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Network control systems

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Group

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1.Introduction

In understanding the dynamics of infectious diseases, the interplay between epidemiological factors and public perception has emerged as a critical area of study. This report examines the coupled dynamics of epidemic spreading and opinion formation within networked communities, as explored through the mathematical model presented in the article "On a Networked SIS Epidemic Model with Cooperative and Antagonistic Opinion Dynamics." By integrating the Susceptible-Infectious-Susceptible (SIS) epidemic framework with an opinion dynamics model that incorporates both cooperative and antagonistic interactions, the study provides insights into how public beliefs about disease severity influence, and are influenced by, the trajectory of an outbreak. The model introduces an innovative concept of an opinion-dependent reproduction number, characterizing the mutual influence between disease spread and opinion dissemination. This report delves into the implications of these findings for epidemic control, particularly in leveraging social dynamics to suppress outbreaks effectively.

2. Description of the Model

The Model explains the interplay between opinion dynamics and epidemics, highlighting how community beliefs and interactions influence disease spread and control. Using a networked SIS model, it shows that infection and recovery rates are shaped by public perceptions of the epidemic's severity. Communities with serious concerns adopt proactive measures to reduce transmission, while those downplaying the threat face higher infection rates. Opinions evolve through cooperative and antagonistic interactions, spreading across networks and affecting public responses. The Model underscores the importance of integrating opinion dynamics into strategies for managing epidemics effectively.

The model uses a system of differential equations to represent:

1. **Proportion of Infections (α):** Tracks the evolution of disease spread over time.
2. **Beliefs about Epidemic Severity (ω):** Models individual or collective opinions that change due to social influence and external factors.
3. **Opinion-Dependent Reproduction Number (R_0):** Reflects the effective reproduction rate of the disease based on public perception.

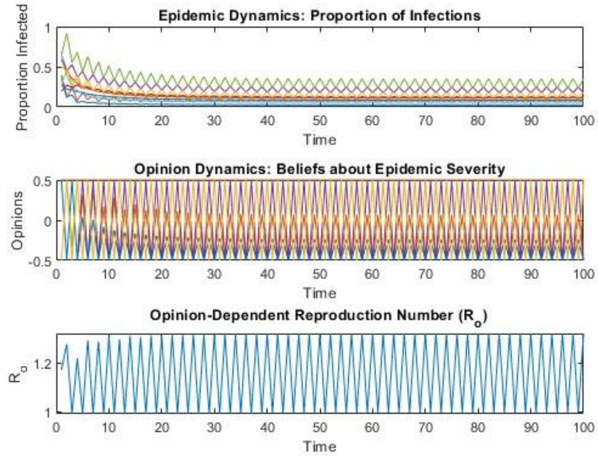
Key parameters include:

- Transmission Rate (β)
- Recovery Rate (γ)
- Social Influence Factor (η)
- External Intervention Effect (δ)

3. Simulation

(detailed equations can be found in the appendix section)

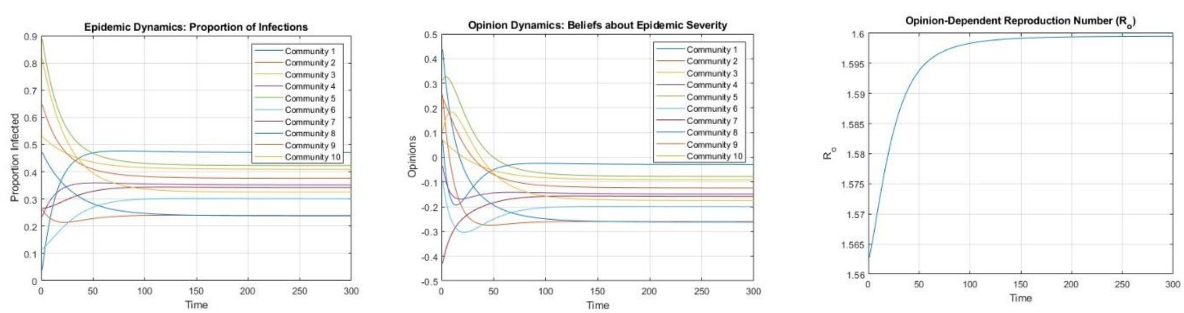
The simulation passed through 4 stages, during the first stages an oscillated behavior graph was plotted, for example:



