vanish in the Maclaurin series A(x)

$$A(x) - \frac{P_m(x)}{Q_n(x)} = 0 \qquad A(x) Q_n(x) - P_m(x) = 0$$

- For exponential function A(x) is given by

$$\exp(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Pade Approximation

- Solving the above equations we get the following recursive formulas

$$P_m(x) = \sum_{k=0}^{m} \frac{(m+n-k)!m!}{(m+n)!k!(m-k)!} x^k$$

$$Q_n(x) = \sum_{k=0}^{n} \frac{(m+n-k)!n!}{(m+n)!k!(n-k)!} (-x)^k$$

$$R\left(F(z)\right)=\frac{1+A_1F(z)+A_2F(z)^2+A_3F(z)^3+A_4F(z)^4}{1-A_1F(z)+A_2F(z)^2-A_3F(z)^3+A_4F(z)^4}$$