ASSIGNMENT 2
SECI1013 DISCRETE STRUCTURE

COURSE CODE : **SECI1013**

COURSE : **DISCRETE STRUCTURE**

FACULTY : **FACULTY OF COMPUTING**

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SECTION : **02**

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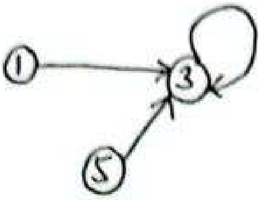
Question 1.

i. $R = \{(1,3), (3,3), (5,3)\}$

ii. Domain $= \{1,3,5\}$

Range $= \{3\}$

iii.



iv. <u>Asymmetric</u>

$\forall x, y \in R, (x,y) \in R \to (y,x) \notin R$

In this sense, a relation is asymmetric if and only if it is both antisymmetric and irreflexive.

$R = \{(1,3), (3,3), (5,3)\}$

IRREFLEXIVE?

$\forall x \in D. (x,x) \notin R$

since $(3,3) \in R$, R is not irreflexive because it has a self-loop.

ANTISYMMETRIC?

$\forall x, y \in D, (x,y) \in R \land x \neq y \to (y,x) \in R$

$\forall x, y \in D, (x,y) \in R \land (y,x) \in R \to x = y.$

$(1,3) \in R$, but $(3,1) \notin R$

$(5,3) \in R$, but $(3,5) \notin R$

$(3,3)$ is allowed $\to (3,3) \in R$ because $x = y$.

since antisymmetric condition is satisfied, R is antisymmetric.

CONCLUSION: Even though R is antisymmetric, it is not irreflexive because it contains a self-loop (3,3). Since ~~assy~~ asymmetric requires a relation to be both antisymmetric and irreflexive, R is not asymmetric.

Irreflexive ✗

Antisymmetric ✓

↓

Asymmetric ✗

Question 2

Since R is an equivalence reaction, R must be reflexive, symmetric and transitive.

$\boxed{\text{REFLEXIVE}}$

$\forall x \in R, (x,x) \in R$

$R = \{(x,x), (y,y), (z,z)\}$

$\boxed{\text{SYMMETRIC}}$

$\forall x,y \in R, (x,y) \in R \rightarrow (y,x) \in R$

Given: $(x,y) \in R \rightarrow (y,x) \in R$

$(y,z) \in R \rightarrow (z,y) \in R$

$\boxed{\text{TRANSITIVE}}$

Let M_R be the matrix of relation R. Let $M_R \otimes M_R = N$. $M_R = [m_{ij}]$, $N = [n_{ij}]$

<u>case 1</u> (rejected).

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- reflexive ✓
  symmetric ✓
  transitive ✗ ($M_R \otimes M_R \neq M_R$).

<u>case 2</u>

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- reflexive ✓
  symmetric ✓
  transitive ✓ ($M_R \otimes M_R = M_R$).

The relation R is transitive if and only if the following is true:

$\forall i \forall j$, if $(n_{ij} = 1)$ then $(m_{ij} = 1)$

$\forall x,y \in R, (x,y) \in R \wedge (y,z) \in R \rightarrow (x,z) \in R$

$(x,y) \in R, (y,z) \in R \rightarrow (x,z) \in R$

$(z,y) \in R, (y,x) \in R \rightarrow (z,x) \in R$

$\therefore R = \{(x,x), (y,y), (z,z), (x,y), (y,x), (x,z), (z,x), (y,z), (z,y)\}$

Question 3

(i) $M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

(ii)

vertex	in-degrees	out-degrees
u	2	2
v	2	2
w	3	2
y	2	3

(iii) $M_R \otimes M_R \neq M_R$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

- Relation R is reflexsive the value on its main diagonal is entirely 1
- Relation R is antisymmetric because all vertex has at most one directed edge.
  $(u,w) \in R$, $u \neq w$, $(w,u) \notin R$
- Relation R is not transitive because (u,w) and $(w,y) \in R$, but $(u,y) \notin R$.
  $M_R \otimes M_R \neq M_R$
- Relation R is not a partial order because it is reflexsive and antisymmetric but not transitive