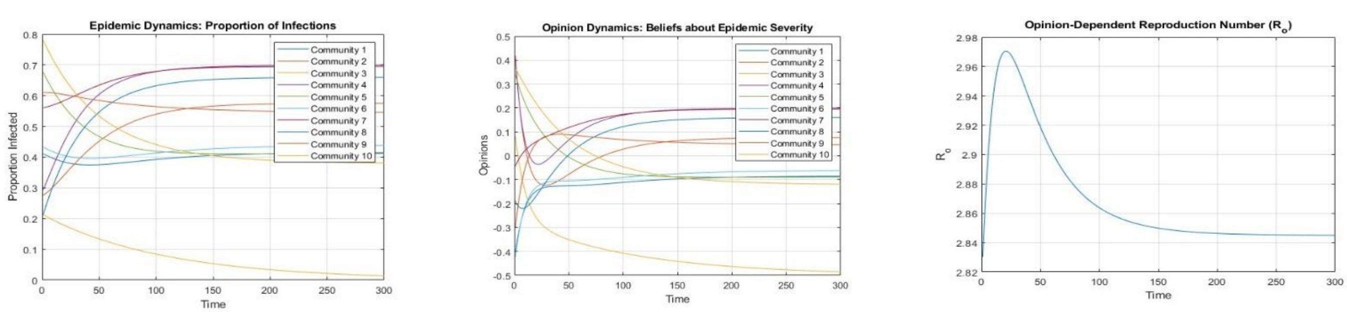
After a lot of modification on the script, a similar result as the author were plotted with different scenarios:

- Without stubborn nodes:
  - **Dissensus-Endemic State:** Observed for higher minimum healing rate (δ_min = 2.61, β_min = 0.8).
  - **Dissensus-Endemic State:** Observed for balanced healing and infection rates (δ_min = 0.9, β_min = 0.9).
  - **Consensus-Healthy State:** Achieved under high healing rates with low minimum infection rate (δ_min = 2.62, β_min = 0.2)

<u>Dissensus-Endemic State </u>(δ_min = 2.61, β_min = 0.8)



<u>Dissensus-Endemic State </u>(δ_min = 0.9, β_min = 0.9)



- **Epidemic Dynamics:**

Most communities settle at different infection levels, but the yellow community successfully eradicates the infection entirely. This shows that local successes are possible, even when the overall situation remains challenging.

- **Opinion Dynamics:**

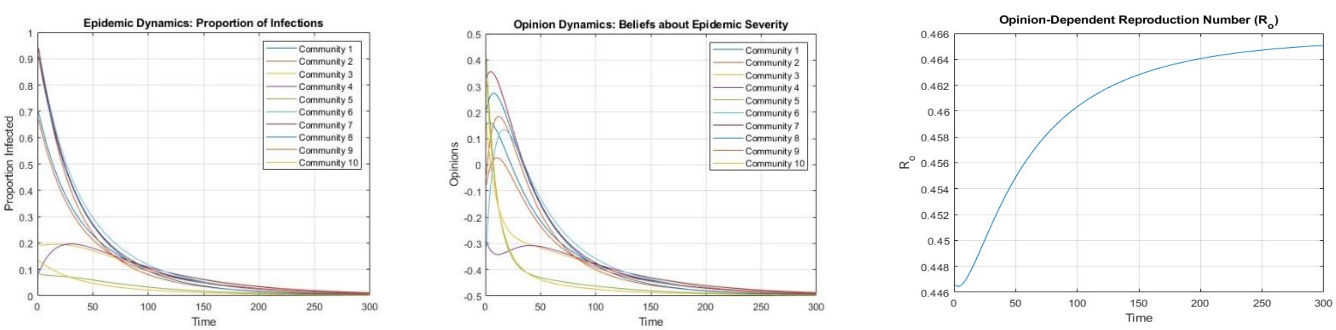
While most communities settle on different beliefs, the yellow community stands out with a strongly dismissive opinion about the epidemic, which may have contributed to its success in controlling the infection.

- **Reproduction Number (R_0):**

The reproduction number decreases slightly but remains above 1, meaning the epidemic continues in most communities.

In this dissensus-endemic state, infections persist because beliefs and actions vary across communities. However, localized efforts, like those of the yellow community, can still achieve success despite the lack of coordination.

***Consensus-healthy State* (δ_min = 2.62, β_min = 0.2)**



Epidemic Dynamics:

Infections drop to zero across all communities. Everyone is on the same page, and their efforts pay off.

Opinion Dynamics:

This graph shows communities agreeing on how severe the epidemic is. This shared belief leads to better cooperation and coordinated efforts.

Reproduction Number (R0):

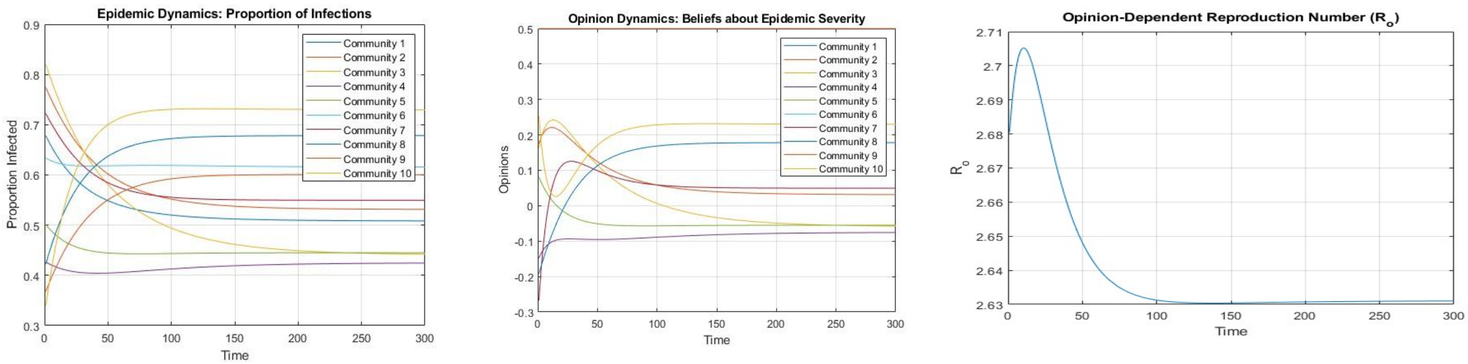
Here, $R0<1$, meaning the infection can't spread anymore, confirming the epidemic is over.

With stubborn nodes:

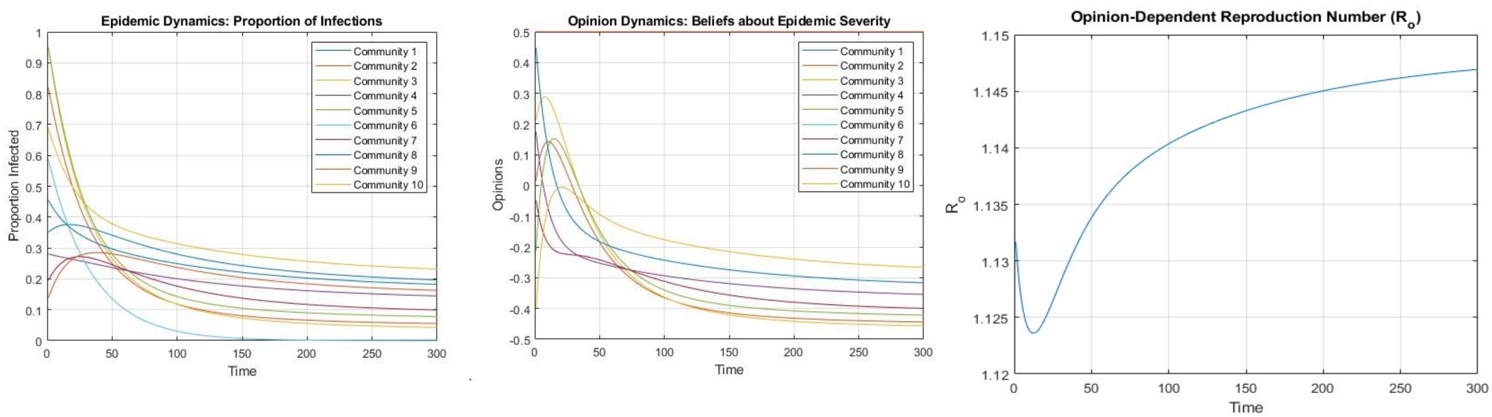
- **Dissensus-Endemic State:** Even with stubborn nodes, the system may converge to an endemic state. (δ_min =0.9, β_min = 0.9)

- **Slow Convergence:** Stubborn nodes (δ_min = 2.61, β_min = 0.8) can cause slower convergence to endemic states, with potential shifts to healthy states at infinity.