Question 4

if $f(x_1) = f(x_2)$

$(x_1 - 1)^2 = (x_2 - 1)^2$

$\sqrt{(x_1-1)^2} = \sqrt{(x_2-1)^2}$

$x_1 - 1 = x_2 - 1$

$x_1 = x_2$

codomain $= [0, \infty] \rightarrow y \geq 0$

$f(x) = (x-1)^2$

$x = \sqrt{y} + 1$

when $y = 0$, $x = 1 \rightarrow x \geq 1$

- Function F is one-to-one function because $f(x_1) = f(x_2)$
- Function F is an onto function because for all $x \in X$, there exist $y \in Y$, where $f(x) = y$
- Function F is a bijection because it is one-to-one function and an onto function.

Question 5

a) let $g^{-1}(y) = x$

$$y = \frac{3}{2}x - 1$$

$$x = \frac{2}{3}(y+1)$$

$$\therefore g^{-1}(y) = \frac{2}{3}(y+1)$$

b) $(g \circ f)(x) = gf(x)$

$$= \frac{3}{2}(9x+4) - 1$$

$$= \frac{27}{2}x + 6 - 1$$

$$= \frac{27}{2}x + 5$$

c) $(f \circ g)(x) = fg(x)$

$$= 9\left(\frac{3}{2}x - 1\right) + 4$$

$$= \frac{27}{2}x - 9 + 4$$

$$= \frac{27}{2}x - 5$$

d) $(f \circ g \circ g)(x) = f[g(g(x))]$

$$g[g(x)] = \frac{3}{2}\left(\frac{3}{2}x - 1\right) - 1$$

$$= \frac{9}{4}x - \frac{3}{2} - 1$$

$$= \frac{9}{4}x - \frac{5}{2}$$

$(f \circ g \circ g)(x)$

$$= 9\left(\frac{9}{4}x - \frac{5}{2}\right) + 4$$

$$= \frac{81}{4}x - \frac{45}{2} + 4$$

$$= \frac{81}{4}x - \frac{37}{2}$$

Question 6

a) $P_t = P_{t-1} + \frac{1}{4} P_{t-2}$, $t \geq 2$, with initial condition $P_0 = 4.0$

b) $P_0 = 4.0\,°F$

$P_1 = 5.0\,°F$

$P_2 = P_{2-1} + \frac{1}{4} P_{2-2} = P_1 + \frac{1}{4} P_0 = 5.0 + \frac{1}{4}(4.0) = 6.0\,°F$

$P_3 = P_{3-1} + \frac{1}{4} P_{3-2} = P_2 + \frac{1}{4} P_1 = 6.0 + \frac{1}{4}(5.0) = 7.25\,°F$

$P_4 = P_{4-1} + \frac{1}{4} P_{4-2} = P_3 + \frac{1}{4} P_2 = 7.25 + \frac{1}{4}(6.0) = 8.75\,°F$

$P_5 = P_{5-1} + \frac{1}{4} P_{5-2} = P_4 + \frac{1}{4} P_3 = 8.75 + \frac{1}{4}(7.25) = 10.5625\,°F$

Question 7

a)
```
input : n
output : s(n)
s(n) {
   if (n = 1)
      return 2
   else
      return s(n-1) * s(n-1) - 1
}
```

b) $n = 4$

S(4)

n = 4
because n ≠ 1
return s(4-1) * s(4-1) - 1 = return s(3) * s(3) - 1 = return 8 * 8 - 1 = return 63

s(3)

n = 3
because n ≠ 1
return s(3-1) * s(3-1) - 1 = return s(2) * s(2) - 1 = return 3 * 3 - 1 = return 8

S(2)

n = 2
because n ≠ 1
return s(2-1) * s(2-1) - 1 = return s(1) * s(1) - 1 = return 2 * 2 - 1 = return 3

S(1)

n = 1
because n = 1
return 2

$\therefore S_4 = 63$