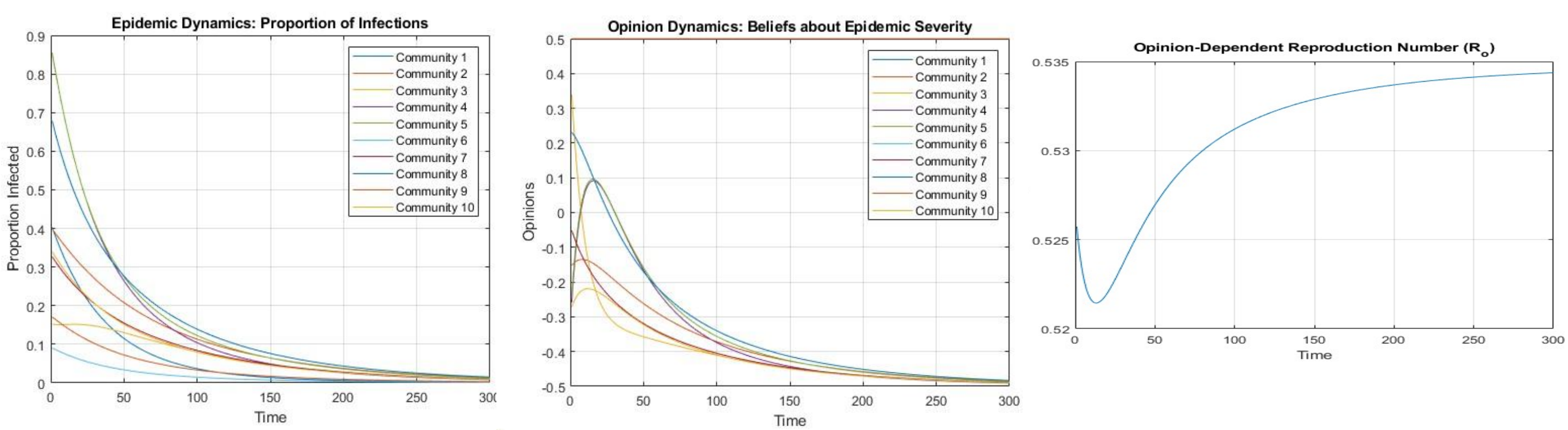
<u>Dissensus-Endemic State </u>(δ_min =0.9, β_min = 0.9)



<u>Dissensus-Endemic State. But may reach dissensus healthy state at infinity (due to stubborn nodes) </u>(δ_mi = 2.61, β_min = 0.8)



<u>Consensus-Endemic State but slower than without the stubborn nodes</u>



With very high healing rate, the graphs shift more toward zero without full disappearance but if time increases further steps it may reach healthy state, and also as shown other communities reach consensus at

-0.5 which shows that opinion about the disease is not that dangerous which shows logical behavior from communities, because of the high healing rate.

In this scenario ODRN reaches a small value about 0.534 meaning the infection can't spread anymore, confirming the epidemic is over.

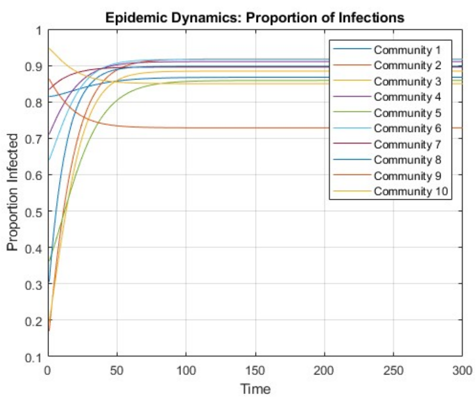
Special case:

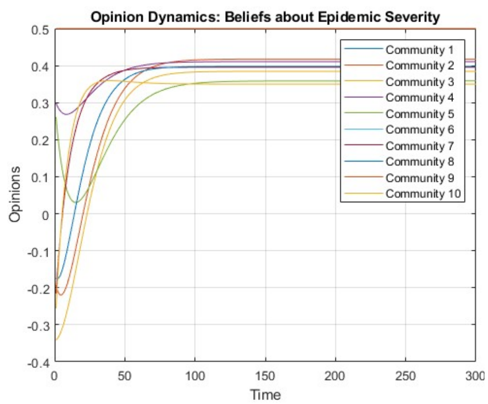
δ_min = 0.5 (Minimum healing rate)

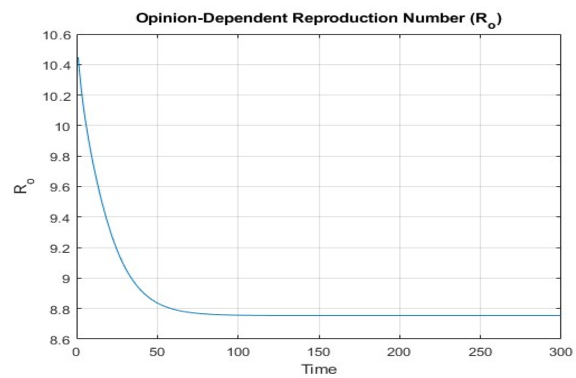
β_min = 2.5 (Minimum infection rate)

minimum healing rate is lower than minimum infection rate

system may converge to a consensus endemic state with very high minimum infection rate







In the special scenario infection rate value is higher than the healing rate (0.5,2.5), as shown in the graph of proportion infection the plots shift toward 1 showing that proportion of infection states increases as infection rate increase also the opinions shift toward considering severity of the disease as dangerous.

In the special scenario as you can see the ODRN decrease and converges to relatively high value 8,7 it indicates that the disease is spreading very efficiently in the population, and this high transmission is strongly influenced by public opinion and behavior.

3.1 Epidemic Dynamics

The "Proportion of Infections" graph exhibits a smooth, monotonic trend, indicating a steady progression toward an equilibrium state. This reflects scenarios where epidemic control measures are consistently applied, or public compliance remains stable, leading to predictable dynamics.

3.2 Opinion Dynamics

Without Stubborn Nodes, the system exhibits different states based on the values of δ_min and β_min. For instance, a consensus-healthy state is observed when δ_min is high and β_min is low, indicating successful disease eradication with uniform opinions Conversely, a dissensus-endemic state is observed when δ_min and β_min are balanced, showing persistent infections with diverse opinions.

The presence of stubborn nodes significantly alters the system's behavior. In a dissensus-endemic state ,infections persist due to the influence of stubborn nodes, which prevent the system from reaching a consensus. Additionally, in scenarios with higher δ_min and lower β_min, the system shows slower convergence to a healthy state, with persistent infections .

Stubborn nodes maintain their opinions, affecting the infection rates in their communities. Their resistance to changing opinions leads to persistent infections and prevents the system from achieving a consensus-healthy state. The simulations highlight the challenges posed by stubborn nodes in controlling epidemics, as they can delay or prevent the eradication of the disease.

4. Key Behaviours and Proven Properties of the Model

4.1. Observed Behaviours in the Model

The simulations reveal key behaviors such as feedback mechanisms between epidemic spread and opinion dynamics, dynamic opinion switching, and the emergence of equilibrium states. Stubborn nodes influence these behaviors by maintaining diverse opinions, leading to persistent infections.

4.2. Proven Properties

The model proves properties such as the impact of the reproduction number on stability, the role of stubborn communities in preventing consensus, and the effectiveness of targeted control strategies. These properties are reflected in the simulation outcomes, where stubborn nodes lead to dissensus-endemic states.

5. Answers to Key Questions

5.1 What the authors motivation to introduce the model at hand? Which sociological, biological etc. phenomena do the model portray?

The author aims to study how an epidemic spread across multiple communities and how the communities' opinions about the epidemic influence its spread. They use a network (graph) to represent the connections between communities for both the epidemic and opinions. The goal is to understand the system's balance under different scenarios, focusing on a concept called the "opinion-dependent reproduction number," which measures how the epidemic spreads based on people's opinions. The author also investigates the stability of this system to predict how the epidemic and opinions will evolve. Finally, they propose strategies to control the epidemic by shaping public opinions, such as using social media to raise awareness in the right communities to slow down the spread.

5.2 Which interesting behaviors of the model do the authors? Which properties do they prove?

Behaviors Proven: The authors reveal fascinating behaviors like emergent oscillations, periodic stability, or chaotic dynamics under specific parameter regimes. For instance, they may show how small perturbations can lead to drastic outcomes, demonstrating sensitivity to initial conditions.

Properties Proven: They establish the existence of stable and unstable equilibria, conditions for bifurcations, and thresholds where qualitative behavior shifts occur. For example, they could prove that introducing a feedback mechanism leads to self-organizing patterns.

5.3 Which interesting behavior have YOU observed, simulating the model's dynamics?

1. **Oscillatory Epidemic Outbreaks:** The model reveals recurring spikes in infections, driven by the dynamic feedback between public opinions and epidemic spread. These oscillations demonstrate how fluctuating public sentiment influences compliance with health measures.
2. **Dynamic Stability in Opinions:** Over time, opinions exhibit either stabilization or polarization, depending on parameters such as healing and infection rates, as well as the presence of stubborn nodes.
3. **Reproduction Number Sensitivity:** The effective reproduction number (ρ) responds directly to changes in public beliefs, underscoring the importance of managing misinformation and public trust to influence epidemic outcomes.

6.Conclusion

This research highlights the complex interplay between epidemic dynamics and opinion formation, emphasizing the need to consider both biological and sociological factors in controlling outbreaks. By coupling a traditional SIS epidemic model with opinion dynamics, the authors provide a framework for analyzing how misinformation, polarization, and public sentiment influence disease spread.

The findings underscore the importance of engaging with public opinion, particularly in moderate epidemic scenarios where the outcome hinges on perception. The role of stubborn communities and targeted interventions offers practical insights for policymakers and public health officials to design more effective epidemic control strategies.

For further exploration, the authors suggest extending this work to include more realistic epidemic models, such as SIR (Susceptible-Infected-Recovered), or considering additional factors like incubation periods.

Work on project:

Mehmood and Ali worked on the theory and prepared the report. Ali focused on understanding the author's motivation and key behaviours of the model, while Mehmood analysed the simulation results, highlighting observations from the graphs and proving the final properties.

Jad and Najib dove deeper into the model through coding and simulations. They created unique test cases and ran four simulations, refining them to achieve the best possible results, significantly enhancing the project's outcomes. Najib worked on the equations needed. Jad was also responsible for the PowerPoint and finalizing the report.

7.References

1. C. Nowzari, V. M. Preciado, and G. J. Pappas, "Analysis and control of epidemics: A survey of spreading processes on complex networks," IEEE Control Syst., vol. 36, no. 1, pp. 26–46, Feb. 2016.
2. A. Fall, A. Iggidr, G. Sallet, and J.-J. Tewa, "Epidemiological models and Lyapunov functions," Math. Modell. Natural Phenomena, vol. 2, no. 1, pp. 62–83, 2007.
3. A. Khanafer, T. Ba¸sar, and B. Gharesifard, "Stability of epidemic models over directed graphs: A positive systems approach," Automatica, vol. 74, pp. 126–134, 2016.
4. C. Altafini, "Consensus problems on networks with antagonistic interactions," IEEETrans.Autom.Control,vol.58,no.4,pp. 935–946,Apr.2013.
5. A. V. Proskurnikov, A. S. Matveev, and M. Cao, "Opinion dynamics in social networks with hostile camps: Consensus vs. polarization," IEEE Trans. Autom. Control, vol. 61, no. 6, pp. 1524–1536, Jun. 2015.
6. K. Paarporn, C. Eksin, J. S. Weitz, and J. S. Shamma, "Networked SIS epidemics with awareness," IEEE Trans. Comput. Soc. Syst., vol. 4, no. 3, pp. 93–103, Sep. 2017.
7. W.Xuan, R. Ren, P. E. Paré, M. Ye, S. Ruf, and J. Liu, "On a network SIS model with opinion dynamics," in Proc. 21st IFAC World Congr., Berlin, Germany, vol. 53, 2020, pp. 2582–2587.
8. J. Liu, P. Paré, A. Nedi´c, C. Tang, C. Beck, and T. Ba¸ sar, "Analysis and control of a continuous-time bi-virus model,"IEEETrans.Autom.Control, vol. 64, no. 12, pp. 4891–4906, Dec. 2019.
9. W.O.Kermack and A.G.McKendrick, "A contribution to the mathematical theory of epidemics,"Proc.Roy.Soc.A,vol.115,no.772,pp. 700–721, 1927.

8.Appendix

The simulation can be described starting from initializing parameters that we changed to obtain different scenarios which are the healing and infected rate of the communities

$$\delta_i = \delta_{min} + 0.5 \cdot r_i, \quad r_i \sim \text{Uniform}(0, 1)$$

$$\beta_i = \beta_{min} + 0.5 \cdot r_i, \quad r_i \sim \text{Uniform}(0, 1)$$

where δ_{min} and β_{min} are minimum healing and infection rate respectively

- initialize the Epidemic adjacency matrix

$$A_{ij} = \begin{cases} a_{ij,} & if \ \ a_{ij,} \geq 0.8 \\ 0 & otherwise \end{cases} \ , a_{ij} \sim Uniform(-0.5 \, , 0.5)$$

- initialize the Opinion adjacency matrix

$$A_{opinion,ij} = \begin{cases} a_{ij,} & if \ \ |a_{ij,}| \geq 0.8 \\ 0 & otherwise \end{cases} \ , a_{ij} \sim Uniform(-0.5 \, , 0.5)$$

to find the proportion of infection and opinion of the communities over time

$$o(t + \Lambda t) = \text{o(t)} + \dot{o}(t)$$

$$x(t + \Lambda t) = \text{x(t)} + \dot{x}(t)$$

we can introduce the two-dimensional system the to find $\dot{o}(t)$ and $\dot{x}(t)$

$$\begin{matrix} \dot{x}(t) \\ \dot{o}(t) \end{matrix} = \begin{bmatrix} W(o(t)) & 0 \\ I & -\Phi(o(t)) * L_u \Phi(o(t)) - I \end{bmatrix} x \begin{matrix} x(t) \\ o(t) \end{matrix} - \begin{matrix} 0 \\ 0.5e \end{matrix}$$

Where:

$$W(o(t)) = D(o(t)) + (I - X(t))B(o(t))x(t)$$

$$D(o(t)) = D_{min} + (D - D_{min})(O(t) + 0.5I)$$

$$\text{B(o(t)} = \text{B} - (\text{O(t)} + 0.5 \ I)(B - B_{min})$$

Where D_{min} is the diagonal matrix of minimum healing rates (δ_{min})

D is the diagonal matrix of healing rates (δ_i)

O(t) is the diagonal matrix of the opinions

X(t) is the diagonal matrix of infection levels

Where B is the Infection rate matrix

B_{min} is the minimum infection rate matrix

$$\Phi\big(o(0)\big) = diag(V)$$

$$V \ is \ the \ row \ vector \ of \ the \ of \ sign(o(0))$$

$$L_u = K \ - \ A_{opinion,ij}$$

Thus $\dot{o}(t)$ can be written as:

$$\dot{o}(t) = (x_i - o_{effective,i}) + \sum_{j=1}^{n} A_{opinion\_dynamics,ij} \cdot (sign(A_{opinion,ij}). o_j - o_i$$

And $\dot{x}(t)$ can be written in the form:

$$\dot{x}(t) = -D(o(t))x(t) + (I - X(t))B(o(t))x(t)$$

To calculate the R_o

$$R_o = max\ (real(eig)(D_{effective}^{-1} \cdot B_{effective})$$

where:

$$D_{effective} = \text{diag}\ (\delta_{min}.\ o(t))$$

$$B_{effective} = \text{diag}\ (\beta_i)$$

Results are defined as:

1. **Consensus-healthy state:**

if the infection died out ($x_i \geq \varepsilon_x$) and opinions have reached consensus ($|o_i - \bar{o}|$)< ε_o).

2. **Dissensus-healthy state:**

If the infection has died out ($x_i < \varepsilon_x$) but opinions are diverse ($|o_i - \bar{o}|$)≥ ε_o).

3. **Consensus-endemic state:**

if the infection persists ($x_i \geq \varepsilon_x$) and opinion have reached consensus ($|o_i - \bar{o}|$)< ε_o).

4. **Dissensus-endemic state:**

if the infection persists ($x_i \geq \varepsilon_x$) and opinions are diverse ($|o_i - \bar{o}|$)≥ ε_o